REPORT ON THE DOCTORAL THESIS

Graph Drawing: Visualization and Geometric Representations of Graphs and Networks

by

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At the Graph Drawing conference in 1996 Don Knuth said: “Graph drawing is the best possible field I can think of. It merges esthetics, mathematical beauty and wonderful algorithms. It therefore provides a harmonic balance between the left and right brain parts.” The thesis of Tomáš Vyskočil is in this field and shows some of the features praised by Knuth but it also contains large technical parts. It is a fairly well written thesis that collects a number of interesting results. With this work the author has clearly proven his ability of mastering formal and technical proofs as well as for creative scientific research. I recommend to graduate the candidate on the basis of this thesis.

After an introductory first chapter the main body of this thesis consists of four chapters dedicated to four loosely connected topics. Each of these chapters corresponds to a publication, see the list on page 4 of the thesis.

Chapter 2 is about segment representations of co-planar graphs. The existence of such a representation for every co-planar graph would imply that MAXCLIQUE is NP-complete for segment intersection graphs. It is not mentioned in the thesis but the complexity question has been resolved by Cabello, Cardinal and Langerman (ESA 2012). The representation problem, however, remains wide open. So far only the result that complements of partial 2-trees have segment representations has been published, this is the main contribution of the chapter. The construction is done with a quite clever induction.

The main characters of Chapter 3 are classes of VPG graphs, where VPG is an acronym for Vertex-intersection graph of Paths on a Grid. If only \( k \) bends are allowed on each of the paths we denote the class of representable graphs as \( B_k \)-VPG. It is shown that \( B_k \)-VPG \( \subseteq \) \( B_{k+1} \)-VPG for all \( k \geq 0 \) and that \( B_1 \)-VPG \( \nsubseteq \) SEG. The key to the proof of these separation results is a technical tool called the Noodle-Forcing-Lemma. This lemma allows to ‘fix a given
representation’ inside of a representation of a larger graph. This general technique is likely to have future impact on questions related to representability with intersecting curves. The lemma is also crucial to the proof of Theorem 3.3.1, where membership in $B_k$–VPG is shown to be NP-complete even for graphs that are given with a $B_{k+1}$–VPG representation.

To describe the representation model studied in Chapter 4 we begin with the extended grid, i.e., the grid with both diagonals added in each of the square cells. An island is a connected induced subgraph of the extended grid. A family of disjoint islands induces a graph where islands are considered adjacent iff there is an edge of the extended grid with ends in both islands. It is observed that the class of island representable graphs is exactly STRING. The main result of the chapter is an NP-completeness proof for recognizing graphs that admit an island representation with islands consisting of at most $k$ vertices. From the proof it is clear that this only holds for some sufficiently large $k$, the LATIN 2010 paper states $k \geq 6$. Unfortunately the technical exposition in the thesis is not accompanied by a formal statement of the result.

The last chapter deals with some notions of “edit distance” for graphs. The complexity of “edits” in form of vertex deletion and edge deletions has already been studied in the 70’s. An FPT algorithm for changing a graph with at most $k$ edge contractions into a bipartite graph has been known to exist. Here the model with edge contractions is studied when the target graph is planar. The result is a FPT algorithm even for the harder problem where the set of contracted edges is required to satisfy a logic statement presented in monadic-second-order-logic (MSOL). The methods developed for this proof are then used to give a shorter proof of Grohe’s theorem which states that crossing number $\leq k$ is in FPT. A nice closing result.

I already gave my recommendation so I will just repeat in brief: a nice thesis, I recommend to award the candidate Mgr. Tomáš Vyskočil the PhD.

(Stefan Felsner)