In our dissertation we deal with the space $\mathbf{H}(K)$ of harmonic functions on a compact space in classical and abstract potential theory. Initially, we prove several equivalent characteristics of this space in classical potential theory. The internal characterization, which describes $\mathbf{H}(K)$ as a subspace of those continuous functions on a compact space K which are finely harmonic on the fine interior of K, is then used as the definition of $\mathbf{H}(K)$ in abstract potential theory.

Further we concentrate on the solution of the Dirichlet problem for open and compact sets mainly with regards to its relation to subclasses of Baire class one functions. The results, proved at first in classical potential theory, are later generalized to abstract potential theory. With a use of more elementary tools we initially prove these results in harmonic spaces with the axiom of dominance and, subsequently, using stronger tools we generalize them to harmonic spaces with the axiom of polarity.

We engage also in a more abstract problem of approximation by differences of lower semicontinuous functions in a more general context of binormal topological spaces.