Abstract of doctoral thesis
Problems in discrete geometry

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This thesis studies three different questions from discrete geometry. A common theme for these problems is that their solution is based on algebraic methods.

First part is devoted to the polynomial partitioning method, which partitions a given finite point set using the zero set of a suitable polynomial. However, there is a natural limitation of this method, namely, what should be done with the points lying in the zero set? Here we present a general version dealing with the situation and as an application, we provide a new algorithm for the *semialgebraic range searching problem*.

In the second part we study Ramsey functions of semialgebraic predicates. Conlon, Fox, Pach, Sudakov, and Suk constructed the first examples of semialgebraic predicates with the Ramsey function bounded from below by a tower function. We reduce the dimension of the ambient space in their construction and as a consequence, we provide a new geometric Ramsey-type theorem with a large Ramsey function.

Last part is devoted to reptile simplices. A simplex S is *k-reptile* if it can be tiled by k simplices with disjoint interiors that are all mutually congruent and similar to S. We show that four-dimensional k-reptile simplices can exist only for $k = m^2$, where $m \geq 1$ is an integer.