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Re: Doctoral thesis  
Andrew Kozlík

This is a report on the PhD thesis of Andrew Kozlík; a very easy note to write. It's worth at least two PhD theses. It is a lovely collection of work. A few details are in order. I will make my comments section by section followed by a summary.

1. Latin directed triple systems. I have spent a lifetime studying Steiner triple systems and Mendelsohn triple systems. The generalization to flexible LDTs is of enormous interest to me. Mr. Kozlík determines the spectrum for both flexible and non-flexible LDTs. There are lots of necessary examples in this section; easy to understand.
2. Basics of DTS quasigroups: algebra, geometry and enumeration. This section is quite technical and gives many carefully explained examples of DTSs I love well thought out examples!
3. Triple systems and binary operations. A hybrid triple system  $(X,B)$  (HTS) consists of both cyclic and transitive triples. If the usual binary operations produce a quasigroup we say that  $(X,B)$  is a latin hybrid triple system (LHTS). A HTS containing both cyclic and transitive triples is called proper. One of the principal results in this sections shows that the spectrum for proper LHTS(n) is  $n \equiv 0 \text{ or } 7 \pmod{3}$ . Again there are lots of carefully explained examples in this section.
4. Cyclic and rotational Latin Hybrid triple systems. This section determines the spectra for LDTs and LHTs admitting a cyclic automorphism. Lots of other interesting results as well.
5. Flexible Latin directed triple systems. The main result in this section is the determination of the spectra for flexible LDTs = all  $n \equiv 0 \text{ or } 1 \pmod{3}$ ,  $n \neq 4, 6, 10, 12$ . (A flexible LDTs satisfies the identity  $x(yx) = (xy)x$ .) Lots of carefully explained examples in this section.
6. Pure Latin directed triple systems. A LDTs is pure if when considered as a 2-fold triple system it contains no repeated blocks. In this section the spectra (modulo a few possible exceptions) is determined for both pure flexible and non-flexible LDTs.
7. Antiflexible Latin directed triple systems. I really like this section! An anti-flexible LDTs-quasigroup is a quasigroup where the identity  $(yx)y = y(xy)$  holds for the least possible

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number of ordered pairs  $(x,y)$ . Quoting from the paper: “In a sense, anti-flexible LDTs are LDTs which are as distant from STs as possible.” The author goes on to show that the spectrum for anti-flexible LDTs is precisely all  $n \equiv 0$  or  $1 \pmod{3}$ . Once again, this section ends with lots of easy to understand examples; necessary for the main result.

8. The centre of a Steiner loop and the maxi-Pasch problem. This topic is not my cup of tea so I will not make any comments.

#### CONCLUDING REMARKS AND RECOMMENDATION

This thesis contains a fantastic collection of results and, as I said on page 1, is worthy of at least two PhDs. Mr. Kozlík might want to consider blending the first 7 sections into a huge survey paper or a monograph. This would be of interest to people in both algebra and design theory.

Regards,



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