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MASTER'S THESIS

Forecasting Jump Occurrence in Czech Day-Ahead Power Market

Author: **Bc. Jana Hortová**

Supervisor: **PhDr. Ladislav Krištoufek Ph.D.**

Academic Year: **2015/2016**

Declaration of Authorship

The author hereby declares that she compiled this thesis independently, using only the listed resources and literature, and the thesis has not been used to obtain a different or the same degree.

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Prague, December 22, 2015

Signature

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Abstract

The very specific features of the spot prices, especially occurrence of severe jumps, create a spot price risk for retailers who purchase electricity at unregulated highly volatile prices but resell it to consumers at fixed price. Therefore, it is of high importance to forecast whether jump is likely to occur during the next hour. However, to the best of our knowledge, such research has not been devoted to the Czech power market yet. Therefore, the aim of this thesis is to forecast the jump occurrence in the Czech day-ahead market. For this purpose we suggest four logit model specifications, each containing various independent variables (for example, electricity demand, outside temperature, lagged price and various dummy variables) where the variable selection is supported by the previous literature and by the characteristic features of the spot prices. Within the in-sample period we compare the suggested models based on the values of pseudo-R squared and Bayesian information criterion. When evaluating the out-of sample performance of suggested models we apply jump prediction accuracy and confidence, but opposed to the previous literature we suggest a kind of sensitivity analysis which, to the best of our knowledge, has not be proposed by any other power research.

JEL Classification	C25, C32, C51, C52, C53, Q41, Q47
Keywords	Electricity spot prices, jumps, forecasting occurrence of jumps, Czech day-ahead market, logit models

Author's e-mail	Janca.Hortova@seznam.cz
Supervisor's e-mail	ladislav.kristoufek@fsv.cuni.cz

Abstrakt

Specifické vlastnosti spotových cen elektřiny, a to především častý výskyt skoků, vytváří vysoké riziko pro obchodníky s elektřinou, kteří ji nakupují za neregulované velmi proměnlivé ceny, ale spotřebiteli ji prodávají za ceny pevně stanovené. Proto je velmi důležité predikovat, jaká je pravděpodobnost výskytu skoku během následujících hodin. Pokud je nám však známo, dosud žádná studie nebyla zaměřena na Českou republiku, a proto je hlavním cílem této práce predikovat pravděpodobnost výskytu skoku na českém denním trhu. Za tímto účelem definujeme čtyři logit modely, kde každý z nich obsahuje různé nezávislé proměnné (jako je spotřeba elektřiny, venkovní teplota nebo dummy proměnné) a kde je výběr proměnných poveden v souvislosti s předchozí literaturou a na základě specifických vlastností cen. „In-sample" porovnání modelů je založeno na hodnotách dvou testů, a to na pseudo-R squared a Bayesovském informačním kritériu. V případě ohodnocování „out-of-sample" kvality modelů vypočítáváme přesnost a důvěryhodnost predikce skoků, statistiky používané v souvislosti s predikováním. Avšak na rozdíl od předchozí literatury se uchýlíme k určitému druhu sensitivní analýzy, která, pokud je nám známo, ještě nebyla v literatuře zaměřené na trh s elektřinou použita.

Klasifikace JEL C25, C32, C51, C52, C53, Q41, Q47

Klíčová slova Cena elektřiny, skoky, předpovídání výskytu skoku, denní trh v České republice, logit modely

E-mail autora Janca.Hortova@seznam.cz

E-mail vedoucího práce ladislav.kristoufek@fsv.cuni.cz

Contents

List of Figures

List of Tables

Acronyms

ACH	Autoregressive conditional hazard model
ARIMA	Autoregressive integrated moving average model
ARMA	Autoregressive moving average model
ARMAX	Autoregressive moving average model with exogenous inputs
BIC	Bayesian information criterion
BV	Bipower variation
CHMU	Czech Hydrometeorological Institute
CV	Continuous component of realized volatility
GARCH	Generalized autoregressive conditional heteroskedasticity process
HAR-RV	Heterogeneous autoregressive model of realized volatility
IV	Integrated volatility
JV	Jump component of realized volatility
MWh	Megawatt hour
OTE	Czech Electricity and Gas Market Operator
PXE	Power Exchange Central Europe
QV	Quadratic variation
RV	Realized volatility
TQ	Tripower quarticity

Master's Thesis Proposal

Author	Bc. Jana Hortová
Supervisor	PhDr. Ladislav Krištoufek Ph.D.
Proposed topic	Jumps Prediction in Electricity Markets

Motivation The dynamic of electricity prices is not only of primary interest for consumers consumption but also for many other agent's behaviors, such as speculation or risk management. The electricity prices possess many characteristic properties, volatility being the most important for our analysis. There are many factors that cause the price to be so volatile. First, electricity is non-storable good. Second, both demand and supply of electricity are dependent on weather. Third, there are technical constraints that may also influence the price especially in case of unexpected technical problems - these may limit the supply of electricity and thus increase the price (Kristoufek & Lunackova, 2013).

Many previous papers studied volatility of electricity prices together with providing the prediction of the future prices. However, more recent studies showed that the total variation in the electricity prices should be divided into two separate parts - jumps and variation. These studies also argued that predicting that omit the separation should provide misleading results, even though this methodology is supposed to work well in other markets, especially in financial markets.

Furthermore, a great deal of literature relating to the predicting of future prices in financial markets can be found. On the other hand, the proportion of literature studying the electricity prices is much lower. Moreover, papers studying the Czech Republic are nearly a rarity.

Even in the Czech Republic, electricity is frequently traded commodity. Therefore, prediction of evolution of its price is very important since price experiences sudden "jump". In 2001 the Czech electricity and gas market operator was established. Among many other things, this company organizes trading in day-ahead electricity market and also publishes continuous data since 2002 (OTE, 2010).

Hypotheses

Hypothesis #1: Jumps form a significant part of the total variation of electricity prices

Hypothesis #2: Occurrence of jumps can be predicted using historical data

Hypothesis #3: Size of jumps can be predicted using historical data

Methodology As it was mentioned above, there exists a source that publishes continuous data on electricity prices since 2002. A big advantage of this source is that prices are available at hourly intervals, moreover, it provides various other characteristics, such as system imbalance, sum of absolute imbalances, etc. (OTE, 2010). Therefore, this source will be exploited to obtain historical data. After that we investigate the data and identify jumps.

Apart from the previous studies, Chan et. al (2008) study variation and jumps separately rather than the total variation alone. Following their work I will apply similar methodology to the Czech Republic. After that models providing the predictions of jumps as well as size of jumps will be considered. For this purpose, contributions of Corsi et. al (2010) will be reviewed to find out whether occurrence and size of jumps can be predicted using historical data.

Expected Contribution During recent years, electricity has become a frequently traded good which encouraged researches to study and predict the evolution of its price. Quite a large number of papers has been written since then. Unfortunately, all these papers are either too general or dedicated to group of countries. To the best of my knowledge, no such studies were devoted to the Czech Republic, therefore, the thesis is supposed to bring new insight into the issue of electricity prices.

At the begging, I will partition the total variation into jumps and variation and expect to detect that both effects matter for explaining the volatility of electricity prices. When predicting the tomorrow' s situation in the electricity market it is expected that the historical data can help us to uncover whether jumps will occur tomorrow. Similarly, historical data are supposed to be useful in predicting the size of jumps.

Outline

1. Introduction: I will provide motivation for my thesis.

2. Literature review: I will try to review the existing literature dealing with jumps prediction in general and then focus on jumps prediction in electricity markets in particular.

3. Total variation partition: the current literature suggests that the total variation can be separated into two main parts, namely jumps and variation, which will be of the main topic in this part.

4. Forecasting: I will provide a model predicting the occurrence of jumps in the following day. Furthermore, the sign and the magnitude of such jumps will be modeled.

5. Results: I will present my findings.

6. Concluding remarks: I will summarize the main findings of the thesis.

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Author　　　　　　　　　　Supervisor

1 Introduction

Historically, the power markets were predominantly centralized and vertically integrated industries with service and infrastructure being a single unit. Consequently, consumers in a given geographical area had to buy electricity from the power company dedicated to this area having no option to switch to a cheaper provider. Nevertheless, during the last two decades the power markets have experienced a considerable change. In particular, most of the countries around the world, including the Czech Republic, deregulated their power markets with the aim to separate the service from the infrastructure and hereby set up competitive industries where consumers can freely decide about their provider. This implies that energy industries have no longer been primary technical businesses and that electricity has become a commodity which can be traded at the market place.

Nevertheless, compared to other commodities, electricity is known to be a very specific one. First, to date there are no efficient means of storing the electricity directly implying that immediate adjustment of supply and demand of electricity is required and so the electricity has to be produced at the moment it is consumed to maintain a balance between production and consumption. Second, the consumption of electricity can be said to depend on human behavior and on weather and climate conditions. Third, the power market is characterized by highly inelastic demand, with absolutely inelastic demand occurring in the short-run which implies that the supply curve determines the electricity price.

More importantly, from these characteristics the very specific features of the spot electricity prices emerge. These include multiple seasonal fluctuations, mean reversion, high volatility and occurrence of severe jumps, and are not common in any other financial or commodity market. Among researchers, occurrence of severe jumps is considered to be the most crucial feature because it hinders the trading position, especially, of the retailers as they are exposed to the spot price risk since they buy the electricity at flexible prices (which are exposed to severe jumps) and resell it to the consumers at fixed price. Consequently, forecasting the occurrence of jumps is of high importance for retailers who can, based on a probability of presence of future jumps, improve their bidding strategies. Despite this general knowledge, most of the previous literature rather deals with forecasting the trajectory of the spot prices with only small fraction of researches being dedicated to the jumps occurrence. Additionally, to the best of our knowledge, no similar study has been devoted to the forecasting the jump occurrence in the Czech Republic by this date. Therefore, it is challenge for the thesis to fill this gap.

In particular, we solely focus on the Czech day-ahead market and hypothesize that jumps form a significant part of the total variation of electricity prices and that occurrence of jumps can be predicted using historical data. Therefore, we first need to show that jumps form a significant part of the total volatility to confirm jumps are not negligible fraction and so their forecasting deserves our attention. For this purpose we employ the quadratic variation theory. Assuming the share of jumps is considerable, we turn to the forecasting of jump occurrence and provided that the dependent variable is kind of binary one (either jump or non-jump), it is necessary to choose the econometric model accordingly in which case we decide for the logit model. Within the analysis we consider various independent variables selected both to capture the important features of the spot prices and also according to the results of previous literature. For example, we include electricity demand and outside temperature as these two are widely suggested by the previous literature, further we consider variable capable of imitating the distance from the last jump or dummy variables which are able to capture the seasonal fluctuations. To provide the most accurate results, the common practise is to split the sample into two parts, namely in-sample and out-of sample period. Within the in-sample, the selection of the best performing model is executed, and then the out-of sample is employed to evaluate the results.

The rest of the thesis is organized as follow: Section 2 reviews the methods used by previous literature which are relevant to this thesis, Section 3 focuses on the structure of the Czech power market and stylized facts of the spot prices. The data description and specification of the econometric models and methods are discussed in Section 4, which is followed by the results in Section 5. Finally, Section 6 concludes the thesis and brings recommendations for further research.

2 Literature review

While the empirical analyses relating to the financial and commodity markets comprise a countless number of articles, the literature dealing with the power markets is much more limited. This is because it is relatively new area emerging only after the power markets worldwide were deregulated which started two decades ago. Additionally, there is neither consensus about the most appropriate method of modeling nor forecasting in the power markets. The focus of this section is, therefore, to provide a brief review of approaches suggested in the previous literature to get a sense about the power markets examined so far and about the methods applied when examining them.

Within the Europe, Nordic Power market is the most popular and the most investigated spot electricity market. This can be attributed to fact that it belongs to the oldest market with history dating back to the early 1990's and, in addition, it is considered to be the most stable market not only in the Europe but worldwide (Weron *et al.*, 2004). This can be documented by number of papers that use Nord Pool for their investigations. For this thesis the most relevant surveys involve Weron *et al.* (2004), Haugom *et al.* (2011), Hellström *et al.* (2012), and Weron & Misiorek (2008) who examine Nord Pool together with California. Nevertheless, there are also other electricity markets that attract researchers. These include, for example, Australia that is studied by Zhao *et al.* (2007), Chan *et al.* (2008), Eichler *et al.* (2012), and Christensen *et al.* (2012), Pennsylvania-New Jersey-Maryland market examined by Haugom & Ullrich (2012), Finnish market studied by Voronin & Partanen (2013) or Germany market investigated by Swider & Weber (2007). To the best of our knowledge, only Krištoufek & Luňáčková (2013) examine the Czech power market with the main focus placed on the properties of the spot prices and especially on their long-term memories.

Regardless of the power market examined, there seems to be consensus in the previous literature regarding the stylized facts of the spot prices. In particular, the authors agree about the following features: multiple seasonal fluctuations, mean reversion, high volatility and occurrence of severe jumps (Simonsen *et al.*, 2004).

These features, described in details in later section, are not common in any other financial or commodity market and therefore the standard methods used for these markets cannot be directly applied to the power markets but modifications and special treatments with respect to such features are necessary.

When regarding seasonality, it is well documented that electricity prices exhibit intra-day, daily and monthly patterns. On the Nord Pool dataset, Weron *et al.* (2004) show that the average daily prices, for period January 1, 1997 to April 25, 2000, have roughly a shape of a sinusoid with a linear trend. As an example illustrating the monthly fluctuation we can consider Chan *et al.* (2008) who show that an Australian electricity prices tend to be higher in winter and summer compared to autumn and be less volatile during spring. This is most likely due to the use of air-conditioning. It is, therefore, obvious that the authors need to use dummy variables depending on the type of seasonality present in particular market. Specifically, Chan *et al.* (2008) include dummy variables for autumn, winter and spring together with dummy for weekend. Contrary, Weron & Misiorek (2008) only use Monday, Saturday and Sunday as dummy variables.

When analyzing the volatility with appearance of sudden and extreme changes of the price, called jumps or spikes, the authors disagree about the method of identifying that a jump occurred on that particular day or hour. According to Weron *et al.* (2004), jump arises when price increases by more than three standard deviations from the long-term mean price. Even simpler method is to specify a threshold of x\$/MWh with defining a jump as price rise above this threshold. Christensen *et al.* (2012) employ precisely this procedure, 100\$/MWh being the threshold for identifying the jumps, with further extension where the authors distinguish two classes of jumps. Prices between 100\$/MWh and 300\$/MWh are regarded as mild jumps whereas those above 300\$/MWh are classified as severe jumps. The third way how to recognize occurrence of jump is to perform a test statistic (the precise definition of this test can be found later in the text) and given that it has exceeded the 99th percentile of the Normal distribution we conclude that a jump occurred, see for instance Chan *et al.* (2008), Haugom *et al.* (2011), Haugom & Ullrich (2012). The least common approach is employed by Hellström *et al.* (2012) where jump is considered to be any price change that does not correspond to their definition of the normal price variation. According to the authors, this method is very practical since two price changes of the same magnitudes do not have to be both classified as jumps. This is the case when one price change occurs in time of low normal variation while the second one in time of high normal variation (when prices are more volatile) and, therefore, the first price change is regarded as jump while the second is not.

More importantly, while the authors are inconsistent in the ways of identifying the jumps, they all regard occurrence of severe and frequent jumps to be the most important feature, especially, to retailers. It is because jumps raise the spot price risk they bear as they buy the electricity at the market place at flexible prices and resell it to the consumers at fixed price which is highly regulated (Christensen *et al.*, 2012). Also, production of some companies requires usage of large amount of electricity and so these companies can, in case they are able to shift production in short-time, save considerable amount of money when they are able to plan the production according to the predicted price development (Eichler *et al.*, 2012).

Consequently, forecasting the occurrence of jumps is of high importance for all market participants. Despite this general knowledge, the bulk of the previously suggested approaches propose models capable of forecasting the trajectory of the spot prices, leaving only minor fraction of the literature to deal with forecasting the probability of jump occurrence only. Nevertheless, to get a sense of modeling and forecasting techniques used when dealing with the power markets, it is worth looking at a very brief summary of literature devoted to the forecasting of the trajectory of the spot prices.

First practice is using a jump-diffusion model which is based on simple relation where realized (total) volatility is a sum of a jump and continuous component. While this model is supported from an theoretical point of view, its estimation is pretty difficult since it is nearly impossible to separate the jump from the continuous component. Given the high frequency of jump occurrence, Chan *et al.* (2008), Haugom *et al.* (2011), and Haugom & Ullrich (2012) suggest to employ the theory of quadratic variation to non-parametrically disentangle these two components and they achieve generally improved results. Probably the first influential model capable of bringing econometrically applicable results is proposed by Corsi (2004). He suggests to forecast the day-ahead realized volatility as a function of today's realized volatility plus realized volatility for the last seven days where the former one has greater weight compared to the later one. He refers to as a heterogeneous autoregressive model of realized volatility (HAR-RV). While Haugom *et al.* (2011) generally accept this approach, they argue that using a seven-day history is meaningless, especially when dealing with Nord Pool exchange market, since the spot electricity markets are in most cases closed during weekdays. Consequently, only a five-day history of jump and continuous components together with additional variables (such as volume or dummy variables for days in a week) are applied. In this thesis, we exploit modified version to compute the jump contribution to the total realized volatility.

The second class of approaches includes the traditional time series models. Such analyses are based on autoregressive moving average models with exogenous inputs (ARMAX), as done by Knittel & Roberts (2005) who capture the seasonal patterns in Californian hourly electricity prices by including seasonal dummy variables and dummy variables capturing the difference between peak and offpeak hours, on autoregressive integrated moving average models (ARIMA), as applied by Conejo *et al.* (2005), or on their combinations with generalized autoregressive conditional heteroskedasticity process (GARCH), as considered by Garcia *et al.* (2005). While these approaches bring considerable improvement, Swider & Weber (2007) argue that they all rest on the assumption of normality of the errors. This assumption is however usually violated in case of the spot price series, as confirmed on the Czech day-ahead market in later section, and therefore the authors suggest, for the German power market, to use combination of autoregressive moving average model (ARMA) with a gaussian mixture approach which allows to cope with the non-normality and reach generally improved results.

The third stream of literature deals with regime-switching models where, broadly speaking, two regimes are considered. The advantage of these models lies in the possibility to switch between two regimes which allow us to have different rates of mean reversion and volatilities in each regime. The first regime, so called base state, accounts for the normal periods in which the spot prices fluctuate around the long-term mean (are mean reverting) while the second regime, so called excited state, comprises the jumps in which the rate of mean reversion and values of volatilities are much higher compared to the former regime. Additionally, the models define matrix of transition probabilities which ensures switching from one regime to another (Mari, 2006). Despite some differences in model specification and implementation, Mount *et al.* (2006), De Jong (2006), Eichler & Tuerk (2013) and Paraschiv *et al.* (2015) consider the regime-switching models, and moreover, Mari (2006) and De Sanctis & Mari (2007) consider even three-regime switching models (they add an extra regime to two regime switching model).

Furthermore, other alternative approaches have been suggested in the literature. These are not that common as the above mentioned and include wavelet techniques applied by Tan *et al.* (2010), Voronin & Partanen (2013), neural networks suggested by Mandal *et al.* (2006) and Pao (2007), their combination used by Voronin & Partanen (2014), data mining techniques employed by Lu *et al.* (2005) and Zhao *et al.* (2007), and others.

Having commented on the techniques regarding the forecasting the trajectory of the spot price we can now review researches related to the forecasting the jump occurrence. Probably the first analysis devoted to this topic is suggested by Christensen *et al.* (2012) who further argue that not only are the above mentioned approaches related to the trajectory of the spot prices but also the vast majority of them treat jumps as memoryless process (meaning they do not depend on the past history). In their work the authors, with the use of Australian half-hourly data, exploit the autoregressive conditional hazard model (ACH) where the day-ahead probability of jump occurrence depends on both the past history of jumps, modeled as the duration between last two jumps, and other independent variables, namely load and temperature.

An influential work of Eichler *et al.* (2012) proposes further extension to this model, such as including lagged variables, and with the use of identical dataset reaches improved results. In addition, they argue that even better results can be achieved when using logit model rather than ACH model in which case they consider various forms, these can be viewed as an alternatives to the ACH models made by

Christensen *et al.* (2012), in which they try to capture additional features of the Australian power market that the former authors omitted. With the use of several comparative measures the authors confirm the overall better performance of logit models over the ACH models.

In summary, traders in the spot electricity markets are exposed to the spot risks resulting from the inconsistency in trading (they buy electricity at flexible and highly volatile prices but sell it at fixed prices). As a result, forecasting the probability of extreme price events is of high importance. Unfortunately, despite the number of researches related to the forecasting in the power markets gradually increases, the proportion of these devoted to the jump forecasting is fairly limited. Moreover, to the best of our knowledge, forecasting the jump occurrence in the Czech Republic has not been studied by any researcher yet and so we intend to fill this gap.

3 Czech power market

In the past, consumers in a given geographical area had to buy electricity from the power company dedicated to this area and so they had no option to switch to a cheaper provider. However, during the last two decades the electricity markets of most of the countries around the world, including the Czech Republic, deregulated their electricity markets with the aim to separate the service from the infrastructure and hereby set up a competitive industry. In the Czech Republic, the precondition guaranteeing the proper functioning of the deregulated system required the establishment of the Energy Act No. 458/2000, the Energy Regulatory Office and the Czech electricity and gas market operator (OTE) (Vitner, 2006).

In this section we provide very brief information regarding the structure of the Czech power market after deregulation and description of the day-ahead market in particular, followed by the stylized facts of the power market, which we should know about before turning to the empirical part.

3.1 Structure of power market and day-ahead market

The process of deregulating the Czech power market took place from 2002 to 2006 during which various phases were aimed at ensuring that the final consumers are no longer dependent on the electricity supplied by locally respective grid operators but rather have the right to select an electricity supplier by their own (Vitner, 2006). Having deregulated the market and thereby having isolated the service from the infrastructure, the electricity has become a commodity which can be traded either over-the-counter or on regulated markets. Trading over-the-counter refers to situations in which the trading parties sign a bilateral agreement determining the price and volume in advance where parties can agree on specifics features of contracts but where the data are not freely available to general public. The second way of trading the electricity, with data being freely available, is on regulated markets where parties can choose to enter long-term or short-term markets but where they have to respect the predetermined rules. While the long-term regulated market is organized by two power exchanges, by Power Exchange Central Europe (PXE) and

Czech Moravian Commodity Exchange Kladno, since 2009 the short-term regulated market is solely organized by OTE (Krejčová, 2012). Furthermore, the short-term regulated market can be subdivided into block market, intra-day market and day-ahead market.

Despite its attractiveness, detailed description of differences between aforementioned parts of the power market is not provided as this is not our primary focus but it can be found in Krejčová (2012) or OTE, a.s. (2015a). Moreover, because of the data availability and, perhaps more importantly, because of the data applicability for further analysis, we only investigate the Czech day-ahead market which is operated by OTE. As a consequence, whenever in the later text we use terms power market and spot price we always have day-ahead market and its spot price in mind.

Foundation of the OTE, which is owned by the state, dates back to 2001. The day-ahead electricity market trading through OTE was launched one year later, and after additional few years trading in the intra-day and block electricity markets was launched (OTE, a.s., 2015a). Until the beginning of 2009, both OTE and PXE organized the short-term electricity trading however since February 1, 2009 these two have been brought together with OTE being a single administrator of the short-term trading (with electricity delivery being in the Czech Republic) from this date, including trading on the day-ahead market (PXE, a.s., 2015).

Currently, the structure of the day-ahead market is kind of daily auction, with the trading currency being Euro, with 1 hour being the traded period and with 1MWh being the minimum tradable volume. Depending on whether the market participant is a buyer or a seller, she sends either bid or ask price, where the minimum allowed price is -500 EUR/MWh and maximum price is 3 000 EUR/MWh, together with her offer of volume, where 99 999 MWh is the ceiling. The offers need to be send for each particular hour of the following day in which the trader wants to participate and also it has to done at least a day in advance and always before 11 a.m. when the market closes (note that unlike others, the Czech day-ahead market is opened also during Saturdays, Sundays and national holidays). Afterwards, one spot electricity price, which is binding for both buyers and sellers regardless of their initial offers, is determined for each particular hour of the next day based on the Business Terms of

OTE that are valid at the time, and it is published online always before 2:30 p.m. but usually around 12 a.m. (OTE, a.s., 2015b).

3.2 Stylized facts of spot prices in Czech day-ahead market

As touched in the literature review, the spot prices are known to have various features that are greatly distinctive from those possessed by other commodities and so to analyze the power market properly, we are forced to account for these features accordingly. Therefore, the rest of this section is dedicated to a detailed description of multiple seasonal fluctuations, mean reversion, high volatility and occurrence of severe jumps, respectively. Since the focus of the thesis is forecasting the probability of jump occurrence in the Czech day-ahead market, they are reviewed and described solely on the Czech data. Nevertheless, they can be generally attributed to any deregulated power market worldwide with some minor differences which are caused by geographical specifications and cultural diversities.

3.2.1 Multiple seasonal fluctuations

Conventional goods are generally well storable and so inventories allow the entrepreneurs to smooth price and quantity fluctuations. In sharp contrast, to date there are no efficient means of storing the electricity directly. Pumped-storage hydropower facilities can be viewed as exceptions but they are not that common (Krištoufek & Luňáčková, 2013), also we can consider indirect storage in form of a pile of coal or possibly water in water reservoir (Simonsen *et al.*, 2004). The non-storability simply implies that immediate adjustment of supply and demand of electricity is required. In other words, to maintain a balance between production and consumption, the electricity has to be produced at the moment it is consumed. When this balance is violated it can lead to overcharge of the electricity grid and consequently to blackouts (Krejčová, 2012).

Furthermore, the consumption of electricity can be said to depend on human behavior, when considering the small scale, and on weather and climate conditions, when considering the large scale. The former one is associated with intra-day, daily and weekly fluctuations while the latter one is associated with monthly fluctuations

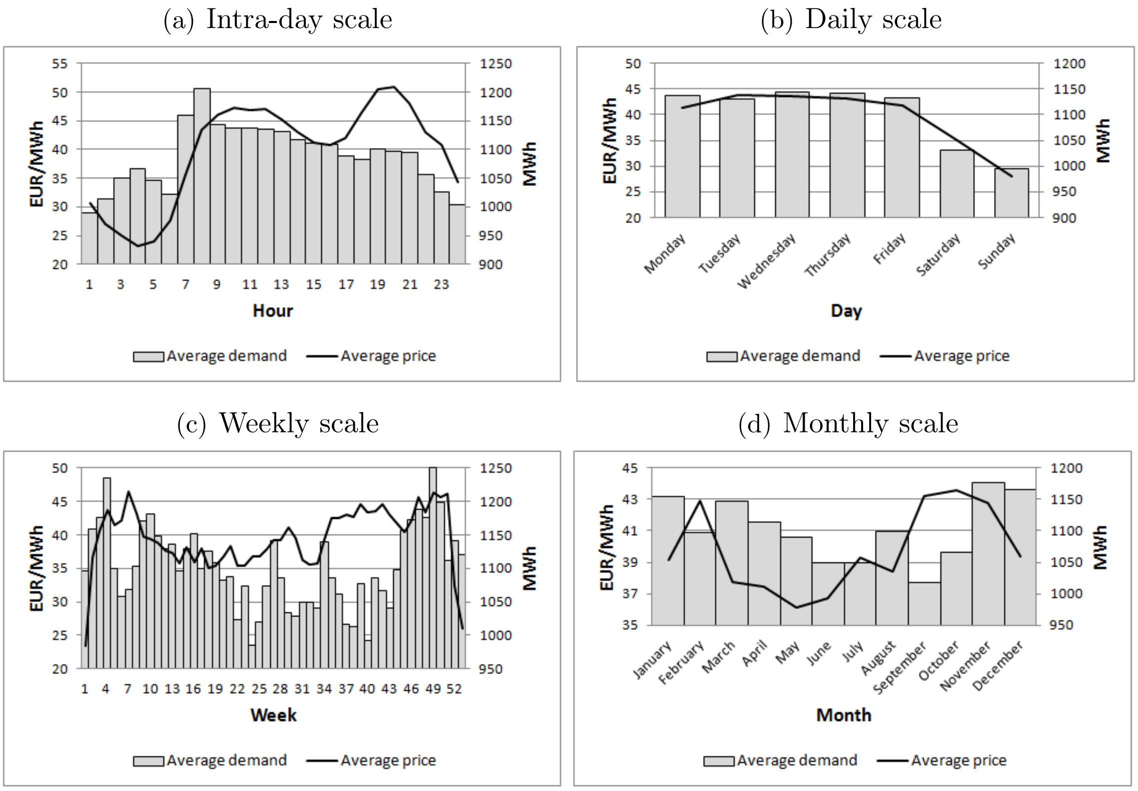
(Simonsen *et al.*, 2004). To observe which fluctuations are present in the Czech day-ahead market, we gather data on hourly spot prices and hourly demand for period from February 1, 2009 to August 31, 2015. Then for each hour, day, week and month we compute averages over this period and depict the results in Figure 3.1.

On a small scale, Figure 3.1 supports the seasonality on intra-day and daily level. According to Panel (a) the prices show double peak structure with first peak being in the morning time and second peak in the afternoon when people return back home and become more active, therefore, use more electricity. Contrary, the lowest prices and consumption occur during night when people are not active and most of the business are closed. In particular, the highest average price is 50.87 EUR which arises during 8 p.m. while the lowest one is 23.27 EUR at 4 a.m. While the graph, and even the current literature, confirms the double peak structure, OTE considers only single peak structure where peak hours are from 8 a.m. to 8 p.m. while 1 a.m. till 7 a.m. and 9 p.m. to midnight are considered as off-peak hours (OTE, a.s., 2015c). Based on Panel (b) we conclude that both consumption and prices sharply decline during the weekends, the average price reaches only 28.05 EUR on Sunday while on Tuesday it climbs to 43.89 EUR. This is in line with expectations since most of the businesses are closed during weekends. Finally, Panel (c) does not provide any evidence suggesting that weekly pattern is strong in the Czech power market, even though other power markets are found to demonstrate it, see for instance Taylor *et al.* (2006) or Eichler *et al.* (2012).

When examining the monthly fluctuations in Panel (d), therefore considering the large scale, it can be seen that consumption and prices tend to be higher during winter and fall, while lower during summer and spring. The average price in May is as low as 36.12 EUR compared to October when the average price is 43.55 EUR. This is as anticipated because large volume of electricity is needed for home heating since winters, and to some extent also falls, in the Czech Republic are usually cold while springs are rather mild. Furthermore, summers do not usually bring prolonged periods of extremely high temperatures and so extensive usage of air-conditioning, which raises the electricity consumption, is not necessary in the Czech Republic. On contrary, in southern countries the trend is usually the other way around with consumption being the highest during summer where this is mainly attributed to such usage of air-conditioning due to very high temperatures (Krištoufek & Luňáčková, 2013).

In general, it can be said that the small scale consumption reflects the human behavior and is supposed to be cultural specific while the large scale consumption is assumed to be geographical specific - higher in winter for northern countries (house heating due to low temperatures) while higher in summer for southern countries (air-conditioning due to high temperatures). Furthermore, the electricity data for the Czech day-ahead market seems to confirm intra-day, daily and monthly fluctuations of the spot prices and electricity demand.

Figure 3.1: Seasonal fluctuations



Note: The graphs capture the average prices (on left axis) and average demands (on right axis) for the Czech day-ahead market computed for the period from February 1, 2009 to August 31, 2015.
Source: Author's computation.

3.2.2 Mean reversion

This characteristic is very specific for electricity price series while it is not common for the financial time series. Saying the spot price series is mean reverting means that no matter how far the actual spot price is from the mean, the time series returns back to the long-term mean in the future. How fast the prices tend to return back to the mean, however, varies with power markets and also from study to study, but usually it does not last more than few days. Econometrically, we can determine whether or not the spot prices are mean reverting based on the value of Hurst exponent or equivalently using the (fractional) integration parameter d (Simonsen *et al.*, 2004).

The Hurst exponent is labeled H and for the stationary processes takes on values between 0 and 1. The threshold $H = 0.5$ identifies uncorrelated processes. The positively correlated process emerges when $H > 0.5$ and it is said to be a persistent one. On contrary, negatively correlated process emerges for $H < 0.5$, which is said as the anti-persistent process. The second important threshold is $H = 1.5$ which is level for the unit root. When $H < 1.5$ the mean-reversion occurs, while when $H > 1.5$ the prices do not return to the mean but rather explode (Krištoufek & Luňáčková, 2013).

For period from January 1, 2009 to November 30, 2012, Krištoufek & Luňáčková (2013) calculate the Hurst exponent to be approximately 1.1. This implies that the prices in the Czech day-ahead market are non-stationary, since H does not lie in the interval from 0 to 1, while they are at the same time strongly persistent since $H \gg 0.5$, and far from the unit-root case as $H < 1.5$. Adding all these statements together implies that the spot prices in the Czech day-ahead market are mean reverting. Furthermore, according to the authors, the prices are pronounced to be mean-reverting at relatively high speed, since H is well below 1.5 characteristic for the unit root process (Krištoufek & Luňáčková, 2013).

As a result, the Czech day-ahead market can be viewed as being in the middle between two extreme cases. Stationary electricity prices, the first extreme, are found by Park *et al.* (2006) in their study of the power markets in the USA. The other extreme, prices nearly resembling the unit-root process, is provided by Simonsen

(2003) when examining the Nord Pool.

3.2.3 High (or excess) volatility

This characteristic refers to the standard deviation of the return (Simonsen *et al.*, 2004). Excess volatility can be, according to the existing literature, mainly attributed to the inability to store electricity which means that reserves cannot be effectively utilized in times of sudden excess demand and/or sudden weather changes (Krištoufek & Luňáčková, 2013).

The daily volatility of the Nord Pool, which is the less volatile power market worldwide, for the period May 1, 1992 to May 1, 2004 is equal to 16%. On contrary, the daily volatility for crude oil is $2-3\%$, for stock indexes it is only around $1-1.5\%$ and for the short-term interest rate it is even just 0.03% (Simonsen *et al.*, 2004). It can be seen, therefore, that when compared to other financial and commodity markets the electricity prices are much more volatile.

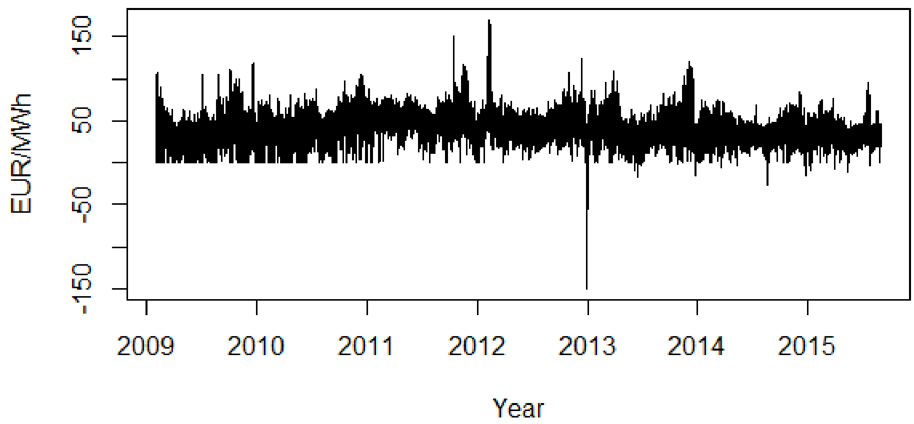
Finally, it should be mentioned that volatility clusters and, furthermore, that it exhibits an inverse leverage effect. The former one relates to fact that volatility tends to be persistently higher in some periods of time compared to the others, while according to the later one the price volatility has tendency to increase by greater extent following the positive shocks relative to the negative shocks (Knittel & Roberts, 2005).

3.2.4 Occurrence of severe jumps

The spot electricity prices are not only highly volatile, as described above, but they also have tendency to change substantially. The prices can change by more than tenfold in just one hour. While these changes occur suddenly and infrequently, the duration is usually short-lived and prices generally return back to "normal" level within only several hours. The feature of sudden sharp price rise followed by more gradual decline is called jump or spike (Simonsen *et al.*, 2004).

Figure 3.2 shows the evolution of hourly spot prices from February 1, 2009 to August 31, 2015 in the Czech day-ahead market. Note that until February 1, 2012 the negative spot prices were not allowed in this market (OTE, a.s., 2015d) and so we can see that the spot prices were bounded from below. Visual inspection of the figure suggests that the price changes substantially in both directions but still fluctuates around the long-term mean. The graph also shows that at the end of year 2011 and at the beginning of year 2012 the spot prices reached the highest values, while the minimum spot price was find at the end of year 2012. In particular, four highest spot prices (170.00, 163.40, 161.86 and 151.60 EUR/MWh) occurred during the first 14 days in February 2012, while four lowest spot prices (-150.00, -120.00, -109.31, -66.45 EUR/MWh) occurred during the last week in December 2012. Although negative spot prices may be counterintuitive it is common finding in each power market worldwide especially in periods when the demand is very low. In such cases, it is cheaper to maintain the production rather than shutting-down and restarting the production later on when the demand suddenly increases (Chan *et al.*, 2008).

Figure 3.2: Time series plot of hourly spot prices

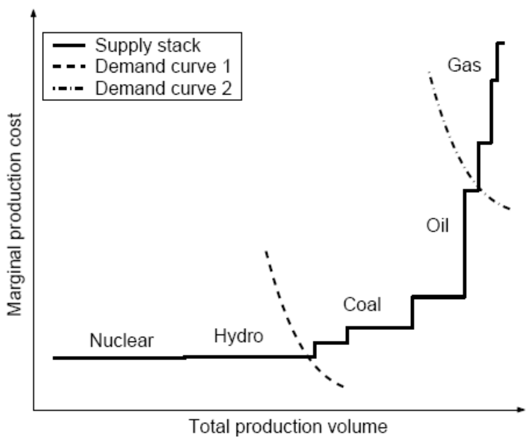


Note: The hourly spot prices for the Czech day-ahead market from February 1, 2009 to August 31, 2015 where negative prices were not allowed before February 1, 2012.
Source: Author's computation.

Occurrence of sudden severe jumps is considered to be the most important stylized fact of the spot prices. Unfortunately, we can hardly find a sole explanation that would clearly clarify why jumps (of such severity) occur so frequently in the power markets. On the other hand, we can fairly say that occurrence of jumps is influenced by the demand curve and the production stack, and by theirs relation to the electricity prices (Simonsen *et al.*, 2004).

To understand it, we need to keep in mind that the power market is characterized by highly inelastic demand, with absolutely inelastic demand occurring in the short-run which implies that the supply curve determines the spot price. Nevertheless, power generations from different sources have different marginal cost of production and so the supply curve is not smooth but has rather shape of upward-sloping stairway (Krištoufek & Luňáčková, 2013), see Figure 3.3. As it is apparent from this figure, power from nuclear plant is much cheaper compared to coal or gas plants.

Figure 3.3: Supply curve and two possible demand curves



Note: The spot price is equal to the vertical distance of the intersection of the supply and demand curve. When demand is low (the bottom dashed line), small increase is not likely to result in high price rise. Contrary, when demand is so high (the upper dashed line) that the plants with higher marginal costs need to be activated, the price must rise, and so jump is more likely to occur.
Source: Simonsen *et al.* (2004), p.26

The spot price is equal to the vertical distance of the intersection of the supply and demand curve. Figure 3.3 further suggests that when demand is low (as captured by the bottom dashed line), small increase is not likely to result in high price rise. The reason is that in the case of low demand, the supply is flat rather that having shape of upward-sloping stairway. When demand is so high (as captured by the upper dashed line) that the plants with lower marginal costs cannot meet the demand, plants with higher marginal costs need to be activated, which simply implies that the price must rise. Additionally, for some market participants the electricity is essential input for their production without which they cannot produce and so they are willing to pay no matter how high the price is. Therefore, even small rise in consumption can be followed by large increase in price, given that more expensive plant needs to be activated to satisfy the increased demand and so higher marginal costs need to be covered. After consumption declines, the price quite quickly returns back to the normal level as the more expensive plants are no longer needed (Simonsen *et al.*, 2004).

When looking at Figure 3.1, these arguments suggest that jumps are more likely to occur in peak hours compared to offpeak hours, during weekdays compared to weekends, and in winters/falls compared to summers/springs, as the demands are always much higher in the former cases as opposed to the latter ones.

Nevertheless, jumps can also appear when the consumption remains constant. This occurs when some cheap plants are not activated (are out of order), which is the case mainly due to the planned maintenance of the transmission grid and/or plant or due to the unexpected technical problems. Under these circumstances, activation of more expensive plants is required in order to satisfy the demand which increases the price (Simonsen *et al.*, 2004).

Such a strong behavior is not common in any other well known market. When modeling and forecasting the spot prices, the trades need to be well aware of existence of jumps. In a later section, we provide more information regarding the jump occurrence, including the distribution over days, seasons and so on, but already this primary analysis of the jump occurrence indicates that the Czech day-ahead market is exposed to frequent jumps which supports the importance of our topic.

4 Data description and econometric models

This section firstly describes the dataset used within the whole econometric analysis, secondly it provides description of the quadratic variation theory together with its modifications proposed in order to be able to separate the jump and continuous component of the realized volatility and hereby calculate the share of jumps, and finally, it presents the models capable of forecasting the occurrence of jumps [1].

4.1 Data

A major part of the dataset used in this thesis is obtained from the OTE web-site (OTE, a.s., 2015c) where the electricity data is readily available at Yearly Reports since 2002. It should be noted that when deciding about the beginning of the observation period the data prior to February 1, 2009 are deliberately discarded, because there were two providers of the day-ahead market before that day with OTE becoming a sole operator and, additionally, with euro becoming a trading currency only after this date. As a consequence, the sample begins on February 1, 2009 and ends on August 31, 2015. The thesis deals with hourly data and so it is important to note that the transitions from summer to winter time and vice versa are retained and so in each year there is one day at the very end of October having 25 hours and one day at the very end of March having 23 hours only. Accordingly, our sample consists of 57 671 observations.

OTE web-site (OTE, a.s., 2015c) for a day-ahead market is investigated to obtain hourly spot prices (in EUR per MWh) and hourly purchases (in MWh) where, by the construction, we can treat purchase as demand. Furthermore, we need to gain detailed meteorological data which are, to the best of our knowledge, not freely accessible at any web-site. For this reason, we connect with the Czech Hydrometeorological Institute (CHMU) and based on the internal communication we acquire historical data on a daily average, maximum and minimum temperature within the whole territory of the Czech Republic (CHMU, 2015).

[1] It also should be mentioned that through the whole thesis, we apply R studio, version 3.2.2, to deal with all the econometric issues.

We begin our data analysis with a detailed description of the most important variable, of the spot prices. Table 4.1, in the first and second column, shows the descriptive statistics of hourly spot prices and returns which are defined as the first difference of the spot prices. The mean spot price is 39.58 EUR/MWh. The maximum price, 170.00 EUR/MWh, is more than four times the mean, while the minimum price is negative, -150.00 EUR/MWh. The rationale behind the negative spot prices has already been commented in Section 3.2.4. For both spot prices and returns the standard deviations are high relative to the mean.

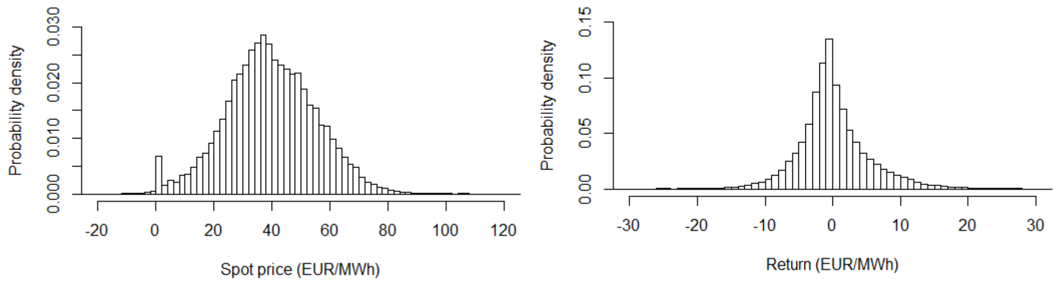
Table 4.1: Summary statistics

	Spot price	Return	Demand	Temperature
Mean	39.58	0.0006	1101.30	-0.25
Std dev	16.20	5.99	577.54	8.09
Median	39.00	-0.40	1119.00	-0.68
Skewness	-0.0099	0.43	0.24	0.18
Kurtosis	6.18	12.70	2.40	2.35
Min	-150.00	-82.31	65.50	-19.64
Max	170.00	99.18	3470.90	23.99
Jarque-Bera test	24274	228090	1426	1302
	(<0.01)	(<0.01)	(<0.01)	(<0.01)
Dickey-Fuller test	-22.15	-44.97	-20.83	-7.85
	(0.01)	(0.01)	(0.01)	(0.01)
Kwiatkowski test	8.12	0.0085	30.68	0.27
	(<0.01)	(0.01)	(<0.01)	(0.01)

Note: Return refers to the first difference of the spot prices and by temperature we mean the deviation (in °C) of daily average temperature from the mean temperature computed over the preceding 365 days. p-values of Jarque-Bera, Dickey-Fuller and Kwiatkowski-Phillips-Schmidt-Shin tests can be found in the parenthesis.

Moreover, according to Figure 4.1, which presents the histograms of hourly spot prices and returns, we can conclude that neither the spot prices nor the returns are normally distributed. These findings can be confirmed by Jarque-Bera test statistics

Figure 4.1: Histograms of spot prices and returns



Source: Author's computation.

which test the null hypothesis of normality against the alternative hypothesis of non-normality (Jarque & Bera, 1980). For both series the p-values appear to be smaller than 0.01 resulting in rejection of the null hypothesis of normality for both spot prices and returns. Moreover, both series exhibit fat tails as described by large excess kurtosis. These results suggest that overall the jumps are more frequent than it would other be the case under a normally distributed series (Swider & Weber, 2007).

A little bit surprising result brings a negative sign on skewness of the spot price as in the literature there seems to a consensus highlighting a positive sign, see for instance Ullrich (2012) among others. According to our results, the spot prices are more likely to move downward. Nevertheless, the magnitude is very close to zero and so we do not regard this finding as considerable obstacle. Moreover, the returns are positively skewed which is in line with previous literature, see for example Mari (2006).

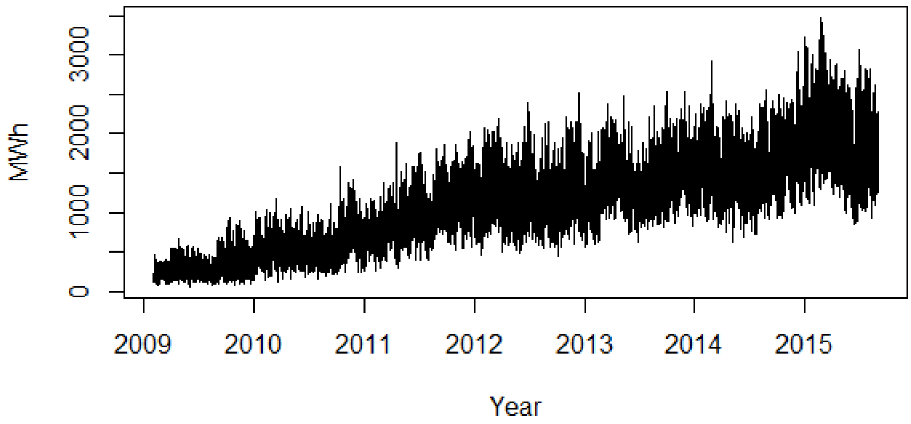
To bring an inference about the dynamics of the spot prices and returns we need to test whether the series are stationary or non-stationary, in which case we follow Krištoufek & Luňáčková (2013) and use results of two tests. The first one is Augmented Dickey–Fuller test where the null hypothesis of unit root in the time series is tested against the alternative of no unit root, the outcome of this test is always a negative number where more negative values indicate a stronger rejection of the null hypothesis (Dickey & Fuller, 1979). As Table 4.1 suggests the null hypothesis is rejected for both series and so neither of them contain a unit root. The second test is Kwiatkowski-Phillips-Schmidt-Shin test which tests the null hypothesis of stationary time series against the alternative of non-stationary series (Kwiatkowski *et al.*, 1992). Again, as the table proposes, the null hypothesis is rejected for both time series implying that both the spot prices and returns are non-stationary. The finding of dealing with non-stationary spot prices that do not contain a unit root has already been touched in Section 3.2.2.

Besides the conclusion about the skewness, all the above mentioned properties are in accordance with characteristics of electricity prices and returns reported in other studies, see for instance Haugom & Ullrich (2012) and Knittel & Roberts (2005). Moreover, the Czech day-ahead market seems to be very attractive as it lies between the Nord Pool, which is the less volatile power market worldwide, and Australia, where the jumps reach extremely high value (sometimes they are even 100 times greater than the mean price).

The second variable that should be commented, but not to the same extent as the spot price, is demand with its descriptive statistics being presented in the third column of Table 4.1. As can be seen, the maximum demand, which is equal to 3 471 MWh, is more than fifty times greater than the minimum demand, which is only 65.5 MWh. This is in line with our previous findings saying that demand is subject to daily fluctuations with demand being lowest during night when most of the businesses are closed and when people are not that active. When depicting the evolution of hourly demand over the entire sample period in Figure 4.2, we immediately reveal that this time series exhibits a linear trend. This feature is, however, not surprising as people are generally known to increase the consumption of goods in time and electricity power is not an exception. Additionally, the linearly trending demand is common finding in the power markets worldwide, see for instance Christensen *et al.* (2012). As a result, before this variable can enter any model it needs to be standardized, which is in detailed described later in the text.

The final variable, whose descriptive statistics are captured in the last column of Table 4.1, is temperature defined as the deviation (in °C) of daily average temperature from the mean temperature computed over the preceding 365 days where the choice of such a form is supported later in the text. Note that while hourly sampling

Figure 4.2: Time series plot of hourly electricity demand



Note: The hourly demand for the Czech day-ahead market from February 1, 2009 to August 31, 2015. As it is obvious, demand exhibits linear trend and so it needs to be treated accordingly.
Source: Author's computation.

frequencies apply to all other variables, temperature is obtained on a daily basic and so the same value is assigned to each hour during one day. This finding, nevertheless, does not cause any difficulty in our further analysis, to verify this statement see for example Huisman (2008).

4.2 Quadratic variation theory

Having described the data, we can turn to the econometric part. The intention of this subsection is to describe the quadratic variation theory and its modifications which are employed in order to decide about the share of jump component in the total realized volatility.

To perform the analysis we first need to define a jump-diffusion model where the spot price, $p(t)$, has the following differential form:

$$dp(t) = \mu(t)dt + \sigma(t)dW(t) + \kappa(t)dq(t) \tag{4.1}$$

where $\mu(t)$ represents the drift, $\sigma(t)$ denotes the instantaneous volatility, $W(t)$ refers to a standard Wiener process, $q(t)$ stands for a jump-counting process, with $q(t) = 1$ provided that a jump occurs at time t, and $\kappa(t)$ refers to the size of the jump given that $dq(t) = 1$ (Chan *et al.*, 2008).

The jump-diffusion model can therefore be viewed as a sum of infrequent jumps and continuous component of price volatility. Though this model is supported from an theoretical point of view, its estimation is pretty difficult since it is nearly impossible to separate jumps from the continuous component. On the other hand, as already mentioned we can overcome such difficulty when using a quadratic variation theory. Accordingly, the quadratic variation (QV) may be defined as follow:

$$QV_t = \int_0^t \sigma^2(s)ds + \sum_{s=1}^{q(t)} \kappa^2(s) \tag{4.2}$$

In order words, the quadratic variation is equal to the integrated volatility of the continuous component plus the sum of squared jumps. It is obvious that given the jump does not occur on a particular day t, the sum of the squared jumps is equal to zero, implying the quadratic variation equals to integrated volatility (IV) only:

$$IV_t = \int_0^t \sigma^2(s)ds \tag{4.3}$$

Nevertheless, we cannot observe the quadratic variation directly and so we need to find a proxy for it, in which case we use the realized volatility (Andersen & Bollerslev, 1998).

Let assume that spot prices are sampled over a period of T days and that within each day the prices are observed at M equally-spaced sampled intervals. In light of our work $T = 2\ 403$ and $M = 24$ [2]. Following the work of Andersen & Bollerslev (1998), the intra-day returns for day t may be expressed as:

$$r_{t,j} = p_{t,j} - p_{t,j-1} \qquad j = 1, \ldots, M \qquad t = 1, \ldots, T \tag{4.4}$$

The limitation of this specification is that the drift, as captured in Equation 4.1, is assumed to be negligible in case of the financial time series provided that the sampling frequency M is high. For that reason when dealing with financial dataset we assume that $\mu(t) \doteq 0$. Nevertheless, the spot prices in all markets exhibit specific features. In particular, in Section 3.2 we show that the spot prices in the Czech day-ahead market have non-zero mean, exhibit intra-day, daily and seasonal

[2] Even though there are some days in the sample having 25 hours, as pointed above, the previous literature only suggests to use $M = 24$, such as Haugom *et al.* (2011) or Haugom & Ullrich (2012).

pattern. Consequently, as opposed to the financial time series we cannot ignore the drift (Chan *et al.*, 2008).

Since mean spot prices are shown to be higher during the peak hours as opposed to offpeak hours, weekdays as opposed to weekend, lowest during summer, and depend on the lagged price we model the spot prices as:

$$\begin{aligned} p_{t,j} = {} & \alpha_1 Peak_j + \alpha_2 OffPeak_j + \alpha_3 Weekend_t + \alpha_4 Fall_t + \alpha_5 Winter_t + \\ & + \alpha_6 Spring_t + \beta p_{t,j-1} + \varepsilon_{t,j} \end{aligned} \tag{4.5}$$

where dummy variables *Peak* and *OffPeak* are only defined during weekdays such that the former one is equal to 1 for hours j from 8 a.m. to 8 p.m. and zero otherwise (OTE, a.s., 2015c), *Weekend* is equal to 1 for days t on Saturday and Sunday and zero otherwise. The seasonal dummy variables *Fall*, *Winter* and *Spring* are equal to 1 for days t belonging to the astronomical fall, astronomical winter and astronomical spring, respectively. Notice that the model involves both *Peak* and *OffPeak*, in addition, $Peak + OffPeak = Weekday$ and $Weekday + Weekend = 1$. Therefore, in order to avoid multicollinearity problem and to be able to run the regression we do not include the dummy variable *Summer* in the regression.

Equation 4.5 has a form of first order Autoregressive–moving-average model with exogenous variables, so called ARMAX(1,0), and therefore it is estimated accordingly. Knittel & Roberts (2005) consent with the model specification and, more importantly, according to this article we can view residuals from Equation 4.5 as the drift $\mu(t,j) \equiv \widehat{\varepsilon_{t,j}}$, where j emphasizes that we use hourly spot prices.

Finally, the common patterns are removed if the estimated drift is subtracted from the intra-day returns, in other words if the returns are demeaned:

$$r^*_{t,j} = r_{t,j} - \mu(t,j) \qquad j = 1,\ldots,M \qquad t = 1,\ldots,T \tag{4.6}$$

where $r^*_{t,j}$ are so-called de-meaned price changes (Chan *et al.*, 2008).

Having calculated the de-meaned price changes, and following Andersen & Bollerslev (1998), for each day t we can calculate the realized volatility (RV):

$$RV_t = \sum_{j=1}^{M} {r^*_{t,j}}^2 \qquad t = 1,\ldots,T \tag{4.7}$$

As already mentioned, realized volatility is consistent estimator of the quadratic variation, therefore, it can be viewed as a sum of continuous and jump component.

In order to be able to accommodate the separation of jump and continuous components we need to apply a bipower variation (BV) which was established by Barndorff-Nielsen & Shephard (2004) and modified by Huang & Tauchen (2005). In light of this work it can be defined as:

$$BV_t = \frac{1}{\mu_1^2(1-2/M)} \sum_{j=3}^{M} |r^*_{t,j}||r^*_{t,j-2}| \qquad t = 1, \ldots, T \tag{4.8}$$

where $\mu_1 = E(|Z|) = \sqrt{(2/\pi)} = 0.79788$ with Z being a standard Normal variable. In other words, the bipower variation is defined as the sum of the multiples of the absolute values of the de-meaned price changes distanced two periods (Huang & Tauchen, 2005).

Let assume that M increases, then RV_t converges to QV_t, and BV_t converges to IV_t. On top of that, $RV_t - BV_t$ converges to $QV_t - IV_t$ (Barndorff-Nielsen & Shephard, 2004) which together with Equation 4.2 and Equation 4.3 imply that the difference between the realized volatility and the bipower variation provides an estimate of jump component.

To test whether or not a significant jump occurs on a day t, Huang & Tauchen (2005) propose to apply the following test statistic:

$$Z_t = \sqrt{M} \frac{(RV_t - BV_t)/RV_t}{\sqrt{(\mu_1^{-4} + 2\mu_1^{-2} - 5)\max(TQ_t/BV_t^2, 1)}} \tag{4.9}$$

where tripower quarticity (TQ) is defined as:

$$TQ_t = \mu_{4/3}^{-3} \left(\frac{M^2}{M-4} \right) \sum_{j=5}^{M} |r_{t,j}|^{4/3} |r_{t,j-2}|^{4/3} |r_{t,j-4}|^{4/3} \tag{4.10}$$

with $\mu_{4/3} = E(|Z|^{4/3}) = 0.8308609$ (Haugom *et al.*, 2011). It can be shown that Z_t converges to a standard Normal variable under the null hypothesis of no jumps, given that M goes to ∞. Accordingly, we conclude that a significant jump occurs on day t if the test statistic Z_t, as stated in Equation 4.9, exceeds the $(1-\alpha)$ percentile of the Normal distribution. Afterwards, for a given significance level α we compute the jump component (JV) of the total variation as:

$$JV_t = I_{\{Z_t > \Phi_{1-\alpha}\}}(RV_t - BV_t) \tag{4.11}$$

where $I_{\{Z_t>\Phi_{1-\alpha}\}}$ is an indicator function that equals to 1 if $Z_t > \Phi_{1-\alpha}$ and 0 otherwise (Eichler *et al.*, 2013).

We have already computed the total realized volatility which is the sum of jump and continuous component of total price change. Therefore, given that we have provided formula for jump component it only remains to subtract it from the realized volatility in order to get the continuous component (CV) of the total price volatility (Chan *et al.*, 2008):

$$CV_t = RV_t - JV_t \tag{4.12}$$

Before moving on, it is worth mentioning that some researchers, such as Haugom *et al.* (2011), do not address the problem of non-zero drift in their analyses and so they only limit themselves in using the intra-day returns from Equation 4.4 rather then the de-meaned price changes. Consequently, when computing the realized volatility, jump and continuous component such analyses would replace $r^*_{t,j}$ with $r_{t,j}$ in Equations 4.7, 4.11 and 4.12, respectively. Nevertheless, we believe that seasonal fluctuations can explain some part of the volatility and, therefore, we rather use the de-meaned price changes in order to get more precise results.

4.3 Logit models

Assuming that jumps form a significant part of total variation, the intention of this subsection is to provide models for forecasting the occurrence of jumps. More precisely, we exploit the historical data in order to determine the probability that a jump will occur.

4.3.1 Jump identification strategy

At the very beginning, few comments concerning a jump identification technique are needed. In previous subsection, the quadratic variation theory is employed in order to econometrically disentangle the jump and continuous component of the total price volatility. Moreover, a significant jump is said to occur on particular day t given that the test statistic, as captured in Equation 4.9, exceeds the $(1-\alpha)$ percentile of the Normal distribution. Therefore, it could seem natural to apply this statistic to identify days where significant jumps occur. While we are aware of the fact that this identification method is solely based on econometric findings and would therefore provide more accurate separation of jump and non-jump days, we do not employ this method when forecasting the occurrence of jumps. The explanation lies in the practical applicability of the results because forecasting based on a such method would only tell us whether or not a jump will occur during a next day but would not be capable to identify the exact hour. Since traders are interested whether a jump will occur in a particular hour of the following day, rather than knowing that the jump will occur in any hour of the following day, we need to go for another method that is able to identify jump in a given hour. Consequently, instead of assigning each observation two indexes, namely t for day and j for hour, we only limit ourselves in using index t which (in the sections dealing with forecasting) indicates the temporal ordering of the time series, where $t = 1, \ldots, 57671$.

To find a proper technique how to identify whether or not a jump occurs on a given hour we review the suggestions of the previous literature. Though each researcher brings slightly distinct method, the approaches of determining a jump can be summarized into 3 different categories:

1. We can treat an outlier price as being jump, in which case the jump is identified whenever the price exceeds mean price plus three standard deviations, both calculated over the horizon of preceding days. The authors however disagree about the length over which to compute these statistics and also about thr number of standard deviations that should be added to the mean price. Literature using this identification strategy comprises, for example, Zhao *et al.* (2007).

2. We can also pre-specify a sufficiently high threshold and then all observations whose magnitude exceeds such threshold are considered as jumps. The literature suggests the threshold to be equal to 90th - 99th percentile of the price series, see for example Christensen *et al.* (2012) or Herrera & González (2014), where the choice of specific percentile basically differs with the power markets.

3. Another source of literature determines jumps based on a comparison of so-

called normal and abnormal price changes. The researchers bring an econometric model estimating normal and jump variations where the former one reflects day-to-day changes that are considered to be normal while the latter one refers to extraordinary changes, which can be thought of as jumps. Consequently, whether or not an observation is regarded as jump heavily depends on volatility of period in which it occurs. When we assume two observations of the same magnitude, then one observation can be regarded as jump, it occurs in less volatile period, while the second one can be thought of as "regular" price change, it occurs in period that is more volatile. This approach can be mainly find in Hellström *et al.* (2012)

In the spirit of these categories and in order not to rely on a single jump identification strategy, we decide to provide and investigate three alternatives. The first one, called *jump_season*, is related to the first category from the above mentioned and, therefore, the jump is said to occur at time t when:

$$p_t > \mu + 2\delta \tag{4.13}$$

where p_t is the spot price, μ is mean and δ is standard deviation. Compared to the previous literature, we do not compute these two statistics over the preceding days but rather allocate observations into 12 groups according to the dummy variables, and then compute means and standard deviations separately for every group, e.g. peak hour during winter weekday. Even though neither means nor standard deviations differ considerably across groups, we believe such modification outperforms the conventional technique as it better reflects the price fluctuations. The determination of groups together with their means and standard deviations can be found in Table A.2 in Appendix.

The second and third jumps identification strategies are based on the second category from the above mentioned and therefore it only remains to decide about a sufficiently high threshold u such as:

$$p_t > u \tag{4.14}$$

The data suggests to choose 90th and 95th percentile of the price series, which correspond to 59.9 EUR and 65.5 EUR, respectively. In the following, we refer to these jump identification strategies as *jump_ 90percentile* and *jump_ 95percentile*, respectively.

Having provided the definitions of individual strategies, it is worth looking at numbers of jumps identified by each of them, since as one would anticipate these are not identical. According to the last raw of Table 4.2, when using *jump_ 90percentile* we identify higher number of jumps then when using *jump_ 95percentile* which is as expected since the latter strategy is much more binding meaning that the price needs to exceed 65.5 EUR while with the former strategy it only has to surpass 59.9 EUR to conclude that a jump occurs. Nevertheless, they both lead to much higher number of identified jumps compared to *jump_ season*. This finding confirms that such identification strategy is, as the only one, able to absorb the seasonal fluctuations meaning that the price changes, which are concluded as jumps under the remaining two strategies, are viewed as normal price changes under *jump_ season*. In other words, since *jump_ season* takes into account the seasonal fluctuations, part of the price change is assigned to such fluctuations and then the remaining part is not that strong to conclude a jumps occurs.

Another interesting feature arising from Table 4.2 relates to the distribution of jumps over hours, days, and seasons. In particular, regardless of the jump identification strategy, we can see that more jumps occur from Monday to Friday compared to weekends, during peak hours compared to offpeak hours and during fall/winter months compared to spring/summer months. None of these relations should be surprising as in all former cases higher demand arises meaning that jumps are more likely to occur (recall the explanation provided in Section 3.2.4).

Before moving on, we examine another important feature of jumps, pointed out by Herrera & González (2014) and Paraschiv *et al.* (2015), namely jump clustering. This feature can be observed in Figure 4.3, which depicts the jump occurrence separately for each jump identification strategy and which, for better visibility, only covers the period from June 1, 2010 to December 31, 2010. Note that for the rest of the sample similar figures appear for all strategies and are available upon request. As visible from the figure and also as the authors suggest, jumps tend to come in groups of more than one, meaning that once a jump occurs it is likely to be followed

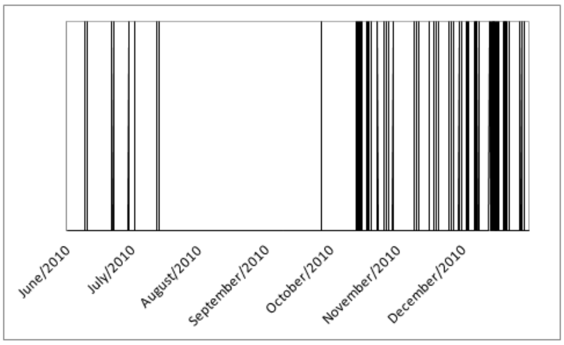
Table 4.2: Number of jumps identified under given specifications

	jump_season	*jump_90percentile*	*jump_95percentile*
Offpeak hour	90	879	386
Peak hour	881	4638	2397
Weekend	25	248	90
Monday	236	1104	584
Tuesday	194	1245	587
Wednesday	192	1186	614
Thursday	231	1082	567
Friday	118	900	431
Saturday	6	130	40
Sunday	19	118	50
Spring	107	1001	449
Summer	91	962	343
Fall	555	2373	1410
Winter	243	1429	671
Total number	996	5765	2873

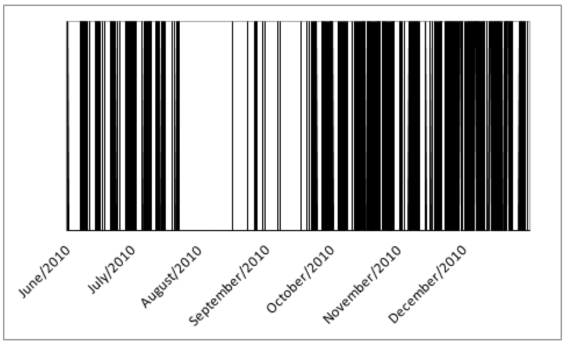
by another jump or by sequence of more jumps. This characteristic is specific for the power market and, therefore, does not have to be omitted when forecasting the probability of jump occurrence.

Figure 4.3: Jump occurrence based on various jump identification strategies

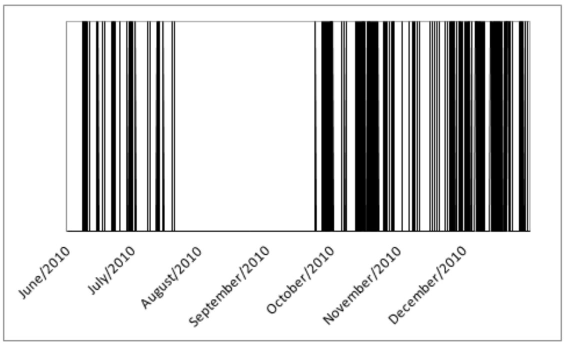
(a) *jump_ season*



(b) *jump_ 90percentile*



(c) *jump_ 95percentile*



Note: The figures only cover the period from June 1, 2010 to December 31, 2010 where each line represents a jump occurrence. For the rest of the sample similar figures appear for all strategies and are available upon request.
Source: Author's computation.

4.3.2 Logit model

When forecasting the jump occurrence and regardless of the choice of the jump identification strategy, the dependent variable can be thought of as binary variable that only takes on two values, it is either equal to 1 if jump occurs or 0 otherwise. Provided that we deal with sort of limited dependent variable, it is necessary to choose the econometric model accordingly in which case we turn to binary response models. For a best understanding of the rest of the analysis, some light on such models needs to be shed. The general form of the binary response models may be expressed as:

$$\text{Prob}(y = 1|\mathbf{x}) = G(\beta_0 + \beta_1 x_1 + \ldots + \beta_k x_k) = G(\beta_0 + \mathbf{x}\beta) \tag{4.15}$$

where $\mathbf{x}$ represents a full set of independent variables, $\beta_0, \ldots \beta_k$ are parameters to be estimated, and for all real numbers z, G stands for a function which can only take on values between 0 and 1, that is $0 < G(z) < 1$. The latter condition implies that the estimated response probabilities can only range from 0 to 1 (Wooldridge, 2009).

According to the econometric literature, various functional forms are proposed for function G. One of the most frequently used form is logistic function which is a cumulative distribution function for a standard logistic random variable:

$$G(z) = \frac{\exp(z)}{1 + \exp(z)} = \frac{1}{1 + \exp(-z)} \tag{4.16}$$

The second most common choice of function G is undoubtedly a standard normal cumulative distribution function which has the form of the following integral:

$$G(z) = \int_{-\infty}^{z} \phi(v) dv \tag{4.17}$$

where standard normal density, $\phi(z)$, is defined as:

$$\phi(z) = \frac{\exp(-z^2/2)}{\sqrt{2\pi}} \tag{4.18}$$

The former functional form leads to logit model that is sometimes referred to as logistic regression while the latter form brings probit model (Wooldridge, 2009). More importantly, the shapes of both functions are considerably similar, to be more specific, both functions $G(z)$ are increasing functions in z, with most quickly increase occurring around $z = 0$. Furthermore, as stated above, $G(z)$ can only takes on values between 0 and 1 which can be confirmed by computing limits at $\pm\infty$. It is clearly visible from Equations 4.16 and 4.17 that for $z \to \infty$ both $G(z) \to 1$ while for $z \to -\infty$ functions $G(z) \to 0$ (Wooldridge, 2009). Consequently, there is no reason to view logit model as a superior one relative to probit model and vice versa.

In the context of forecasting the occurrence of jumps, we decide to exploit logit model for the analysis as this choice is supported by Eichler *et al.* (2012). As obvious from above, Equations 4.15 and 4.16 together define a logit model and, therefore, we only need to perform slightly changes in order to make the model capable of forecasting.

Let assume a binary process J_t that is equal to 1 when a significant jump is concluded to occur at time t and 0 otherwise, in other words such a binary process can be thought of as occurrence of jump. Further assume that H_{t-1} represents all relevant information which is available up to time $t-1$. Then, following Eichler *et al.* (2012), the probability of jump occurrence at time t, given the past information is available up to time $t-1$, so called hazard rate, can be expressed as:

$$h_t = \text{Prob}(J_t = 1|H_{t-1}) = \frac{1}{1 + \exp(-\beta z_t)} \tag{4.19}$$

where we can choose either *jump_ season*, *jump_ 90percentile* or *jump_ 95percentile* to identify a jump occurrence at time t and, additionally, where βz_t refers to various definition of the model specification. Therefore, in order to save space, in the reminder of this subsection we do not rewrite the entire Equation 4.19 but rather restrain ourselves to provide only various forms of βz_t which define particular logit model specifications.

In Figure 4.3 we show that jumps tend to cluster and so once a jump occurs it is likely to be followed by another jump or by sequence of jumps. Having this in mind, we prose first model, to which we refer *logit_ 1*, where the probability of jump at t solely depends on occurrence of jump at $t-1$:

$$\beta z_t = \beta_0 + \beta_1 J_{t-1} \tag{4.20}$$

The resulting probabilities of this simple logit model can be interpreted as probability of jump occurrence provided that jump does not occur previously, which can be expressed as $\text{Prob}(J_t = 1|J_{t-1} = 0)$, and probability of jump occurrence conditional on previous jump, which can be similarly expressed as $\text{Prob}(J_t = 1|J_{t-1} = 1)$. The biggest advantage of this logit model can be seen in its ability to adjust right after the sequence of jumps is either initiated or terminated. On the other hand, assuming that the threshold probability, to conclude that jump occurs, is set to 0.5, problems arise when $\text{Prob}(J_t = 1|J_{t-1} = 0) < 0.5$ as we are unable to forecast the termination of sequence of jumps and we therefore always forecast one extra jump occurrence, and also when $\text{Prob}(J_t = 1|J_{t-1} = 1) \geq 0.5$ as we are unable to forecast the initiation of the sequence of jumps (Eichler *et al.*, 2012).

Attentive reader also immediately uncovers the biggest disadvantage of this model which is related to the very specific feature of the day-ahead market, commented in Section 3.1. As already stated, traders have to submit their offers for each particular hour of the following day, in which they want to participate, a day in advance and always before 11 a.m. Then, around 12 a.m., the resulting spot prices for each hour of the following day are published online. Let assume that the trader wants to submit an offer for 6 p.m., then it is obvious that she can never know the spot price nor whether a jump occurs at 5 p.m. simply because these data has not been published yet (note that the same apply to demand). Even thought we are aware of this obstacle, it may be interesting to set *logit_ 1* as benchmark model with which to compare the forecasting performance of further logit model specifications.

A tremendous amount of previous studies, though considering different approaches how to model jump occurrence or electricity prices as a whole, show that electricity demand (to which some authors refer to as load) and weather variables have extensive explanatory power when regarding the forecasting in power market, see for instance Becker *et al.* (2007), Christensen *et al.* (2012) or Herrera & González (2014). As already mentioned, electricity cannot be effectively stored implying that at each point in time the amount of electricity taken from the grid by the consumers, generally known as demand, needs to be balanced with amount of electricity generated by the producers, which is referred to as load. Consequently, assuming that no unusual events (such as a blackouts) take place, the demand and load can be easily interchanged (Becker *et al.*, 2007).

Before providing further model specifications, several issues related to the aforementioned variables need to be resolved. First, we consider the variable demand. When making the decision about its exogeneity we refer to Christensen *et al.* (2012) according to which the regulatory framework ensures the electricity is supplied to consumers at regulated prices and therefore protects consumers against unexpected jumps, which consequently guarantee that demand is exogenous. In Figure 4.2 we show the evolution of demand over time and conclude that it exhibits a linear trend. Therefore, in order to take this nonstationarity into account we need to standardize demand in which case we employ a generally used method that removes the linear trend while at the same time preserves the seasonal fluctuations, which is applied for instance by Christensen *et al.* (2012) or Eichler *et al.* (2012). For each $t = 1, \ldots, 57671$, we subtract the mean and divide the difference by the standard deviation, where the last two statistics are computed over the preceding 8 760 hours[3]:

$$\textit{Standardized Demand}_t = \frac{\textit{Demand}_t - \textit{Mean}^{8760}}{\textit{Standard Deviation}^{8760}} \tag{4.21}$$

where the upper indexes indicate the statistics are computed over the preceding 8 760 hours. For simplicity, whenever we refer to *Demand* in the following text we always have standardized demand in mind.

Second, we review the weather variable. Even though the previous studies agree about its considerable effect, they disagree about its precise form. Some researchers propose to employ the absolute deviations (in °C) of maximum and minimum daily temperatures from the mean temperature computed over the previous year, such as Eichler *et al.* (2012), relative deviation of daily temperature from the mean temperature, such as Huisman (2008), average of the daily temperatures over several cities, such as Knittel & Roberts (2005), maximum daily temperature, such as Herrera & González (2014) and other suggestions. In this thesis we decide to use deviation (in °C) of daily average temperature from the mean temperature computed over the preceding 365 days and refer to it as *Temperature* in the remainder. Note that since we only have daily data on temperature instead of hourly data, we have to compute the averages over the preceding 365 days, while in the case of *Demand* specification in terms of hours needs to be employed.

Since all the upcoming suggested logit model specifications include *Demand* and *Temperature* as independent variables, reliable forecasts of both *Demand* and *Temperature* are needed, in order to be able to forecast the probability of jump occurrence. Forecasting *Temperature* is of independent interest and is beyond the scope

[3]We enlarge the sample period of demand by obtaining data from February 1, 2008 to January 31, 2009, which are available from the OTE web-site (OTE, a.s., 2015c), in order to have properly defined standardized demand for even the first 8 760 hours of the sample.

of this thesis, nevertheless, contemporary meteorology agencies are able to forecast temperature up to few-days ahead. They base the forecasts on a broad spectrum of data and apply advanced modeling and, more importantly, they are able to provide considerably more accurate forecasts compared to our situation with limited data availability and limited modeling capability. Since the temperature forecasts are readily accessible by all market participants, we assume that data on *Temperature* are always available one day in advance. The idea of using the actual rather than forecasted weather data coincide with Christensen *et al.* (2012), Weron & Misiorek (2008) and many others. On the other hand, forecasting *Demand* is surely within the scope of this thesis but in order not to distract the reader by inserting the model of forecasting *Demand* inside the logit model specifications we set it aside and touch it in the following subsection. Nevertheless, since this variable enters all further logit model specifications, it needs to be at least mentioned that $Demand^F$ is used to label the forecasts of standardized demand in the following text.

Since there seems to be a consensus of opinions saying electricity demand and weather variable have the greatest impact on the probability of jump occurrence, we provide second logit model specification that only captures these two as independent variables which yields:

$$\beta z_t = \beta_0 + \beta_1 Demand_t^F + \beta_2 Temperature_t \tag{4.22}$$

With this model specification, which we call *logit_2*, we are able to model a great deal of short-term and long-term fluctuations in jump occurrence where $Demand^F$ captures the short-term while *Temperature* captures the long-term fluctuations (Eichler *et al.*, 2012). Nevertheless, the model completely omits the information about the past jumps. Eichler *et al.* (2012) resolve such problem by including a dummy variable for a lagged jump, Christensen *et al.* (2012) involve, albeit not in logit model, a variable counting the distances between jumps. Eichler *et al.* (2013) examine their suggestion and bring rather more complicated method which models the probability of jump as a function of elapsed time from the last jump and t. The problem behind all these methods relies on the fact that we are only able to identify jumps which occurred no later than today (the explanation is analogous to the one we provided when considering the lagged jump).

Therefore, we propose modification of theirs suggestions and define variable $Duration_t$ in such a way to correctly depict the day-ahead market. The consequent extension, to which we refer as *logit_ 3*, has the following form:

$$\beta z_t = \beta_0 + \beta_1 Demand_t^F + \beta_2 Temperature_t + \beta_3 Duration_t \tag{4.23}$$

To understand how $Duration_t$ is defined, let assume that time t belongs to day D and that the last jump occurs at day $D - 1$. Then $Duration_t$ represents the time elapsed between the hour when the last jump occurs and the first hour of day D. For clarity, when the last jump occurs at 5 p.m., then $Duration_t$ is equal to 7. When the last jump occurs at day $D - 2$ we need to add 24, therefore, assuming that the jump occurs at 5 p.m. on day $D - 2$, then $Duration_t$ is equal to 31. Analogously for jumps occurring before $D - 2$. It should also be mentioned that this variable is same for all hours within one day. Further note that it changes according to which jump identification strategy is used as each of them bring unequal number of jumps and also different jump location within the sample.

Furthermore, motivated by the already mentioned characteristics of jumps, meaning that jumps are less likely to occur during offpeak hours and also during weekends as opposed to peak hours and weekdays, we allow for more precise forecasting by adding additional variables and define *logit_ 4*:

$$\begin{aligned} \beta z_t = \beta_0 + \beta_1 Demand_t^F + \beta_2 Temperature_t + \beta_3 Duration_t + \beta_4 OffPeak_t + \\ + \beta_5 Weekend_t + \beta_6 p_{t-24} \end{aligned} \tag{4.24}$$

where dummy variables *OffPeak* and *Weekend* are defined already in the previous section. When studying the previous literature, we find out that some researchers use price lagged by one hour, see Eichler *et al.* (2012). Once again, since we cannot know this variable at time we submit the offer, we should not include such variable in the model. Moreover, the previous studies stress that both price lagged by one hour and by 24 hours should be included, such as Eichler *et al.* (2013), nevertheless, we find non zero correlation between these two. Therefore, in order to obtain more accurate results and to account for the memory properties of the spot prices we decide to use price lagged by 24 hours, p_{t-24}, which is readily available at the time we submit the offer. The biggest advantage of this model specification over those defined earlier is its ability to capture the sequences of jumps and at the same time, on the basis of further information, to forecast first and last jumps in the sequence.

4.3.3 Estimation technique and initial expectations

Having identified all the logit model specifications, it only remains to comment on the estimation techniques and given the previous literature and findings uncovered in Section 3.2 to provide our suggestions about the expected signs on coefficients.

We estimate models *logit_ 1* through *logit_ 4* via Maximum Likelihood. While *logit_ 1* through *logit_ 4* capture different bundles of independent variables, we can still hypothesize about the effect of each variable. Notice that the dependent variables of all model specifications are binary ones, equal to 1 for jump hour and 0 otherwise. Consequently, the magnitudes of estimated coefficients cannot be treated in the same way as in the case of linear models because they do not represent the marginal effects of independent variable on the dependent variable. Fortunately, this does not provide an obstacle as we are primarily interested in the direction of the effect of the independent variable and so only signs of the coefficients are relevant for our analysis. In particular, positive sign indicates that the variable increases the probability while negative sign indicates it decreases the probability of jump occurrence (Wooldridge, 2009).

First, the lagged jump J_{t-1}, which is included only in *logit_ 1*, is definitely assumed to have a positive effect on probability of jump occurrence as sequences of jumps are shown to be present in the data and so past jump should increase the probability that another jump will occur in the next period.

Second, and perhaps most importantly, $Demand^F$ is expected to raise the probability of jump occurrence where there seems to be two main justifications behind it. The first one relies on necessity to activate plants with higher marginal costs in order to meet demand in times of unexpectedly high demand which may translate in sudden price increase, in jump, (Simonsen *et al.*, 2004). The second possible justification relates to the capacity constraint in production. When demand is high, the system works very close to capacity constraint, and so when even very small negative supply or positive demand shock hits the power market, it is very likely to be followed by jump (Hellström *et al.*, 2012).

Third, *Temperature* is supposed to have positive effect on the probability of jump occurrence. When recalling the definition of this variable (deviation of the daily temperature from the average temperature computed over the preceding 365 days) and when keeping the average temperature computed over the preceding 365 days constant, it immediately follows that the higher is the *Temperature*, the lower is the daily temperature. In Table 4.2 we show that, no matter which jump identification strategy is used, the highest number of jumps occurs during fall and winter when the daily temperatures are low. As a result, the positive sign on *Temperature* is expected, as higher *Temperature* means lower daily temperature which is expected to transform into higher probability of jump occurrence.

Fourth, *Duration* is anticipated to have a negative effect on the probability. The explanation lies behind the fact, that the shorter is the period elapsed from the last jump, the higher is the probability that another jump will occur as jumps tend to occur in sequences of more than one.

Fifth, we already presented a table showing the distribution of jump occurrences within a week indicating that these are less likely during offpeak hours and weekends. It is therefore not surprising that both *OffPeak* and *Weekend* are assumed to reduce the probability of jump occurrence.

Finally, the lagged price, p_{t-24}, is expected to raise the probability of jump occurrence as the long-term memory of the spot prices is revealed in the Czech day-ahead market by Krištoufek & Luňáčková (2013) and so the higher is the lagged price, the higher is the current price and so the higher is the probability of jump occurrence.

4.3.4 Forecasting demand

Since forecasting the electricity demand is not of the primary interest but at the same time it is crucial part of models for forecasting the jump occurrence we attempt to implement a simple method only.

When reviewing the previous literature to gain a sense of various methods we surprisingly find out stable periodic weekly patterns in the electricity demand which seem to hold across different power markets, see for instance Taylor *et al.* (2006) or Eichler *et al.* (2012). Nevertheless, as found out earlier such patterns are not evident in the Czech day-ahead market. Perhaps more importantly, to the best of our knowledge, no literature dealing with the forecasting of the electricity demand in the Czech power market exists.

Therefore, after deep examination of the electricity demand we decide to use the following model to forecast demand, which is simply estimated by Ordinary Least Squares:

$$\begin{aligned} Demand_t^F = {} & \delta_0 + \delta_1 Average\ Demand_{t-24} + \delta_2 Demand_{t-24} + \delta_3 OffPeak_t + \\ & + \delta_4 Weekend_t + \delta_5 February + \delta_6 March + \delta_7 April + \delta_8 May + \\ & + \delta_9 June + \delta_{10} July + \delta_{11} August + \delta_{12} September + \delta_{13} October + \\ & + \delta_{14} November + \delta_{15} December + \varepsilon_t \end{aligned} \tag{4.25}$$

Note that we already mentioned we do not know the demand at the time we submit the offer for the following day. Therefore, to forecast the demand accurately with data being available at that time, we decide to use, among others, *Average* $Demand_{t-24}$ and $Demand_{t-24}$, where the latter one simply refers to the demand lagged by 24 hours, and given that the time t belongs to day D, the former one refers to the average daily demand for day $D-1$ (which is same for each hour of the particular day). The dummy variables *OffPeak* and *Weekend* are added to emphasize that demand is considerably lower during offpeak hours and weekends as opposed to peak hours and weekdays, respectively. Finally, the seasonal dummy variables *February* through *December* enter the equation to correct for the seasonal fluctuations in demand (needless to say that *February* is equal to 1 for observations falling to February and similarly for others).

5 Empirical results

In this section empirical results and their discussion can be found. In the first part, we apply the quadratic variation theory to the spot price data, as described in Section 4.2, and step by step provide the results until we reach conclusion about whether or not jumps form a significant part of the total variation of electricity prices. In the second part, we focus on forecasting the probability of jump occurrence and, as described in Section 4.3, we concentrate on four logit model specifications and based on various measures we decide, at the end, about the best model.

5.1 Share of jump component

Before turning to the descriptive statistics of the total realized volatility, and jump and continuous components, in Table 5.1 we provide estimation results of ARMAX(1,0) as captured in Equation 4.5. When interpreting the results we need to keep in mind that variables *Peak* and *OffPeak* are only defined during weekdays. Therefore, according to the coefficients it can be concluded that the mean spot prices vary considerably during peak and offpeak hours, in particular the mean spot price during the peak hours is 40.89 EUR/MWh while it is only 37.99 EUR/MWh during offpeak hours. Additionally, the coefficient of variable *Weekend* suggests that the mean spot price is equal to 39.74 EUR/MWh during weekends.

To test whether the spot price at peak hours is significantly different from offpeak hours, and whether the price during weekends is significantly different from weekdays, we perform ARMAX(1,0) excluding the variable *Peak* but at the same time including the intercept (the model specification and its results can be found in Appendix in Equation A.1 and Table A.1, respectively). According to the results, the mean price is estimated to be significantly lower during offpeak as opposed to peak hours and also significantly lower during weekends as opposed to weekdays. To sum up, the results indicate that the mean spot price varies considerably within the day which confirms the intra-day patterns and which also corresponds to Figures 3.1(a) and 3.1(b).

The last three rows in Table 5.1 capture the fluctuation of the spot prices across seasons. The positive signs on *Fall* corresponds to our findings in Figure 3.1(d) in which case the spot prices in fall months are, on average, much higher compared to summer months. The negative sign on *Spring* also conforms with our previous finding that the spot prices tend to be lowest during spring months. *Winter* is the only variable that comes in with an opposite sign than expected, nevertheless it is far from significant so it does not cause any obstacles. More importantly, from the estimates on the seasonal binary variables we conclude the spot prices exhibit seasonal fluctuations.

Table 5.1: Results of ARMAX(1,0)

Variable	Coefficient	Standard Error	z statistic
AR(1)	0.9271	0.0016	579.4375***
Peak	40.8901	0.6158	66.4016***
OffPeak	37.9919	0.6133	61.9467***
Weekend	39.7384	0.6301	63.0668***
Fall	4.2230	0.8658	4.8776***
Winter	-0.5675	0.8757	-0.6481
Spring	-2.7937	0.8246	-3.3879**

Note: *** significance at 1%, ** significance at 5%, * significance at 10%

Having estimated the ARMAX(1,0) model and thereby gathered the drifts, we can calculate the de-meaned price changes and, consequently, compute the realized volatility as captured in Equation 4.7. Before moving on, we need to decide about the significance level α. Since relative contributions of jump and continuous components to the total realized variation depend on chosen level of α, a bit caution is required in setting it to avoid two extreme cases. First, when α is too large, we classify too many jumps though some of them may be regarded as large price changes only. Then the jump component is higher and continuous component is lower than would other be the case. Second, when α is too small, some jumps are omitted and rather viewed as higher price volatility. Then jump and continuous component move in an opposite directions.

Therefore, in setting the significance level α we follow Tauchen & Zhou (2011). In their simulation experiment they show that we should set α relatively loosely, around $\alpha = 5\%$, in case the contribution of jump component to the total variation is in the order of 10%, and set α relatively tightly, around $\alpha = 0.1\%$, in case the contribution is in the order of 80%. The former case fits to our dataset and so we set $\alpha = 5\%$ for the analysis. To verify that choosing a proper significance level matters, in the first column of Table 5.2 numbers of detected jump days for different significance levels are presented (note that the total number of days is 2 403). It is clearly visible in the table that the higher is the significance level, the more days with significant jumps are revealed. Moreover, the second column verifies that using the de-meaned price changes, as captured in Equation 4.6, brings improvement compared to situation in which only returns, as captured in Equation 4.4, are used for computation of realized volatility, jump and continuous components. In particular, it can be seen that the number of days with significant jumps are lower for all levels of α meaning that seasonal fluctuations have considerable effect. To sum up, demeaning brings more accurate results since it takes into account the possibility in which the higher price change is result of seasonal fluctuation and therefore such day is not concluded as a jump day.

Table 5.2: Number of days with significant jumps for different significance level α

Significance level	ARMAX	Returns
$\alpha = 1\%$	164	290
$\alpha = 5\%$	316	585
$\alpha = 10\%$	424	800

Note: The first column labeled as "ARMAX" uses de-meanded price changes, $r^*_{t,j} = r_{t,j} - \mu(t,j)$, to compute RV, JV and CV, while the second column labeled as "Returns" uses for this computation returns only, $r_{t,j} = p_{t,j} - p_{t,j-1}$.

Having identified days t during which significant jumps occur, for each day the total realized volatility can be partitioned into its jump component, according to Equation 4.11, and into its continuous component, according to Equation 4.12. Furthermore, for each day t, the relative contribution of the jump component to the total realized volatility is calculated as JV_t/RV_t and a mean of these values brings the overall jump contribution to the total realized volatility over the entire period.

Table 5.3 shows the results for total realized volatility RV, jump and continuous components, JV and CV, respectively, for $\alpha = 5\%$ from February 1, 2009 to August 31, 2015. Note that means of jump and continuous components are lower compared to the mean of realized volatility which is due to the construction and, further, note the excess kurtosis for RV and CV. Nevertheless, the last row is of our primary interest and it indicates that the jump contribution to the total realized volatility is 5.2%. This is in line with previous studies both dealing with electricity and financial markets. For example, Tauchen & Zhou (2011) conclude in their study that the jump contribution is most likely to be in the order of 10% and they also confirm it for the equity index, with jump contribution being 13%. Huang & Tauchen (2005) study stock market and find 7% jump contribution to the total stock market price variance and Chan *et al.* (2008) deal with five Australian electricity markets and detect jump contribution from 5.21% to 10.85%.

Table 5.3: Summary statistics for RV, JV, CV

	RV	JV	CV
Mean	44.047	11.541	42.609
Standard deviation	45.120	4.912	45.695
Skewness	13.920	1.543	13.473
Kurtosis	346.909	2.685	331.317
RV/JV	5.2%		

Note: RV stands for realized volatility, JV and CV refer to jump and continuous component of realized volatility, respectively. For the computation of the summary statistics we apply $\alpha = 5\%$.
Source: Author's computation.

We also perform the whole analysis for $\alpha = 1\%$ and $\alpha = 10\%$ to validate our findings. The results, not included but available upon request, are qualitatively the same and indicate that in the former case the jump contribution to the total realized volatility is 3.2% while in the latter case it is 6.3% which is in accordance with our previous explanation.

In summary, the particular share of jumps depends on a significance level α we choose but in general we can conclude that according to the results jumps form a significant part of the total variation of electricity prices and, therefore, we can continue in our analysis and move to the forecasting the probability of jump occurrence.

5.2 Forecasting probability of jump occurrence

In Section 4.3 we define four logit models designed to forecast the probability of jump occurrence with majority of them including the forecasts of demand. Therefore, we need to partition this part accordingly and at the very begging provide and comment on the forecasts of demand, which is then followed by the forecasts of jump occurrence.

Two sorts of tests are conducted to evaluate the forecasting performance of logit models which require us to divide the analysis and thereby the dataset into two parts. In the first so-called in-sample period, which is from February 1, 2009 to August 31, 2013, we make models comparison and selection of the best performed model. In the second so-called out-of-sample period, which is from September 1, 2013 to August 31, 2015, evaluation of forecasting performance of the models is executed based on several measures.

5.2.1 Demand forecasting

To be able to forecast the probability of jump occurrence we first need to forecast the demand as it enters all the logit model specifications but the benchmark model. On the other hand, since demand forecasting is not the primary interest of this thesis we shorten the interpretation of the results to the most important ones.

The estimates of the model, captured by Equation 4.25, correspond to our expectations and can be found in Table 5.4. In particular, both lagged demand and lagged average demand are statistically significant and positive meaning that higher past demand results, on average, in higher value of future demand. Further, the significant and negative signs on *OffPeak* and *Weekend* suggest that observations during offpeak hours and weekends are expected to have lower demand compared to those falling into peak hours and weekdays, respectively. Finally, the positive/negative signs on the seasonal variables suggest whether demand is higher/lower compared to January.

Then, we perform 1-hour ahead forecasts starting from the first hour on February 1, 2010. In every step, we apply a rolling window of the length of 8 760 observations within which the parameter estimates are attain and subsequently applied to generate 1-hour ahead forecast. Then the window is rolled one hour forward and the whole procedure is repeated until the forecast for the last hour of the sample is reached.

Table 5.4: Forecasting the demand

Variable	Coefficient	Standard Error	t statistic
Average Demand$_{t-24}$	0.1960	0.0070	28.078***
Demand$_{t-24}$	0.3896	0.0052	74.294***
Offpeak	-0.0956	0.0089	-10.740***
Weekend	-0.5239	0.0095	-55.423***
February	0.0796	0.0189	4.219***
March	0.1070	0.0185	5.795***
April	0.0440	0.0186	2.369*
May	-0.0655	0.0184	-3.558***
June	-0.1234	0.0186	-6.634***
July	-0.2011	0.0185	-10.853***
August	-0.1627	0.0185	-8.788***
September	-0.0537	0.0193	-2.784**
October	0.0068	0.0191	0.358
November	0.1467	0.0193	7.588***
December	0.0059	0.0191	0.311
Constant	0.4628	0.0148	31.313***

Note: *** significance at 1%, ** significance at 5%, * significance at 10%

Source: Author's computation.

5.2.2 In-sample estimation and model selection

The purpose of the in-sample analysis is to choose the best performing jump identification strategy, from those captured in Equations 4.13 and 4.14, and more importantly, given the chosen specification, to examine the in-sample performances of four logit model specifications, stated in Equations 4.20, 4.22, 4.23 and 4.24. For an in-sample period from February 1, 2009 to August 31, 2013, evaluation and comparison of their in-sample performances is done with the help of two evaluation criteria, namely pseudo R-squared measure and Bayesian Information Criterion (BIC) (Kauppi & Saikkonen, 2008). Following Estrella (1998), pseudo, in our case so-called McFadden, R-squared measure is given by:

$$\text{pseudo } R\text{-squared} = 1 - \frac{\log L_u}{\log L_c} \tag{5.1}$$

where L_u is the likelihood of the unconstrained model, meaning the model which includes a set of all independent variables, while L_c is the likelihood of the constrained model, in other words of model including an intercept only. Following Wagenmakers & Farrell (2004) the Bayesian Information Criterion for model M_i $(i = 1, 2, 3, 4,)$ can be written as:

$$\text{BIC}_i = -2 \log L_i + V_i \log n \tag{5.2}$$

where L_i refers to the maximum likelihood for the candidate model i, n captures number of observations, and V_i counts free parameters to be estimated, for more information see Wagenmakers & Farrell (2004). When making the decision about the best performing model, the one with the highest value of pseudo R-squared and with the lowest possible value of BIC should be preferred.

First of all, the decision about the best jump identification strategy needs to be made. For this reason, we estimate *logit_ 1* with dependent variable being first defined by *jump_ season*, then by *jump_ 90percentile* and finally by *jump_ 95percentile*. The same procedure is applied to *logit_ 2*, *logit_ 3* and *logit_ 4* but note that in case of *logit_ 3* and *logit_ 4* the independent variable *Duration* changes accordingly as each identification strategy brings unequal number of jumps and also different jump location within the sample. Overall, the results, not present here but available upon request, are very much the same. In particular, regardless of the definition of the dependent variable, each logit model specification brings the same sign and statistical significance for a given variable. Furthermore, for each logit model specification the lowest value of BIC is reached when *jump_ season* is used to define the dependent variable compared to cases when either *jump_ 90percentile* or *jump_ 95percentile* is applied, with *jump_ 90percentile* performing the worst. Such finding implies that *jump_ season* is employed to identify jump hours in our further analysis.

The results of the in-sample estimations of our logit model specifications with *jump_ season* being the jump identification strategy are summarized in Table 5.5. Interestingly, none independent variable is statistically insignificant with absolute majority of them being significant at even 1% significance level and also the sign of every independent variable across all model specifications is in accordance with our expectations and with previous literature. Note that only signs of estimates are of our interest as their magnitudes do not have useful interpretation.

The lagged jump, J_{t-1}, which is only included in the benchmark model *logit_ 1*, has positive sign indicating that the past jump occurrence increases the probability of future jump occurrence. In the second and third rows statistically significant estimates of *Demand* and *Temperature* are obtained for all three model specifications implying that they still have a predictive power even when other variables are included in the model which confirms the previous literature in the sense that these belong to the main drives of the movement in the spot prices. As mentioned earlier, the positive signs on *Demand* and *Temperature* indicate that higher values are associated with higher probability of a jump occurrence.

The negative effect of *Duration* is consistent with the characteristic feature of jumps, meaning that jumps tend to occur in sequences of more than one, as the shorter the time elapsed since the last jump (in terms of short duration), the more the jumps are likely to occur. Further, the negative signs on dummy variables *Off-Peak* and *Weekend* indicate that the probability of jump occurrence is depressed for observations falling into offpeak hours and weekend while increases for observations falling into peak hours and weekdays, respectively. Finally, the positive sign of lagged price, p_{t-24}, suggests that not only the memory property applies to the spot prices but also that higher past prices raise the probability of future jump.

When looking at the last two rows of Table 5.5 we can compare the in-sample performances of our logit models with the benchmark model, *logit_ 1*. It can be seen that the benchmark model is not performing the worst as one would expect and it outperforms *logit_ 2* and *logit_ 3*. Perhaps more surprising is the finding that *logit_ 2* has the poorest performance according to both evaluating criteria and, further, that *logit_ 3* is by far worse compared to *logit_ 4*. These detections suggest that models based solely on *Demand* and *Temperature* cannot bring reliable forecasts of jump occurrence and, furthermore, that adding dummy variables *OffPeak* and *Weekend* and lagged price p_{t-24} can bring very promising results.

In summary, *jump_ season* seems to dominate all other jump identification strategies. Moreover, when comparing the in-sample performances of *logit_ 1* through *logit_ 4* where *jump_ season* is applied to identify the jump hours, then according to the values of both pseudo R-squared and BIC, *logit_ 4* ranks first meaning that it has the best in-sample performance, while *logit_ 2* has the poorest performance.

5.2.3 Out-of-sample evaluation

Making conclusions solely based on the in-sample outcomes can bring misleading results as the model having the best in-sample predictability may not be identical with the one having the best out-of sample predictability. According to the literature, sensitivity to outliers and data mining may be serious problem when relying only on the in-sample outcomes (Becker *et al.*, 2007). Consequently, the last two years of the sample period, from September 1, 2013 to August 31, 2015, are left for the out-of-sample evaluation.

To obtain 1-hour ahead forecasts starting from the first hour on September 1, 2013, we continue in the following manner: each time we re-estimate the model iteratively with rolling window being equal to 8 760 observations where each window produces parameter estimates which are then used to provide 1-hour ahead forecast (this corresponds to out-of-sample forecast). After that the window is rolled one hour forward and the whole procedure is repeated until 1-hour ahead forecast for the last hour of August 31, 2015 is computed. It worth noting that such window

Table 5.5: In-sample comparison of different logit model specifications

Variable	*Logit_1*	*Logit_2*	*Logit_3*	*Logit_4*
$Jump_{t-1}$	5.1031	-	-	-
	(0.0867)			
$Demand^F$	-	0.8729	0.6818	0.2514
		(0.0610)	(0.0626)	(0.0883)
Temperature	-	0.1060	0.0799	0.0553
		(0.0050)	(0.0530)	(0.0056)
Duration	-	-	-0.0030	-0.0011
			(0.0003)	(0.0002)
OffPeak	-	-	-	-1.2617
				(0.1454)
Weekend	-	-	-	-4.3369
				(0.4045)
$Price_{t-24}$	-	-	-	0.0942
				(0.0038)
Constant	-4.6939	-4.9275	-4.1187	-8.1039
	(0.0532)	(0.0837)	(0.0969)	(0.2663)
Pseudo R-squared	0.3855	0.3025	0.3269	0.5323
BIC	5278.5	5998.7	5800.2	4073.9

Note: Table contains parameter estimates with standard errors in parenthesis and two evaluation criteria for each logit model specification, the in-sample period is from February 1, 2009 to August 31, 2013.

Source: Author's computation.

length is chosen deliberately the as same window length is applied to the demand forecasting. To be able to compare the forecasting performances of our logit models, we apply the aforementioned procedure to *logit_ 1* through *logit_ 4*.

The forecasted probabilities can, theoretically, take on values from 0 to 1. When examining the previous literature, researchers usually apply a simple rule according to which a jump is forecasted to occur whenever the value of the forecasted probability exceeds the cutoff value 0.5, in other words whenever there is 50% probability that a jump will occur (Eichler *et al.*, 2012). We, however, believe that one rule does not fit all, meaning that the power markets are not identical and so each of them may be sensitive to different cutoff value. The idea of varying sensitivity across the power markets is mentioned in the work of Eichler *et al.* (2013), more deep investigation is however not presented. In this thesis we, therefore, decide to provide a kind of sensitivity analysis in which we propose various cutoff values, labeled as c, for which we compare the out-of-sample performances of the logit model specifications. To the best of our knowledge, no similar method has been applied in the energy researcher.

Comparison based simply on counting numbers of forecasted jumps (those events that exceed the cutoff value c) is simple but at the same time not very effective method. As one would anticipate, the fact that the model is able to forecast the same number of jumps as actually occur does not indeed mean that the hour in which a jump is forecasted to occur matches the hour in which it really happens so. The previous literature takes this possibility into account and suggests two measures which together can asses the performance of the forecasting models. In the spirit of Zhao *et al.* (2007) or Voronin & Partanen (2013), these can be defined as:

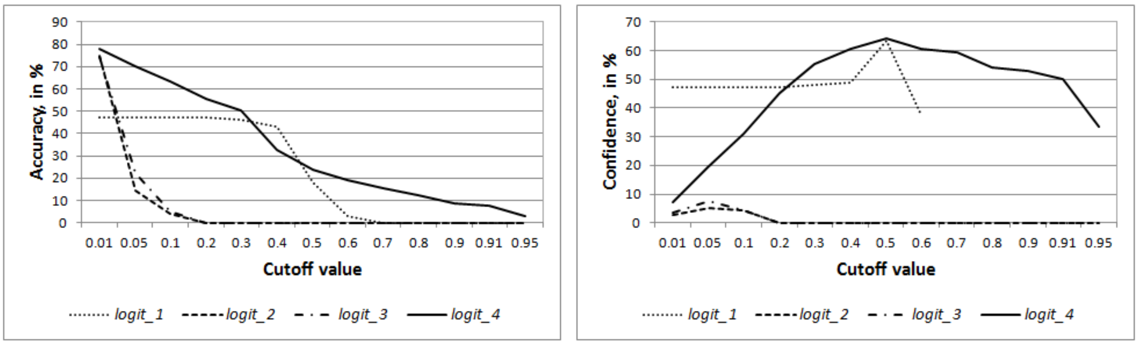
$$\textit{Jump prediction accuracy} = \frac{\textit{Number of correctly forecasted jumps}}{\textit{Number of actual jumps}} \cdot 100\% \tag{5.3}$$

$$\textit{Jump prediction confidence} = \frac{\textit{Number of correctly forecasted jumps}}{\textit{Number of forecasted jumps}} \cdot 100\% \tag{5.4}$$

where *Number of correctly forecasted jumps* calculates occasions in which the forecasted probability of jump occurrence for time $t+1$ is greater than $c\%$ (meaning the jump is forecasted to occur) while at the same time jump actually occurs at $t+1$, *Number of actual jumps* refers to observations being classified as jumps with the use of *jump_ season* during the out-of-sample period, and *Number of forecasted jumps* counts events in which the forecasted probability of the jump occurrence exceeds the threshold $c\%$ (regardless whether or not it matches actual jump). The denominator of Equation 5.4 comprises correctly and incorrectly forecasted jumps, where the latter one refers to occasions in which the forecasted probability of jump occurrence for time $t+1$ is greater than $c\%$ while at the same time the jump actually does not occur at $t+1$, and so the jump prediction confidence takes into account the possibility of model to misclassify some non-jump observations as jumps. Thus, the first measure is capable of assessing the ability of individual model to correctly forecast occurrence of jumps while the latter one is capable of assessing the percentage of cases in which the individual model make a false alarm (and hence relates to a reliability of forecasting). An ideal model should have both jump prediction accuracy and confidence as high as possible, however, there is tradeoff between their values as the higher is the accuracy, the more it is likely that more misclassifications are generated and so the lower is the confidence (Zhao *et al.*, 2007).

Figure 5.1: Jump prediction accuracy and confidence of *logit_ 1* through *logit_ 4*

(a) Jump prediction accuracy (b) Jump prediction confidence



Note: The graphs depict jump prediction accuracy and confidence, computed based on Equations 5.3 and 5.4, respectively, for various cutoff values. An ideal model should have both accuracy and confidence as high as possible. Also note that logit_ 1 is not defined for cutoff value $c \geq 0.7$.
Source: Author's computation.

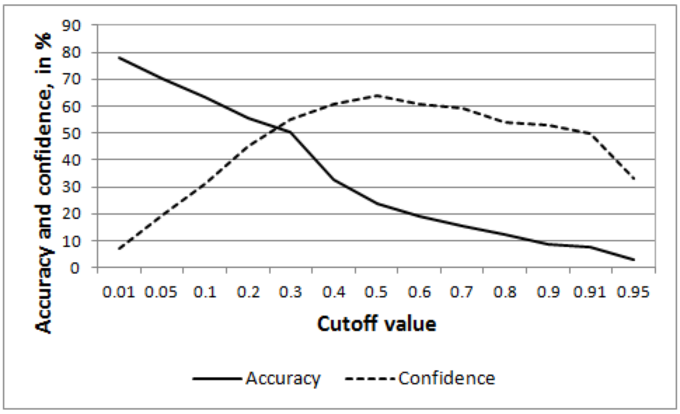
We define the cutoff values to be $c = 0.01, 0.05, 0.1, 0.2, \ldots, 0.9, 0.91, 0.95$. Just for clarity, when $c = 0.91$, for example, then jump is forecasted to occur at $t+1$ when the forecasted probability is higher than 0.91 (91%). For each c we compute the jump prediction accuracy and confidence for *logit_ 1* through *logit_ 4* and depict the resulting values in Figure 5.1, with Panel (a) representing the accuracy and Panel (b) representing the confidence. The graphical illustrations are chosen as they provide easier comparison of values across models, nevertheless, interested readers can still find the exact values in Tables A.3 and A.4 in Appendix.

When examining the accuracy, it is clearly visible from Panel (a) that its value decreases for all logit model specifications as the cutoff value rises. This is in line with expectations since higher cutoff value means that fewer number of forecasted probabilities overcomes this value and so the number of correctly (and falsely) forecasted jumps is lower which suppresses the numerator of Equation 5.3. When looking on the graph, it can be seen that the benchmark model takes on lowest values for $c = 0.01$ while taking on the highest values for $c = 0.4$ but then, as cutoff value reaches 0.7, it gains zero accuracy. Furthermore, *logit_2* and *logit_3* have very poor performance as both of them are zero from $c \geq 0.2$. The zero accuracy is attributed to the fact, that these models are not able to correctly forecast any jump, and so the numerators of Equation 5.3 are zeros. *logit_4* achieves the highest accuracy for each cutoff value except for $c = 0.4$ where the difference is negligible anyway. Consequently, we can say that *logit_4* surpasses other models in terms of accuracy.

When regarding the confidence, one could expect that steadily growing trend should be observed in the data. Panel (b) however suggests that it increases at first but after reaching some value, which is not identical for all model specifications, it starts declining. The explanation lies in changing speeds at which the numerator and denominator decrease. For each model, at lower cutoff values the rate at which denominator decreases surpasses the rate at which declines the numerator, but at higher cutoff values these rates revert. It is because at lower cutoff values, the models forecast rather high number of jumps with large fraction of them being false alarms. As the cutoff value increases, the number of false alarms sharply declines and so larger fraction of forecasted jumps is left for correct ones. Furthermore, as can be seen in Panel (b), the benchmark model has the highest confidence for $c \leq 0.2$ with other models performing rather poorly in this range, but after c reaches 0.3, *logit_4* is not beaten by any other model specification. Additionally, for $c \geq 0.2$ *logit_2* and *logit_3* gain zero confidence, and for $c \geq 0.7$ *logit_1* fails to forecast any single jump implying that the denominator of Equation 5.4 is not defined which explains the termination of its graphical illustration for higher cutoffs. As a results, we can claim that *logit_4* surpasses other models also in terms of confidence.

Showing that *logit_4* has the best out-of sample performance, it is worth finding also the best cutoff value. For this purpose we provide Figure 5.2 which put together both its accuracy, represented by the solid line, and its confidence, depicted by the dashed line. Based on the statement saying we are looking for the highest values of both measures, it seems reasonable to find the optimum as the intersection of the lines, that corresponds to the cutoff value of 0.3, at which accuracy and confidence are slightly higher than 50%. This result suggests that the Czech day-ahead market is more sensitive and confirms that different power markets may be sensitive to different cutoff values.

Figure 5.2: Jump prediction accuracy and confidence of *logit_4*



Note: The optimum cutoff value, which can be found at the intersection of the accuracy and confidence, is equal to 0.3, at which accuracy and confidence are slightly higher than 50%.
Source: Author's computation.

In summary, the out-of-sample evaluation reveals that, in terms of both accuracy and confidence, *logit_4* has the best performance while *logit_2* and *logit_3* have the poorest one, with *logit_1* lying in between. Therefore, based on the inferences of the in-sample and out-of sample evaluations, we conclude that *logit_4* is the best candidate (out of the model suggested) to forecast the occurrence of jumps.

6 Conclusion

In this thesis we investigate the Czech day-ahead market for the period from February 1, 2009 to August 31, 2015 where the observations from September 1, 2013 are left for the out-of sample evaluation. The deep examination of the spot prices reveals that the prices exhibit all stylized facts commonly known in the power market, in particular, multiple seasonal fluctuations, mean reversion, high volatility and occurrence of severe jumps. This finding, especially the fact that the day-ahead market is exposed to frequent jumps, confirms the importance to deal with the forecasting of jump occurrence in the Czech power market, topic which has not been covered by this data in the Czech Republic. Moreover, we can say that the contribution of this thesis to the existing literature is threefold.

First, modifying the generally known theory of quadratic variation, in a light of previous findings, and applying it to the Czech hourly spot prices we disentangle the jump and continuous component of the total volatility with the aim to compute the percentage share of the jump component to the total spot price volatility. We successfully find out that jumps form a significant part of the total volatility, exactly they account for 5.2% which is in line with previous literature.

Second, having demonstrated that the share of jump is not negligible, we propose various models to forecasting the occurrence of jump in the Czech day-ahead power market. The first model, to which we refer benchmark or *logit_ 1*, includes only binary variable reflecting lagged jump. In the second, so called *logit_ 2*, model we involve demand and temperature to account for the generally agreed facts suggesting that these two variables are the most important when dealing with forecasting in the power market. On top of these two, the third model, *logit_ 3*, adds variable reflecting the time elapsed from the last jump, in order to capture that jumps occur in a sequence of more than one. To the last model, *logit_ 4*, offpeak and weekend dummy variables are added, as jumps are shown to be less likely during offpeak hours and also during weekends as opposed to peak hours and weekdays, together with lagged price, which has been shown be previous literature to influence the probability. The in-sample selection of the best performing model is executed based on the values of the pseudo R-squared and Bayesian Information Criterion and, according to the results, we find out that *logit_4* ranks first meaning that it has the best in-sample performance, while *logit_2* and *logit_3* have the poorest performance. Then, with the use of rolling window, for each model we forecast the probabilities of jump occurrence for hour ahead for the rest of the sample.

Third, when evaluating the out-of sample performance of suggested models we apply two criteria, namely jump prediction accuracy and confidence, but opposed to the previous literature we apply a kind of sensitivity analysis which, to the best of our knowledge, has not be proposed by any other power research. In other words, when computing these criteria, the previous researchers suggest to define each observation with the forecasted probability higher than 50% as jump. We however believe that one rule does not fit all, meaning that the power markets are not identical and so each of them may be sensitive to different cutoff value. To take this into account, we define various cutoff values for which we compute the accuracy and confidence. It follows that *logit_4* has the best out-of sample performance while *logit_2* and *logit_3* perform the worst, in terms of both accuracy and confidence. The results further imply that the optimal cutoff is 30%, which is considerably lower compared to previously suggested 50%, at which the accuracy and confidence are slightly higher than 50%.

Since the worldwide deregulation of the power markets took place, the research has made a noticeable step forward but still some important topics remain for future investigation. Econometrics models with enhanced out-of-sample performance, in terms of higher jump prediction accuracy and confidence, should be proposed. Furthermore, when evaluating such models, the researchers could also consider the sensitivity analysis proposed in this thesis, or its modification, to take into account the different nature of various power markets and thereby provide more accurate results. Finally and perhaps more importantly, even though this thesis toughes the Czech power market, there is still lot of space left for further analyses which would help the market participants to improve their trading positions.

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Appendix

$$
\begin{aligned}
p_{t,j} = {} & \alpha_0 + \alpha_1 \mathit{OffPeak}_j + \alpha_2 \mathit{Weekend}_t + \alpha_3 \mathit{Fall}_t + \alpha_4 \mathit{Winter}_t + \\
& + \alpha_5 \mathit{Spring}_t + \beta p_{t,j-1} + \varepsilon_{t,j}
\end{aligned}
\tag{A.1}
$$

Table A.1: Results of ARMAX(1,0) given by Equation A.1

Variable	Coefficient	Standard Error	z statistic
AR(1)	0.9271	0.0016	579.4375***
OffPeak	-2.8982	0.1035	-28.0019***
Weekend	-1.1517	0.2493	-4.6197***
Fall	4.2230	0.8658	4.8776***
Winter	-0.5675	0.8757	-0.6481
Spring	-2.7937	0.8246	-3.3879**
Intercept	40.8901	0.6158	66.4016***

Note: *** significance at 1%, ** significance at 5%, * significance at 10%

Source: Author's computation.

Table A.2: Threshold values of individual groups

Group	Mean	Std dev	Mean + 2 Std dev
Offpeak, weekday, fall	41.30	16.3404	73.9760
Offpeak, weekday, winter	40.20	16.2970	72.7988
Offpeak, weekday, spring	39.76	16.2227	72.2022
Offpeak, weekday, summer	39.92	16.1808	72.2788
Peak, weekday, fall	41.30	16.3396	73.9784
Peak, weekday, winter	40.21	16.2930	72.7948
Peak, weekday, spring	39.76	16.2237	72.2061
Peak, weekday, summer	39.92	16.1761	72.2717
Weekend, fall	41.26	16.3620	73.9799
Weekend, winter	40.22	16.3321	72.8876
Weekend, spring	39.75	16.2242	72.2011
Weekend, summer	39.90	16.1797	72.2641

Note: Each observation in our sample can be assigned to one group according to the dummy variables, for example assume the following observation: October 28, 2010, 11 p.m. For this particular observation *Fall* = 1, *Offpeak* = 1, *Weekday* = 1. Therefore, this observation belongs to the first group. Analogously for other observations. Recall that offpeak and peak hours are only defined during weekdays implying that we only have 12 groups instead of 16, as one could expect.
Source: Author's computation.

Table A.3: Jump prediction accuracy of *logit_1* through *logit_4* for different cutoff values

Cutoff value	*logit_1*	*logit_2*	*logit_3*	*logit_4*
0.01	47.12	75.00	74.04	77.88
0.05	47.12	14.42	22.12	70.19
0.10	47.12	3.85	4.81	63.46
0.20	47.12	0	0	55.77
0.30	47.12	0	0	50.23
0.40	43.28	0	0	32.69
0.50	18.27	0	0	24.04
0.60	2.88	0	0	19.23
0.70	0	0	0	15.38
0.80	0	0	0	12.50
0.90	0	0	0	8.65
0.91	0	0	0	7.69
0.95	0	0	0	2.88

Note: For each logit model specification, and for different cutoff values, the table provides values of jump prediction accuracy computed according to Equation 5.3.

Source: Author's computation.

Table A.4: Jump prediction confidence of *logit_1* through *logit_4* for different cutoff values

Cutoff value	*logit_1*	*logit_2*	*logit_3*	*logit_4*
0.01	47.12	2.56	3.39	7.36
0.05	47.12	5.12	7.62	19.52
0.10	47.12	4.26	4.42	30.99
0.20	47.12	0	0	45.33
0.30	48.00	0	0	55.16
0.40	48.91	0	0	60.71
0.50	63.33	0	0	64.10
0.60	37.50	0	0	60.61
0.70	ND	0	0	59.26
0.80	ND	0	0	54.17
0.90	ND	0	0	52.94
0.91	ND	0	0	50.00
0.95	ND	0	0	33.33

Note: ND stands for not defined. For each logit model specification, and for different cutoff values, the table provides values of jump prediction confidence computed according to Equation 5.4.
Source: Author's computation.