Dirichlet’s theorem on arithmetic progressions says that there are infinitely many primes in any arithmetic progression \( a_n = kn + \ell \) with coprime \( k, \ell \). The original proof of this theorem was analytic using a lot of non-elementary methods. The goal of this thesis is to give sufficient and necessary conditions on \( k \) and \( \ell \) under which a more elementary algebraic proof of the theorem can exist, and give the proof in these cases.