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Report on the dissertation of **Pavel Paták** entitled:

Using algebra in geometry

The dissertation of Pavel Paták deals with classical and very hard problems of geometric topology and geometric combinatorics. The problems considered in the thesis were, and still are, in the focus of study of many generations of well known mathematicians. In this thesis, as a contrast to classical approaches to non-embeddability and multiple coincidence problems, the author develops a new framework based on the homological algebra view point. The results emerging from this approach are impressive and include progress on:

- Kühnel conjecture: Theorem 1.1 gives a new condition on the k-the $\mathbb{Z}/2\mathbb{Z}$-Betti number of the $2k$-manifold M which does not allow an embedding of the k-th skeleton of a simplex.
- Affine hull version of the optimal colored Tverberg theorem: Theorems 1.3 and 1.4 give interesting generalizations of the topological optimal colored Tverberg theorem. Unlike the topological version that holds only for prime number of intersections these theorems do not have such a restriction.
- Van Kampen–Flores theorem for maps to manifolds: Theorem 1.5 is a new version of multiple intersection Van Kampen–Flores type result.
- Generalized Helly type theorems: Theorems 1.9 and 1.10 offer new estimates of the Helly number for families of subsets of the Euclidean space $\mathbb{R}^d$.

The dissertation of Pavel Paták contains important and deep results on well known open problems. Therefore, in my opinion, it deserves acceptance with maximal grade **summa cum laude.**

The dissertation begins with extensive introduction where the main results of the thesis are presented. Multiple connections of the obtained results with previous work are well explained.

The second chapter treats colorful algebraic Tverberg type results. The central result of this chapter is Theorem 2.12 that is a version of the topological optimal colored Tverberg theorem. The most interesting part of this theorem for me presents the polynomial time algorithm for finding disjoint rainbow collections of points whose affine hulls intersect. Proof of Theorem 2.12 is done by reducing it to Theorem 2.15 which is then proved in details.

In the third chapter the author makes preparations for the non-embeddability results that will follow in the next chapter. The main result of this chapter is Theorem 3.25 that is a consequence of much stronger algebraic version given in Theorem 3.26. These theorems provide conditions for the existence of almost embeddings between k-skeletons of different simplices with desired properties with respect to continuous maps to manifolds.

In the next chapter the author utilizes the results he presented in the previous chapter and in combination with a result of Volovikov obtains a non-embeddability result (Theorem 4.2) as well as a Van Kampen–Flores type result for maps to manifolds (Theorem 4.4).

The chapter 5 brings a view on almost-embeddings from the angle of homological algebra. A notion of homological almost-embedding for a chain map is introduced in Definition 5.4. Some results on the non-existence of homological almost-embeddings are given in Corollaries 5.7 and 5.8. Section 5.3 presents an algebraic analogue of the classical obstruction theory approach towards the embeddability problem.

The final chapter contains a proof of a general Helly type theorem 1.8 for families of subsets of the Euclidean space $\mathbb{R}^d$. The proof is a neat combination of the developed methods from homological algebra (described in the previous chapter) and an involved Ramsey-type argument.

In summary, the dissertation of Pavel Paták presents a remarkable work containing a wealth of new and original ideas as well as new and relevant results. The results presented in this thesis testify that the author is talented and already formed young mathematician who is able to act independently and creatively when facing old and very hard open problems. Thus, as already stated, I suggest acceptance of Pavel Paták's thesis with maximal grade **summa cum laude**.

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