Reviewer report on doctoral thesis
Thermal Convection in Terrestrial Planetary Mantles
submitted by **Nina Benešová**
Charles University in Prague, Faculty of Mathematics and Physics

The dissertation is devoted to the numerical modelling of the thermal convection in the mantle of the Earth and other terrestrial planets. Motivation of the research and its position in the context of recent study of convection in the planetary mantles is described concisely in the Introduction.

The problem, solved in the extended Boussinesq approximation, is introduced in Section 1.1. The energy equation includes, besides adiabatic cooling/heating and viscous dissipation, which are essential for extended the Boussinesq approximation, also radioactive heat sources and latent heat associated with the phase transition. The model enables high variability in description of viscosity. The problem is solved both in 2D axisymmetric as well as full 3D geometry. The method is based on spectral decomposition in angular coordinates and finite differences in radial direction. The algorithm is described in Section 1.2, whereas mathematical background is presented in appendix.

According the section 1.2.2.2, the viscous dissipation was omitted in the thermal equation in 3D model while adiabatic heating term was kept. The importance of adiabatic heating/cooling for realistic modelling of convection is out of discussion, however, I am missing any estimation, how much can be the solution influence by violation of energy conservation due to the dropping of viscous heating.

The numerical implementation was verified by several benchmarks. The 3D numerical code was tested against seven codes based on various numerical principles (spectral, finite elements, finite volumes) under Boussinesq approximation and one code with adiabatic heating included. The difference between models is about 1%, which approves the correctness of the code. The 2D code was compared with one numerical code with similar results. Benchmark tests are of great importance as the problem is too complicated to allow for check by a simplified analytical solution and thus only comparison of the results with other codes independently developed can mutually confirm their correctness.

Time efficiency of the code was also tested. The test of dependence on the lateral resolution confirmed the power law. The test of parallelisation was performed on Intel Quad Core processor using OpenMP library and the efficiency on 4 cores was about 53%. The efficiency could be in the future improved by employment of MPI library (and implementation on more powerful computers).

My feeling is that the quality and significance of the numerical code was not sufficiently accented in the thesis. The sentence on page 7 saying that “The extended Boussinesq approximation used here is widely used when simulating mantle convection” could lead to the conclusion that there exists a plenty of similar numerical codes and one could ask, why a new one has been developing. However, many of these codes were either 2D or in classical Bousinesq approximation or constrained by other conditions. I would appreciate a short survey of existing numerical codes and comparison with the new one in the thesis defence presentation.

The numerical modelling was focused on two problems of different nature.

Part I is aimed at constraining the internal structure of Venus and Mercury using their geoid and surface topography data obtained from satellite observations. The description of modelling is in both cases preceded by a coherent survey of the recent knowledge and theories. Four basic radial viscosity profiles with a broad range of Rayleigh number were used for Venus and power spectra of the geoid and topography were compared with the spectra of the observed data. The results are presented in the form of graphs. Besides the spectra, also the fit of the shape of the geoid and topography above the assumed mantle plumes on Venus were taken into consideration. The complex analysis suggested that the geoid and the surface topography of Venus are consistent with a radially symmetric viscosity model with a strong 200 km thick lithosphere.

According to the recent knowledge, the Mercury's mantle is very thin (400 km or even less). Two models with constant viscosity and two models with temperature-dependent viscosity were computed. Contrary to Venus, the results are inconsistent with the observed geoid and topography, which supports recently published analysis that suggests other mechanisms to be important for Mercury's geoid and topography.

Part II of the thesis is devoted to the possible influence of post-perovskite (PPV) on the Earth's mantle convection and its long-term evolution. As this high-pressure phase of perovskite was discovered only in 2004, such studies are recently of great interest of research community. Calculations were carried-out in 2D geometry using extended Boussinesq approximation. The selection of model parameters was substantiated by recently published papers. As there exist quite high uncertainty about the viscosity of PPV, models with viscosity of 1 and 2 orders weaker than perovskite were computed and compared with models without PPV. The integration period covered 4.5 Ga. Several characteristics were selected to evaluate the influence of PPV on the thermal history. They are snapshots of temperature and viscosity in 5 stages of evolution and graphs of time evolution of temperature, heat flux, Rayleigh number and thickness of the PPV layer. These characteristics, together with the accompanying commentary, provide the reader with a very good insight into the problem and make the conclusions straightforward.

The thesis represents an original contribution to the recent research of the thermal evolution of the Earth and terrestrial planets. The bibliography contains more than 150 items, about 100 of them were published in the last ten years. The text is concise, understandable and without unneeded details. I have not found any mistakes or incorrectness that should be mentioned here and I have noticed minimum of misprints.

Substantial parts of the thesis have been already published or submitted for publication.

Conclusion:

Nina Benešová showed excellent understanding of applied mathematics, skills in programming and deep insight into the dynamics of the planetary interior. She proved ability to independent research work. I recommend that the candidate be awarded the PhD degree.

RNDr. Pavel Hejda, CSc.

Geofyzikální ústav AV ČR, v. v. i.
Boční II/1401
141 31 Praha 4

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