Report on the thesis "Thermal Convection in Terrestrial Planetary Mantles" submitted by Mgr. Nina Benešová to Charles University in Prague, Faculty of Mathematics and Physics, for the degree of Doctor of Philosophy in specialization 4f-7 Geophysics.[1]

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Thesis summary

Mgr. Benešová's thesis is concerned with numerical modeling of mantle convection in mantles of terrestrial planets Venus, Mercury, and Earth. One part of the thesis deals with dynamic geoid and topography inversion for Venus and Mercury. In the other part the effect of post-perovskite phase on thermal evolution of the Earth is investigated.

The Introduction (3.5 pages) opens the thesis by discussing basic facts about terrestrial planets, geophysical methods of studying their internal dynamics and evolution, and highlights the importance of numerical modeling of mantle convection and the related geoid and topography inversions.

Chapter 1 (22 pages including figures & tables) describes the mathematical model (Section 1.1), the method of numerical solution (1.2), and the benchmarks of the numerical code that Mgr. Benešová developed (1.3). The extended Boussinesq approximation of the governing equations, with include phase changes, in a spherical shell domain with impermeable free-slip isothermal boundary conditions, and calculation of the dynamic topography and the geoid constitute the standard mathematical description of planetary mantle convection developed largely in the 1980's and 1990's. The equations are clearly presented in both dimensional and scaled form. To solve the spherical shell model numerically, the equations are discretized into a finite spherical harmonic series laterally (Appendix A reviews the spectral formalism) and using finite differences in radial direction, then solved either in 2-D axisymmetric case with variable viscosity (coupling across spherical harmonic degrees) or in 3-D with purely depth-dependent viscosity (degree/order decoupled set). Specific methods of solution are described. The new numerical code is carefully benchmarked against published results and against independent codes; efficiency of an OpenMP parallelization is quantified (four cores maximum; I presume these were single-machine, multi-core calculations).

Part I, consisting of Chapters 2 & 3, focuses on constraining the internal structure of convecting Venus and Mercury using observed geoid and topography. The introduction to Part I (which has no heading) reviews the geoid and topography calculations. In my opinion, it would better fit as a section of the "Theory and Method" Chapter 1.

Chapter 2 (22 pages) is a study of thermal convection in Venusian mantle using both 2-D and 3-D models and mainly focuses on fitting the observed geoid and topography. Chapter 3 (14 pages) is a similar albeit more limited investigation of Mercury.

Part II, consisting of Chapters 4 & 5, focuses on the effects of post-perovskite phase in the lowermost mantle on thermal evolution of the Earth. Chapter 4 (26 pages) is a study of thermal convection and thermal evolution of the Earth's mantle coupled with cooling of the core. Effects of depth-dependent thermal conductivity and thermal expansivity, of the initial core-mantle boundary temperature, and of convective vigor are investigated in addition to the viscosity decrease of the post-perovskite phase.

Chapter 5 (2.5 pages) summarizes the findings of Velímský et al. (2012) on the detectability of PPV layer by 1-D inversions of electromagnetic induction signals, including the construction of synthetic PPV distributions based on a 3-D thermal convection model.

The thesis closes with a 1.5-page "Conclusions" section.

Journal articles

A search on *Web of Science* returns one first-authored and one co-authored publication by Mgr. Benešová:

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[1] Following the language choice of the dissertation, I complete this report in English.

- Benešová & Čížková: "Geoid and topography of Venus in various thermal convection models," *Stud. Geophys. Geodaet.*, **56**(2), 621–639, doi:10.1007/s11200-011-0251-7, 2012.
- Velímský, Benešová, & Čížková: "On the detectability of 3-D postperovskite distribution in D'' by electromagnetic induction," *Phys. Earth Planet. Int.*, **202**, 71–77, doi:10.1016/j.pepi.2012.02.012, 2012.

The first publication constitutes Chapter 2 of the thesis and the second publication is summarized in Chapter 5. Content of Chapter 4 of the thesis is reported as submitted to *Phys. Earth Planet. Int.*:

- Benešová & Čížková: "Effect of post-perovskite rheology on thermal evolution of the Earth."

Assessment, comments, questions

I expect a doctoral dissertation to be a well-written, largely self-contained piece of work that addresses some important issues in the chosen field of study and offers some new and original perspectives, methods or results.

On the positive side, I acknowledge the amount of dedicated careful work to construct a new spectral-finite difference spherical-shell parallelized code to numerically model the thermal convection in planetary mantles. However, in some aspects, described below and documented in my comments/questions, the presented work perhaps falls short of the standard for a PhD thesis. These will need to be discussed during the defense.

Reading the text, I sometimes find the presentation rather too brief, the presented arguments imprecise or incomplete or stylistically awkward, with a higher than negligible number of typos. (Examples: "This energy is manifested on the surface by the plate tectonics." "Character of the convection is *determined* by the parameters of the mantle material." "Model A *may be considered* the best." "radioactive elements" instead of "radionuclides"; use of the colloquial "like" in place of "such as"; "losses" or "looses" instead of "loses"; "ant" instead of "any"; "succesful" instead of "successful") Of course, much of this text quality assessment is subjective.

I present a list of specific questions and/or comments to the candidate. I suggest that during the defense Mgr. Benešová pays foremost attention to the first four points on the list.

1. What new information have we learned about Venus from the study presented in Chapter 2? The conclusions that viscosity profile "A" best fits the observations and that mantle viscosity is constrained to ~10^{21} Pa s do not strike me as particularly exciting, surprising or novel results. Of the four viscosity profiles tested, "A" is the only geophysically plausible one, from what we know about Venus and planetary interiors in general. Additionally, it is taken from an earlier study (Pauer et al. 2006) and no effort is made to refine or improve it. You constructed a fully time-dependent convection model but mainly focus on geoid and topography, that is, looking either at a snapshot in time or a time-averaged quantity. Was there perhaps something more to learn from the time dependence? It is stated in the intro of Chapter 2 that Venus probably experiences resurfacing with a characteristic time of 300–800 Myr, i.e., a very non-steady-state behavior. Is there any hint in your convection runs of such behavior?
2. What is the goal of the short exercise on Mercury in Chapter 3? I could see this section as possibly interesting if the study were further developed. As it stands, only a very basic investigation is performed with an isoviscous model (2-D and 3-D) and a 2-D model with temperature-dependent viscosity. How do you interpret your conclusion that "none of the models is able to fit the observed spectra" [somewhat shortened from the original statement]? Based on your study, is Mercurian mantle currently convecting or not or we cannot say?
3. I find the post-perovskite (PPV) study (Chapter 4 of the thesis) the most interesting part of this work. That said, with a time stamp of 2015, the findings trail the main wave of similar studies on this topic—which arrive at essentially the same conclusions—by some five years. The first part of "Concluding remarks" (Section 4.3, pp. 103–104) summarizes results from models without PPV that agree with existing work. Models with PPV also yield result in agreement with previous studies. The findings make sense, as far as I can tell, and are interesting, but sadly, come a day late and a dollar short.

4. Figure 4.1, Table 4.1 (pp. 80–81) and the discussion of internal heating are confusing. Are you suggesting that given present-day abundances of K, Th, U in the mantle, you have two different "models" of radiogenic heating where the power agrees at present and differs by a factor of ~two at 4.5 Ga? That can't be. An Earth with present day radiogenic power output equal to *P* and with K:Th:U ≈ 12000:3.9:1 had power output ≈ 5*P* at 4.5 Ga. Your MH2 model seems consistent with that – although equation 4.3 makes no sense to me; where does it come from? Equation 4.2 for the MH1 model (or some variation of it) is how one calculates time evolution of radiogenic heating, but the MH1 decay of heating with time is too gentle. Where is the problem? Table 4.1 lists elements, but really shows values for specific radionuclides (e.g., potassium has three naturally occurring isotopes of which only ^{40}K is radioactive). What present-day mantle abundances of K, Th, U did you use to calculate the heating rate? How would formation of continental crust—which is not included in the model—affect the radiogenic heating history? After eqn. 4.2: Neglecting ^{235}U heating merely based on its natural isotopic fraction is an incomplete argument; one also has to consider the energy released per decay in addition to the half-life. [Btw, "Ga" is commonly used as a unit of age, not time (which would be "Gy"), therefore present is 0 Ga and nascent Earth is 4.5 Ga.]
5. Below eqn. 1.3 (p.7): How coarse an approximation is the omission of self-gravitation for each of the three studied planets?
6. Is the radial discretization equidistant across the mantle or refined in the thermal boundary layers? Unclear from description in section 1.2 (p.13).
7. Figure 1.1 (p.19): Giving the non-dimensional value of temperature isosurface between the boundary values 0 and 1 would be more informative than "1800 K". Same comment applies to Figure 3.4a.
8. Table 1.4 (p.22): Why not show relative differences in percent, as in the other benchmarks?
9. Why do you call density anomalies $\Delta\varrho$ in eqn. (1.6), and then change to ϱ in eqn. (I.2)? Are deflections (topography) of phase boundaries accounted for in the geoid calculation?
10. p. 39, on radiogenic heat production in Venus: 3 TW to 16 TW is a large span. Why did you choose 6 TW? (I am just curious.) In perspective of results shown in Figure 2.10, it would have been interesting to look at several different heating rates…
11. p. 48, paragraph beginning "Four 3D models": The first figure reference should be to 2.7, not to 2.8.
12. Figure 2.6 top panels (p.47) are probably mislabeled: The text refers to blue line as Model C but the label states "3B6x10^{21}", i.e., Model B.
13. Figure 2.13a and all other cross-sectional plots of temperature: It would be better not to color the entire core all the way up to CMB in order to better see the thermal features and temperature variation in the bottom boundary layer of the mantle.
14. Figure 3.7 (p.68): Why the even-odd degree oscillation of the spectra in model 2DLV4?
15. Top of page 87: Looking at Figure 4.5, I see the green and blue Rayleigh numbers being higher, not lower as you write, than the red one. Please explain.
16. Figure 4.4 and Figure 4.7 (pp. 88 & 92) show the same graphic (this was checked by extracting the pages from the PDF version available at geo.mff.cuni.cz). It seems to me that while 4.7 shows the appropriate images, the actual 4.4 plots are not present and therefore it is impossible to visually follow the written description of section 4.2.1. As Figure 4.4 is supposed to include the reference model to which all other models should be compared, this technical glitch is particularly unfortunate. [The correct version of Figure 4.4 was provided by Mgr. Benešová upon email request.]
17. Figure 4.20 (p.103): What is the color scale in Figure 4.20, going from −6 to +6? This figure is only briefly mentioned in the text (bottom of page 100) and one has to guess what is plotted.

Conclusion

In my opinion, this thesis proves the author's ability for creative scientific work, and in particular attests to her proficiency in numerical modeling of geophysical phenomena. Given the above-mentioned questions regarding the novelty and originality of this work as well as the overall quality of the manuscript, however, I have reservations about recommending this thesis to be accepted as doctoral dissertation at Charles University in Prague. My hesitation may be lifted depending on Mgr. Benešová's ability to defend her work during the dissertation defense.

Ondřej Šrámek
In Rio en Medio, New Mexico, USA on July 31, 2015