

POSUDEK OPONENTA BAKALÁŘSKÉ PRÁCE

Název: Filippovy dynamické systémy a jejich aplikace
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1 Summary and General Evaluation

The thesis contains an introduction to the theory Filippov systems, and several modeling exercises with corresponding numerical experiments. The modeling exercises were originally intended to be Filippov systems, but apparently, during the work on the thesis, Filippov systems turned out to be too restrictive, and hence, some models eventually ended up as general hybrid dynamical systems.

The thesis is difficult to evaluate since problem statement (“zadání”) consists just of a few vague keywords (6 words altogether). Especially, the bare keyword “numerical experiments” is problematic. In general, numerical experiments should not be a goal in itself, but should serve a purpose, for example, insight into a certain mathematical phenomenon, or into the behavior of certain numerical solvers. The problem statement does not formulate any such goal for the experiments. The experiments contained in the thesis then are really just runs with certain solvers, without any specific motivation, and without any conclusions drawn from the experiments.

The structure of the thesis is confusing, with the introduction at the beginning introducing an issue (discretization of contact problems by finite elements) that is never mentioned again in the thesis, and with the conclusion being more a comment on the issues faced while working on the thesis than a conclusion describing some gained insights.

The evaluation of the thesis is also difficult due to several mistakes that are not severe in themselves, but that makes understanding the material difficult, since those mistakes occur at key places. For example, the first sentence in Section 2.1 refers to Problem 2.5 and 2.6, but I cannot find those problems in the thesis. There are Formulas 2.5 and 2.6 later in the thesis, but they do not describe problems. Also, the meaning of the variables of the model introduced in this section is never described. As a result I am neither able to understand the model introduced in Section 2, nor to evaluate this section.

2 Originality

Section 1 is a survey of Filippov systems. While the material itself is not new, the structure of the presentation seems to be original.

Section 2 seems to be an original modeling exercise. However, I am not able to evaluate it due to the problem described above. The section also says that the numerical experiments were done using the algorithm by Piironen and Kuznetsov for solving Filippov systems, but then says that they were done using standard integration algorithms in Matlab. It is not clear, whether the student implemented the Piironen/Kuznetsov algorithm on top of that, or not.

Section 3 refers to a large extent to an older bachelor thesis. Especially, the software used originates from the older thesis. The present thesis does not describe the precise functionality of this software package, which again makes it difficult to evaluate this section. In general, this section ignores the fact that there exist well-known methods and software for simulating hybrid systems (often this is built into ODE solvers under the keyword "root finding"). So the motivation for using software from the referenced bachelor thesis is not clear to me.

Section 4 contains numerical experiments that are original (with the deficiencies described in the first section of this review). Still, in Section 4.2, it is not clear which solver was used, since this case is not a Filippov system.

3 Mathematical Quality

In general, the mathematical quality is good, with some formal problem here and there. Here are examples:

- The notion of solution of a Filippov system is used without definition (page 6). Apparently, here the notion is used informally, with the rest of Section 1.2 explaining how it can be formalized. Here, the notion "Filippov's convex method" is used, unfortunately, again without any formal definition.
- Definition 1: α_{x_0} and δ_{x_0} are undefined
- Remark 1.8 is strange. The formula looks like a differential inclusion, however, in this case $F_1(x)$ and $F_2(x)$ must be sets, whereas they have been introduced as functions. Moreover, the notion of solution of a differential inclusion is also not defined.
- Section 2.3: the function Sign is formulated in a formally sloppy way: First it is said that it is a function in $\mathbb{R} \rightarrow \mathbb{R}$, but then, in the case $z = 0$, it is defined to be the interval $[-1, 1]$, that is, a *set* of real numbers, not a real number.
- Formula 4.3 has a serious typo: To model friction, it has to be λ_ν instead of u_ν . Later (Formulas 4.6, and 4.7), indeed λ_ν is used.

- The physical motivation for the case μ'_τ in formulas 4.6 and 4.7 is not clear to me. Does the model distinguish static and dynamic friction? How? The variables u_τ seems to model horizontal position, which means that u'_τ is horizontal speed. But the text after Formula 4.7 says that $u'_\tau = 0$ does *not* mean that speed is zero, but that speed does *not change*, which would correspond to $u''_\tau = 0$, not $u'_\tau = 0$.
- The left-hand of Figure 4.2 side is strange: the caption says that it is a phase portrait of variables x_3 and x_4 . Earlier, x_3 is defined as u_τ and x_4 as u'_τ . However the phase portrait definitely does not show any trajectory for which $x_3 = x'_4$ holds. For example, for the points where $x_3 \geq 1$, x_4 is decreasing although x_3 is positive.

4 Handling of Literature

In general, citations of the literature are correct, with the following exceptions:

- The reference Ligurský/Renard 2011 is incomplete, lacking a title and year which makes it difficult to find the actual article. This is all the more problematic since this reference is key to Section 4 of the thesis.
- Figure 4.1 is copied from Ligurský/Renard 2011. This should have been explicitly mentioned.
- The existing literature on simulation of hybrid systems is completely ignored.

5 Conclusion

I consider this thesis to be *average* and recommend to accept it as a bachelor thesis provided the student is able to clarify key issues mentioned in this review, especially

- What are Problems 2.5 and 2.6 referred to in Section 2.1 and how does the model introduced in this section correspond to them?
- Which numerical experiments were done by precisely which software packages? What is the precise functionality of those software packages?

If asked, the student should also be able to clarify how the issues mentioned in Section 3 can be fixed.