The thesis is motivated by problems of contact mechanics with friction. At the beginning we describe a class of piecewise smooth systems with discontinuous vector field called Filippov systems. We also show how to solve them. The rest of this thesis is focused on applications, especially dry friction model and finite element model of Coulomb friction with one contact point. We propose a technique for simulation of the second mentioned model which combines sovling methods for Filippov systems and impact oscillators.