The report on the dissertation

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Symplectic spin geometry

The main topic of the thesis is a study of properties of invariant first order differential operators on a symplectic vector space acting on fields with values in the space of the Konstant symplectic spinor. There are two such operators - the symplectic Dirac operator and the symplectic twistor operator. The first one was defined and studied on symplectic manifolds by members of the group of Thomas Friedrich in Berlin, in particular by Katharina Habermann, who is a co-author of a monograph on this topic. There was not, up to now, such a more detailed study of local properties of solutions of the symplectic twistor operator available up to now. The author is focusing mainly on the flat case, she considers the mentioned operators mostly on a vector space with a fixed symplectic stucture, the exception being the case of dimension 2 and the operators acting on symplectic spinors defined on elliptic curves.

The first 5 chapters of the dissertation contain an introductory material about symplectic stuctures, the Schwartz space, the Kostant spinor representation, fibre bundles over symplectic manifolds and the corresponding covariant derivatives, and the fifth chapter contains the definition of both invariant first order operators and a description of spinor-valued differential forms.

The rest of the dissertation contains a lot of new and interesting material. Chapter 6 describes solutions of the twistor operator in dimension 2, while the next Chapter 7 contains a discussion of properties of solutions of the twistor operator in higher dimensions. The next four chapters (8 - 11) are devoted to the symplectic Dirac operator from the different point of view. It contains a discussion of its symmetries including applications in the description of a basis of the space of solutions, and a discussion of a analogue of the Fourier transform in the symplectic Clifford setting. Another topic treated here is a description of the reproducing kernel for symplectic spinor fields. The part of the dissertation reproduced in Chapters 6 - 11 was taken from paper already published in journals.

The last chapters contain new results not yet published. Chapter 12 describes an inductive procedure how to construct higher dimensional symplectic monogenic functions using lower dimensional ones. The last two chapters treat an interesting cases of invariant operators of both types for

symplectic spinor fields on elliptic curves.

The summarize, the dissertation is very nicely and carefully written with minimum of misprints. It contains a substantial amount of new and interesting results, sometimes with very technical and complicated proofs containing long non-trivial computations. A large part of them was already published in good international research journals. At the same time, the results contained in dissertation are just a beginning of a systematic treatment of this research direction and they open the room for further interesting development. It is also probable that the future development will bring some simplifications and better understanding of the structure of the kernels of the twistor operator and the symplectic Dirac operator and their relations.

In a sense, the dissertation is not a thesis of a classical type, which is usually supposed to be a systematic, pedagogical and polished treatment of a chosen topic (containing new results). It is usually supposed that the presented results will be diggested and described in a style of a small monograph. In the present case, it resembles more a habilitation thesis.

A bigger part of new results contained in the thesis (Chapters 6 - 11) are the papers already published and reprinted here word by word. As a consequence the notation is not uniform and changes from one chapter to another chapters (e.g., in Chapter 8, variables in the symplectic vector space are denoted by $(x_1, \dots, x_n, y_1, \dots, y_n)$, while in Chapter 7, notation is $(x_1, \dots, x_n, x_{n+1}, \dots, x_{2n})$, it would be convenient for the reader to have the same notation for all chapters). The last three chapters then contains new unpublished results and the first five chapters contain a summary of needed notions and results as is expected for dissertation.

Conlusion.

The dissertation presents a solid foundation of a new direction in Clifford analysis centered around the symplectic twistor and Dirac operators. It contains a lot of new basic results, often with very complicated and computational proofs. The thesis is systematically and carefully written. There is no doubts that it satisfies comfortably all criteria expected for a a good PhD dissertation.

Praha, 17.5.2016 prof. Vladimír Souček