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Report on the PhD thesis of Tomáš Gavenčiak "**Structural and complexity questions of graph theory**".

Tomáš Gavenčiak's Ph.D. dissertation "Structural and complexity questions of graph theory" is a contribution to the areas of Graph Theory, Graph Algorithms and Combinatorial Games. More precisely, the thesis is devoted to the study of cops and robber games on graph and hypergraphs,and in my opinion, title ``Pursuit-evasion games on graphs and hypergraphs" would have a closer relevance to the contents of the thesis.

The dissertation is written in English and consists of four chapters. The first chapter is introductory; it provides the reader with various definitions from Graph Theory, defines graph classes used in the thesis and gives a brief hysteric overview. The remaining three chapters cover the main research contributions of the thesis. Each of the research chapters is self-contained and contains a historical overview as well as a conclusion with open problems.

In what follows, I will give a brief chapter-by-chapter overview of the thesis, highlight the most interesting, in my opinion, parts, provide some critics and questions, and then jump to the conclusion.

Chapter 2. This chapter is devoted to the study of the game of Cops and Robber. This is the classical game on graphs introduced in 1980s by Nowakowski-Winkler and Quilliot. While on general graphs the minimum number of cops required to capture a robber can be arbitrary large, it was proven by Aigner and From in 1990s that on planar graphs three cops always suffices. This result was later extended to classes of graphs embeddable in a surface of a fixed genus and graphs excluding some fixed graph as a minor. The natural question, which is addressed in this chapter, if such type of bounds can be obtained for other ``geometric" classes of graphs. The main result of the chapter is that on string graphs 15 cops are always suffice to capture the robber. This result is also generalized to genus-g string graphs. Pipelined with the standard arguments about games on graphs, the obtained combinatorial bound implies that the optimum number of cops required to win the game on these classes of graphs is computable in polynomial time. (The problem is known to be EXPTIME in general.) The main result of the chapter is pretty non-obvious and it also yields bounds for various classes of intersection graphs.

Chapter 3. This chapter is devoted to a variant of the Cops and Robber game with Robber able to move arbitrarily fast along the edges of the graph. This variant of the game was introduced by me, Golovach and Kratochvil in 2008. In particular, we left open the question about the complexity of computing the minimum number of the cops required to win on an interval graph. I am delighted to see that this problem is finally resolved. The solution is technical, and it is based on the study of another problem, defensive domination, which is interesting in its own. A section with an informal discussion of the main ideas used in the proof, would greatly improve the readability of this technical chapter.

Chapter 4 provides a generalization of the tree-depth of a graph to hypergraphs and hypergraph pairs. It also discusses the corresponding pursuit-evasion game characterization in terms of marshals and robbers. While on graphs, the tree-depth plays an important role in obtaining a number of combinatorial and algorithmic results, the significance of this parameter on hypergraphs is still to be seen. This issue can be discussed and clarified at the defense.

Specific comments and questions

General comment on Chapter 2. The proofs in this chapter for different geometric graphs are build on the same idea which was used by Aigner and From for planar graphs. The idea is to show that a fixed number of cops can always patrol the border of a region, and then ``unemployed” cops are used to shrink the guarded region. It would be very helpful to try to abstract this property. A ``dream” theorem here would say that if a graph class poses some combinatorial property (like having separators of certain property) then a constant number of cops will suffice. Then instead of providing a cop strategy for every class of graphs, the only thing one has to do is to prove that the graph class posses the required property. I do not know if such a theorem would be ever proven but this can be an interesting topic for discussion at the defense.

p.28 Proof of Theorem 16. When an outer-string representation is fixed, I think a formal explanation why such representation exists, would be useful.

p.30 Theorem 19. The theorem claims upper and lower bounds, but only the upper bound is proved. While upper bound is much harder, the formal proof of the lower bound is required.

p.32 Corollary 23, the proof is by the figure, which is not a rigorous proof.

p.35 proof of Theorem 19 is based on the statement proved in Lemma 23, but I did not find this lemma.

p.37 Lemma 32 and its proof. The arguments have to be slightly more careful here. The reason is that when we cut a surface along a non-contractible curve, we can switch between orientable and non-orientable surfaces. A standard way out of this, see Mohar-Thomassen book, is to work with the Euler genus.

p.61 A reference to the relevant work *Archontia C. Giannopoulou, Paul Hunter, Dimitrios M. Thilikos: LIFO-search: A min-max theorem and a searching game for cycle-rank and tree-depth. Discrete Applied Mathematics 160(15): 2089-2097 (2012)* is missing.

Summary

The scope of Tomáš Gavenčiak thesis is the study of full information pursuit-evasion games played on graphs and hypergraphs. This is definitely an important and challenging topic and the thesis provides an original treatment of several important problems in this area. The quality of the research is high, and the work raises interesting questions for future research.

Conclusion

The work presented by Tomáš Gavenčiak is of high quality, clearly fulfilling the requirements for the degree of PhD in Computer Science.

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