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ESSAYS ON VERTICALLY DIFFERENTIATED MARKETS FOR COMPLEMENTARY GOODS

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Contents

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Abstract

The purpose of this thesis is to shed light on how product complementarity affects the variety of possible equilibrium outcomes in a vertically differentiated market. Complementarity is not uncommon. Many vertically differentiated goods have value for the consumer as complements, that is only if they are used in combination with other goods which can also be of different qualities (e.g. piano with tuning service, business trip with hotel accommodation, computing platform with web browsing application, etc.).

Complementarity between goods brings an exogenous expense that the consumer must pay on top of the price of any of the goods available in a vertically differentiated market. However, firms are only partially able to compensate consumers for the exogenous expense by charging lower prices. Some might also be prompted to increase the qualities of their goods. Then, however, the general validity of the maximum-differentiation choice cannot be taken for granted as in the classical no-complementarity case. How many firms will have positive market shares and whether they will serve all consumers at equilibrium cannot be decided based only on the distribution of the consumer identification characteristic (income or taste). By taking this into account, this thesis reveals a set of possible equilibrium outcomes that have been (with few exceptions) mostly omitted in the existing literature.

In the first essay, exogenous expense is modeled as a lump-sum tax imposed on consumer incomes. The aim is to show that the maximum-differentiation principle does not need to be an optimal quality-choice strategy at any size of the exogenous expense. The larger the tax, the lower the ability (income) or willingness (taste-driven valuation) of consumers to pay for a given level of quality. Accordingly, there is a critical size

of exogenous expense, above which the entrant with smaller quality choice is forced to increase the quality of its good, in order to keep consumers interested in buying. Hence, its good should not be maximally differentiated from the higher-quality good at equilibrium.

In the second essay, a multi-market setting is introduced in which two complementary types of goods are sold independently by single-product firms but consumed in a fixed one-to-one ratio. That is, any two goods of a different type form a pairwise combination. As a result, the good of each actual entrant could be present in more than one combination. In this new setting it is shown that the well-known maximum-differentiation principle could take a new form. Specifically, firms prefer to choose prices at which the so called ”mixed-quality combinations”, consisting of one high-quality good and one low-quality good each, remain unsold.

In the third essay, again a dual-market setting is analyzed, but one of the firms is assumed to be a potential entrant in both markets. It could face competition from a potential superior-quality entrant in one of the markets, and from a potential inferior-quality entrant in the other market. The aim is to check if, at equilibrium, the multi-market entrant could effectively deter entry in either of the two markets by selling its goods only together in a bundle. The results imply that, on the one hand, the entry of the superior-quality entrant should never be excluded, as long as it could enhance the quality of the multi-product firm's bundle, because both firms would then gain. On the other hand, the entry of the inferior-quality entrant can be deterred via bundling. However, the entry-deterrence outcome at equilibrium appears to depend in the first place on the variability of consumer tastes and the quality differentiation of the available combinations, rather than on the pricing or bundling strategies of the multi-product firm.

Abstrakt

Cílem této práce je objasnit, jak komplementarita statků ovlivňuje rovnováhu na vertikálně diferencovaných trzích. Komplementarita není neobvyklá. Mnohé diferencované statky představují z hlediska spotřebitele komplementy, to znamená, že jsou užívány pouze v kombinaci s jinými statky, které také mohou mít různou kvalitu (např. klavír a služby ladiče, pracovní cesta a ubytování, počítač a internetový prohlížeč apod.).

Komplementarita mezi statky představuje exogenní náklad pro spotřebitele, který je třeba zaplatit navíc k ceně jakéhokoli dostupného statku na vertikálně diferencovaných trzích. Schopnost firem kompenzovat tyto zvýšené náklady spotřebitele tím, že účtují nižší ceny, je však pouze omezená. Některé firmy jsou také nuceny zvýšit kvalitu zboží, které nabízí. Pak ovšem již neplatí obecný princip výběru maximalizujícího diferenciaci. To, kolik firem bude mít kladné tržní podíly a zda budou obsluhovat všechny zákazníky, není možné určit pouze na základě rozdělení charakteristik zákazníků (příjmu nebo preferencí). Při zohlednění těchto faktů tato práce diskutuje množství možných rovnovážných řešení, které byly (až na pár výjimek) v existující literatuře dosud ignorovány.

V prvním článku jsou exogenní náklady spotřebitele modelovány jako jednorázová daň uvalená na příjem spotřebitele. Cílem je ukázat, že princip maximální diferenciace nemusí být optimální strategií při kvalitativním výběru pro libovolnou úroveň exogenních nákladů. Čím je daň vyšší, tím nižší je schopnost (příjem) nebo vůle (daná preferencemi) spotřebitelů platit za kvalitu. Z toho plyne existence kritické úrovně exogenních nákladů, nad kterou je firma nabízející nižší kvalitu nucena zvýšit kvalitu svého zboží, aby udržela

zájem spotřebitele toto zboží koupit. Proto v rovnováze dané zboží nutně nemusí být maximálně diferencované od zboží vyšší kvality.

V druhém článku je představen rámec více trhů, na kterých jsou prodávány dva komplementární druhy statků, nezávisle prodávané firmami produkujícími jediný statek, ovšem konzumované v poměru jedna k jedná. To znamená, že každé dva statky různého druhu tvoří vzájemnou kombinaci. Ve výsledku tak statek nabízený každou z firem může figurovat ve vice než jedné kombinaci. V tomto novém uspořádání je ukázáno, že princip maximální diferenciace může nabýt nové podoby. Firmy dávají přednost cenám, při nichž takzvané "kombinace smíšené kvality", obsahující jeden statek s vysokou a jeden s nízkou kvalitou, nejsou prodávány.

Ve třetím článku je opět analyzován rámec více trhů, ovšem jedna z firem je od počátku přítomna na obou trzích. Na jednom z trhů čelí hrozbě vstupu potenciálního konkurenta nabízejícího zboží vyšší kvality, na druhém z trhů čelí hrozbě vstupu potenciálního konkurenta nabízejícího zboží nižší kvality. Cílem je ukázat, zda by taková firma mohla na kterémkoli z trhů odradit konkurenta od vstupu na trh tím, že bude své zboží prodávat pouze v kombinaci. Výsledek ukazuje, že na jedné straně nikdy nelze zabránit vstupu konkurenta s vyšší kvalitou, pokud tento vstup posílí úroveň kvality kombinace nabízené původní firmou, protože v takovém případě benefitují obě firmy. Na druhé straně je kombinací zboží možné zabránit vstupu konkurenta s nižší kvalitou. Tato schopnost zabránit vstupu konkurenta však záleží především na různorodosti spotřebitelských preferencí a na diferenciaci kvality spíše než na cenové a nebo kombinační strategii původní firmy.

1 Exogenous Expenses in Industries With Vertical Product Differentiation and Quality Constraints

In this paper we study how an exogenous expense of owning a market good affects the equilibrium outcome in a market with vertical product differentiation; i.e. consumers differ by income but have identical preferences for the good's quality. For simplicity, the additional expense is modeled as a lump-sum tax on the consumption of the market good. Another novelty is that each firm is assumed to face an individual constraint representing the best quality it could produce.

We identify three possible subgame-perfect equilibrium outcomes dependent on the amount of the exogenous expense.

First, at a small exogenous expense tending to zero, quality choice is characterized by maximal product differentiation, and all consumers buy one of the two qualities in the market. Second, at a medium exogenous expense, some low-income consumers refrain from buying, which incentivizes the producer of the low-quality good to decrease the quality difference from the high-quality good. Third, at a large exogenous expense at which the consumers of the low-quality good cannot afford it, the market is monopolized by the high-quality firm.

The results imply that a lump-sum tax of a moderate size might stimulate the low-quality producer in a vertically differentiated market to raise the quality of its good, which has interesting implications about the potential welfare-improving role such a tool could have in an economy with vertically-differentiated industries.

Keywords: vertical product differentiation, commodity taxation, market participation, blockaded entry

JEL classification: L11, L13, L15

1.1 Introduction

In industrial organization theory, markets in which consumers differ by income[1] but have the same preferences for the distinct qualities of the goods offered are defined as vertically differentiated markets. Gabszewicz and Thisse (1979) as well as Shaked and Sutton (1982, 1983) suggest three standard assumptions that need to hold for an industry to be modeled as a vertically differentiated market: heterogeneous consumer incomes, unanimously agreed ranking of the goods by quality, and indivisible and mutually exclusive purchases (i.e. consumers buy only one unit of the good or none). Examples of industries that possess these three characteristics are the automotive industry, real estate, furniture manufacturing, travel, and electronics. Most of these industries, however, share also a characteristic which is not assumed in the existing models of vertical product differentiation. Namely, consumers encounter extra expenses of owning and using the goods supplied by these industries. For instance, in an example given by Gabszewicz and Thisse (1979) about how a pianist makes a buying decision in the market for pianos, the expenditure incurred by the pianist actually includes not only the producer's price of the piano itself as assumed in the existing models, but also sales tax, the transportation fee to move the piano from the shop to her (or his) home, the fee for the piano technician to tune it after transport, the warranty against hidden defects that might be revealed after the sale, insurance against damage due to external factors like fire or flood, spare keys and strings, etc.

In this paper, we address the issue with the previously unexamined effect of exogenous

[1] In the first model of vertical product differentiation developed by Mussa and Rosen (1978) consumers differ not by income but by taste for quality. However, Tirole (1988) shows that the consumer's taste variable introduced by Mussa and Rosen (1978) gives the reciprocal value of the marginal rate of substitution between income and quality. Intuitively speaking, wealthier consumers value money (income) less and are therefore more willing to pay for an additional unit of quality. See Tirole, 1988, pp. 96–97.

expenses in vertically differentiated markets by adding a fourth assumption to the three standard ones. We introduce a stylized model of a vertically differentiated market where consumers are assumed to face a fixed exogenous expense of owning the good purchased in the market.

Another novelty of our approach is the imposition of exogenous constraints on the quality choices of firms. The range from which a firm can choose the quality of its good is restricted individually from above and universally from below. This assumption reflects better the actual choice of a firm. Manufacturers are never free to increase the quality of their goods infinitely and usually differ in their technological frontiers. That is, for some period of time the highest feasible quality for a firm might be unachievable for others in the market due to patent protection or limited access to scarce high-quality resources necessary for its production. In many industries there are also minimal (safety, hygiene etc.) requirements for the quality of a product in order for it to be approved for sale. Distinct from the technological quality restrictions, however, these minimal requirements must ideally apply the same to all firms in the industry. Further, the restriction of the quality choices of firms makes it possible to unambiguously identify each potential market entrant with the quality rank that its good would have after entry. This makes it possible to explicitly describe how the size of the exogenous expense that consumers face affects the optimal quality choices of firms, that is, the optimal differentiation of the goods offered in the market.

The solution of our model allows us to explore the effect of the size of the exogenous expense on the corresponding equilibrium outcome while using the standard outcome in the case without exogenous expense as a benchmark.

The standard outcome of the solutions of the models of vertical product differentiation is that the structure (respecting the number of firms) and the coveredness[2] of the market at equilibrium do not depend on the quality differences between goods. Instead, they are fully determined by the length of the support interval of the distribution of the consumer incomes. Particularly, a sufficiently narrow spread of consumer incomes[3] ensures that even the lowest-income consumer will prefer to buy the second-best quality in the market rather than to choose the worse quality of a potential third entrant or another non-market option. Thus, at equilibrium the producer of the second-best quality does not face a competition from a lower-quality good. Accordingly, it would be optimal to maximally differentiate the quality of its good from that of the best-quality producer, which is its only competitor. This solution holds at equilibrium because the quality differentiation strictly relaxes the competition between the best producer and the second-best producer. Thus, the larger is the difference between the qualities of the products of the two firms, the higher will be both their profits. Accordingly, maximal differentiation implies corner solutions in qualities. That is to say, one of the entrants chooses to produce the highest possible quality while the other chooses to produce the lowest possible quality that still distinguishes its good from its best non-market substitute.

Our main objective is to show that the presence of an exogenous expense could lead to an equilibrium outcome where the lowest-income consumers would find the outside option more attractive than the second-best quality in the market. We identify a range of values of the size of the exogenous expense at which even when the spread of consumer

[2]By "coveredness" of the market we mean a characterization whether at equilibrium all consumers willing to buy the market good purchase one of the available qualities so that market (demand) is "covered" or there are consumers who choose the outside option (the best free non-market substitute) instead, so that the market (demand) is "non-covered".

[3]Shaked and Sutton (1982) prove that to have a covered duopoly market at equilibrium given the uniform distribution of consumer income, it is necessary and sufficient for consumer income to have a support interval with a right endpoint which is more than two times but at most four times as large as its left endpoint. The duopoly result (i.e. with finite equilibrium number of entrants) holds even for the case of free entry; that is when sunk cost is assumed to tend to zero.

incomes is still sufficiently narrow to prevent the entry of a third entrant, a non-covered duopoly equilibrium exists. That is, at these values the producer of the second-best quality faces a competition not only from the best-quality good in the market but also from a worse-quality non-market good, the free outside option. As a result, the maximal differentiation will no longer be an optimal choice for both firms in the market. One of the entrants, which we call the high-quality producer, will still choose to produce the best possible quality, but the other, which we call the low-quality producer, will deviate from the corner solution in favor of an internal solution. In particular, at equilibrium the low-quality producer will not choose a quality for its good which is the closest possible to that of the outside option, but one that is significantly higher though still below the quality of the best good in the market. This solution will hold because by choosing a quality in-between the other two available options in the market, the low-quality producer does indeed distinguish its product not only from the best good in the market but also from the outside option. So, here we do not have the classical single-competitor story, but one where the low-quality producer needs to differentiate the quality of its good from that of two competitors. This two-competitor differentiation relaxes the competition which the low-quality firm faces, thus allowing it to maximize its profit.

In our paper, we have chosen to model the exogenous expense in a vertically differentiated market as a lump-sum tax on consumption of the market good. The motivation for our choice comes from the fact that the expenses consumers pay on top of the producer's price can be divided into two categories: expenses that vary with the quality of the good (e.g. sales tax, insurance premium, spare keys and strings) and expenses that are constant in the good's quality (e.g. transportation fee, tuning fee). Accordingly, the first category could be modeled as an ad valorem tax or specific tax on consumption or production which is already explored in the existing literature, whereas the second category is more appropriately modeled as a lump-sum tax on consumption.[4]

The competition effect of proportional taxation on production in vertically differentiated markets is extensively studied in the literature. Particularly, Cramer and Thisse (1994) show that by its proportional negative effect on the marginal revenue, an ad valorem tax will mitigate the quality advantage of the high-quality producer at a covered-market equilibrium, while Constantatos and Sartzetakis (1999) show that if the market is not covered, this will result in increased entry of lower-quality producers. Brécard (2008) compares the effects of the ad valorem tax to the effects of the specific tax imposed on the producer per unit of its good sold. Both tax regimes have influence on firms' revenue because ad valorem tax is proportional to price, while specific tax is proportional to the market share. The results imply that these taxes negatively affect demand, qualities, differentiation and profits of the two firms in the market.

To our knowledge, the competition effect of lump-sum taxation on consumption in a vertically differentiated market has not been studied in the existing literature. In our model, because we impose a lump-sum tax on consumption, the equilibrium price does not change due to an increase in firm's cost, that is by shifting the individual supply curves up, instead, it negatively affects the buyer's net income, which shifts the residual demand curves down. As a result, we are able to study the equilibrium solution in a setting largely consistent with the original model of Shaked and Sutton (1982), so that we do not need to relax even its simplifying assumption of zero unit production cost. This allows us to identify a direct relationship between the size of the tax and the shift from a covered to non-covered duopoly market, given free entry. It also enables us to describe more

[4]We are grateful to Jakub Steiner for pointing out the similarity between exogenous expense and lump-sum consumption tax in his comments to an earlier version of the present paper.

precisely the competition mechanism that leads to a decrease in the quality differentiation between the two goods in the market. As noted above, at a tax size at which the market is not covered, the low-quality producer faces competition from the outside option, which gives it an incentive to lessen the difference of its products from that of the high-quality producer.

Our results suggest the existence of three distinct equilibrium outcomes dependent on the amount of the lump-sum tax imposed on consumers.

The first equilibrium outcome is similar to the case of proportional taxation in the model of Cramer and Thisse (1994). For a small lump-sum tax tending to zero, the equilibrium outcome is a covered duopoly. Only the demand share of the high-quality firm decreases proportionally to the amount of the exogenous expense in favor of the market share of the low-quality producer.

The second equilibrium outcome corresponds to a medium-sized lump sum tax which does not tend to zero but is still below the level at which all consumers who would otherwise buy the low-quality good cannot afford it. As a result, some (although not all) of the consumers of the low-quality firm switch to the free outside option and the market is a non-covered duopoly at equilibrium. This is the equilibrium outcome, which is of primary interest in our paper because it implies an internal solution for the quality choice of the low-quality firm.

The third equilibrium outcome occurs at and above the lump-sum threshold, where only consumers who prefer the high-quality good buy in the market, while the rest choose the outside option. As a result, the producer of the low-quality good cannot make positive sales. Therefore, the low-quality firm loses incentive to enter the market and there is a non-covered monopoly structure at equilibrium.

The paper is organized as follows. In section 1.2 we introduce the model of a vertically differentiated market with lump-sum taxation on consumption. In section 1.3 we present each of the three equilibrium solutions and derive the respective conditions for their existence in terms of the tax size, income distribution and quality constraints. Section 1.4 summarizes the results and discusses their implications.

1.2 The Model

Here we introduce a model of a market for vertically differentiated goods with a lump-sum tax imposed on consumers who buy them. To allow for direct comparability, the notation of the common terms is held the same as in Shaked and Sutton (1982).

Consider N independent firms who are potential entrants to the market. They face the following three-stage game:

Stage 1: Firms simultaneously decide to enter the market or not.

Before making an entry decision each potential entrant is assigned an individual exogenously defined upper bound on its quality.[5] Each potential entrant is identified by the sequential number $n \in [1,N]$ of its quality upper bound $(\bar{u}_n)$ in descending order:

$$\bar{u}_1 > \bar{u}_2 > \bar{u}_3 > ... > \bar{u}_n > \bar{u}_{n+1} > ... > \bar{u}_N \tag{1}$$

where:

$\bar{u}_n$ - upper bound on quality choice with rank n, $n = 1,2,...,N$.

Based on its assignment, each firm independently decides whether to enter the market or not. In the real world firms may face quality constraints as in (1) if, for example,

[5]This assumption ensures only distinct quality choices in equilibrium. For a solution without quality constraints which also allows for classical Bertrand homogeneous-good equilibrium, see Shaked and Sutton, 1982, p.10.

quality-enhancing technologies are patent-protected[6].

Stage 2: Entrants make a quality choice simultaneously.

Let K entrants be in the market after the first stage, $K \leq N$. Each actual entrant is identified by the sequential number $k \in [1,K]$ of its quality upper bound $(\bar{u}_{n_k})$ in a subsequence of the initial sequence in (1). The subsequence is formed by arranging in descending order the quality bounds of the actual entrants only.[7] Accordingly, each entrant k chooses the quality $u_k\,(k = 1,2,...,K)$ of its good so that the following inequality must hold:[8]

$$u_k \leq \bar{u}_{n_k}, k = 1,2,...,K, K < N \tag{2}$$

where:

u_k - quality choice of entrant with upper bound re-ranked to k

$\bar{u}_{n_k}$ - k-th element of sub-sequence from sequence $\{\bar{u}_n\}_{n=1}^{N}$

Stage 3: Firms compete in prices simultaneously

Each firm makes a take-it-or-leave-it offer in the form of quality-price pair. For simplicity, as in Shaked and Sutton (1982), the production costs are assumed to be zero. Therefore, each firm sets the price that maximizes its revenue as follows:

$$\max_{p_k,u_k} R_k = p_k D_k\left(p_1,...,p_k,...,p_K\right), k = 1,...,K \tag{3}$$

where:

[6] For a stylized model of patent race see Loury (1979).

[7] Alternatively, qualities could be indexed in ascending order as in Shaked and Sutton (1982). Here, we arrange them in descending order, to make indices independent on the number of entrants so that the entrant with highest quality is always indexed by number 1, the entrant with the second-best quality is indexed by 2 and so on till the last entrant, who is indexed by K.

[8] In our model, the rank k of a good in the quality constraint sequence is meant to play the same role of a unique quality identifier as its brand does in the real world. Hereby, we are able to unambiguously determine which potential entrant will enter as a quality leader, and which one as its immediate follower.

R_k - revenue of the entrant providing good with quality rank k in the market, $k = 1, 2, ..., K$

p_k - price of good with quality rank k

$D_k(p_1, ..., p_k, ..., p_K)$ - demand for good with quality rank k

The demand for a quality-ranked good coincides with the number of consumers who choose to buy it. This is implied by the standard assumption that consumers make indivisible and mutually exclusive purchases (i.e. buy one or none) from among the K qualities available in the market.[9] Consumers are also assumed to form a continuum so that there is no difference between market share and demand for quality. To explicitly derive the market shares of the qualities in the market, the consumer's side of the market is defined below.

Consumer's preferences are given by the following utility function of a simplified Cobb-Douglas form:

$$U(k, x) = u_k x \tag{4}$$

where:

$U(k, x)$ - utility function of a consumer who buys a good with quality rank k

x - the consumption of all the other ('non-quality') goods represented by the quantity of a composite good that is accordingly taken to be a numéraire good with a unit price.

We have twofold reason to choose the functional form in (4). Intuitively, consuming a better quality good should enhance the comfort and enjoyment of the other goods consumed together with it. Therefore, it makes sense to assume that the quality variable

[9] "...this is necessarily the case if the choice of a consumer concerns indivisible products which, by their very nature, are either bought in a single unit of a single brand or not bought at all. So are cars, TV's, washing machines, stereo chains, pianos, a.s.o." (Gabszewicz and Thisse, 1979, p. 340).

u_k is an augmenting factor of the quantity variable x. The second argument to choose the functional form in (4) is that the exact indirect utility function introduced by Shaked and Sutton (1982) could be derived from it, as shown below.

We make a simplifying assumption common for the non-representative consumer models of product differentiation. Specifically, that the identifying characteristics of consumers, which in our model is their gross income, is assumed to be uniformly distributed according to a density that equals unity on some support $0 < a \leq t \leq b$. Respectively, each consumer maximizes her/his utility subject to the following budget constraint:

$$p_k + x \leq t - T \tag{5}$$

where:

t - gross consumer income, $t \sim U\left[a,b\right]$

T - amount of the lump-sum tax imposed on the consumption of the market good.

So long as both prices are positive and smaller than the income, the consumer problem has an internal solution, so that the budget constraint is binding. This allows the amount of the numéraire good consumed to be expressed through the prices and consumer's budget as follows:

$$x = t - T - p_k \tag{6}$$

Substituting for x in the objective function yields the following unconstrained consumer's optimization problem:

$$\max_{k} U(k,t) = u_k (t - T - p_k) \tag{7}$$

This reduced-form utility function is directly assumed to represent the common consumer preferences by Gabszewicz and Thisse (1979) and Shaked and Sutton (1982). The value function that corresponds to the solution gives the indirect utility function $V_k(t, p_k)$.

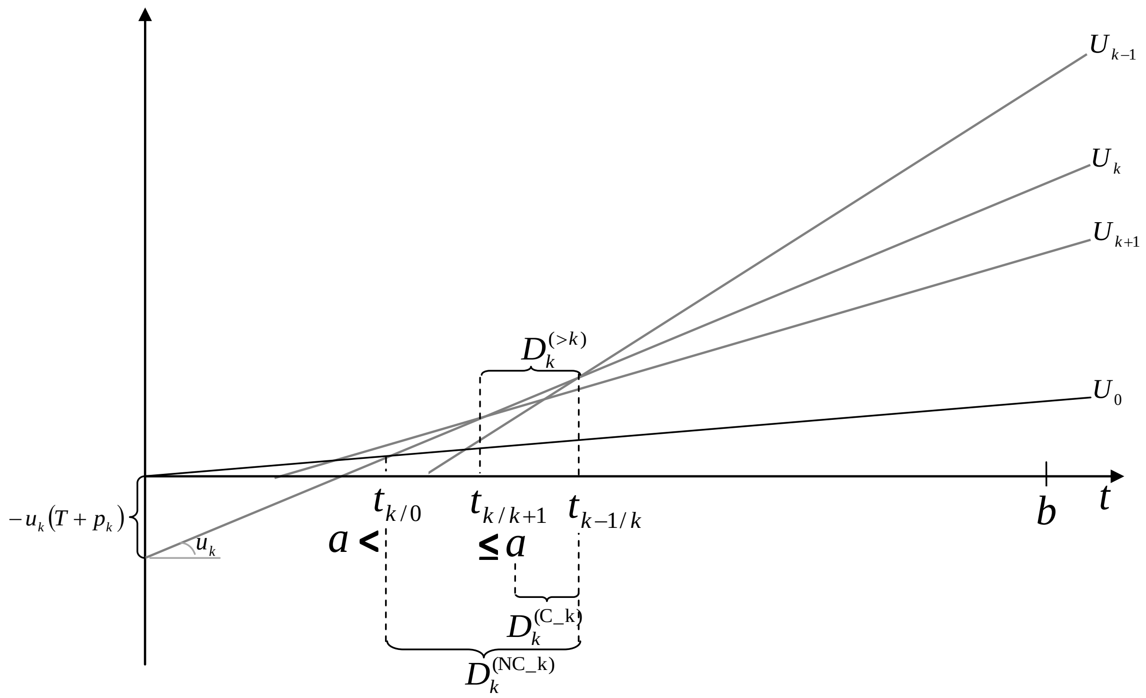
To rule out a situation where a consumer participates in the market although her/his budget cannot cover the price plus tax of either of the available qualities, a free non-taxable outside offer is also made available to the consumers. It has a quality of u_0 and zero price $p_0 = 0$ at which the utility function in (7) takes the following reduced form:

$$U(k=0,t) = u_0 t \tag{8}$$

Note that the utility from the outside option is always non-negative even when the consumer who chooses it has zero gross income.

The derivation of the quality demands follows from the solution of the optimization problem in (7). Note that (7) implies that the utility from a good with quality rank k increases linearly in the gross income of the consumer. Therefore, on figure 1, the curve representing the utility from quality u_k as a function of a consumer's gross income is a straight line.[10]

[10] A graph similar to Figure 1 has been employed by Giannakas (2011) to illustrate the derivation of market shares in a model of a vertically differentiated market without additional expense and outside option.



<u>Figure 1:</u> Deriving the quality demands as solutions to the utility-maximization problems

The market share of quality u_k is given by the intersection of the two subsets $\left[t_{k/k+1}, t_{k-1/k}\right]$ and $[\,a,b\,]$ that gives the subset of the feasible consumer incomes at which its utility (line) is the highest. $D_k^{(\mathrm{C\_k})}$ represents the market share of u_k when it covers the market $\left(t_{k/k+1} \leq a\right)$. $D_k^{(\mathrm{NC\_k})}$ represents the market share of u_k when it is the lowest quality but the market is non-covered $\left(t_{k/0} > a\right)$. $D_k^{(>k)}$ represents the market share of u_k when there is a lower quality u_{k+1} $(K > k)$.

On Figure 1, the crossing point between any two utility curves gives the marginal gross income at which a consumer would be indifferent between the qualities they represent. Particularly, at marginal income $t_{k-1/k}$ the consumer is indifferent between quality u_{k-1} and quality u_k while at marginal income $t_{k/k+1}$ the consumer is indifferent between quality u_k and quality u_{k+1}.

The assumptions for unit-consumption and uniform distribution of incomes in the interval $[\,a,b\,]$ imply that the market share of quality u_k is given by the intersection of the two subsets of the set of possible consumer incomes, $\left[t_{k/k+1}, t_{k-1/k}\right]$ and $[\,a,b\,]$.

Accordingly, there are three possibilities.

First, if the marginal incomes $t_{k-1/k}$ and $t_{k/k+1}$ belong to the support interval of the consumer income distribution, $t_{k/k+1} \in [\, a, b\,] \bigwedge t_{k-1/k} \in [\, a, b\,]$, both u_k and u_{k+1} would have positive market share so that u_k is not the lowest quality in the market, $(K > k)$. To distinguish the market share in this case, we denote it by $(> k)$.

Second, if $t_{k-1/k}$ belongs to the support interval of the consumer income distribution and $t_{k/k+1}$ is excluded from it, $t_{k-1/k} \in [\, a, b\,] \bigwedge t_{k/k+1} \notin [\, a, b\,]$, u_k will be the lowest quality with positive market share. Given that the lowest-income consumer prefers it to the quality of the outside option u_0, i.e. $t_{k/0} \leq a$, it will also cover the market. Therefore, we denote this case by (C_k).

Third, when u_k is the lowest quality with positive market share but the lowest-income consumer prefers the outside option u_0 to it, i.e. $t_{k/0} > a$, the market is not covered. We denote this case by (NC_k).

Hence, the market share of quality $u_k, k = 2, \ldots, K$, is represented by the following conditional equation:

$$D_k = \begin{cases} D_k^{(C\_k)} = t_{k-1/k} - a, & \text{if } t_{k/k+1} < t_{k/0} \leq a < t_{k-1/k} < b \\ D_k^{(NC\_k)} = t_{k-1/k} - t_{k/0}, & \text{if } t_{k/k+1} \leq a < t_{k/0} < t_{k-1/k} < b \\ D_k^{(>k)} = t_{k-1/k} - t_{k/k+1}, & \text{if } \max[t_{k+1/0}, a] < t_{k/k+1} < t_{k-1/k} < b \\ 0, & \text{if } t_{k/k+1} < t_{k-1/k} \leq a < b \end{cases} \tag{9}$$

where:

D_k – demand share of good with quality rank k, $k = 2, \ldots, K$

$D_k^{(C\_k)}$ – demand share of good with quality rank k when it covers (C) the market, i.e. $t_{k/k+1} \leq a$, $k = 2, \ldots, K$

$D_k^{(NC\_k)}$ – demand share of good with quality rank k when it has the lowest quality but the market is non-covered (NC) $\left(t_{k/0} > a\right)$

$D_k^{(>k)}$ – demand share of good with quality rank k when its quality is not the lowest in the market $(K > k)$

We ignore the case when $t_{k/k+1}$ belongs to the support interval of the consumer income distribution and $t_{k-1/k}$ is excluded from it, $t_{k/k+1} \in [\,a,b\,] \cup t_{k-1/k} \notin [\,a,b\,]$, because by definition this cannot be an equilibrium outcome. The higher quality rank (as unanimously agreed by all consumers) of u_{k-1} relative to u_k implies that the former cannot be efficiently undercut by the latter at equilibrium. On the contrary, even if quality u_{k-1} is priced slightly higher than u_k, it would still be preferred by the consumer at any income level.

When $k = 1$, however, there is no better quality in the market, so its demand share is bounded from above by the maximum income b itself, as shown below:

$$D_1 = \begin{cases} D_1^{(C\_k)} = b - a, & \text{if } t_{1/2} < t_{1/0} \leq a < b \\ D_k^{(NC\_k)} = b - t_{1/0}, & \text{if } t_{1/2} \leq a < t_{1/0} < b \\ D_k^{(>1)} = b - t_{1/2}, & \text{if } \max[t_{2/0}, a] < t_{1/2} < b \\ 0, & \text{if } a < b < t_{1/2} \end{cases} \tag{10}$$

Finally, the demand functions $D_k\left(p_1, ..., p_k, ..., p_K\right)$, $k = 1, .., K$, in (3) can be derived in explicit form by expressing the demand shares in (10) and (11) in terms of prices. For the purpose, we need to first express the marginal incomes. The general expression for the marginal consumer's income $t_{k/k+1}$ can be derived by equalizing the utility function as defined in (7) for u_k and u_{k+1}:

$$U\left(k, t_{k/k+1}\right) = U\left(k+1, t_{k/k+1}\right) \tag{11}$$

which after being solved for $t_{k/k+1}$ in explicit terms yields the following result:

$$t_{k/k+1} = \frac{u_{k+1}p_{k+1} - u_k p_k}{u_{k+1} - u_k} + T \tag{12}$$

where:

$t_{k/k+1}$ - the marginal gross income that would make a consumer indifferent between buying quality u_k or quality u_{k+1} at the market prices at which these qualities are supplied given the lump-sum tax T.

The expression for the income at which the utility curves of u_k and u_0 cross each other can be derived in the same way:

$$t_{k/0} = \frac{u_k p_k}{u_k - u_0} + \frac{u_k}{u_k - u_0}T \tag{13}$$

The result in (13) differs from (12) because the outside option is free and non-taxable.

The corresponding double-kinked demand curve of u_k is given by the following expression:

$$D_k = \begin{cases} \frac{u_{k-1}p_{k-1} - u_k p_k}{u_{k-1} - u_k} + T - a, & \text{if } p_k \leq \hat{p}_k \qquad \text{(C\_k)} \\ \frac{u_{k-1}p_{k-1} - u_k p_k}{u_{k-1} - u_k} - \frac{u_k p_k}{u_k - u_0} - \frac{u_0}{u_k - u_0}T, & \text{if } \hat{p}_k < p_k < \hat{\hat{p}}_k \quad \text{(NC\_k)} \quad \text{k}= 2,\ldots,\text{K} \\ \frac{u_{k-1}p_{k-1} - u_k p_k}{u_{k-1} - u_k} - \frac{u_k p_k - u_{k+1}p_{k+1}}{u_k - u_{k+1}}, & \text{if } \hat{\hat{p}}_k < p_k < \bar{p}_k \quad (> \text{k}) \\ 0, & \text{if } p_k \geq \bar{p}_k \end{cases} \tag{14}$$

where:

$\hat{p}_k = \frac{u_k - u_0}{u_k} a - T$ – price level at which there is a kink in the demand for the good with quality rank k because below that price it covers the market.

$\hat{\hat{p}}_k = \frac{u_{k+1}(u_k - u_0)p_{k+1} + u_0(u_k - u_{k+1})T}{u_k(u_{k+1} - u_0)}$ – price level at which there is another kink in the demand for the good with quality rank k because below that price it undercuts the goods with worse qualities in the market.

$\bar{p}_k = \frac{u_{k-1}(u_k - u_{k+1})p_{k-1} + u_{k+1}(u_{k-1} - u_k)p_{k+1}}{u_k(u_{k-1} - u_{k+1})}$ – price level at and above which for any consumer's income the good with quality rank k is less preferred to its nearest competitor by rank, which excludes it from the market.

The if-conditions in (14) can be also imposed on T. For example, the condition for the market to be covered by the last entrant K is:

$$T \leq \frac{u_K - u_0}{u_K} a - p_K^{(C\_K)} \tag{15}$$

where:

$p_{\mathrm{K}}^{(C\_K)}$ – optimal price of the last entrant with quality rank K if it is expected to cover the market

Similarly, the condition for the market to be non-covered by the last entrant K is:

$$T > \frac{u_K - u_0}{u_K} a - p_K^{(NC\_K)} \tag{16}$$

where:

$p_{\mathrm{K}}^{(NC\_K)}$ – optimal price of the last entrant with quality rank K if it is expected not to cover the market

The demand function for the best quality in terms of prices is also double-kinked:

$$D_1 = \begin{cases} b-a, & \text{if } p_1 \le \hat{p}_1 \quad \text{(C\_1)} \\ b - \frac{u_1 p_1}{u_1-u_0} - \frac{u_1 T}{u_1-u_0}, & \text{if } \hat{p}_1 < p_1 \le \hat{\hat{p}}_1 \quad \text{(N\_1)} \\ b - \frac{u_1 p_1 - u_2 p_2}{u_1-u_2} - T, & \text{if } \hat{\hat{p}}_1 < p_1 < \bar{p}_1 \quad (>\ 1) \\ 0, & \text{if } p_1 \ge \bar{p}_1 \end{cases} \tag{17}$$

where:

$\hat{p}_1 = \frac{u_1-u_0}{u_1} a - T$ – price level at which there is a kink in the demand curve of the good with quality rank 1 because below that price it covers the market.

$\hat{\hat{p}}_1 = \frac{u_2(u_1-u_0)p_2 + u_0(u_1-u_2)T}{u_1(u_2-u_0)}$ – price level at which there is another kink in the demand curve of the good with quality rank 1 because below that price it undercuts the goods with worse qualities in the market.

$\bar{p}_1 = \frac{u_2 p_2 + (u_1-u_2)(b-T)}{u_1}$ – price level at and above which for any consumer's income the good with quality rank 1 is less preferred to its nearest competitor by rank, which excludes it from the market.

Note that both the expressions in (14) and (17) consist of three distinct linear functions that are related differently to price and tax. For example, the (C_k) segment of the conditional demand function in (14) is less sensitive to price p_k (i.e. steeper) than the (NC_k) segment. The former is positively related to the amount of the lump sum (i.e. parallelly shifting to the right when T increases) while the latter is negatively related to the amount of the lump sum (i.e. parallelly shifting to the left when T increases). Therefore, at different amounts of the lump sum, the residual demand curves change not only their locations but also their shapes, which implies a difference in the segment to which the optimal solution belongs.

In the next section, we apply the concept of Selten's (1975) subgame perfect equilibrium and solve the model by backwards induction to identify three distinct optimal outcomes characterized by the structure and the coveredness of the market at different lump sums of the tax on consumption.

We first derive the conditions on a, b and T for having a covered duopoly outcome at equilibrium and show that it is not independent of the quality choices of the entrants. Further, we specify the conditions on the individual quality upper bounds for having alternative equilibrium solutions at larger amounts of the lump sum, respectively non-covered duopoly and non-covered monopoly.

1.3 Equilibrium Solutions

1.3.1 Covered Duopoly Equilibrium With Small Lump-sum Tax

Given K entrants in the market, we can explicitly express their revenue-maximization problems by substituting for the demand function from (14) and (17) in (3):

$$\max_{p_1,u_1} R_1 = p_1\left(\bar{\theta} - \frac{u_1 p_1 - u_2 p_2}{u_1 - u_2}\right) \tag{18}$$

$$\max_{p_k,u_k} R_k = p_k\left(\frac{u_{k-1}p_{k-1} - u_k p_k}{u_{k-1} - u_k} - \frac{u_k p_k - u_{k+1}p_{k+1}}{u_k - u_{k+1}}\right), k = 2, ..., K-1 \tag{19}$$

$$\max_{p_K,u_K} R_K = \begin{cases} p_K\left(\frac{u_K p_K - u_{K+1}p_{K+1}}{u_K - u_{K+1}} - a\right), & \text{if } p_{\mathrm{K}} \le \frac{u_K - u_0}{u_K}a - T \\ p_K\left(\frac{u_K p_K - u_{K+1}p_{K+1}}{u_K - u_{K+1}} - \frac{u_K p_K}{u_K - u_0}\right), & \text{if } p_{\mathrm{K}} > \frac{u_K - u_0}{u_K}a - T \end{cases} \tag{20}$$

The optimal solution for prices in the last subgame implies the following sequence of inequality relationships between the marginal incomes $t_{k/k+1}$ and the endpoints a and b of the support interval of the consumer income distribution:[11]

$$b \geq 2t_{1/2} - T \geq 4t_{2/3} - 3T \geq \ldots \geq 2^k t_{k/k+1} - \left(2^k - 1\right) T \geq$$
$$\geq \begin{cases} 2^{K-1}a - (2^{K-1} - 1)T, & \text{if } p_{\text{K}}^{(C\_K)} \leq \frac{u_K - u_0}{u_K} a - T \\ 2^K t_{K/0} - \left(\frac{(2^K-1)u_K - (2^{K-1}-1)u_0}{u_K - u_0}\right)T, & \text{if } p_{\text{K}}^{(NC\_K)} > \frac{u_K - u_0}{u_K} a - T \end{cases} \tag{21}$$

Hence, if the maximum income b satisfies the constraint below:

$$b \leq 4a - 3T \tag{22}$$

the following inequality would characterize the equilibrium with more than two entrants:

$$a > t_{k-1/k} > t_{k/0}, k = 3, \ldots, K \text{ for } K > 2 \tag{23}$$

That is, when (22) holds for all firms that decide to enter the market in the first stage, at most two would have positive market share and therefore would remain in the market at equilibrium. Note that this result is not affected by the coveredness of the market. So, (22) would ensure efficient foreclosure of a potential third entrant ($a > t_{2/3}$) even in the case of non-covered equilibrium market outcome, which is studied in the next subsection.

In this subsection, we are interested in the conditions for having a covered duopoly at equilibrium. Therefore, we assume that condition (22) holds and the market is covered by two entrants with ranks 1 and 2, respectively. The corresponding subgame equilibrium is

[11] The result in (21) is derived from the system of revenue-maximizing first-order conditions with respect to prices as in Shaked and Sutton, 1982, p.5.

given by the following expressions for the optimal prices in terms of the qualities of the goods the two firms offer:

$$
\begin{aligned}
p_1^{(C\_2)} &= \frac{(2b-a-T)\left(u_1^{(C\_2)} - u_2^{(C\_2)}\right)}{3u_1^{(C\_2)}} \\
p_2^{(C\_2)} &= \frac{(b-2a+T)\left(u_1^{(C\_2)} - u_2^{(C\_2)}\right)}{3u_2^{(C\_2)}}
\end{aligned}
\tag{24}
$$

where:

$p_1^{(C\_2)}$ – price choice of firm 1 in the case of a covered duopoly equilibrium

$p_2^{(C\_2)}$ – price choice of firm 2 in the case of a covered duopoly equilibrium

$u_1^{(C\_2)}$ – quality choice of firm 1 in the case of a covered duopoly equilibrium

$u_2^{(C\_2)}$ – quality choice of firm 2 in the case of a covered duopoly equilibrium

The optimal expressions for demands are:

$$
\begin{aligned}
D_1^{(C\_2)} &= \frac{1}{3}(2b-a-T) \\
D_2^{(C\_2)} &= \frac{1}{3}(b-2a+T)
\end{aligned}
\tag{25}
$$

where:

$D_1^{(C\_2)}$ – market share of firm 1 in the case of a covered duopoly equilibrium

$D_2^{(C\_2)}$ – market share of firm 2 in the case of a covered duopoly equilibrium

Both expressions in (25) are positive if the maximum income b satisfies the constraint below:

$$
b > 2a - T \tag{26}
$$

Combining conditions (22) and (26) gives us the following inequality:

$$2a - T < b \leq 4a - 3T \tag{27}$$

that must be satisfied for having two entrants with positive market shares at equilibrium provided that the market is covered. Note that in case of zero additional expense to the consumer, T=0, the inequality in (27) coincides with Shaked and Sutton's (1982) condition for a covered duopoly equilibrium. Distinct from this, however, there are some sufficiently large positive values of T for which (27) would not hold. For its right-hand side to be larger than its left-hand side, the tax should not exceed the minimum consumer income a.

$$T < a \tag{28}$$

Condition (22) is not sufficient to have a covered market equilibrium at $T > 0$. For the purpose, we introduce an additional condition on T which is not independent of the quality choices of the firms at the second stage. Before deriving it, we need to first present the solution for the quality-choice subgame equilibrium at the second stage. We express the revenues of the two entrants in terms of their qualities by taking the pairwise products of the respective expressions in (24) and (25):

$$\begin{aligned} R_1^{(C\_2)} &= \frac{(2b-a-T)^2\left(u_1^{(C\_2)} - u_2^{(C\_2)}\right)}{9u_1^{(C\_2)}} \\ R_2^{(C\_2)} &= \frac{(2b-a-T)^2\left(u_1^{(C\_2)} - u_2^{(C\_2)}\right)}{9u_2^{(C\_2)}} \end{aligned} \tag{29}$$

where:

$R_1^{(C\_2)}$ – profit/revenue of firm 1 in the case of a covered duopoly equilibrium

$R_2^{(C\_2)}$ – profit/revenue of firm 2 in the case of a covered duopoly equilibrium

Both expressions in (29) are increasing in the higher quality u_1 and decreasing in the lower quality u_2 which implies the following set of optimal quality choices in the second stage:

$$
\begin{aligned}
u_1^{(C\_2)} &= \bar{u}_1 \\
u_2^{(C\_2)} &= \underline{u}_2
\end{aligned}
\tag{30}
$$

where:

$\underline{u}_2$ – the lowest quality that the entrant with quality constraint of rank 2 could choose so that its good is still better than the outside option, $\underline{u}_2 = u_0 + \varepsilon, \varepsilon \to 0, \varepsilon > 0$.

We derive the condition for a covered market by substituting for the optimal price $p_2^{(C\_2)}$ from (24) in (15) for K=2:

$$T \leq \frac{a\left(2\bar{u}_1 + \underline{u}_2 - 3u_0\right) - b\left(\bar{u}_1 - \underline{u}_2\right)}{\bar{u}_1 + 2\underline{u}_2} \tag{31}$$

For simplicity, we denote the right-hand side of (31) by $T^{(C\_2)}$ that stands for the upper bound on the lump-sum tax size below which a covered duopoly equilibrium exists.

If we substitute for b by the left-hand side of (27) in the right-hand side of (31), we derive an expression that must still be larger than $T^{(C\_2)}$:

$$T^{(C\_2)} < \frac{3a\left(\underline{u}_2 - u_0\right) + T\left(\bar{u}_1 - \underline{u}_2\right)}{\bar{u}_1 + 2\underline{u}_2} \tag{32}$$

because $T^{(C\_2)}$ is strictly decreasing in b. However, the inequality in (32) is valid only if the following necessary condition holds:

$$T < \frac{\underline{u}_2 - u_0}{\underline{u}_2} a = \frac{\varepsilon}{u_0 + \varepsilon} a \tag{33}$$

That is, if (33) does not hold we cannot have a covered market at equilibrium because the inequality condition in (31) will be violated. This result is especially interesting because it implies that the market can be a covered duopoly at equilibrium only if T is extremely small, almost zero, as by definition ε tends to zero.

The joint validity of conditions (2), (27) and (31) ensures the existence of a unique covered duopoly equilibrium which is established in proposition 1 below.

Proposition 1. Let the consumer income distribution satisfy condition (27) and the restrictions from above on the quality choices in (2) and on the amount of the lump-sum tax in (31) hold. Then, a unique subgame perfect equilibrium exists at which exactly the entrants with quality ranks 1 and 2 enter the market, make the quality choice in (30) and set the corresponding prices in (24). The firms' market shares and profits are given by the respective expressions in (25) and (29). The market is covered.

The intuition behind proposition 1 can be illustrated by plotting the residual demand curves of firm 1 and 2 according to the expressions for their demand functions in (14) and (17) as shown on figure 2 below. Since marginal cost is assumed to be zero for both entrants, the equilibrium solutions are given by the points of unit elasticity in their residual demand curves. That is, if we take the segment to which the unit elasticity point belongs as a linear demand curve itself, the unit elasticity point would be equally distant from its intersections with the horizontal and vertical axes.

Generally, the amount of expense T is negatively related to the net income of the buyers in the market. Therefore, its imposition leads to a shift in the residual demand curves. However, the assumption of indivisibility and mutual exclusivity of goods does not imply the classical parallel shift of the overall demand curve to the left. As the expressions in

(14) and (17) show, the magnitude and the direction of the demand curve shift vary across its segments. As a result, at different values of T the unit elasticity point would move from one segment to another, thus implying distinct equilibrium outcomes.

What we have shown so far is that in the particular case when the support interval of the distribution of consumer incomes complies with the inequalities in (27) and the additional expense T satisfies the restriction from above in (31), the unit elasticity points belong to the highest segment of the residual demand curve of firm 1 and to the middle segment of the residual demand curve of firm 2, respectively.

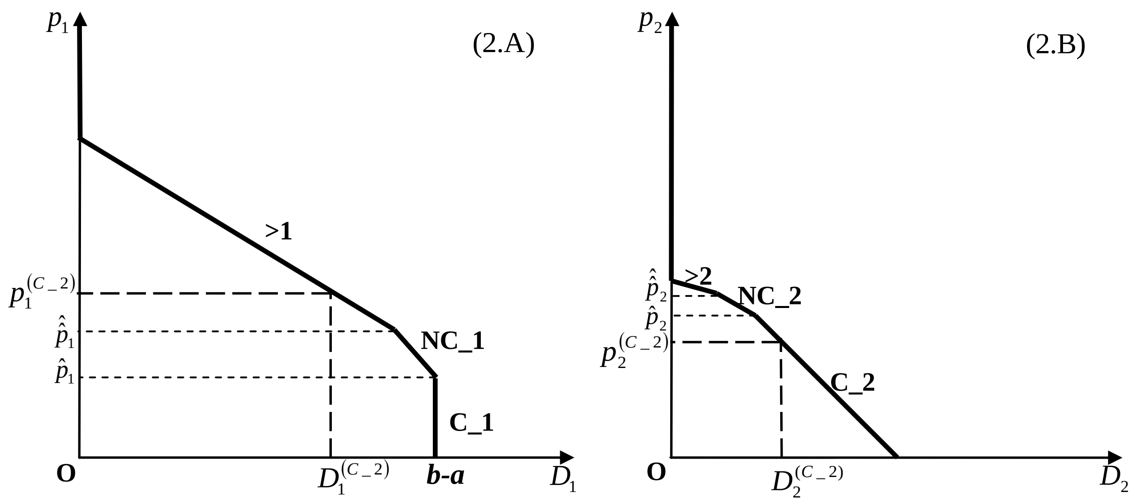


Figure 2: Solution for equilibrium with covered duopoly outcome

Panel (2.A) represents the residual demand curve of a good with quality rank 1 as given in (17) at the small tax established in (31). The optimal solution is given by the unit-elastic point with coordinates $(D_1^{(C\_2)}, p_1^{(C\_2)})$ which belongs to the segment (>1) above the price $\hat{\hat{p}}_1^{(C\_2)}$ at which at least one more entrant with a worse good is accommodated in the market. Panel (2.B) illustrates the demand for the good with quality rank 2 as given in (14) for $k = 2$ at the small tax established in (33). The optimal solution is given by the unit-elastic point with coordinates $(D_2^{(C\_2)}, p_2^{(C\_2)})$ which belongs to the bottom segment (C_2) below the price $\hat{p}_2$ at which the market is exactly covered by the second-best quality.

1.3.2 Non-Covered Duopoly Equilibrium With Middle Lump-sum Tax

In this subsection, we establish the conditions for a non-covered duopoly outcome at equilibrium.

Let the condition in (22) hold, so that there are at most two entrants. Now, however, the market is assumed to be non-covered. The corresponding subgame equilibrium at the pricing stage is given by the following set of optimal prices for the two entrants:

$$\begin{aligned} p_1^{(NC\_2)} &= \frac{\left(2(u_1^{(NC\_2)} - u_0)b - (2u_1^{(NC\_2)} - u_0)T)\right)\left(u_1^{(NC\_2)} - u_2^{(NC\_2)}\right)}{\left(4u_1^{(NC\_2)} - u_2^{(NC\_2)} - 3u_0\right)u_1^{(NC\_2)}} \\ p_2^{(NC\_2)} &= \frac{\left((u_2^{(NC\_2)} - u_0)b - (u_2^{(NC\_2)} + u_0)T\right)\left(u_1^{(NC\_2)} - u_2^{(NC\_2)}\right)}{\left(4u_1^{(NC\_2)} - u_2^{(NC\_2)} - 3u_0\right)u_2^{(NC\_2)}} \end{aligned} \tag{34}$$

where:

$p_1^{(NC\_2)}$ – price choice of firm 1 in the case of a non-covered duopoly equilibrium

$p_2^{(NC\_2)}$ – price choice of firm 2 in the case of a non-covered duopoly equilibrium

$u_1^{(NC\_2)}$ – quality choice of firm 1 in the case of a non-covered duopoly equilibrium

$u_2^{(NC\_2)}$ – quality choice of firm 2 in the case of a non-covered duopoly equilibrium

Substituting for the optimal prices from (34) into (16) for K=2 yields the following expression of the condition for having a non-covered duopoly at equilibrium:

$$T > \frac{\left[\left(4u_1^{(NC\_2)} - u_2^{(NC\_2)} - 3u_0\right)a - \left(u_1^{(NC\_2)} - u_2^{(NC\_2)}\right)b\right]\left(u_2^{(NC\_2)} - u_0\right)}{3u_1^{(NC\_2)}u_2^{(NC\_2)} - \left(u_1^{(NC\_2)} + 2u_2^{(NC\_2)}\right)u_0} \tag{35}$$

The optimal expressions for demands are:

$$
\begin{aligned}
D_1^{(NC\_2)} &= \frac{2\left(u_1^{(NC\_2)} - u_0\right)b - \left(2u_1^{(NC\_2)} - u_0\right)T}{4u_1^{(NC\_2)} - u_2^{(NC\_2)} - 3u_0} \\
D_2^{(NC\_2)} &= \frac{\left(\left(u_2^{(NC\_2)} - u_0\right)b - \left(u_2^{(NC\_2)} + u_0\right)T\right)\left(u_1^{(NC\_2)} - u_0\right)}{\left(4u_1^{(NC\_2)} - 3u_0 - u_2^{(NC\_2)}\right)\left(u_2^{(NC\_2)} - u_0\right)}
\end{aligned}
\tag{36}
$$

where:

$D_1^{(NC\_2)}$ – market share of firm 1 in the case of a non-covered duopoly equilibrium

$D_2^{(NC\_2)}$ – market share of firm 2 in the case of a non-covered duopoly equilibrium

The conditions for both firms to have positive market shares are respectively:

$$
\begin{aligned}
u_1^{(NC\_2)} &> \frac{(2b-T)}{2(b-T)}u_0 \\
u_2^{(NC\_2)} &> \frac{b+T}{b-T}u_0
\end{aligned}
\tag{37}
$$

It is trivial that the condition on $u_2^{(NC\_2)}$ is stricter and should also be valid for $u_1^{(NC\_2)}$ which by definition is set to be bigger than $u_2^{(NC\_2)}$:

$$
u_1^{(NC\_2)} > u_2^{(NC\_2)} > \frac{b+T}{b-T}u_0 \tag{38}
$$

Hence, the optimal choice of the producer of the lower-quality good $u_2^{(NC\_2)}$ must vary in the interval $(\frac{b+T}{b-T}u_0,\ u_1^{(NC\_2)})$. Otherwise, it will either have no sales or it will not be the lower-quality producer.

Note also that the inequality condition in (38) can hold only as long as the lump-sum tax is sufficiently small for the expression on the right-hand side to be smaller than the upper-bound constraint $\overline{u}_2$:

$$T \geq \frac{\overline{u}_2 - u_0}{\overline{u}_2 + u_0} \tag{39}$$

For simplicity we will denote the right-hand side of (39) by $\overline{T}^{(NC\_2)}$ that stands for upper bound of the lump-sum tax above which firm 2 would not have positive demand because its customers cannot afford to pay such a high tax.

After substituting for the optimal demands from (36) and for the optimal prices from (34) in the expressions for the firm's revenues, the optimal expressions of the latter are as follows:

$$\begin{aligned} R_1^{(NC\_2)} &= \frac{\left(2\left(u_1^{(NC\_2)}-u_0\right)b-\left(2u_1^{(NC\_2)}-u_0\right)T\right)^2\left(u_1^{(NC\_2)}-u_2^{(NC\_2)}\right)}{\left(4u_1^{(NC\_2)}-u_2^{(NC\_2)}-3u_0\right)^2\ u_1^{(NC\_2)}} \\ R_2^{(NC\_2)} &= \frac{\left(\left(u_2^{(NC\_2)}-u_0\right)b-\left(u_2^{(NC\_2)}+u_0\right)T\right)^2\left(u_1^{(NC\_2)}-u_0\right)\left(u_1^{(NC\_2)}-u_2^{(NC\_2)}\right)}{\left(4u_1^{(NC\_2)}-u_2^{(NC\_2)}-3u_0\right)^2\left(u_2^{(NC\_2)}-u_0\right)u_2^{(NC\_2)}} \end{aligned} \tag{40}$$

where:

$R_1^{(NC\_2)}$ – profit/revenue of firm 1 in the case of a non-covered duopoly equilibrium

$R_2^{(NC\_2)}$ – profit/revenue of firm 2 in the case of a non-covered duopoly equilibrium

As we will show below, only firm 1 has a corner solution for its optimal quality $u_1^{(NC\_2)}$, while firm 2 has an internal solution for its optimal quality $u_2^{(NC\_2)}$. This is a key difference that distinguishes the solution for qualities in case of a non-covered duopoly from the maximal differentiation solution in case of a covered duopoly.

As a first step to show that the non-covered solution in qualities is different, we compare the results for the optimal market shares in (36) with the corresponding results for the optimal prices in (34) to note that the latter could be considered to be transformations of the former:

$$\begin{aligned} p_1^{(NC\_2)} =& D_1^{(NC\_2)} \frac{(u_1^{(NC\_2)} - u_2^{(NC\_2)})}{u_1^{(NC\_2)}} \\ p_2^{(NC\_2)} =& D_2^{(NC\_2)} \frac{(u_2^{(NC\_2)} - u_0)}{(u_1^{(NC\_2)} - u_0)} \frac{(u_1^{(NC\_2)} - u_2^{(NC\_2)})}{(u_2^{(NC\_2)})} \end{aligned} \tag{41}$$

The latter observation should not come as a surprise, given that by construction the demand functions of the two goods in (14) and (17) are linear in their own prices.

Note also that for at least a consumer to remain solvent, the lump-sum tax should not exceed the highest consumer income, i.e. $b > T$. Hence, when (38) holds, the expression of the market share of the high-quality producer in (36) must be strictly decreasing in its quality choice u_1:

$$\frac{\partial D_1^{(NC\_2)}}{\partial u_1} = -\frac{2\left[(b-T)\,u_2^{(NC\_2)} - (b+T)\,u_0\right]}{\left(4u_1 - u_2^{(NC\_2)} - 3u_0\right)^2} < 0 \tag{42}$$

The additional operator that transforms the market share expression of the high-quality producer into an expression of its optimal price is still strictly increasing in its quality choice u_1:

$$\frac{\partial \frac{(u_1-u_2)}{u_1}}{\partial u_1} = \frac{u_2}{(u_1)^2} > 0 \tag{43}$$

Hence, it is not straightforward to conclude whether the optimal price is increasing or decreasing in the quality of firm 1. To identify the sign of the correlation between the two, we analyze the first order condition of the expression of the optimal price in (34) with respect to u_1:

$$\frac{\partial p_1^{(NC\_2)}}{\partial u_1} = \frac{4(b+T)u_0u_1^2 - \left(3(b-T)\left(u_2^{(NC\_2)}\right)^2 + 2(b+T)u_0u_2^{(NC\_2)} + 3(b+T)u_0^2\right)u_1 + 2(2b-T)u_0\left(u_2^{(NC\_2)}\right)^2}{\left(4u_1 - u_2^{(NC\_2)} - 3u_0\right)^2\left(u_2^{(NC\_2)}\right)^2} \tag{44}$$

The quadratic function of u_1^* in the numerator of (44) has no real roots:

$$u_1^{(1)/(2)} = \frac{4(2b-T)u_2u_0 \pm \sqrt{-6(2b-T)u_2^{(NC\_2)}u_0\left(u_2^{(NC\_2)}-u_0\right)\left((b-T)u_2^{(NC\_2)}-(b+T)u_0\right)}}{2\left(3(b-T)u_2^{(NC\_2)}+(b+T)u_0\right)} \tag{45}$$

because its discriminant is strictly negative when (38) holds.

Note also that the coefficient before the squared argument in the numerator of (44) is strictly positive, which implies that the squared function in the numerator has an absolute minimum. Furthermore, the minimum value must be positive, because otherwise the function would have had real roots. Since the minimum value of the numerator is positive, the whole expression in (44) must also be strictly positive. So, we can conclude that the price of the high-quality good is strictly increasing in its quality:

$$\frac{\partial p_1^{(NC\_2)}}{\partial u_1} > 0 \tag{46}$$

It is trivial that the transforming operator in (41) is smaller than 1 for any feasible u_1. Therefore, the optimal price cannot exceed the value of the optimal market share.

Analogously, note that we could represent the revenue of firm 1 as a product of the squared demand and the transforming operator:

$$\begin{aligned} R_1 = D_1^{(NC\_2)}p_1^{(NC\_2)} &= \left(D_1^{(NC\_2)}\right)^2 \frac{(u_1 - u_2^{(NC\_2)})}{u_1} = \\ &= \frac{(2(b-T)u_1 - (2b-T)u_0)^2}{\left(4u_1 - u_2^{(NC\_2)} - 3u_0\right)^2} \frac{(u_1 - u_2^{(NC\_2)})}{u_1} \end{aligned} \tag{47}$$

Since the transforming operator is smaller than 1, the revenue must approach the squared market share from below as u_1 tends to infinity. As we have already shown in (42) the demand for good 1 is positive and strictly decreasing in u_1 when (38) holds. Therefore, after being squared, it should still be strictly decreasing, approaching a certain positive limit as u_1 tends to infinity. Accordingly, the revenue of firm 1 must be approaching the same limit but from below:

$$\lim_{u_1 \to \infty} R_1^{(NC\_2)} = \frac{(b-T)^2}{4} = \lim_{u_1 \to \infty} \left(D_1^{(NC\_2)}\right)^2 \tag{48}$$

In other words, the revenue of firm 1 must be strictly increasing in u_1. So, as in the solution for covered duopoly, the optimal quality choice of firm 1 in a non-covered duopoly setting is again given by the upper bound on its quality:

$$u_1^{(NC\_2)} = \bar{u}_1 \tag{49}$$

Now we move to analysis of the optimal quality choice of firm 2 in case of non-covered market duopoly, which leads to the key implication of the current paper. Specifically, firm 2 has an incentive to increase the quality of its good when the market is not covered, so that the outside option reveals itself as an external inferior competitor. To demonstrate this, we substitute for the optimal quality choice of firm 1 in the expression for the optimal market share of firm 2 in (36) and then take first derivative of it with respect to u_2:

$$\frac{\partial D_2^{(NC\_2)}}{\partial u_2} = \frac{\left((b-T)u_2^2 - 2(b+T)u_0u_2 + \left(8T\bar{u}_1^{(NC\_2)} - 5Tu_0 + bu_0\right)u_0\right)\left(\bar{u}_1^{(NC\_2)} - u_0\right)}{\left(4\bar{u}_1^{(NC\_2)} - u_2 - 3u_0\right)^2 (u_2 - u_0)^2} \tag{50}$$

The quadratic function of u_2 in the numerator of (50) has no real roots:

$$u_2^{(1)/(2)} = \frac{(b+T)u_0 \pm 2\sqrt{-\left[2(b-T)\bar{u}_1^{(NC\_2)} - (2b-T)u_0\right]\,Tu_0}}{b-T} \tag{51}$$

because when the condition in (38) holds its discriminant is negative.

Note also that the coefficient before the squared argument in the numerator of (50) is strictly positive, which implies that the squared function in the numerator has an absolute minimum. Furthermore, the minimum value must be positive, because otherwise the function would have had real roots. Since the minimum value of the numerator is positive, the whole expression in (50) must also be strictly positive. So, we can conclude that the market share of the low-quality good is strictly increasing in its quality:

$$\frac{\partial D_2^*}{\partial u_2} > 0 \tag{52}$$

Since the revenue of firm 2 is given by the product of its market share and its price, when both are increasing in the quality u_2 the whole revenue must be increasing in it as well. Note however that the price of good 2 in (34) cannot be positively correlated with its quality for any value in the interval defined by (38). At the endpoints of the interval, the price of good 2 would be zero, while in-between it would be strictly positive. Hence, up to some critical value of u_2 in the interval defined by (38), the price must be increasing, but then it should start decreasing. To show this explicitly, we analyze the first derivative of the expression for the price of good 2 in (34) with respect to u_2:

$$\frac{\partial p_2^*}{\partial u_2} = \frac{(2(2b-T)u_0-3(b-T)\bar{u}_1)u_2^2-2(b+T)\bar{u}_1u_0u_2+(b+T)(4\bar{u}_1-3u_0)\bar{u}_1u_0}{(4\bar{u}_1-u_2-3u_0)^2u_2^2} \tag{53}$$

The sign of the coefficient in front of the squared term in the numerator of (53) depends on the relationship between the maximum income b and the size of the lump-sum tax T as well as on the qualities of good 1 and the outside option. It is negative when the lump-sum tax is large enough relative to the maximum income b:

$$b \leq 5T \Rightarrow \bar{u}_1 > \frac{b+T}{b-T}u_0 > \frac{2}{3}\frac{2b-T}{b-T}u_0 \tag{54}$$

or alternatively if the upper bound on the quality choice of firm 1 is sufficiently larger than u_0

$$b > 5T \quad \wedge \quad \bar{u}_1 > \frac{2}{3}\frac{2b-T}{b-T}u_0 > \frac{b+T}{b-T}u_0 \tag{55}$$

So, when (54) and (55) hold, the quadratic function in the numerator of (53) is concave with roots:

$$\begin{aligned} u_2^{(1)} &= \frac{-2(b+T)\bar{u}_1u_0+\sqrt{4(b+T)^2\bar{u}_1^2u_0^2+4(b+T)(4\bar{u}_1-3u_0)(3(b-T)\bar{u}_1-2(2b-T)u_0)\bar{u}_1u_0}}{2(3(b-T)\bar{u}_1-2(2b-T)u_0)} > 0 \\ u_2^{(2)} &= \frac{-2(b+T)\bar{u}_1u_0-\sqrt{4(b+T)^2\bar{u}_1^2u_0^2+4(b+T)(4\bar{u}_1-3u_0)(3(b-T)\bar{u}_1-2(2b-T)u_0)\bar{u}_1u_0}}{2(3(b-T)\bar{u}_1-2(2b-T)u_0)} < 0 \end{aligned} \tag{56}$$

The second root is strictly negative. Therefore, for $u_0 < u_2 < u_2^{(1)}$, the price of good 2 is increasing in its quality while for $u_2^{(1)} < u_2 < u_1$, it is decreasing.

Alternatively, when neither (54) nor (55) hold:

$$b > 5T \quad \wedge \quad \frac{2}{3}\frac{2b-T}{b-T}u_0 > \bar{u}_1 > \frac{b+T}{b-T}u_0 \tag{57}$$

the quadratic function in the numerator of (53) is convex and both its roots in (57) are positive.

However, the larger root $u_2^{(2)}$ exceeds the quality of good 1 as long as (57) holds:

$$\bar{u}_1 - u_2^{(2)} = \frac{-3(b-T)(\bar{u}_1+u_0)\bar{u}_1-\sqrt{6}\sqrt{(b+T)(\bar{u}_1-u_0)(2(b-T)\bar{u}_1-(2b-T)u_0)\bar{u}_1u_0}}{2(2b-T)u_0-3(b-T)\bar{u}_1} < 0 \tag{58}$$

so that again for $u_0 < u_2 < u_2^{(1)}$, the price of good 2 is increasing in its quality, while for

$u_2^{(1)} < u_2 < u_1$, it is decreasing.

Hence, $u_2^{(1)}$ is the critical value up to which the price of good 2, respectively the revenue of firm 2, is increasing in its quality within the interval defined by (38). At the upper bound of the interval, however, the optimal revenue of firm 2 is zero because as discussed above, its optimal price is zero. This implies that there must be a critical value in the interval between $u_2^{(1)}$ and $\overline{u}_1$ beyond which, within the expression of firm 2's revenue, the positive effect of the quality of good 2 on the market share of firm 2 will be dominated by its negative effect on the price of good 2. Accordingly, if exceeded by the upper bound on the quality choice of firm 2 [12], this critical value which we denote by u_2^* will in fact be the revenue-maximizing quality choice of firm 2:

$$u_2^{(NC\_2)} = u_2^* \{= \operatorname*{argmax}_{u_2} R_2 < \overline{u}_2\} \tag{59}$$

After substituting for the optimal quality choices of firm 1 and firm 2 in (35), the condition on the size of the lump-sum tax T for the existence of a non-covered duopoly equilibrium takes the following final form:

$$T > \frac{[(4\overline{u}_1 - u_2^* - 3u_0)\,a - (\overline{u}_1 - u_2^*)\,b]\,(u_2^* - u_0)}{3\overline{u}_1 u_2^* - \left(\overline{u}_1 + 2u_2^*\right) u_0} \tag{60}$$

Keep in mind that u_2^* is not a parameter but a variable which represents the optimal solution for the quality of good 2 as a function of the lump-sum tax T. However, since the price of good 2 decreases as the quality difference from the leader is shrinking, i.e., it is negatively related to the quality of good 2 at $u_2^*(> u_2^{(1)})$, the right-hand side of the

[12] If u_2^* is larger than the upper bound on the quality choice of firm 2, the latter will be the optimal solution, which still implies a shift from the maximal differentiation outcome in the case of covered-market duopoly equilibrium. So, we could consider it a special case of the internal solution. Accordingly, for the sake of brevity, we discuss the internal solution only.

expression in (60) must be strictly increasing in u_2^*. So, the critical value of the tax above which there exists a non-covered duopoly equilibrium has a fixed upper bound:

$$\underline{T}^{(NC\_2)} = \frac{\left[(4\overline{u}_1 - \overline{u}_2 - 3u_0)\,a - (\overline{u}_1 - \overline{u}_2)\,b\right](\overline{u}_2 - u_0)}{3\overline{u}_1\overline{u}_2 - (\overline{u}_1 + 2\overline{u}_2)\,u_0} \tag{61}$$

where by $\underline{T}^{(NC\_2)}$ we denote the fixed lower bound on the lump-size for having a non-covered duopoly equilibrium. The right-hand side of (61) is derived from (60) where u_2^* is substituted by $\overline{u}_2$.

It is straightforward to show that the lower-bound constraint on the lump-sum tax whose maximum value is represented by $\underline{T}^{(NC\_2)}$ is strictly smaller than its upper-bound constraint $\overline{T}^{(NC\_2)}$:

$$\overline{T}^{(NC\_2)} - \underline{T}^{(NC\_2)} = \frac{(\overline{u}_2 - u_0)(4\overline{u}_1 - \overline{u}_2 - 3u_0)((b-a)\overline{u}_2 - au_0)}{(\overline{u}_2 + u_0)(3\overline{u}_1\overline{u}_2 - (\overline{u}_1 + 2\overline{u}_2)\,u_0)} > 0 \tag{62}$$

provided that the exogenous constraint on the quality of good 2 is sufficiently larger than the quality of the outside option:

$$\overline{u}_2 > \frac{a}{b-a}u_0 \tag{63}$$

It is also trivial that the lower-bound condition on the lump-sum tax for having a non-covered duopoly solution as given by the right-hand side of (60) does strictly exceed the upper-bound constraint $T^{(C\_2)}$:

$$\underline{T}^{(NC\_2)} - \overline{T}^{(C\_2)} > 0 \tag{64}$$

because the general functional form of the right-hand sides of (15) and (16) is strictly increasing in u_2 while u_2^* is larger than $\underline{u}_2$ by definition.

Hence, (39) and (60) define a range of feasible values of T for which a unique non-covered duopoly equilibrium would exist, provided that the inequality conditions in (2), (27), (59) and (63) hold. The result is established in proposition 2 below.

Proposition 2. Let the consumer income distribution satisfy (27), the restrictions from above on the quality choices in (2) as well as the restrictions from below on the upper quality constraint of the low-quality firm 2 in (59) and (63) hold. Then, given a lump-sum tax sufficiently large for (60) to hold but still not so high as to exceed the upper bound in (39), a unique subgame perfect equilibrium exists at which the entrants with quality ranks 1 and 2 enter the market, make the quality choices in (49) and (59) respectively, and set the corresponding prices in (34). The firms' optimal market shares and profits are given by the expressions in (36) and (40). The market is non-covered.

The intuition behind proposition 2 is illustrated in figure 3 below. Note that the price at the kink $\hat{p}_2$ exceeds the unit-elasticity point only so long as T is too small to satisfy the condition in (32). Otherwise, for larger T's the middle segment shifts parallelly to the left while the bottom segment shifts parallelly to the right so that the kink between the two segments is moved below the optimal unit-elastic point. That is, at the equilibrium prices the lowest-income consumer would prefer the free outside option to paying the high tax implied by the inequality in (60). However, the upper-bound constraint in (39) ensures that the optimal price of firm 1 still belongs to the top segment of its residual demand curve, so that firm 2 faces positive demand. Therefore, the corresponding equilibrium solution implies a non-covered duopoly outcome.

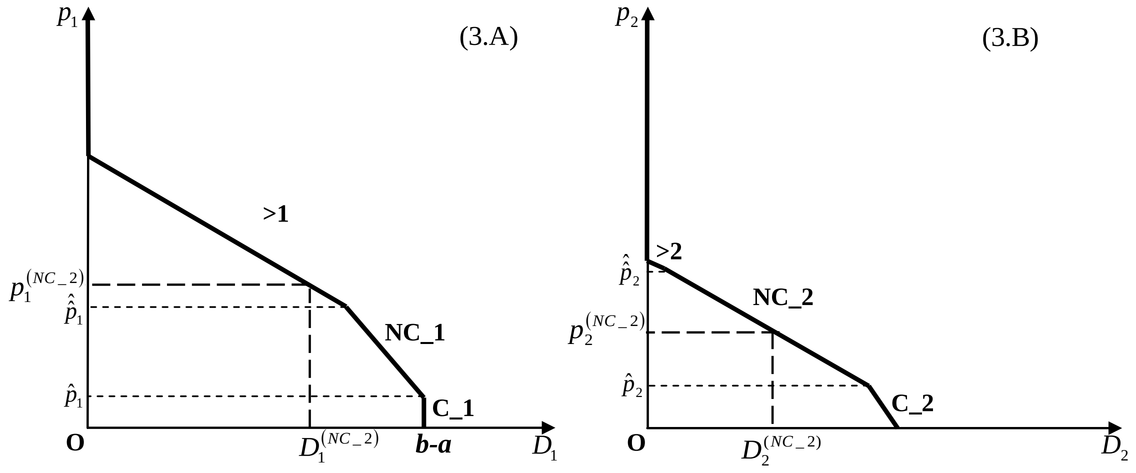


Figure 3: Solution for equilibrium with non-covered duopoly outcome

Panel (3.A) represents the residual demand curve of a good with quality rank 1 as given in (17) at a middle lump-sum payment that satisfies (60) and (39). The optimal solution occurs at the unit-elastic point with coordinates $(D_1^{(NC\_2)}, p_1^{(NC\_2)})$ which belongs to the segment (>1) above the price $\hat{\hat{p}}_1$ even though the larger lump-sum tax as defined by (60) relative to (33) reduces its length in favor of the middle segment. Panel (3.B) illustrates the demand for the good with quality rank 2 as given in (14) for $k = 2$ at the middle lump-sum payment that satisfies (60) and (39). The optimal solution occurs at the unit-elastic point with coordinates $(D_2^{(NC\_2)}, p_2^{(NC\_2)})$ which belongs to the middle segment (NC_2) above the price $\hat{p}_2$ at which the market would otherwise be exactly covered by the second-best quality good. Therefore, the market is not covered at the depicted equilibrium.

Before closing the section, note that (60) is never consistent with (22) if $T = 0$. Therefore, in Shaked and Sutton (1982), the inequalities in (27) are not only necessary but also sufficient conditions for having a covered duopoly outcome, which is the unique equilibrium solution in the standard case of vertical product differentiation without exogenous expense. Accordingly, figure 4 below illustrates how the imposition of a lump-sum payment, large enough to satisfy condition (60), makes the existence of a non-covered market equilibrium possible. For the purpose, we use the original graphical representation from Shaked and Sutton (1982), where firms' best price responses are expressed implicitly through their impact on the relation between the marginal taste parameters $t_{1/2}$ and $t_{2/0}$:

$$t^H_{1/2} = \frac{1}{2}[b - t_{2/0}(\bar{V} - 1) + \bar{C}_1 T] \tag{65}$$

$$t^L_{1/2} = \begin{cases} a + t_{2/0}(\underline{V} - 1) - (\underline{C}_1 - 1)T, \text{ if } t_{2/0} \leq a \\ t_{2/0}(\bar{V} + 1) - (\bar{C}_1 - 1)T, \text{ if } t_{2/0} > a \end{cases} \tag{66}$$

where:

$t^H_{1/2}$ – implicit best-response function of the high-quality producer, i.e. firm 1

$t^L_{1/2}$ – implicit best-response function of the low-quality producer, i.e. firm 2

For expositional simplicity and easier comparability to Shaked and Sutton's (1982) results, we borrow the following supplementary notations, which here are especially adjusted to reflect the internal solution for the optimal quality choice of firm 2 at the non-covered market equilibrium:

$$\begin{aligned} &\underline{C}_1 = \frac{\bar{u}_1}{\bar{u}_1 - \underline{u}_2}; \bar{C}_1 = \frac{\bar{u}_1}{\bar{u}_1 - u^*_2} > \underline{C}_1 \\ &\underline{C}_2 = \frac{\underline{u}_2}{\underline{u}_2 - u_0}; \bar{C}_2 = \frac{u^*_2}{u^*_2 - u_0} < \underline{C}_2 \\ &\underline{V} = \frac{\underline{C}_1 - 1}{\underline{C}_2} = \frac{\bar{u}_1 - u_0}{\bar{u}_1 - \underline{u}_2}; \bar{V} = \frac{\bar{C}_1 - 1}{\bar{C}_2} = \frac{\bar{u}_1 - u_0}{\bar{u}_1 - u^*_2} > \underline{V} \end{aligned} \tag{67}$$

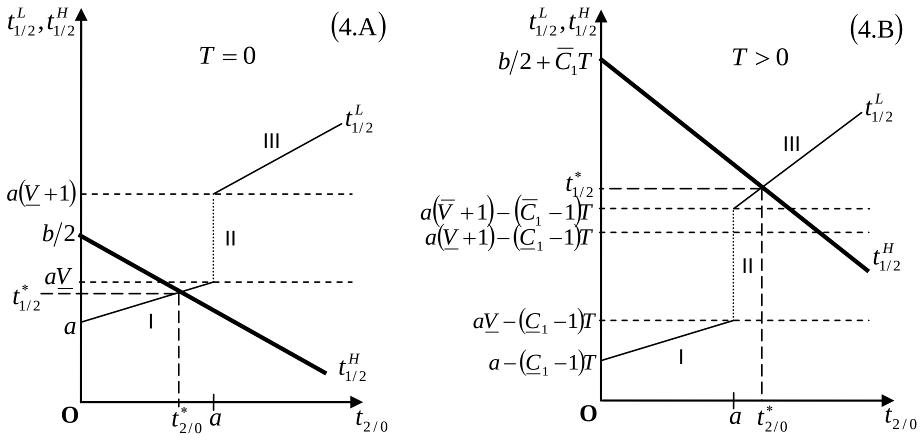


Figure 4: Implicit solutions for covered market equilibrium at $T = 0$ and for non-covered duopoly equilibrium at $T > 0$ when conditions (35) and (39) hold

The equilibrium marginal incomes, $t^*_{2/0}$ and $t^*_{1/2}$, which demarcate the market shares of firm 1 and firm 2 are given by the coordinates of the point of intersection of their implicit best-response curves, $t^H_{1/2}$ and $t^L_{1/2}$, respectively. Panel (4.A) represents the solution without tax suggested by Shaked and Sutton (1982) . When the condition in (22) holds as a given that $T = 0$, the vertical intercept of the downward-sloping linear curve $t^H_{1/2}$ cannot be larger than twice the value of a. At the same time, $\underline{V}$ is trivially larger than 1 so that the lowest point of segment III exceeds more than twice a. Therefore, the two best-response curves cannot intersect at a point that belongs to segment III i.e. there cannot be a non-covered market at equilibrium when $T = 0$. Panel (4.B) represents the solution for positive tax that satisfies conditions (35) and (39). The vertical intercept of $t^H_{1/2}$ increases in T, while the lowest point of segment III shifts down as T increases, which makes it possible for the two best-response curves to intersect at a point that belongs to segment III; i.e. there is a non-covered market equilibrium. [13]

1.3.3 Non-Covered Monopoly Equilibrium With Large Lump-sum Tax

This final subsection represents the equilibrium solution at an amount of the lump sum T exceeding the upper-bound constraint $T^{(NC\_2)}$ in (39):

$$T \geq \frac{2(\bar{u}_2 - u_0)}{2\bar{u}_2 - u_0} a \tag{68}$$

[13] Note also that as the lump sum of the tax T increases, the best-response curve of firm 2 shifts down, moving the equilibrium solution below the 45-degree bisector where $t^*_{2/0}$ would exceed $t^*_{1/2}$ so that condition (38) would be violated and firm 2 could not have a positive market share. This gives firm 2 an incentive to raise the quality of its good, that is, to decrease the quality differentiation, which shifts up segment III of firm 2's best-response curve and makes it and the best-response curve of firm 1 steeper.

provided that the condition in (22) for having at most two entrants with positive market shares holds.

Since firm 2 cannot make positive sales, it has no incentive to enter the market. So, only firm 1 enters at the first stage and acts as a monopolist. The corresponding subgame-perfect equilibrium is defined by the following price-quality pair:

$$\begin{aligned} p_1^{(NC\_1)} &= \frac{(u_1^{(NC\_1} - u_0)b - u_1^{(NC\_1)}T}{2u_1^{(NC\_1)}} \\ u_1^{(NC\_1)} &= \bar{u}_1 \end{aligned} \tag{69}$$

where:

$p_1^{(NC\_1)}$ – the optimal price of firm 1 given non-covered monopoly outcome

$u_1^{(NC\_1)}$ – the optimal quality choice of firm 1 given non-covered monopoly outcome

The optimal expressions for the market share and the revenue of firm 1 are:

$$\begin{aligned} D_1^{(NC\_1)} &= \frac{\left[(\bar{u}_1 - u_0)b - \bar{u}_1\right]T}{2(\bar{u}_1 - u_0)} \\ R_1^{(NC\_1)} &= \frac{\left[(\bar{u}_1 - u_0)b - \bar{u}_1\right]^2 T^2}{4\bar{u}_1(\bar{u}_1 - u_0)} \end{aligned} \tag{70}$$

where:

$D_1^{(NC\_1)}$ – the optimal market share of firm 1 given non-covered monopoly outcome

$R_1^{(NC\_1)}$ – the optimal revenue of firm 1 given non-covered monopoly outcome

A non-covered equilibrium would exist so long as b satisfies the following condition:

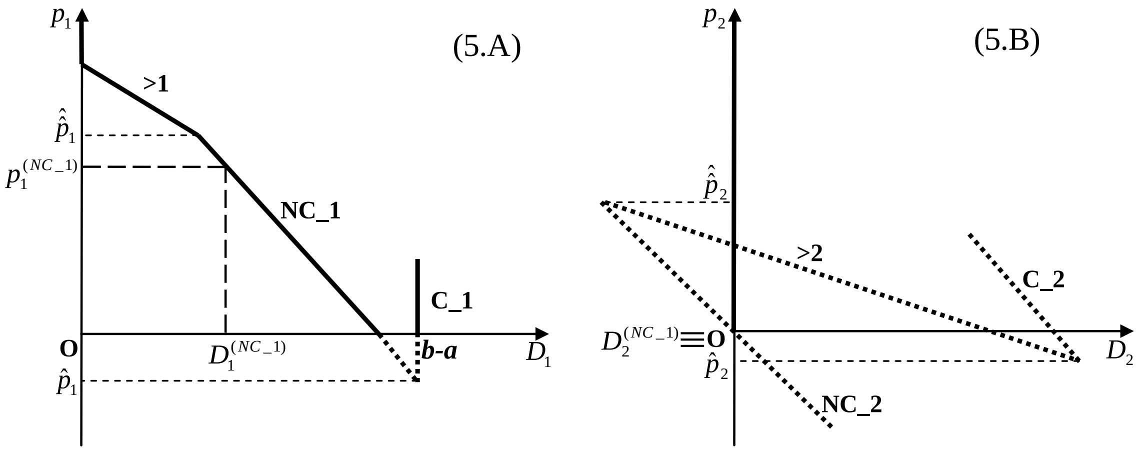
$$b > 2a - \frac{u_1}{u_1 - u_0}T\{< 2a - T\} \tag{71}$$

which is trivially satisfied when the left-hand side inequality condition in (27) holds.

The subgame equilibrium is established in proposition 3 below.

Proposition 3. Given that the consumer income distribution satisfies (27) the restrictions from above on the quality choices in (2) as well as the restriction from below on the lump-sum tax in(68) hold. Then, a unique subgame perfect equilibrium exists at which only the entrant with quality rank 1 enters the market, makes the quality choice and sets the price in (69). Firm 1's market share and profit are given by the respective expressions in (70). The market is non-covered.

The intuition behind proposition 3 is illustrated in figure 5 below. The middle segment of the residual demand curve of firm 1 experiences a parallel shift further to the left when (68) holds. Therefore, the optimal price of firm 1 belongs to the middle segment where consumers buy either the best quality or the free outside option.



<u>Figure 5:</u> Solution for equilibrium with non-covered monopoly outcome

Panel (5.A) represents the residual demand curve of a good with quality rank 1 as given in (17) at a high tax that exceeds $\overline{T}^{(NC\_2)}$ as implied by the condition in (68). The optimal solution is given by the unit-elastic point with coordinates $(D_1^{(NC\_1)}, p_1^{(NC\_1)})$. It belongs to the middle segment (NC_1) that is shifted to the left and down so that it has no positive border price $\hat{p}_1$ with the bottom segment. As a result, the market is non-covered and no other firm with lower quality is accommodated in the market. Correspondingly, in Panel (5.B) the optimal solution of a potential second entrant coincides with the origin of the coordination system – it could not make positive sales even if its good was priced at zero and its quality was set at its upper-bound constraint $\overline{u}_2$.

1.3.4 Summary of the Effects of the Lump-sum Tax Size

Based on the equilibria defined in the three propositions of the current section we could make a general conjecture about the effect of the size of the lump-sum tax on the structure and the coveredness of the market when the distribution of consumer incomes is determined by the condition in (27). The propositions identify three ranges of the lump-sum tax T, for which we have three unique pure-strategy equilibria that imply distinct market outcomes characterized by the number of firms in the market and its coveredness. These ranges are illustrated in figure 6 below.

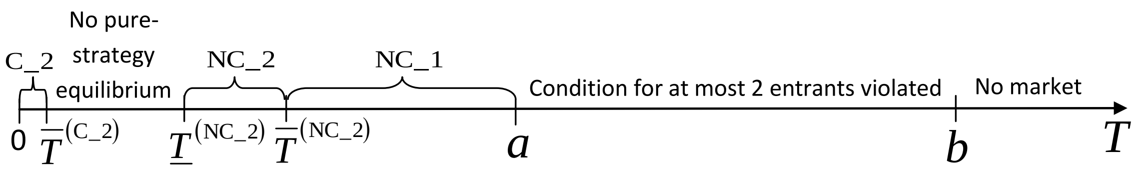


Figure 6: Effect of the size of the lump-sum tax T on the structure and the coveredness of the market

For a small lump-sum tax not exceeding $\overline{T}^{(C\_2)}$, we have exactly two firms with positive market shares and covered market (denoted by C_2). For a moderate tax size within the range between $\underline{T}^{(NC\_2)}$ and $\overline{T}^{(NC\_2)}$, again exactly two firms enter the market but they do not cover the whole market, so some of the low-income consumers prefer the outside option (denoted by NC_2). Accordingly, for very large tax size at least equal to $\overline{T}^{(NC\_2)}$ but not exceeding the lowest consumer income a, there is only one entrant and the market is not covered at equilibrium (denoted by C_1).

There are also two intervals of the values of T for which we cannot provide an explicit solution based on the analysis so far. For values of T within the closed interval between $\overline{T}^{(C\_2)}$ and $\underline{T}^{(NC\_2)}$ an equilibrium in pure strategies does not exist. If firms play their pure quality-price strategies expecting a covered duopoly, the outcome will be a non-covered market, and vice versa when they expect a non-covered market, the market will be a covered duopoly. Therefore, instead, an equilibrium in mixed strategies must exist where firm 2 chooses better quality than in case of maximal differentiation but still lower than at the pure-strategy non-covered duopoly equilibrium established in proposition 2. Accordingly, the prices of the two firms will be higher but still below the optimal price at the pure-strategy covered duopoly equilibrium established in proposition 1. Then, there must be a critical value of T within the interval between $\overline{T}^{(C\_2)}$ and $\underline{T}^{(NC\_2)}$ below which the outcome of the mixed-strategy equilibrium will be a covered duopoly, while above it the market will be a non-covered duopoly. Finally, in the interval of T between the two bounds of the consumer income distribution, a and b, the condition in (22) for having at most two entrants will be violated. However, this does not imply that there will be more entrants into the market because the latter cannot be covered when the lump-sum tax exceeds the lowest consumer income. Hence, entrants should compete with the non-taxable outside option, which cannot be beaten by any other quality than the best one when the lump-sum tax is above $\overline{T}^{(NC\_2)}$. As a result, the equilibrium will again be a non-covered monopoly as defined in proposition 3. Finally, for values of T above the highest consumer income b, even the best quality will be unsalable so that the market will not exist.

1.4 Conclusion

This paper explores the effect that the amount of an exogenous expense can have on the structure and the coveredness of a market for vertically differentiated goods. The exogenous expense is modeled as a lump-sum tax imposed on the incomes of the consumers of the market good.

The results confirm the existence of the covered-duopoly market equilibrium established by Shaked and Sutton (1982) when exogenous expenses are small and tending to zero. At a greater exogenous expense, however, some low-income consumers prefer the outside option. That is, the equilibrium outcome is a non-covered duopoly. In turn, the producer of the low-quality good is prone to divert from maximal product differentiation to an internal solution for quality. However, when the amount of the exogenous expense exceeds the incomes of all the consumers who prefer the low-quality good to the high-quality good at the equilibrium prices, even the minimal product differentiation and zero pricing cannot save the positive market share of the low-quality firm. Such an exogenous expense is unaffordable for these consumers and therefore they choose the outside option instead. Accordingly, a monopoly outcome prevails at equilibrium.

The suggested solutions indicate not only the effect of the lump-sum tax on the market structure and product differentiation, but also on the market coveredness. A clear negative relationship is identified between the size of the tax and the share of consumers being served by the firms in the market.

The presented work also opens new perspectives for future research on the topic. Specifically, one of the equilibrium outcomes suggests the existence of a potential for a lump-sum tax to perform as a tool for social welfare improvement. This would be the case, if, at a medium-sized lump-sum tax, the positive effect on the average quality in the market which increases consumer surplus dominates the negative effect from the parallel exclusion of the lowest-income consumers from the market. To be able to explore further this issue, however, we need to modify the model in a way that allows for explicit derivation of the optimal quality choice of the low-quality firm at the non-covered duopoly equilibrium established in the paper.

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2 Why Mixed Qualities May Not Survive at Equilibrium: The Case of Vertical Product Differentiation

This paper explores price competition in a vertically differentiated market where a firm's product is consumed not separately but in a fixed one-to-one ratio with another complementary type of good supplied by a different producer in another vertically differentiated market.

When there is only a single entrant in the one market, a monopoly outcome is shown to prevail at equilibrium in the other market as well. The result holds for any number of potential entrants in the second market. It is independent of the distribution of the consumer tastes, provided that consumers value the monopoly good more. A further necessary condition is that the best good in the adjacent market must be of a sufficiently higher quality than any other good of its type (i.e. at least two and a quarter times better).

In a setting with more than two entrants in each of the two markets, certain conditions on the quality choice of firms and the consumer taste distribution are identified, at which a particular duopoly equilibrium exists in both markets. A key characteristic of this equilibrium is that the optimal prices are such that the combinations consisting of both high- and low-quality goods, which we call "mixed-quality combinations", will remain unsold. This equilibrium outcome represents a further form in which the well-known maximum-differentiation principle could be implemented in a multi-market setting with exogenous constraints imposed on firms' quality choices.

Keywords: complementary goods, vertical product differentiation, market exclusion
JEL classification: L11, L13, L15

2.1 Introduction

The principles that drive firms' behavior in a single vertically differentiated market are well-established in the existing industrial organization literature (Gabszewicz and Thisse (1979), Shaked and Sutton (1982), and Tirole (1988)). Little is known, however, about the way these principles hold in a broader multi-market environment. In the present paper, we make first steps in this direction by modeling two adjacent vertically differentiated markets for final goods consumed as complements.

Adjacent vertically differentiated markets are not uncommon. There are many examples of vertically differentiated goods that are complements to other goods which are also vertically differentiated. In this particular case, consumers always purchase them both in a fixed one-to-one ratio even though they are sold by different firms, in different markets and at different prices. For instance, when buying a new flat or a house from a real-estate company, consumers also often buy furniture from a home-furnishing shop or cabinet-maker. When going on a business trip, employees should be provided not only with transportation from an airline company but also with accommodation from a hotel at their destination.[1] Or, when buying a computer, a consumer actually pays for a central-processing unit (CPU) manufactured by a computer hardware firm, screen from a TV producer, an operation platform from a system software provider, and application software from a number of application software developers.

One of the most interesting features of single vertically differentiated markets is the

[1]Note that transportation and accommodation are also demanded when going on a holiday trip, but the two are mostly offered by holiday agencies in a pre-selected bundle. In our paper, however, we are interested in studying only vertically differentiated markets where consumer choice is in no way pre-selected by firms but represents an optimal consumers' decision given firms' optimal quality and price choices. For a thorough discussion of the competition and welfare effects of the existing practices of firms to restrict free consumer choice by providing them with pre-selected bundles of goods in the form of take-it-or-leave it offers, see Whinston (1990), Carlton and Waldman (2002), and Nalebuff (2003a, 2003b).

so-called maximum-differentiation principle that drives firms' optimal quality choice. Mussa and Rosen (1978) first show that an uninformed multi-product monopolist could benefit from broadening the range of the quality spectrum of its product line, which would allow it to imperfectly price discriminate between consumers who have different tastes for quality and therefore heterogeneous willingness to pay for it. Since the consumers with a low willingness to pay are inclined to compromise on quality, they can be easily distinguished from the consumers with a high willingness to pay by strategic deterioration of the lower edge in the quality spectrum offered by the monopolist. Gabszewicz and Thisse (1979) apply the same principle to a duopoly market with single-product firms where consumers differ by income. They show that duopolists could gain from differentiating the quality of their products, which leads to segmentation of the market and raises the market power of each duopolist in its respective segment. Maskin and Riley (1984) extend the model of Mussa and Rosen (1978) for the case of multiple-product monopolist facing non-unit consumption. Donnenfeld and White (1988) introduce a discrete heterogeneity of consumers to show that a multi-product monopolist would prefer to broaden the quality spectrum of its product line upwards when the relationship between absolute and marginal willingness to pay for quality is allowed to be negative. Champshur and Rochet (1989) consider the case of a multi-product duopoly and prove that even though duopolists still prefer to broaden their quality spectrums there will always be a gap between their product lines at equilibrium. The competition relaxing effect of vertical product differentiation is further explored by Moorthy (1988), Choi and Shin (1992) and Wauthy (1996).

In this paper, we propose a model of two adjacent vertically differentiated markets to explore how the maximum-differentiation principle might drive outcomes in this new setting. We assume complementary goods are consumed in a fixed one-to-one proportion and are supplied by single-product firms, while consumers are not restricted in their choice of how to combine the available goods from the two markets. Particularly, the solution of the model implies that a duopoly equilibrium exists at which firms choose not only to maximize the differentiation between their products within each market but also to charge prices at which the quality/price ratio of the less-differentiated combinations is not optimal for any of the consumers. That is, only the most differentiated combinations that consist of the two top-quality and the two bottom-quality goods will be sold, while the remaining combinations which we call "mixed-quality combinations" will remain unsold.

In some vertically differentiated markets for complementary goods it is not uncommon to observe self-selection bias in consumers' choices so that only the combinations of goods with similarly ranked qualities and prices are purchased by consumers while the mixed-quality combinations are ignored. For instance, it would be very unusual to see consumers who buy an expensive luxurious house and furnish it with cheap do-it-yourself furniture. Or vice versa; normally consumers who buy small cheap flats do not equip them with high-design expensive furniture. Likewise, businessmen who order business-jet charter flights naturally choose to be accommodated in five-star hotels, which is not the case of small-firm entrepreneurs or lower-level managers whose business-trip budget only allows them to travel in the business or economy class on public airline flights. [2]

One straightforward explanation for the observed self-selection choices of consumers could be that the valuations for the both types of goods are identically distributed among consumers so that the buyers of the high-quality good in one of the markets at equilibrium will also choose the high-quality good in the other market, while the rest purchase low-quality goods in both markets. Therefore, in the end, no-one finishes with a

[2] We are grateful to Michael Kunin for suggesting both the house-furniture and the flight-hotel examples.

mixed-quality combination. However, this explanation is implied by the assumption that consumers choose each of the complementary goods in its single market (Kováč 2007).

Given that two goods are complements, it is perhaps too strong an assumption to make that consumers purchase them in single markets. This is our argument for why, in the present paper, we follow a different approach by assuming instead that consumers do not choose from the goods available in each market but from the one-to-one combinations that can be formed out of the goods in both markets. In the setting with two duopoly markets, there are four possible combinations of the two goods in each of the markets. The best combination is apparently given by the pair of the higher-quality goods, while the worst combination involves the lower-quality goods only. The remaining two mixed-quality combinations are ranked second and third respectively, based on how substitutable the qualities in one market are by the qualities in the other market, while determining the qualities of the combinations in which they take part. To derive a determinable solution, we also introduce an individual upper bound on the quality choice of each potential entrant as well as a general lower bound on the quality choice of all firms in each market. Intuitively, the upper constraint on the quality choice of the firms could be considered a contemporary maximal technological frontier which varies across firms, whereas the lower restriction may represent the minimal social (safety or hygienic) requirements for a good to be acceptable for sale in a given market.

In the two-market setting that we suggest, each good participates in two of the possible combinations and therefore by choosing the price of its good a firm is also affecting the demand for the two possible combinations in which it takes part. If the combination of the two high-quality goods and the combination of the two low-quality goods are the only two combinations that have positive demand at equilibrium, this should not be only because maximum differentiation quality choices were made by the entrants in the both markets. In addition, it must be the case that the high-quality producers find it suboptimal to set prices low enough for their goods to be sold also in mixed-quality combinations with their low-quality counterparts from the other market.

This new approach that we apply further allows us to identify how the characteristics of the well-known single-market duopoly equilibrium changes in the context of a two-market setting. The existing literature uses three key properties to characterize the equilibrium duopoly outcome in a single vertically differentiated market. We take these properties as a benchmark when analyzing the optimal solution of the two-market setting. Accordingly, before presenting our results, we will discuss below the single-market properties as initially described by Shaked and Sutton (1982, 1983).

First, Shaked and Sutton (1982) explore a simple setting where firms have zero unit production cost and consumer incomes follow a continuous uniform distribution, to show the positive correlation between the market segmentation and the dispersion in consumers' incomes. This property follows straight from the definition of vertical product differentiation, which states that, independent on how consumers differ by income, they all unanimously rank the qualities of the available goods in the market. That is to say, no matter how low is the income of a consumer, s/he will always value and be willing to pay more for the top-ranked good than for the second-best ranked good. Similarly, her (or his) valuation will be higher for the second-best ranked good than for the third-best ranked good, and so on down the mutually agreed quality ranking of goods. The firm offering the top quality, therefore, can efficiently drive all competitors out of the market. For this purpose, it is sufficient to charge a price for its good that exceeds the valuation of the lowest-income consumer for the second-best ranked good but is still below her (or his) reservation price for the top-quality good. Whether the top-quality firm will, however, find it optimal to charge such an exclusionary price would depend on how low the lowest consumer income is relative to the highest consumer income. The lower the ratio between the two the more likely it would be for the market size to exceed the profit-maximizing scale of the top-quality firm. Then, there will be more space for further segmentation through accommodating the market entry of other lower-quality firms. Shaked and Sutton (1982) prove that if, and only if, the lowest consumer income is more than two times but at most four times smaller than the largest consumer income, exactly two firms can have positive market shares at equilibrium.

Second, Shaked and Sutton (1982) demonstrate that the same condition that ensures at most two market entrants is also sufficient for the market to be covered; that is, each consumer will buy one or the other of the two available goods at the single duopoly equilibrium. Moreover, the result does not depend on quality differentiation in the market. Quality difference positively affects only the prices of the two entrants but not their market shares.

Third, Shaked and Sutton (1983) derive a general condition for having the so-called finiteness property of the vertically differentiated markets, even in the case of positive unit production costs. As opposed to the classical (monopolistic competition) outcome of the representative-consumer model of market differentiation described by Chamberlin (1933), Shaked and Sutton (1983) state that when the entry barriers tend to zero in a vertically differentiated market, the number of firms with positive market share remains finite. For the finite upper bound on the number of entrants at equilibrium to exist, the difference in the price markups of any two firms whose goods have neighboring quality ranks should be larger than the difference in their unit production costs. Analogously to the case without production cost, the result again follows straight from the definitional assumption that there is unanimous consensus among consumers regarding how the available goods should be ranked by quality. Accordingly, since the finite number of entrants is endogenously determined, Shaked and Sutton (1983) call the corresponding market structure a “natural oligopoly”.

In our paper, we start by analyzing the solution for the case when the number of entrants in one of the markets is exogenously restricted to one. For expositional convenience we identify the consumers by taste as in the classical model of Mussa and Rosen (1978). This does not affect the comparability of our results with the respective results of Shaked and Sutton (1982), because, as shown by Tirole (1988), richer consumers are also more demanding, so that consumers’ identification by taste could be considered to be equivalent to that by income. Accordingly, we assume a dispersion of consumer tastes which satisfies the condition on the consumer income distribution, proven by Shaked and Sutton (1982) to be necessary and sufficient for having a covered-duopoly outcome in a single-market setting. Our results, however, suggest the existence of a unique non-covered monopoly equilibrium at the two-market setting instead. This outcome is also surprising in the context of the literature on market foreclosure (Whinston 1990). It implies that leveraging the market power from an adjacent market with a lower level of competition might appear not as a consequence but rather as a cause of market exclusion of competitors. The condition for blockading all but the best-quality entrant in the adjacent market that is its quality choice upper bound must be at least a nine-fourth of the upper bound on the quality choice of the entrant with the second-best good.

We further show that the difference in the market power of firms between the two adjacent markets is decisive for the ranking of the mixed-quality combinations, given two entrants in each market. Due to the positive relationship between quality differentiation and the market power of firms, there is a clear dependence between the quality ranking of the goods in the more differentiated of the two markets and the ranking of the mixed-quality combinations in which these goods take part. That is, the mixed-quality combination based on the higher-quality good in the more differentiated market has higher rank than the mixed-quality combination based on the lower-quality good in the same market. This is demonstrated for three different possible relations between the qualities of any pair of available complementary goods with regard to how they define the quality of the combination they form. Particularly, we consider linear (perfectly substitutable qualities), multiplicative (non-perfectly substitutable qualities) and Leontief (perfectly complementary qualities) functional relationships between the combined goods' qualities.

Next, we analyze the solution for the case when the number of entrants in both markets is exogenously restricted to two. Our results imply that, for having the mixed-quality combinations unsalable at equilibrium, it is sufficient for the quality of the second-best ranked combination to be at most equal to the median between the qualities of the best and worst combinations. Then, the higher-quality firm in the more differentiated market does not benefit from decreasing the price of good as low as is necessary to sell it as a part of the second-best ranked combination; that is, together with the lower-quality good. We further show that the condition for having this equilibrium outcome could not hold when the qualities are linearly related, because the higher-quality good perfectly substitutes its lower-quality counterpart in the second-best combination. The exclusionary outcome for the mixed-quality combinations, however, is proven to be feasible at equilibrium in the case of less substitutable (multiplicative and Leontief) relationships between the qualities of the combined complementary goods.

Additionally, we show that, distinct from the single-market solution, the dispersion of consumer tastes, for which Shaked and Sutton (1982) prove a covered duopoly outcome, is neither necessary nor sufficient for both markets to be covered at equilibrium in the two-market setting. The reason is again the difference in quality differentiation between the two markets. The lower-quality firm in the more differentiated market has higher market power, which makes it less willing than its counterpart in the adjacent market to decrease its price in order to cover the market. Therefore, the consumers must be more demanding than at the single-market equilibrium to be still willing to pay the higher optimal price of the lowest-quality combination, instead of giving it up for an inferior free non-market option. Furthermore, the larger the differentiation between the best and the worst combinations, the higher the lowest consumer taste for quality should also be for the market to be covered at equilibrium. This implies that quality differentiation of the combinations has a relaxation effect on the competition of firms, which resembles the well-recognized competition relaxing effect of the product differentiation in a single-market setting. Indeed, when firms can combine their goods with complementary goods from another vertically differentiated market, this extends their ability to gain market power through direct quality differentiation of their goods by also allowing them to implicitly restrict the salability of their mixed-quality combinations through exclusionary pricing.

Finally, we derive the conditions for effective deterrence of a potential third entrant in any of the two markets. It is also not independent of the quality choice of the firms. Specifically, the upper bound on the quality choice of the entrant needs to be at least twice as small as the upper bound of the higher-quality firm in the market targeted by the entrant. Otherwise, the quality of the best combination involving the good of the third entrant is so close to the quality of the worst combination of the incumbent goods that the suppliers of the latter find it optimal to accommodate the entrant at equilibrium by setting prices at which the lowest-income consumers buy the entrant's good.

The paper is organized as follows. Section 2.2 describes the model, section 2.3 presents the particular solution for the proposed subgame-perfect equilibrium and establishes the sufficient conditions for it to hold. Section 2.4 summarizes the results and provides brief comments on their implications.

2.2 The Model

We start with the introduction of a stylized model of a market for combinations consisting of two types of goods, each offered independently by a separate firm. The two types of goods are denoted by A and B, respectively. Since goods are assumed to be consensually ranked by quality, they can be identified by their rank starting from the best-quality good being ranked at 1, the second-best at 2 and so on, as shown below:

$$f(A_1) > f(A_2) > .... > f(A_m) > 0 \tag{1}$$

where $f(A_i)$ denotes the value that consumers assign to a mutually-agreed mix of characteristics of the A type product, based on which it is ranked at i-th place, $i = 1, \ldots, m$;

$$g(B_1) > g(B_2) > .... > g(B_n) > 0 \tag{2}$$

where $g\left(B_j\right)$ denotes the value that consumers assign to a mutually-agreed mix of characteristics of the B type product, based on which it is ranked at *j*-th place, $j = 1, ..., n$.

The assumption that the two types of goods are complements in quantities (i.e. consumed in fixed one-to-one combinations) does not imply that they should also be perfect complements in qualities.[3] There are various ways in which the consumer valuations of the two types of goods could interact in defining the qualities of the combinations these goods form. Therefore, in our model the qualities of the combinations $A_iB_j,\ i = 1, ..., m,\ j = 1, ..., n$ that might be formed by any pair of the available A and B-type goods are given as a function of the consumer valuations of these goods. Specifically, we assume a simplified version of a CES-form quality-aggregation function:[4]

$$\chi_{ij} = \left\{ \left[f\left(A_i\right)\right]^{\rho} + \left[g\left(B_j\right)\right]^{\rho} \right\}^{\frac{1}{\rho}} \tag{3}$$

where:

χ_{ij} - the aggregate quality of the combination $A_iB_j,\ i = 1, ..., m; j = 1, ..., n$

$\rho = 1 - \frac{1}{\sigma}$ - a coefficient which is increasing in the elasticity of substitution σ

To reveal the impact of substitutability between the qualities of the two types of goods, we compare the following three cases of interaction between the qualities:

1. linear function: $\chi_{ij} = f\left(A_i\right) + g\left(B_j\right)$ when $\rho \to 1$ i.e. the two goods are perfect substitutes in qualities
2. simple Cobb-Douglas function: $\chi_{ij} = f\left(A_i\right) g\left(B_j\right)$ when $\rho \to 0$ i.e. the two goods

[3]We are grateful to Avner Shaked for pointing out this issue in his comments to an earlier draft of the paper.

[4]We are grateful to Vahagn Jerbashian for suggesting the CES form of the quality aggregation function for its property to encompass the full spectrum of possible qualitative substitutability between two complementary goods.

are non-perfect substitutes in qualities

3. Leontief function: $\chi_{ij} = \min\{f(A_i), g(B_j)\}$ when $\rho \rightarrow -\infty$ i.e. the two goods are perfect complements in qualities

In compliance with the classical approach of Mussa and Rosen (1978), we assume that consumers differ by taste and model their preferences by the following utility function:

$$u(\chi_{ij}, \theta) = \theta\chi_{ij} - p(\chi_{ij}) = \theta\chi_{ij} - p_i^A - p_j^B \tag{4}$$

where:

θ - taste variable by which consumers are identified, $\theta \sim U[\underline{\theta}, \overline{\theta}], 0 < \underline{\theta} < \overline{\theta}$

$p(\chi_{ij}) = p_i^A + p_j^B$ - total consumer expenditure for the combination of goods A_i and B_j

$p_i^A = p[f(A_i)]$ - price of good A_i as a function of its quality $f(A_i)$, $i = 1, ..., m$

$p_j^B = p[g(B_j)]$ - price of good B_j as a function of its quality $g(B_j)$, $j = 1, ..., n$

The demand side of the market is represented by a continuum of consumers who make individual and mutually exclusive purchases (i.e. buy one combination or do not buy any good at all) from all possible $m \cdot n$ combinations of the available goods of the A and B types. We follow the approach of Mussa and Rosen (1978) to identify consumers by their taste for quality. The latter is assumed to be uniformly distributed on the support interval $[\underline{\theta}, \overline{\theta}], 0 < \underline{\theta} < \overline{\theta}$. These simplification assumptions, common for models of vertical product differentiation, imply that the demand for a good coincides with the market share of its producer. Thus, they facilitate the representation of the solution of the producer problem.

Like Kováč (2007), we assume that each consumer has the same taste for the quality of both types of goods. Alternatively, it could be argued that the taste variable θ is type-specific. That is, a different support interval can be defined for each type of good. Then, instead of modelling consumer valuation as a product of a type-irrelevant taste variable with a quality aggregation function, a consumer valuation aggregating function, e.g. $\chi_{ij} = \left\{ \left[\theta_A f(A_i)\right]^{\rho} + \left[\theta_B g\left(B_j\right)\right]^{\rho} \right\}^{\frac{1}{\rho}}$, could be directly proposed to replace the minuend in (4). The adoption of such an approach[5] would change the form rather than the intuition behind our results in the sense that the conditions will be defined by means of four $\{\underline{\theta}_A, \overline{\theta}_A, \underline{\theta}_B, \overline{\theta}_B\}$ instead of only two $\{\underline{\theta}, \overline{\theta}\}$ consumer taste endpoint parameters. Furthermore, there is a well-established proof in the literature that consumer taste could be regarded as a measure of the inverse marginal rate of substitution between consumer income and quality (see Tirole, 1988, p.96). Hence, the taste for the quality of a good should be strictly positively correlated with the income of its consumer. Therefore, given that consumer incomes do not vary across the products for which they are spent, we find it reasonable to assume the same for consumer tastes. Finally, there is also a technical argument for assumption of a type-irrelevant consumer taste variable. Namely, the single-dimensional consumer taste space is preferable because it enables the direct comparison of our equilibrium solution with that of the single-market case which we take as a benchmark in our paper.

Together with the similarities, our model has also inevitable divergences from the single-market models. The function in (4) includes two modifications of the original function of Mussa and Rosen (1978), to make it applicable to our model of a vertically differentiated market for combinations of complementary goods. The first modification is prompted by the perfect complementarity of the goods being combined. It implies that consumers could only enjoy combinations containing both types of goods in a one-to-one

[5] We are grateful to Eugen Kováč and Martin Peitz for pointing out the adoption of a multi-dimensional consumer taste space as a less restrictive alternative to the approach we follow in this paper.

ratio, but gain nothing from buying only a single good. We reflect this difference from the single-market model in the utility function of (4) by replacing the single-good quality parameter (χ_i) of Mussa and Rosen (1978) by the combination quality parameter χ_{ij} defined by the function in (3). The second modification we make to the utility function suggested by Mussa and Rosen (1978) is inspired by another trivial difference from the single-good case. Namely, the total expenditure $p\left(\chi_{ij}\right)$ which consumers incur when purchasing given combination A_iB_j is not set by any particular firm alone, but is instead given by the sum of the prices, p_i^A and p_j^B, set by the two firms supplying the goods included in that combination.

To avoid implausible negative utilities, in the single-market models, consumers are commonly assumed to have an outside option: not to buy any quality available in the market, but to consume a free good of inferior non-negative quality. Here, for simplicity and without loss of generality, we assume that the outside option is of zero quality. In addition, since we do not have a single type of good but combinations of paired goods of two different types we introduce positive lower bounds, $\underline{F}$ and $\underline{G}$, on the quality choice of firms producing goods of type A and type B, respectively:

$$\begin{aligned} &f(A_i) \geq \underline{F} > 0 \text{ for } \forall i \in \{1, ..., m\} \\ &g\left(B_j\right) \geq \underline{G} > 0 \text{ for } \forall j \in \{1, ..., n\} \end{aligned} \tag{5}$$

The inequality conditions in (5) impose minimal quality requirements for a good of a given type to be considered a market good. Accordingly, the minimal quality requirement is the same for all producers of a particular type of good.

To establish the producers' decision problem, we next need to derive the demand functions that correspond to the consumer preferences expressed by the utility function in (4). Particularly, note that in our model the market space is given by the support interval of the consumer taste distribution. Accordingly, the demand shares of the possible combinations are represented by subintervals within the support interval, enclosed by the so-called marginal taste variables. Each marginal taste variable, generally denoted by θ_{ij/i^*j^*}, corresponds to a taste for quality at which a consumer would be indifferent between buying combination A_iB_j or combination $A_i^*B_j^*$, where A_iB_j is of higher quality than $A_i^*B_j^*$. For easier identification, from now on, we will call it marginal taste variable of combinations $A_i^*B_j^*$ and A_iB_j.

By making use of the utility functional form defined in (4), we could show that the marginal taste variable is given by the ratio between the price difference and the quality difference of the two combinations:

$$u\left(\chi_{ij},\theta_{ij/i*j*}\right)=u\left(\chi_{i*j*},\theta_{ij/i*j*}\right)\Rightarrow\theta_{ij/i*j*}=\frac{p\left(\chi_{ij}\right)-p\left(\chi_{i*j*}\right)}{\chi_{ij}-\chi_{i*j*}},$$
$$\text{for any } \; i=1,...,m;\;\; j=1,...,n;\;\; i^*=1,...,m;\;\; j^*=1,...,n;$$
$$\text{such that } \; (i^*\neq i)\wedge(j^*\neq j) \tag{6}$$

where:

θ_{ij/i^*j^*} – marginal taste variable of combinations $A_i^*B_j^*$ and A_iB_j at which consumers are indifferent between buying one or the other, given that $\chi_{i^*j^*}<\chi_{ij}$

Similarly, we could also derive the marginal taste $\theta_{ij/0}$ at which a consumer would be indifferent between buying combination A_iB_j or not purchasing in the market at all, as follows:

$$u\left(\chi_{ij},\theta_{ij/0}\right)=u\left(\chi_0=0,\theta_{ij/0}\right)\Rightarrow\theta_{ij/0}=\frac{p_i^A+p_j^B}{\chi_{ij}}$$
$$\text{for any } \; i=1,...,m;\;\; j=1,...,n; \tag{7}$$

where:

$\theta_{ij/0}$ – marginal taste variable of combination A_iB_j and the free outside option O at which consumers are indifferent between buying the former and switching to the latter

As in the single-market case, when a marginal taste is within the range of variation of consumer tastes, it plays the role of a boundary which divides the consumer population into two groups. Or technically speaking, in our model, any marginal taste variable belonging to the support interval $\left[\underline{\theta},\ \overline{\theta}\right]$ divides the support interval into two subintervals as shown in figure 1 below. One of the group consists of the consumers with tastes lower than $\theta_{ij/i*j*}$. If asked at the given prices to choose only between the two combinations, A_iB_j and $A_{i^*}B_{j^*}$, they will strictly prefer the latter to the former. Therefore, we call the subinterval of the tastes of the consumers belonging to this group, the lower-quality subinterval of $\theta_{ij/i*j*}$. On the contrary, the other group of consumers with taste (variables)[6] larger than $\theta_{ij/i*j*}$, will strictly prefer A_iB_j to $A_{i^*}B_{j^*}$. In turn, we call the subinterval of the consumer taste variables belonging to this second group, the higher-quality subinterval of $\theta_{ij/i*j*}$.

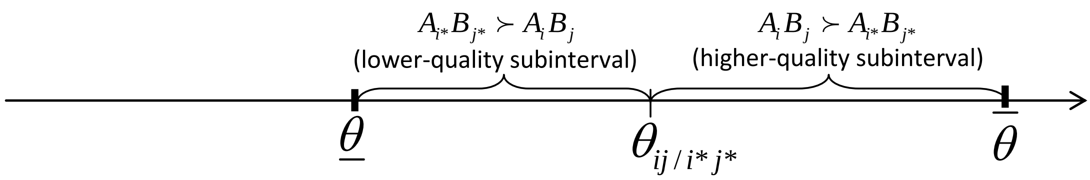

Figure 1. Division of the support interval $\left[\underline{\theta},\ \overline{\theta}\right]$ into the lower-quality and upper-quality subintervals of $\theta_{ij/i*j*}$.

Alternatively, if the marginal taste does not belong to the range of variation of consumer tastes, at the given prices all consumers will prefer either the one or the other combination. Nobody will be indifferent. In technical terms, if a marginal taste variable $\theta_{ij/i*j*}$ is smaller than the lower endpoint $\underline{\theta}$ of the support interval, at the given prices, its higher-quality subinterval will coincide with the support interval, whereas its

[6]Throughout the paper we use "consumer taste" as a brief form of "consumer taste variable".

lower-quality subinterval will be empty. Similarly, if a marginal taste variable $\theta_{ij/i*j*}$ is larger than the upper endpoint $\overline{\theta}$ of the support interval of consumer taste distribution, at the given prices, its lower-quality subinterval will coincide with the support interval, whereas its higher-quality subinterval will be empty.

In order to generally define the demand for any combination A_iB_j, $i = 1,...,m$; $j = 1,...,n$, it is also useful to categorize its marginal taste variables based on whether A_iB_j is demanded by their lower- or higher-quality intervals. The first category includes the marginal taste variables in the indices of which the combination A_iB_j is positioned after the slash. These are the marginal taste variables whose lower-quality subintervals represent the consumers who at the given prices prefer A_iB_j to other higher-quality combinations. Therefore, from now on we will refer to them as lower-quality marginal taste variables of combination A_iB_j. In a similar manner, we will call higher-quality marginal taste variables of A_iB_j the marginal taste variables in the indices of which the combination A_iB_j is positioned before the slash. That is, the higher-quality subinterval of each higher-quality marginal taste variable of A_iB_j represents the consumers who, at the given prices, prefer A_iB_j to another lower-quality combination.

The demand for any combination $A_iB_j, i = 1,...,m; j = 1,...,n$, is given by the intersection of all its higher- and lower-quality subintervals and the support interval of the consumer taste distribution. In particular, it is equal to the difference between the smallest higher-quality marginal taste variable $\overline{\theta}_{ij}$ and the largest lower-quality marginal taste variable $\underline{\theta}_{ij}$ of any particular combination A_iB_j provided that the difference is positive. Otherwise, if the difference is non-positive, the demand for the combination is zero:

$$D_{ij} = max\{(\overline{\theta}_{ij} - \underline{\theta}_{ij}),\ 0\}, i = 1,2,\ \dots\ ,m;\ j = 1,2,\ \dots\ ,n; \tag{8}$$

where for any $i = 1,2,\ \dots\ ,m$ and $j = 1,2,\ \dots\ ,n$ we have:

$\overline{\theta}_{ij}$ – the smallest higher-quality marginal taste variable of combination A_iB_j,

i.e. $\overline{\theta}_{ij} = min\{\theta_{\psi(1)/ij}, \dots, \theta_{\psi(r_{ij}-1)/ij}, \overline{\theta}\}$, so that:

r_{ij} – quality rank of combination A_iB_j; $r_{ij} \in \{1, 2, \dots, m \cdot n\}$

$\psi(r_{ij})$ – inverse rank-translating function which takes the quality rank of any combination A_iB_j as an argument and returns the index ij formed by the ordered pair of quality ranks of the goods A_i and B_j of which the combination is formed, see table 1 below

$\underline{\theta}_{ij}$ – the largest lower-quality marginal taste variable of combination A_iB_j,

i.e. $\underline{\theta}_{ij} = max\{\theta_{ij/\psi(r_{ij}+1)}, \dots, \theta_{ij/\psi(m \cdot n)}, \theta_{ij/0}, \underline{\theta}\}$

Since the best-ranked combination A_1B_1 does not have a higher-quality marginal taste variable $(r_{11} = 1)$, the minuend of the difference term in its demand function is given by the right endpoint $\overline{\theta}$ of the support interval of the consumer taste distribution. Similarly, if the smallest higher-quality marginal taste variable of a combination exceeds the right endpoint $\overline{\theta}$, the latter replaces it as a minuend of the difference term in the demand function of the combination. Also, if the largest low-quality marginal taste variable of a combination is smaller than the left endpoint $\underline{\theta}$ of the support interval of the consumer taste distribution, the latter replaces it as a subtrahend of the difference term in the demand function of the combination.

In the current paper, we explore two particular market settings. First, we define the equilibrium in a market with single type A entrant ($M = 1$) that is assumed to have higher quality than any of the N potential type B entrants $(f(A_1) > g(B_1))$. Second, we solve for an equilibrium where the mixed-quality combinations are effectively excluded from a market with two actual type A and B entrants, respectively.

Below, we provide a graphical illustration of how the intersection of the higher- and lower-quality subintervals give the equilibrium demands for the best ranked combination A_1B_1 in the first setting (Figure 2) and for the worst-ranked combination A_2B_2 in the second setting (Figure 3).

The two equilibrium outcomes depicted in figure 2 and figure 3 represent two mechanisms in which a combination might be efficiently excluded from the market. In figure 2 we have an equilibrium outcome where the price of one of the goods which is present in all the combinations is so high relative to the qualities of the goods excluded that consumers prefer to switch to the free outside option rather than to buy any of these low-ranked combinations. In figure 3, the qualities of the excluded combinations do not differ sufficiently from the quality of the worst-ranked combination. Accordingly, at the optimal prices that firms choose, the less-differentiated combinations are effectively excluded in favor of the lower-quality, but more differentiated, worst-ranked combination. Neither of the two exclusionary outcomes can occur at equilibrium in a single-market setting.

There is also a third mechanism for excluding a good, which Shaked and Sutton (1982) prove to be working in a single-good market. It requires the lowest consumer identification variable, in our model $\underline{\theta}$, to be sufficiently low so that even when a good is priced at zero, its lower-quality taste-variable is below $\underline{\theta}$. In the current paper, this third mechanism plays a role in preventing the entry of a third type A and/or B firm. It enters the solution by defining the necessary condition for having only two actual entrants in the extended version of the second setting, allowing for more than two potential entrants of a type. We do not provide graphical illustration of this outcome below because it does not directly refer to the definition of the demand expression in (8).

In the first setting, since we have a single potential type A entrant whose good's quality exceeds the quality of any other type B good, for any of the three special cases of the aggregation function in (3), the ranking of possible combinations follows the ranking of the type B goods, as shown in table 1 below:

Table 1 Ranking of combinations given single type A entrant and *n* type B entrants

rank (r_{ij})	**combination** (A_iB_j)	**index** ($\psi(r_{ij})$)
1	A_1B_1	11
2	A_1B_2	12
3	A_1B_3	21
…	…	…
n	A_1B_n	1n

Then, as shown in figure 2 below, at equilibrium only two marginal taste variables, $\theta_{11/12}$ and $\theta_{11/0}$, have values that exceed the left endpoint $\underline{\theta}$ of the support interval of the consumer taste distribution.

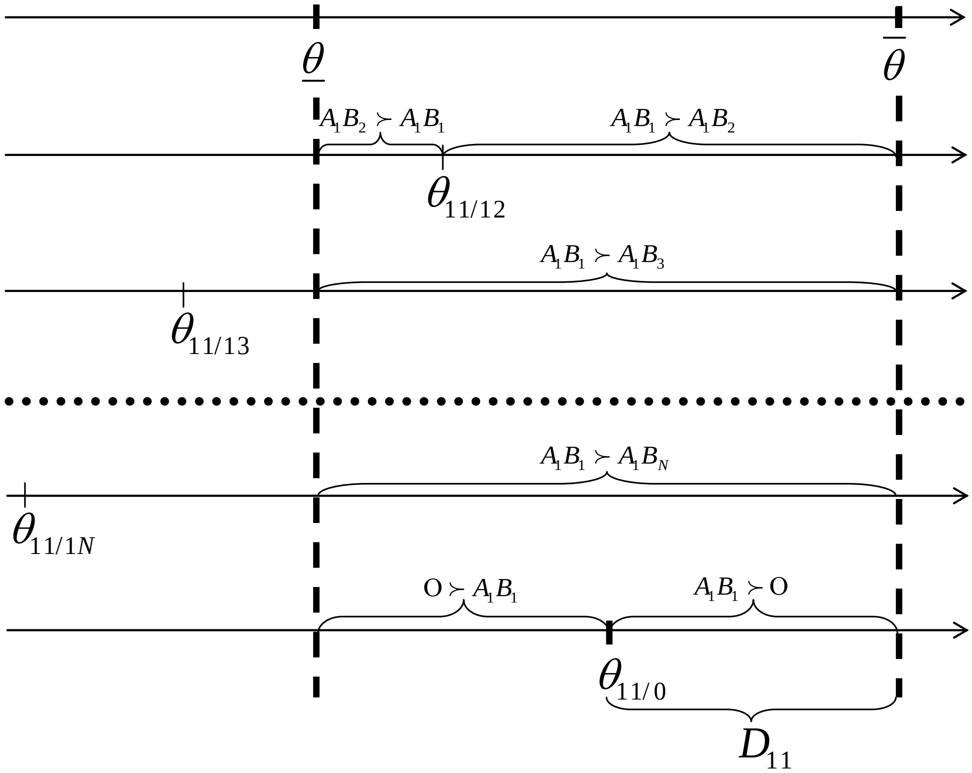

Figure 2. Demand D_{11} for combination A_1B_1 defined by the intersection of the subintervals of its marginal taste variables and the support interval of the

consumer-taste distribution given a single potential entrant of type A and N potential entrants of type B.

The value of the marginal taste variable $\theta_{11/0}$ is so high that it exceeds not only $\underline{\theta}$ but also the marginal taste variable $\theta_{11/12}$. As a result, there are consumers with tastes lower than $\theta_{11/0}$ and higher than $\theta_{11/12}$ who prefer the free outside option to combination A_1B_1 and, by transitivity, to combination A_1B_2, as well. Accordingly, the consumers with tastes lower than $\theta_{11/12}$ will also by transitivity prefer the outside option to A_1B_2. Thus, the equilibrium outcome presented in figure 2 implies that all consumers prefer to buy either the best combination A_1B_1 or the free outside option, which makes good B_2 unsalable. By induction we could show that this would be the case with any other B-type producer whose good is not going to be ranked the best after entry. Therefore, at any positive spread of consumer tastes, provided that there is a single entrant of type A, A_1B_1 will be the only salable combination, with its demand given by the difference between $\overline{\theta}$ and $\theta_{11/0}$. The right endpoint $\overline{\theta}$ of the support interval takes the place of the minimal higher-quality marginal variable $\theta_{i-j-/ij}$ in (8) because A_1B_1 is the best-ranked combination and as such cannot have a higher-quality marginal variable. Accordingly, $\theta_{11/0}$ is the largest lower-quality marginal variable of A_1B_1 and as such it takes the place of the subtrahend in the general demand function (8) for $i = j = 1$.

Such a market, where not all consumers are served by the incumbent firms, but in which some low-taste consumers instead switch to the outside option, is said to be not covered by those (incumbent) firms. Accordingly, the market coveredness and the number of entrants with positive market shares are commonly recognized in the existing literature as defining characteristics of the equilibrium outcome in any market with product differentiation. Since in our setting we have only single-product firms, neither of which alone is able to define the amount $p(\chi_{ij})$ spent for any particular combination of goods A_iB_j, in what follows we prefer to talk not about firms, but about combinations covering the market.

In the second setting that we consider, there are two actual entrants of each type offering goods of distinct qualities, so there could be two different rankings of their combinations as follows:

Table 2 Possible rankings of the combinations given two entrants of both types A and B

rank	**ranking 1**	**ranking 2**
1	A_1B_1	A_1B_1
2	A_1B_2	A_2B_1
3	A_2B_1	A_1B_2
4	A_2B_2	A_2B_2

In figure 3 below, a particular equilibrium outcome is depicted for the case when ranking 1 holds.

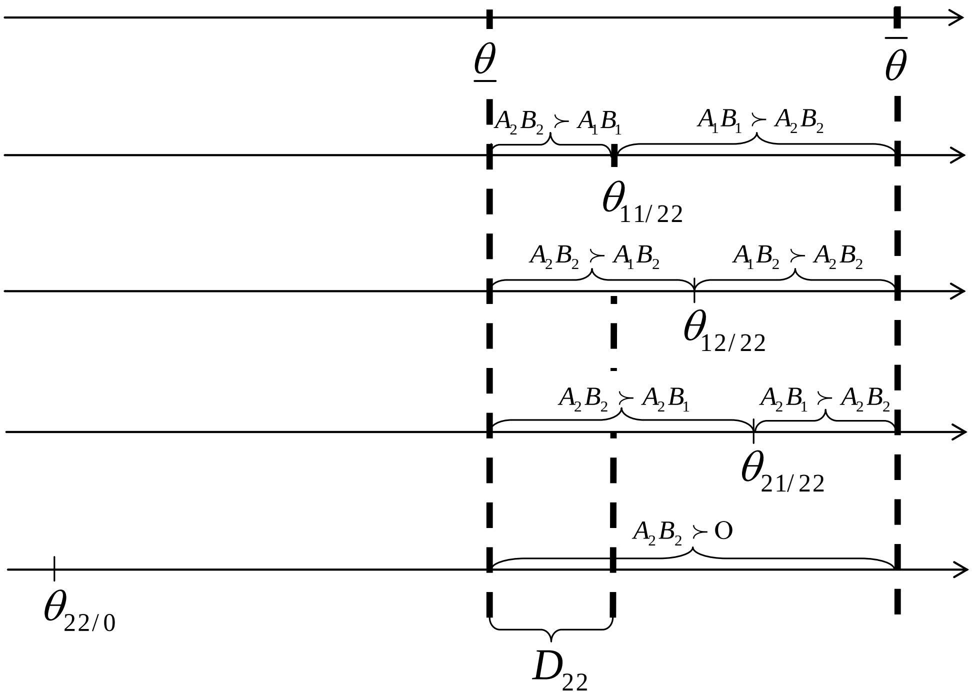

Figure 3. Demand D_{22} for combination A_2B_2 defined by the intersection of the subintervals of its marginal taste variables and the support interval of the consumer-taste distribution given two entrants of each type, A and B.

As we will show in the next section, under certain conditions sufficient for having the equilibrium outcome in figure 3, the outcome is the same at the both rankings 1 and 2. Namely, $\theta_{11/22}$ is of lower value than $\theta_{11/12}$ and $\theta_{11/21}$. Accordingly, there are consumers with tastes larger than $\theta_{11/22}$ and smaller than $\theta_{11/12}$ and $\theta_{11/21}$ who at the equilibrium prices prefer to buy A_1B_1 to A_2B_2, and A_2B_2 to A_1B_2 and A_2B_1, respectively. By transitivity, this implies that all consumers with tastes larger than $\theta_{11/22}$ buy A_1B_1 but not A_1B_2 or A_2B_1. By the same reasoning, all consumers with tastes smaller than $\theta_{11/22}$ buy A_2B_2 but not A_1B_2 or A_2B_1. Thus, only the best- and the worst-ranked combinations are salable at equilibrium.

Based on the consumers' demand for the possible combinations of the available goods of each type, producers solve a decision problem which can be represented by the following 3-stage game:

Stage 1: Firms simultaneously decide to enter the market or not.

There are *M* potential entrants of type A and *N* potential entrants of type B. There are no multi-product firms. Accordingly, each entrant faces a different technological frontier which is a restriction from above on the characteristics of the best good that it is capable of producing. This is important for the entry decision of each potential entrant because, upon entry, besides the other features the restriction from above will also constrain the mutually-agreed mix of characteristics by which consumers value the type of good the entrant offers. Therefore, in the model we introduce an individual upper bound on the quality choice of each firm, which is already known by potential entrants before entry. We will call this upper bound maximal quality constraint.

Since all consumers agree on the ranking of the characteristics of the goods of the same type, ranking the potential entrants in descending order of their maximal quality constraints should not vary among the consumers. Hence, the rank of the maximal quality constraint of a potential entrant is a unique identifier that allows us to distinguish it from the others. Indeed, it represents the rank of the best potential product a firm could produce in a kind of a virtual pre-market competition with the other potential entrants of the same type. Thus, the rank of a potential entrant's maximal quality constraint plays the same role as a firm's brand does in the real world. It sends a signal to the consumers (as well as to the other external stakeholders of the potential entrant) about what is the best quality that a firm can produce. Therefore, below we rank in descending order the maximal quality constraints of the potential entrants of type A and B and denote them by $\bar{F}_k,\ k = 1,\ \dots,M$ and $\bar{G}_l,\ l = 1,\ \dots,N$, respectively:

$$\bar{F}_1 > \bar{F}_2 > \bar{F}_3 > \ \dots\ > \bar{F}_M \tag{9}$$

$$\bar{G}_1 > \bar{G}_2 > \bar{G}_3 > \ \dots\ > \bar{G}_N \tag{10}$$

<u>Stage 2:</u> Entrants choose the qualities of their goods simultaneously.

Let m entrants of type A and n entrants of type B be in the market after the first stage, $m \in \{1,\dots,M\}$, $n \in \{1,\dots,N\}$. We identify actual entrants in the same way as the potential entrants. Each actual entrant of type A is associated with the rank i of its maximal quality constraint $\bar{F}_{k_i},\ i = 1,\dots,m$ within the subsequence of sequence (9) which is formed by arranging in descending order the maximal quality constraints of the actual entrants of type A, as follows:

$$\bar{F}_{k_1} > \bar{F}_{k_2} > \bar{F}_{k_3} > \ \dots\ > \bar{F}_{k_m} \tag{11}$$

Similarly, each actual entrant of type B is associated with the rank j of its maximal quality constraints $\bar{G}_{l_j},\ j = 1,\ \dots,n$ within the subsequence of sequence (10) which is formed by arranging in descending order the maximal quality constraints of the actual entrants of type B, as follows:

$$\bar{G}_{l_1} > \bar{G}_{l_2} > \bar{G}_{l_3} > \ ... \ > \bar{G}_{l_n} \tag{12}$$

To clarify the transition from potential entrants' ranking to actual entrants' ranking, imagine a situation in which the first two best-ranked potential entrants of type A decide not to enter in the first stage of the game but the third best-ranked potential entrant decides to enter the market. Then, the third best-ranked potential entrant will in fact have the best-ranked maximal quality constraint from all the actual type A market entrants, so that in stage 2 its constraint from above will have rank 1, that is $\bar{F}_{k_1} = \bar{F}_3$. As we will show later, this will never be the case at equilibrium. In fact, at equilibrium the m actual entrants are the m best-ranked potential entrants. It cannot be optimal for either of them to stay out of the market, given that at the same time market entry is an optimal choice for any lower-ranked potential entrant. Similar examples and comments about the equilibrium outcome after the entry stage can also be made for the type B entrants. That is, the rankings in (11) and (12) characterize only the transition to the equilibrium outcome, but not the equilibrium itself. The equilibria established in the next section are therefore definable by solely using the notation for the maximal quality constraints of the potential entrants in (9) and (10).

After observing how many firms of each type have entered the market in the first stage, every actual entrant i of type A chooses the quality of its good $f(A_i)$ from within the interval $\left[\underline{F}, \overline{F}_{k_i}\right]$, while every actual entrant j of type B chooses the quality of its good $g\left(B_j\right)$ from within the interval $\left[\underline{G}, \overline{G}_{l_j}\right]$. Entrants make their quality choices simultaneously. The chosen qualities represent the same mix of characteristics of the goods of each type on whose ranked value consumers are assumed to mutually agree according to (1) and (2).

Stage 3: Firms compete in prices.

After making their quality choices in the second stage, firms observe the available qualities in the market and simultaneously choose the prices of their goods. Possibilities for product extension mergers, collusions and tying practices imposing pre-selected bundles on consumers are precluded in the model.

The payoffs of the firms are given by their profits. For simplicity, production costs are assumed to be zero for all the producers of both types so that, for each firm, profit coincides with revenue. The demand share of a good is given by the sum of the demand shares of the combinations in which it takes part. Accordingly, dependent on the type of its good, the decision of a market entrant at the first and second stage can be jointly represented by one of the following profit-maximization problems:

$$\max_{f(A_i),p_i^A} \Pi_i^A = R_i^A = p_i^A D_i^A = p_i^A \sum_{j=1}^{n} D_{ij},\ i = 1,...,m \tag{13}$$

$$\max_{g(B_j),p_j^B} \Pi_j^B = R_j^B = p_j^B D_j^B = p_j^B \sum_{i=1}^{m} D_{ij},\ j = 1,...,n \tag{14}$$

where:

Π_i^A - profit of the producer of type A good with quality rank i

Π_j^B - profit of the producer of type B good with quality rank j

R_i^A - revenue of the producer of type A good with quality rank i

R_j^B - revenue of the producer of type B good with quality rank j

D_i^A - demand share of type A good with quality rank i

D_j^B - demand share of type B good with quality rank j

If a firm decides not to enter the market in the first stage, its profit is zero.

We solve the model first with exogenously set number of entrants. We start with a model setting in which there is a single type A entrant and n potential type B entrants. We assume that the consumer taste distribution satisfies the condition of Shaked and Sutton (1982) for at least two entrants with positive market shares at a single-market equilibrium. Our aim is to show that only the firm offering the best-ranked type B good could still have positive market share at the equilibrium with only one type A entrant. Then, we continue by analyzing a setting with two type A and two type B entrants to define the conditions expressed in terms of the constraints on the quality choices of the firms for having the mixed-quality combinations unsalable at a covered-market equilibrium. Again the consumer taste distribution is assumed to satisfy the single-market condition of Shaked and Sutton (1982) of exactly two entrants with positive market shares. Finally, we define sufficient conditions of at most two entrants of a type at equilibrium and show that these conditions are stricter than the single-market conditions of Shaked and Sutton (1982). Under these more restrictive conditions, we establish an equilibrium at which only the best and the worst combinations in the market have positive market shares and cover the market. The results are formally presented and discussed in the next section.

2.3 Equilibrium Solutions

In this section we discuss the results of the solutions of our model settings. Here we present only the intuition behind the equilibrium outcome, while rigorous solution are provided in the appendix. We are interested in exploring the conditions for having a covered-duopoly market where only the best and worst combinations have positive market shares. However, we reach it gradually.

First, we establish the equilibrium solution for the case with only a single type A entrant who faces higher maximal quality constraint than the n type B entrants. Accordingly, for any of the three special cases of the aggregation function in (3), the ranking of possible combinations follows the ranking of the type B goods as shown in the previous section (table 1). We show that the variability of the consumer identification variable (the taste variable in our model) should satisfy the same condition as in Shaked and Sutton (1982) for having at most two type B entrants with positive market shares. This is formally stated in proposition 1 below.

Proposition 1. Given that the number of type A entrants is restricted to one, let the parametric endpoints $\underline{\theta}$ and $\overline{\theta}$ of the support interval, on which consumers' tastes are assumed to be distributed, satisfy the following condition:

$$\frac{\overline{\theta}}{4} < \underline{\theta} \tag{15}$$

Then, of any n potential type B entrants at most two will have positive market share at equilibrium.

Proof: see section A of the appendix

Distinct from the single-market equilibrium outcome, at certain conditions the optimal solution of the particular case with a single entrant offering a complementary type A good implies not two but only one type B entrant with positive market share. Furthermore, the result is independent of the distribution of the taste variable. As shown in figure 2, A_1B_1 is the only possible combination demanded by consumers at the optimal price chosen by the single type A entrant. By being the sole type A entrant, the supplier of good A_1 is in a similar position to that of a classical monopolist. It maximizes its revenue by setting a price at which only the consumers with elastic demand would buy its good. However, when the best-ranked type B good is sufficiently differentiated from the goods of the rest of the potential entrants, all the consumers with elastic demand buy A_1 only at equilibrium. That is, the taste at which consumer demand for A_1B_1 is exactly unit-elastic is actually the one at which consumers would be indifferent between buying the best-ranked combination or switching to the outside option. Accordingly, if there are consumers with lower tastes, they will strictly prefer the outside option. That is, the market will not be covered at equilibrium. The result is established in proposition 2 below.

Proposition 2. Given that the number of potential type A entrants is restricted to one and the maximal-quality constraints of the two-best-ranked type B goods are related as follows:

$$\bar{F}_1 > \bar{G}_1 > \frac{9}{4}\bar{G}_2 \tag{16}$$

of any n potential entrants of type B exactly one will have positive market share at equilibrium. The result holds for any $\underline{\theta} < \bar{\theta}$ and for $\sigma \to \{0,1\}$. Furthermore, the market will not be covered as long as the following condition holds:[7]

$$\underline{\theta} < \frac{\overline{\theta}}{2} \tag{17}$$

Proof: see section B of the appendix.

This situation resembles a standard case in the literature on applying product tying as a market power leverage device where the producers of the two types of goods are assumed to offer them in different markets. As a result, the firm operating as a monopolist in one of the markets is in a position to decide whether to exclude the lower-quality producer in the other duopoly market (Whinston, 1990, p. 841). Therefore, the monopoly market is usually designated as core market, while the competitive market is designated as adjacent market whereas the good sold on the core market is called bottleneck good (Rey and Tirole, 2007, p. 2183). Since, in our setup, firms make decisions independently, we could consider the type A entrant as operating in a separate market, let us call it market A, from the market in which the type B entrants operate, let us call it market B. Accordingly, good A_1 plays the role of a bottleneck good. Distinct from the situation

[7] In a single-good model, condition (17) is sufficient for at least two goods of distinct qualities to have positive market share. Together with condition (15), it ensures covered duopoly at equilibrium. See (Shaked and Sutton, 1982, p.7).

described in (Whinston, 1990), however, the market-power leverage outcome is achieved in our solution without the entrant in core market A to have to tie its good to the good produced by any of the potential adjacent market B entrants. Indeed, the consumers are free to buy the bottleneck good A_1 in combination with the good of any of the n type B entrants. However, the consumers who value A_1 sufficiently high to buy it at its monopoly price, are too demanding to compromise with the quality of the best-ranked type B good, which discourages the lower-ranked firms of type B from entering the market.

The result established in proposition 2 implies that the presence of a complementary good of superior quality offered by a monopolist in market A might change the equilibrium outcome in market B from a covered duopoly into a non-covered monopoly. The condition for having such an outcome at equilibrium is that the best-ranked good in market B must be sufficiently differentiated from the rest of the type B goods as required by the second inequality in (16). Furthermore, in section B of the appendix we explore the possible equilibrium solutions if the best-ranked type B good is not so differentiated. We show that a non-covered equilibrium with more than one actual entrant in market B cannot exist. The single entrant in market A will always apply exclusionary pricing, given a non-covered market. An outcome with more than one entrant could occur only at a covered-market equilibrium. Still, we could have at most three entrants, because the existence of a covered-market equilibrium requires the consumer taste spread to be restricted from below $\left(\frac{\overline{\theta}}{7} < \underline{\theta}\right)$. Otherwise, the type A producer again prefers to charge an exclusionary price for its good. Accordingly, for having exactly three entrants at equilibrium the condition in (15) also needs to be relaxed $\left(\underline{\theta} < \frac{\overline{\theta}}{4}\right)$.

In the second setting for which we provide a solution of our model, there are two potential type A entrants and two potential type B entrants. With this analysis, we move towards answering the key question in the present paper. Namely, we are interested to define the conditions under which an equilibrium exists at which only the most differentiated possible combinations of the available goods are actually sold.

In contrast to the first setting, here, there is no asymmetry between the two types of goods in terms of the number of potential entrants that offer them. We have two potential type A and two potential type B entrants. Therefore, the ranking of the possible combinations cannot be deduced directly from the difference in the qualities of the best-ranked goods only. Indeed, as shown in table 2 of the previous section, when there are two potential entrants of each type, their goods have four possible combinations, for which two possible rankings exist. For any of the three special cases of the aggregation function in (3), combination A_1B_1 formed by the best goods of both types has the lowest rank 1, whereas the combination A_2B_2 formed by the worst goods of both types has the highest rank 4. The only uncertainty is which of the two mixed-quality combinations is better, A_1B_2 or A_2B_1. The determinant of the preference relation between the two combinations is given by the relation between the qualities of type A and type B. The particular condition, however, depends on the elasticity of substitution σ between the qualities of the two types which is present through the parameter ρ in the quality-accumulation function in (3). The rigorous expressions of the conditions for each relation in the three special cases of substitutability between the qualities of the two types of goods are presented in part C of the appendix. Proposition 3 below establishes general sufficient conditions for ranking 1 in table 2 to hold at any of the three values which ρ is assumed to approach.

Proposition 3. Given two entrants of each type in the market and $\sigma \to \{-\infty, 0, 1\}$, the combinations formed by the better type A good would be of higher rank than the combinations formed by the worse type A good if the following relation between the qualities of the available goods holds:

$$f(A_1) > g(B_1) > g(B_2) > f(A_2) \tag{18}$$

Proof: see section C of the appendix

Since ranking 2 is a mirror image of ranking 1, a general sufficient condition for it could also be derived. It is given by trading the places of $f(A_i)$ and $g(B_j)$ for $i = j$ in (18):

$$g(B_1) > f(A_1) > f(A_2) > g(B_2) \tag{19}$$

The quality relations in (18) and (19) imply that the type of good with greater differentiation is the one whose quality matters more for the ranking of the combinations. That is, from the two mixed-quality combinations the one with the better-ranked good of the more differentiated type is also of higher rank.

No matter which of the two mixed-quality combinations is of higher rank, we cannot have both mixed-quality combinations salable at a covered-market equilibrium if the quality difference between the combination with rank 2 and the best combination exceeds the quality difference between the combination with rank 3 and the worst combination:

$$\begin{aligned} &\chi_{11} - \chi_{12} > \chi_{21} - \chi_{22}, \text{ if ranking 1 or,} \\ &\chi_{11} - \chi_{21} > \chi_{12} - \chi_{22}, \text{ if ranking 2} \end{aligned} \tag{20}$$

When the condition in (20) holds, the denominator of the marginal taste variable of the former pair of combinations is larger than that of the latter pair. At the same time, the general expression in (7) implies that the two marginal taste variables have the same numerator given by the difference of the prices of the goods of the less differentiated type. Hence, the marginal taste variable of the higher-ranked pair of combinations is smaller than that of the lower-ranked pair of combinations:

$$\begin{aligned} \theta_{11/12} &= \frac{p_1^B - p_2^B}{\chi_{11} - \chi_{12}} < \frac{p_1^B - p_2^B}{\chi_{21} - \chi_{22}} = \theta_{21/22}, \text{ if ranking 1 or,} \\ \theta_{11/21} &= \frac{p_1^A - p_2^A}{\chi_{11} - \chi_{21}} < \frac{p_1^A - p_2^A}{\chi_{12} - \chi_{22}} = \theta_{12/22}, \text{ if ranking 2} \end{aligned} \tag{21}$$

The inequalities in (21) imply that it is not possible for both the former marginal taste variable $\theta_{11/12}$ (resp. $\theta_{11/21}$) to be larger and the latter marginal taste variable $\theta_{21/22}$ (resp. $\theta_{12/22}$) to be smaller than the marginal taste of the two mixed-quality combinations $\theta_{12/21}$ (resp. $\theta_{21/12}$). There is either only one of the combinations salable at equilibrium or neither of the two.

To understand the intuition behind the possible equilibrium outcomes when the rank-related conditions in (20) hold, note that when the worst-ranked combination A_2B_2 is of sufficiently higher quality than the free outside option to cover the market, the rank 3 combination is the least differentiated from its neighbors by rank. Accordingly, the suppliers of the goods in that combination are subjected to strong competitive pressure to decrease their prices in order to make the combination salable. However, the firm offering the lower-ranked good of the more differentiated type in the rank 3 combination prefers to keep its price high because the sales of the worst combination, in which its good is also present, are not threatened by the outside option. Similarly, the firm offering the higher-ranked good of the less differentiated type is better-off selling its good to the highest-taste consumers, who buy it as a part of the best-ranked combination. Indeed, the resulting additional revenue compensates for the lost sales of the good as a part of the mixed-quality rank 3 combination.

The competitive pressure experienced by the suppliers of the goods in the rank 2 combination is not as strong as that faced by the firms supplying the goods in the rank 3 combination. On the one hand, the inequalities in (20) ensure that the differentiation of the rank 2 combination from the best-ranked combination is larger than the differentiation of the rank 3 combination from the worst-ranked combination. On the other hand, one of the goods in the rank 2 combination takes part also in the worst-ranked combination A_2B_2 sold to the lowest-taste consumers. Therefore, its supplier should not necessarily lose when the rank 2 combination has no demand. Indeed, the revenue from the worst-ranked combination would compensate for the lost sales of the rank 2 combination if it is at least as differentiated from the best-ranked combination than from the worst-ranked one:

$$\begin{aligned}&\chi_{11}-\chi_{12}>\chi_{12}-\chi_{22}, \text{ if ranking 1 or,}\\&\chi_{11}-\chi_{21}>\chi_{21}-\chi_{22}, \text{ if ranking 2}\end{aligned} \tag{22}$$

The condition in (22) is necessary and sufficient for both mixed-quality combinations to be unsalable at a covered market equilibrium. To establish this equilibrium, however, the condition on the combination qualities in (22) as well as the condition for a covered market need to be translated into a condition on the constraints of the good quality choices of the market entrants of each type. The result is formally presented in proposition 4 below.

Proposition 4. Given the distribution of consumer's taste according to the conditions in (15) and (17), let two firms of each type enter the market. Then, if whichever of the following alternative relations characterize their quality-choice ranges:

$$\frac{3}{2}\frac{\underline{F}}{\bar{F}_1}\underline{G}>\bar{F}_1>\bar{G}_1>\frac{6}{5}\underline{G}>\frac{6}{5}\underline{F} \tag{23}$$

$$\frac{3}{2}\frac{\underline{G}}{\bar{G}_1}\underline{F}>\bar{G}_1>\bar{F}_1>\frac{6}{5}\underline{F}>\frac{6}{5}\underline{G} \tag{24}$$

for $\sigma \to \{0,1\}$ the following subgame equilibria exist in the last two stages:

1. at the quality-choice stage the maximum differentiation outcome prevails: see the expressions in (D6) in the appendix

2. at the pricing stage the optimal prices are given by the expressions in (D2) in the appendix.

Proof: see section D the appendix.

There are three moments in proposition 4.

First, the condition in (22) for both mixed-quality combinations to be unsalable holds only if the two goods are not perfectly substitutable in quality. That is, the negative impact of the quality of the lower-ranked good on the aggregate quality of any of the two mixed-quality combinations should not be perfectly compensable by the respective positive effect of the quality of the higher-ranked good. Otherwise, the quality of the rank 2 combination will be more similar to the quality of A_1B_1 than to the quality of A_2B_2. So, there will be positive demand for the rank 2 combination at the equilibrium prices. In the other two forms of the quality aggregation function we consider, namely, non-perfectly substitutable ($\sigma \to 1$) and perfectly complementary ($\sigma \to 0$) qualities, for the condition in (22) to be satisfied, the quality of the higher-ranked good of the less differentiated type is sufficient to exceed that of the lower-ranked good by at least six-fifths.

Second, if there is demand only for the best-ranked and the worst-ranked combinations, the optimal quality choice of the suppliers of each of the two types of goods is to maximize the difference between their products. That is, the firm with the higher maximal quality constraint of one of the types will choose that maximal quality for its good whereas the other firm offering the same type of good will choose the minimal quality constraint. Thus, by setting the range marked by the higher maximal quality constraint and the minimal quality constraint of a type to be a subinterval of the range marked by the corresponding quality constraints of the other type, the latter type will be more differentiated at the equilibrium established in proposition 4. In addition, we require the higher maximal quality constraint of the less differentiated type to exceed the minimal constraint by at least six-fifths to ensure the validity of the sufficient condition for (22) to hold as discussed in the previous paragraph.

Third, as the equilibrium established in proposition 2 for the setting with a single type A entrant, the equilibrium with two type A entrants and two type B entrants in proposition 4 is defined for distribution of the consumer identification variable that satisfies the conditions in (15) and (17). They are those proven by Shaked and Sutton (1982) to be necessary and sufficient for exactly two entrants with positive market shares to cover a single market at equilibrium. As discussed above, for the market to be covered at this setting, the quality of the worst-ranked combination needs to be sufficiently greater than the quality of the outside option, respectively not much smaller than the best-ranked combination. This is ensured by the first left-hand inequality condition in (23) and (24), respectively.

Finally, we establish the conditions for having exactly two entrants of a type at the entry-stage subgame equilibrium at a setting where there are more than two potential entrants. The key question is if two goods of each type could fit the market, whether a third good of either of the two types cannot enter and have a positive market share at equilibrium. In this sense, note that the optimal prices are negatively related to the number of market entrants. The more entrants, the stronger the price competition between them is. Therefore, if we can establish a condition at which the entry of just a third firm is effectively deterred because it faces too low best-response prices from its competitors, the condition will imply that these prices will be even lower in case of more than three entrants. The effective deterrence of a third entrant again depends on the relations between the qualities of the two types of goods and the corresponding ranking of the combinations they form. The equilibrium at which a potential third entrant is effectively excluded from the market is established in proposition 5 below.

Proposition 5. For $\sigma \to \{0,1\}$, let the consumer taste distribution be characterized by the following inequality condition:

$$\frac{4\left(\chi_{11}-\chi_{22}\right)}{6\chi_{11}-\chi_{22}}\bar{\theta} \leq \underline{\theta} < \frac{2\bar{\theta}}{3} \tag{25}$$

Then, if the quality-choice ranges of the three potential entrants with highest maximal quality constraints satisfy whichever of the following alternative relations:

$$\bar{F}_1 > \bar{G}_1 > 2\bar{G}_3 > 2\max\{\bar{F}_3,\underline{G}\} > 2\min\{\bar{F}_3,\underline{G}\} > 2\underline{F} \tag{26}$$

$$\bar{G}_1 > \bar{F}_1 > 2\bar{F}_3 > 2\max\{\bar{G}_3,\underline{F}\} > 2\min\{\bar{G}_3,\underline{F}\} > 2\underline{G} \tag{27}$$

a subgame-perfect equilibrium exists, which is established by the solution of the three-stage game below:

1. Only the two firms with the highest maximal quality constraints of each type enter the market, the rest do not enter.
2. The optimal quality choices of the two entrants imply maximum product differentiation (see the expressions in (D6) in the appendix)
3. The optimal prices are given by the expressions in (D2) in the appendix.

Proof: see section E of the appendix

The effective market exclusion of a third entrant of whichever type requires a wider range of quality choices $\left(\bar{G}_1 > 2\underline{G}\right)$ or $(\bar{F}_1 > 2\underline{F})$ than is sufficient for having the subgame equilibria with an exogenously set number of entrants in proposition 4. As a result, the condition for a covered market in (25) is more relaxed than in (15) and stricter than in (17). It restricts the lowest consumer taste from below instead of the higher maximal constraints from above. The optimal quality and price choices remain the same as in proposition 4.

2.4 Conclusion

In this paper we introduce a stylized model of price competition in two adjacent vertically differentiated markets for complementary products manufactured and offered by different firms. The setting analyzed here differs from the case considered in the classical literature on vertical differentiation where goods are assumed to be offered independently in a single market. By introducing vertical product differentiation in a multi-market setting we also suggest an alternative market structure, natural (endogenous) duopoly, instead of the classical (exogenous) duopoly structure commonly assumed in the existing literature on adjacent markets.

We show that when the two types of goods combined are complements, consumed in a fixed one-to-one ratio, the producers of the high-quality goods of each type might find it optimal under certain conditions to charge prices at which the combinations they could form with the low-quality goods of the other type are unsalable. As a result, if there is only a single entrant in one of the markets, at equilibrium it would charge a price for its good which makes all the combinations except the best one too expensive to be sold in the other market, which thus becomes a monopoly. This monopoly outcome in the adjacent market will prevail even if the distribution of consumer tastes satisfies the condition for a covered-duopoly market in a single-market setting. Specifically, we assume that the smallest consumer taste is more than two times but at most four times smaller than the largest consumer taste. Furthermore, with this condition fulfilled, neither market will be covered at equilibrium, which is to say that there will be at least one consumer who will prefer the free outside (non-market) option to the only available top-quality combination.

In a setup with two entrants of each type, the producers of the type of good with the more differentiated qualities will have more market power. Again, as in the single-entrant market setting, these producers might find it optimal under certain conditions to charge prices at which the combinations formed by high-quality good of the one type and low-quality good of the other type will not have positive market shares. Such an outcome where these mixed-quality combinations are effectively driven out of the market could occur at equilibrium only if the two types of goods are not perfect substitutes in qualities. In addition, the differentiation between the high and low quality of one of the types should be sufficiently larger than the differentiation between the high and low quality of the other type of good for the quality of the second-best combination to exceed the median between the best combination and the worst. The discrepancy between the two quality differentiations should also be restricted from above for the market to be covered by the worst-quality combination.

Further, we explore the conditions for having exactly two entrants of a type at equilibrium, given free market entry for the both types of firms. We show that a stricter condition on the consumer taste distribution must be imposed than that derived for the single-market case. In particular, for the efficient foreclosure of a third entrant, the upper bound on its quality choice needs to be exceeded at least twice by the upper bound on the choice of the firm supplying the best-quality good of the same type. The resulting market exclusion of the mixed-quality combinations provides a new explanation of the self-selection bias in consumption observed in some industries where there are no tying arrangements (e.g. business-trip transportation and accommodation, real-estate and furniture, PC hardware and software).

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Appendix (Mathematical Proofs)

A. Proof of Proposition 1

The proof of the sufficient condition for having at most two entrants of type B at equilibrium follows the procedure suggested by Shaked and Sutton (1982). However, here we apply it to the model setting with two types of complementary goods, single entrant of type A and consumers identified by taste but not by income.

Suppose that all n entrants of type B in the market have positive market shares. The respective profit-maximization problems that they solve at the pricing stage could be derived from the expressions in (14) by substituting sequentially for the marginal tastes from the expressions in (6), (7) and the demands in (8), given the ranking in table 1 as follows:

$$\max_{p_1^B} \Pi_1^B = p_1^B D_1^B = p_1^B \left(\overline{\theta} - \theta_{11/12}\right) = p_1^B \left(\overline{\theta} - \frac{p_1^B - p_2^B}{\chi_{11} - \chi_{12}}\right)$$

$$\max_{p_2^B} \Pi_2^B = p_2^B D_2^B = p_2^B \left(\theta_{11/12} - \theta_{12/13}\right) = p_2^B \left(\frac{p_1^B - p_2^B}{\chi_{11} - \chi_{12}} - \frac{p_2^B - p_3^B}{\chi_{12} - \chi_{13}}\right)$$

$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots$$

$$\max_{p_n^B} \Pi_n^B = p_n^B D_n^B = \begin{cases} p_n^B \left(\theta_{1n-1/1n} - \underline{\theta}\right), & \text{if } \theta_{1n/0} \leq \underline{\theta} \\ p_n^B \left(\theta_{1n-1/1n} - \theta_{1n/0}\right), & \text{if } \theta_{1n/0} > \underline{\theta} \end{cases} = \tag{A1}$$

$$= \begin{cases} p_n^B \left(\frac{p_{n-1}^B - p_n^B}{\chi_{1n-1} - \chi_{1n}} - \underline{\theta}\right), & \text{if } p_n^B \leq \chi_{1n}\underline{\theta} - p_1^A \\ p_n^B \left(\frac{p_{n-1}^B - p_n^B}{\chi_{1n-1} - \chi_{1n}} - \frac{p_1^A + p_n^B}{\chi_{1n}}\right), & \text{if } p_n^B > \chi_{1n}\underline{\theta} - p_1^A \end{cases}$$

The corresponding first-order optimality conditions could be represented by the following system of equations:

$$
\begin{aligned}
&\overline{\theta} - 2\theta_{11/12} - \frac{p_2^B}{\chi_{11} - \chi_{11}} = 0 \\
&\theta_{11/12} - 2\theta_{12/13} - \frac{p_2^B}{\chi_{11} - \chi_{12}} - \frac{p_3^B}{\chi_{12} - \chi_{13}} = 0 \\
&\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \\
&\begin{cases} \theta_{1n-1/1n} \quad -\underline{\theta} - \frac{p_n^B}{\chi_{1n-1} - \chi_{1n}} = 0, & \text{if } p_n^B \leq \chi_{1n}\underline{\theta} - p_1^A \\ \theta_{1n-1/1n} \quad -\theta_{1n/0} - \frac{p_n^B}{\chi_{1n-1} - \chi_{1n}} - \frac{p_n^B}{\chi_{1n}} = 0, & \text{if } p_n^B > \chi_{1n}\underline{\theta} - p_1^A \end{cases}
\end{aligned}
\tag{A2}
$$

The system of first-order optimality conditions in (A2) implies the following inequality relations of the marginal taste variables at equilibrium:

$$
\overline{\theta} > 2\theta_{11/12} > 4\theta_{12/13} > \dots > 2^{k-1}\theta_{1k-1/1k} > \dots > \begin{cases} 2^{n-1}\underline{\theta}, & \text{if } p_n^B \leq \chi_{1n}\underline{\theta} - p_1^A \\ 2^{n-1}\theta_{1n/0}, & \text{if } p_n^B > \chi_{1n}\underline{\theta} - p_1^A \end{cases}
\tag{A3}
$$

Hence, when condition (15) holds, we should have the following inequality fulfilled:

$$
\theta_{12/13} < \underline{\theta}
\tag{A4}
$$

which implies zero market share for the entrants with quality rank larger than two. Therefore, when condition (15) is satisfied at most two entrants of type B will enter the market at equilibrium. Q.E.D.

B. Proof of Proposition 2

Suppose one entrant of type A and n entrants of type B all with positive market shares. The profit-maximization problem that the single entrant of type A solves at the pricing stage could be derived from (13). As in appendix A above, we derive it by substituting for the demand from (8) and for the marginal tastes from the expressions in (6) and (7) according to the ranking in table 1 as follows:

$$\max_{p_1^A} \Pi_1^A = p_1^A D_1^A = p_1^A \left(\bar{\theta} - \theta_{1n/0}\right) = p_1^B \left(\bar{\theta} - \frac{p_1^A + p_n^B}{\chi_{1n}}\right) \tag{B1}$$

We first assume non-covered market at equilibrium to show that when such an equilibrium exists there cannot be more than a single entrant with positive market share.

The corresponding first-order optimality condition looks as follows:

$$\overline{\theta} - 2\theta_{1n/0} + \frac{p_n^B}{\chi_{1n}} = 0 \tag{B2}$$

The equation in (B2) implies the following inequality that must hold for $\theta_{1n/0}$ at equilibrium:

$$\theta_{1n/0} \geq \frac{\overline{\theta}}{2} \tag{B3}$$

which implies non-covered market outcome only if (17) holds.

Furthermore, combining (B3) with (A3) yields the following result for the marginal tastes at equilibrium:

$$\theta_{1n/0} > \theta_{11/12} > 2\theta_{12/13} > \ldots > 2^{k-2}\theta_{1k-1/1k} > \ldots > 2^{n-1}\theta_{1n-1/1n} \tag{B4}$$

which precludes the existence of a non-covered market equilibrium for $n > 1$. Exactly one entrant of type B could have positive market share if a non-covered equilibrium exists.

The condition in (17), however, is only necessary but not sufficient for the existence of a non-covered equilibrium. The sufficient condition is the payoff of the entrant of type A to be higher in case of non-covered market than in the case of covered market. So, now we derive these payoffs explicitly.

Since the existence of a non-covered market equilibrium implies a single entrant in market B, there is only one combination available, A_1B_1. Accordingly, the entrant in market B faces the same profit-maximization problem as the entrant in market A:

$$\max_{p_1^A} \Pi_1^A = p_1^A \left(\overline{\theta} - \frac{p_1^A + p_1^B}{\chi_{11}} \right) \max_{p_1^B} \Pi_1^B = p_1^B \left(\overline{\theta} - \frac{p_1^A + p_1^B}{\chi_{11}} \right) \tag{B5}$$

Therefore, the solution for their prices is also symmetric:

$$p_1^{A*} = p_1^{B*} = \frac{\overline{\theta}\chi_{11}}{3} \tag{B6}$$

By substituting for the prices from (B6) in the expression for the higher-quality marginal taste variable of combination A_1B_1 we show it to be given by a fixed two-thirds share of the highest consumer taste $\overline{\theta}$:

$$\theta_{11/0}^* = \frac{p_1^{A*} + p_1^{B*}}{\chi_{11}} = \frac{2}{3}\overline{\theta} \tag{B7}$$

Hence, the necessary condition in (17) for the non-covered assumption to hold at equilibrium could be strengthened as follows:

$$\underline{\theta} < \frac{2}{3}\overline{\theta} \tag{B8}$$

Finally, we derive the payoff of the entrant of type A by substituting for the optimal prices from (B6) in the expression for Π_1^A in (B5):

$$\Pi_1^{A*} = \frac{\overline{\theta}^2}{9}\chi_{11} \tag{B9}$$

The existence of a covered-market equilibrium where all the *n* entrants of type B have positive market shares implies the following inequality to hold:

$$\theta_{1n/0} = \frac{p_1^{A**} + p_n^{B**}}{\chi 1n} \leq \underline{\theta} \tag{B10}$$

where p_1^{A**} stands for the optimal price of the good of the entrant of type A given the market is covered by combination A_1B_n while we denote by p_n^{B**} the optimal price of the good of the lowest-ranked actual entrant in market B.

We could re-write the inequality in (B10) by leaving only p_1^{A**} on the left-hand side:

$$p_1^{A**} \leq \underline{\theta}\chi 1n - p_n^{B**} \tag{B11}$$

After setting the price of the lowest-ranked good of type B to be equal to the zero production cost of its producer, the right-hand side of (B11) gives us the value of the maximal price the entrant of type A could charge its good while still preserving the coveredness of the market:

$$\overline{p}_1^{A**} = \underline{\theta}\chi_{1n} \tag{B12}$$

Since at a covered-market equilibrium, all consumers buy the good of the entrant of type A, its maximal revenue is given by the following expression:

$$\Pi_1^{A**} = \frac{\overline{\theta}^2}{9}\chi_{11} \tag{B13}$$

Comparing the expressions for the profit of the entrant of type A in (B9) and (B13) allows us to derive the condition for the former to exceed the latter:

$$\chi_{1n} < \frac{\overline{\theta}^2}{9\left(\overline{\theta}-\underline{\theta}\right)\underline{\theta}}\chi_{11} \tag{B14}$$

Note that the fraction in front of χ_{11} on the right-hand side of the inequality in (B14) is strictly decreasing in $\underline{\theta}$ for $\underline{\theta}$ satisfying the condition in (17):

$$\frac{\partial\left(\frac{\overline{\theta}^2}{9(\overline{\theta}-\underline{\theta})\underline{\theta}}\right)}{\partial\underline{\theta}} = -\frac{\overline{\theta}^2(\overline{\theta}-2\underline{\theta})}{9(\overline{\theta}-\underline{\theta})^2\underline{\theta}^2} < 0, \text{ for } \underline{\theta} < \frac{\overline{\theta}}{2} \tag{B15}$$

Hence, we could derive the lowest value this fraction could have by substituting for $\underline{\theta} = \frac{\overline{\theta}}{2}$ in (B14). Then, the condition for the profit of the entrant of type A to be larger at a non-covered equilibrium in (B14) could be re-written as follows:

$$\chi_{1n} < \frac{4}{9}\chi_{11} \tag{B16}$$

Now for proving the result in proposition 2, it only remains to check for which of the three cases of substitutability between the qualities of the two types of goods the condition in (16) is sufficient for the inequality in (B16) to be satisfied.

Beforehand, however, note that since $\chi_{1n} < \chi_{11}$ by definition, the inequality in (B14) would always hold as long as the numerator of the fraction in front of χ_{11} exceeds its denominator. This implies that we could have covered-market equilibrium only if the spread of consumer tastes is restricted from below as follows:

$$\overline{\theta}^2 \leq 9\left(\overline{\theta}-\underline{\theta}\right)\underline{\theta} \;\; \Rightarrow \underline{\theta} \;\; \leq \frac{\overline{\theta}}{7} \tag{B17}$$

Then, the result in (A3) implies that when (B16) holds at most three entrants could have positive market shares. Thus, since the inequality in (B16) is a necessary condition for having a covered-market equilibrium, we cannot have more than three actual entrants at such an equilibrium.

Below, we translate the sufficient condition in (B16) for having non-covered monopoly equilibrium outcome in market B into a particular condition on the maximal-quality constraints of the potential entrants for each of the three cases of substitutability between the qualities of the two types of goods.

First, if the two types of goods are perfect substitutes in qualities ($\sigma \to \infty$), the inequality in (B16) takes the following form:

$$9f(A_1)+9g(B_2) < 4f(A_1)+4g(B_1) \;\; \Rightarrow \;\; f(A_1) < \frac{4g(B_1)-9g(B_2)}{5} \tag{B18}$$

which can never be satisfied as long as the condition $(f(A_1) > g(B_1))$ for having the ranking of the combinations in table 1 holds.

Second, if the two types of goods are imperfect substitutes in qualities ($\sigma = 1$), the inequality in (B16) takes the following form:

$$9f(A_1)g(B_2) < 4f(A_1)g(B_1) \;\; \Rightarrow \;\; g(B_1) > \frac{9}{4}g(B_2) \tag{B19}$$

Third, if the two types of goods are perfect complements in qualities ($\sigma \to 0$), the inequality in (B16) takes the following form:

$$9g(B_2) < 4g(B_1) \;\; \Rightarrow \;\; g(B_1) > \frac{9}{4}g(B_2) \tag{B20}$$

The inequality conditions in (B19) and (B20) are trivially satisfied if the condition in (16) holds. Hence, condition (16) is sufficient for having a monopoly non-covered market outcome at equilibrium provided that the spread of consumer tastes satisfies the necessary condition in (B8). When the latter condition does not hold $\left(\text{i.e. if } \underline{\theta} \geq \frac{2}{3}\overline{\theta} > \frac{1}{2}\overline{\theta}\right)$, however, the result in (A3) implies that the market will be covered by the best-ranked combination A_1B_1 at equilibrium. Therefore, condition (16) is sufficient for having a monopoly market outcome independent on whether the necessary condition in (16) is satisfied. That is, its weaker statement in (17) just ensures that the monopoly outcome will occur at a non-covered market equilibrium. Q.E.D.

C. Proof of Proposition 3

In this section of the appendix, we derive the conditions for the following inequality to hold:

$$\chi_{12} > \chi_{21} \tag{C1}$$

i.e. A_1B_2 to be preferred to A_2B_1, at all the three special cases of the quality aggregation function.

<u>Case 1:</u> $f(A_i)$ and $g(B_j)$ - perfect substitutes ($\rho \to 1$)

First, let's suppose that the qualities of goods of type A and the goods of type B are perfect substitutes. That is, the quality of any combination A_iB_j is given as a linear sum of the qualities of the goods of types A and B that form it: $\chi_{ij} = f(A_i) + g(B_j)$ Substituting for the combination qualities in (C1) yields the following result:

$$f(A_1) + g(B_2) > f(A_2) + g(B_1) \tag{C2}$$

which after rearrangement takes the form below:

$$f(A_1) - f(A_2) > g(B_1) - g(B_2) \tag{C3}$$

i.e. the difference between the qualities of the goods of type A should exceed the difference between the qualities of the goods of type B.

Case 2: $f(A_i)$ and $g(B_j)$ – non-perfect substitutes $(\rho \to 0)$

Second, let's assume that the goods of type A and the goods of type B are non-perfect substitutes. That is, the qualities of the combinations that any pair of goods of types A and B form are given by the product of their qualities (particular Cobb-Douglas functional form): $\chi_{ij} = f(A_i)\, g(B_j)$

Substituting for the combination qualities in (C1) yields the following result:

$$f(A_1)\, g(B_2) > f(A_2)\, g(B_1) \tag{C4}$$

which after rearrangement takes the form below:

$$\frac{f(A_1)}{f(A_2)} > \frac{g(B_1)}{g(B_2)} \tag{C5}$$

i.e. the ratio between the high quality and the low quality of type A should exceed the respective ratio of the qualities of the goods of type B.

Case 3: $f(A_i)$ and $g(B_j)$ – perfect complements $(\rho \to -\infty)$

Third, let's assume that the goods of type A and the goods of type B are perfect complements. That is, the qualities of the combinations that any pair of goods of types A and B form are given by the smaller of their qualities (particular Leontief functional form): $\chi_{ij} = \min\left[f(A_i), g(B_j)\right]$

Substituting for the combination qualities in (C1) yields the following result:

$$\min[f(A_1), g(B_2)] > \min[f(A_2), g(B_1)] \tag{C6}$$

which is equivalent to the following sufficient condition:

$$g(B_2) > f(A_2) \tag{C7}$$

i.e. the low quality of type B should exceed the low quality of type A.

The quality ranking in (18) is sufficient for all the three conditions (C3), (C5) and (C7) to be fulfilled. Hence, for having (C1) satisfied, it is sufficient the quality ranking in (18) to hold. Q.E.D.

The corresponding combination rankings are represented graphically by iso-quality curve mapping for each of the three cases on figure 4 below.

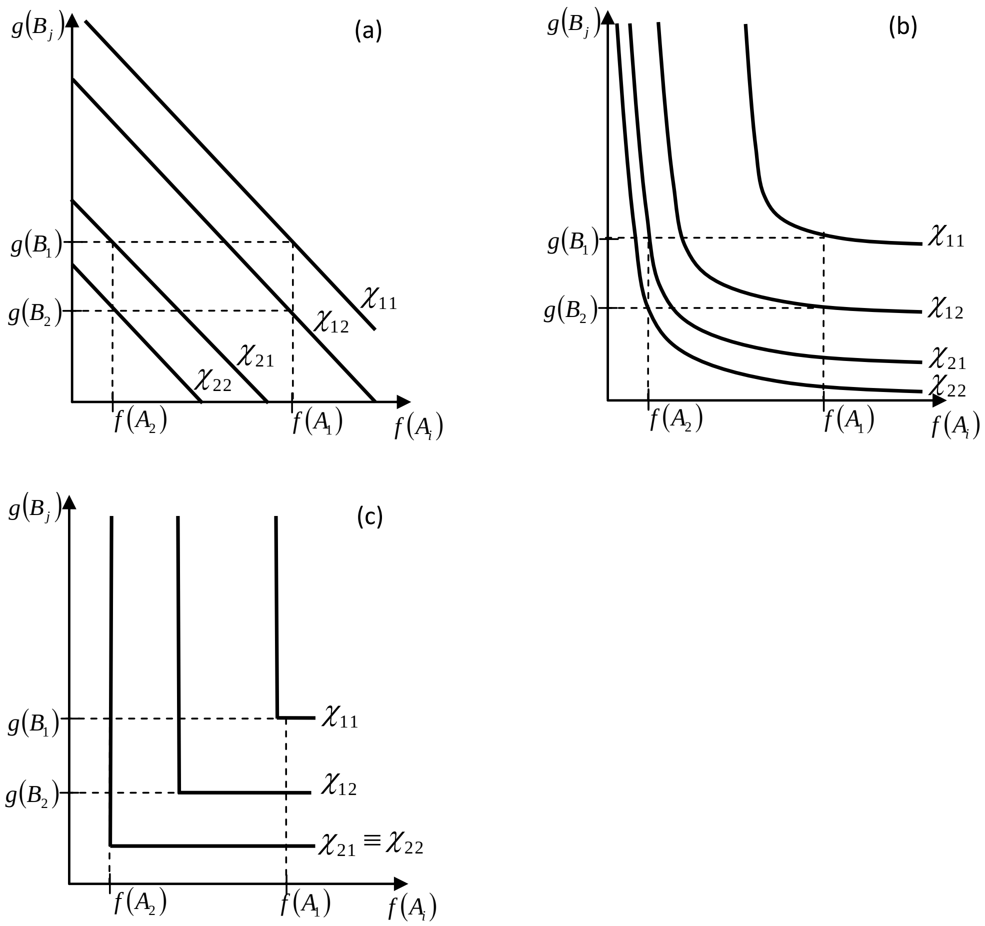

Figure 4. Iso-quality curve maps of the combination quality ranking corresponding to the good quality ranking in (18) for: (a) perfect substitutes, (b) non-perfect substitutes, (c) perfect complements.

For any given elasticity of substitution between the two types of goods, each iso-quality curve represents the set of pair values $\left(f(A_i), g(B_j)\right)$ which characterize combinations with equal quality (rank). The more up and to the right an iso-quality curve is located, the higher is the quality (rank) of the combinations formed by the pair values that belong to the curve. As long as the values of the two qualities of type B lay in the interval $(f(A_2), f(A_1))$, qualities of type A are determinative for the ranks of the combinations they form. That is, A_1B_2 is better than A_2B_1 $(\chi_{12} > \chi_{21})$, at any of the three considered cases of constant elasticity of substitution between the two types of goods.

Note that if we swap the values between the axes of any of the graphs on figure 4, the two types of goods will trade their roles. The qualities of type B will become determinative for the ranks of the combinations they form and A_2B_1 will be better than A_1B_2 $(\chi_{21} > \chi_{12})$.

D. Proof of Proposition 4

Here, we derive the solutions for the subgame equilibria at the quality-choice and at the pricing stages of the game, respectively as established in proposition 4. The equilibrium solutions are derived by backward induction. Therefore, we start with the solution for the pricing stage.

The number of entrants is exogenously set to two of each type. For now we assume that the qualities of goods are related according to the condition in (18). Latter we will show how the results change if the condition in (19) holds instead. The condition in (18) implies that the combinations formed by the four entrants are ranked by quality according to ranking 1 in table 2. We are interested in establishing an equilibrium at which only the best-quality and worst-quality combinations, A_1B_1 and A_2B_2, have positive market shares at equilibrium. At the equilibrium established in proposition 4 the market is covered, i.e. $\theta_{22/0} < \underline{\theta}$. All the assumptions together imply the following expressions for the profit-maximization problems with respect to prices:

$$\begin{aligned}
&\max_{p_1^A} \Pi_1^A = p_1^A D_1^A = p_1^A \left(\bar{\theta} - \theta_{11/22}\right) = p_1^A \left(\bar{\theta} - \frac{p_1^A + p_1^B - p_2^A - p_2^B}{\chi_{11} - \chi_{22}}\right) \\
&\max_{p_1^B} \Pi_1^B = p_1^B D_1^B = p_1^B \left(\bar{\theta} - \theta_{11/22}\right) = p_1^B \left(\bar{\theta} - \frac{p_1^A + p_1^B - p_2^A - p_2^B}{\chi_{11} - \chi_{22}}\right) \\
&\max_{p_2^A} \Pi_2^A = p_2^A D_2^A = p_2^A \left(\theta_{11/22} - \underline{\theta}\right) = p_2^A \left(\frac{p_1^A + p_1^B - p_2^A - p_2^B}{\chi_{11} - \chi_{22}} - \underline{\theta}\right) \\
&\max_{p_2^B} \Pi_2^B = p_2^B D_2^B = p_2^B \left(\theta_{11/22} - \underline{\theta}\right) = p_2^B \left(\frac{p_1^A + p_1^B - p_2^A - p_2^B}{\chi_{11} - \chi_{22}} - \underline{\theta}\right)
\end{aligned} \tag{D1}$$

The two high-quality producers of type A and B face identical problems. The same holds true also for the low-quality producers of type A and B. Therefore, the solutions for the optimal prices are symmetric:

$$p_1^{A*} = p_1^{B*} = \frac{3\bar{\theta} - 2\underline{\theta}}{5}(\chi_{11} - \chi_{22})$$
$$p_2^{A*} = p_2^{B*} = \frac{2\bar{\theta} - 3\underline{\theta}}{5}(\chi_{11} - \chi_{22}) \tag{D2}$$

Substituting for the prices back in the expressions for the profits yields the following expressions for the optimal profits:

$$\Pi_1^{A*} = \Pi_1^{B*} = \frac{\left(3\bar{\theta} - 2\underline{\theta}\right)^2}{25}(\chi_{11} - \chi_{22})$$
$$\Pi_2^{A*} = \Pi_2^{B*} = \frac{\left(2\bar{\theta} - 3\underline{\theta}\right)^2}{25}(\chi_{11} - \chi_{22}) \tag{D3}$$

Note that the condition in (17) is stricter than the necessary condition for A_2B_2 to have positive market share:

$$D_{22} = \theta_{11/22} - \underline{\theta} = \frac{2\bar{\theta} - 3\underline{\theta}}{5} > 0 \; for \; \underline{\theta} < \frac{2\bar{\theta}}{3} \tag{D4}$$

Both the expressions in (D3) are increasing in χ_{11} and decreasing in χ_{22}. Also, for any ρ both χ_{11} and χ_{22} are strictly non-decreasing in the qualities of the goods that form them. Hence, the equilibrium solutions for the optimal qualities are given by the upper-bound (maximal quality) constraints for the high-quality goods and by the lower-bound constraints for the low-quality goods:

$$f^*(A_1) = \bar{F}_{m_1}; f^*(A_2) = \underline{F}$$
$$g^*(B_1) = \bar{G}_{n_1}; g^*(B_2) = \underline{G} \tag{D5}$$

Without loss of generality, we could assume that the two entrants of each type are exactly the potential entrants with highest maximal quality constraints. Then, we could mark the optimal quality choices above by the indices of the initial sequence of maximal quality constraints assigned at the entry stage:

$$f^*(A_1) = \bar{F}_1; f^*(A_2) = \underline{F}$$
$$g^*(B_1) = \bar{G}_1; g^*(B_2) = \underline{G} \tag{D6}$$

Accordingly, by substituting for the optimal quality choices from (D6) in condition (18), it takes the following form:

$$\bar{F}_1 > \bar{G}_1 > \underline{G} > \underline{F} \tag{D7}$$

The results so far are based on the assumption that only the best-quality and worst-quality combinations, A_1B_1 and A_2B_2, have positive market shares at equilibrium. To hold at equilibrium they should imply that the mixed-quality combinations, A_1B_2 and A_2B_1, have no positive market shares at the prices expressed in (D2).

Note that the symmetric prices for the high-quality and low-quality goods of both types imply that the two middle quality cost the same:

$$p_1^{A*} + p_2^{B*} = p_2^{A*} + p_1^{B*} \tag{D8}$$

However, when (D7) holds, A_1B_2 is strictly preferred to A_2B_1. Hence, nobody would buy A_2B_1 at the optimal prices of the goods that form it.

Then, for the sales of A_1B_2 to be foreclosed, the following condition must hold:

$$D_{12} = \theta_{11/12} - \theta_{12/22} \leq 0 \tag{D9}$$

Substituting for the optimal prices from (D2) in the expressions for $\theta_{11/12}$ and $\theta_{12/22}$

yields the following explicit expressions in terms of the combination qualities:

$$\begin{aligned}\theta^*_{11/12} &= \frac{p_1^{B*}-p_2^{B*}}{\chi_{11}-\chi_{12}} = \frac{\left(\bar{\theta}+\underline{\theta}\right)\left(\chi_{11}-\chi_{22}\right)}{5\left(\chi_{11}-\chi_{12}\right)} \\ \theta^*_{12/22} &= \frac{p_1^{A*}-p_2^{A*}}{\chi_{12}-\chi_{22}} = \frac{\left(\bar{\theta}+\underline{\theta}\right)\left(\chi_{11}-\chi_{22}\right)}{5\left(\chi_{12}-\chi_{22}\right)}\end{aligned} \tag{D10}$$

For (D9) to hold, the combination qualities must satisfy the following inequality:

$$\chi_{12} \leq \frac{\chi_{11}+\chi_{22}}{2} \tag{D11}$$

Next, we need to check how this condition on the combination qualities translates into corresponding condition on the good qualities for different values of the constant elasticity of substitution σ. We consider again the three standard cases.

<u>Case 1:</u> $f(A_i)$ and $g\left(B_j\right)$ - perfect substitutes $(\rho \to 1)$

When $\sigma \to \infty$ i.e. $\rho \to 1$ and the quality accumulation function is linear, the inequality in (D12) takes the following form:

$$f(A_1)+g(B_2) \leq \frac{f(A_1)+g(B_1)+f(A_2)+g(B_2)}{2},\ \sigma \to \infty \tag{D12}$$

which holds only when the inequality below is satisfied:

$$f(A_1)-f(A_2) \leq g(B_1)-g(B_2) \tag{D13}$$

The condition in (D13) contradicts not only (18) but also (C3). Therefore, the sales of combination A_1B_2 cannot be efficiently foreclosed if $\sigma \to \infty$ and the two types of good are perfect substitutes.

<u>Case 2:</u> $f(A_i)$ and $g\left(B_j\right)$ – non-perfect substitutes $(\rho \to 0)$

When $\sigma = 1$ i.e. $\rho \to 0$ and the quality accumulation function gives the combination quality as a product of the qualities of the goods that form it, the inequality in (D13) takes the following form:

$$f(A_1)\,g(B_2) \leq \frac{f(A_1)\,g(B_1)+f(A_2)\,g(B_2)}{2},\ \sigma=1 \tag{D14}$$

which is satisfied only when the following inequality holds:

$$g(B_1) \geq \left(2-\frac{f(A_2)}{f(A_1)}\right) g(B_2) \tag{D15}$$

<u>Case 3:</u> $f(A_i)$ and $g\left(B_j\right)$ – perfect complements ($\rho \to -\infty$)

When $\sigma=0$ i.e. $\rho \to -\infty$ and the quality accumulation function gives the combination quality as equal to the smaller of the qualities of the goods that form it, the inequality in (D11) takes the following form:

$$\min\left[f(A_1),g(B_2)\right] \leq \frac{\min\left[f(A_1),g(B_1)\right]+\min\left[f(A_2),g(B_2)\right]}{2},\ \sigma=0 \tag{D16}$$

which is consistent with the condition in (18) only if the following inequality holds:

$$g(B_1) \geq 2g(B_2)-f(A_2) \tag{D17}$$

Substituting for the optimal quality choices from (D6) in the conditions in (D15) and (D17) yields the following inequality constraints that should hold for the bounds of the quality choices:

$$\bar{G}_1 \geq \left(2-\frac{\underline{F}}{\bar{F}}\right)\underline{G} \tag{D18}$$

$$\bar{G}_1 \geq 2\underline{G} - \underline{F} \tag{D19}$$

Finally, we need to check the validity of the assumption for covered market at equilibrium. The condition for covered market is given by the inequality:

$$\theta_{22/0} = \frac{p_2^{A*} + p_2^{B*}}{\chi_{22}} \leq \underline{\theta} \tag{D20}$$

which after substituting for the optimal prices from (D1) and (D2) takes the following form in terms of the combination qualities:

$$\frac{4\left(\chi_{11} - \chi_{22}\right)\bar{\theta}}{6\chi_{11} - \chi_{22}} \leq \underline{\theta} \tag{D21}$$

For the restriction on $\underline{\theta}$ in (15) to be stricter than the constraint in (D21), the following condition must hold:

$$\chi_{11} \leq \frac{3}{2}\chi_{22} \tag{D22}$$

Below, we translate it into conditions on the qualities of the goods only for the case of $\sigma = 1$ and $\sigma = 0$.

When $\sigma = 1$ i.e. $\rho \to 0$ and the quality accumulation function gives the combination quality as a product of the qualities of the goods that form it, the inequality in (D22) takes the following form:

$$f\left(A_1\right)g\left(B_1\right) \leq \frac{3f\left(A_2\right)g\left(B_2\right)}{2},\ \sigma = 1 \tag{D23}$$

which is satisfied only when the following inequality holds:

$$g(B_1) \leq \frac{3}{2}\frac{f(A_2)}{f(A_1)}g(B_2) \tag{D24}$$

When $\sigma = 0$ i.e. $\rho \to -\infty$ and the quality accumulation function gives the combination quality as equal to the smaller of the qualities of the goods that form it, the inequality in (D22) takes the following form:

$$\min[f(A_1), g(B_1)] \leq \frac{3\min[f(A_2), g(B_2)]}{2},\ \sigma = 0 \tag{D25}$$

which is consistent with the condition in (18) only when the following inequality holds:

$$g(B_1) \leq \frac{3}{2}f(A_2) \tag{D26}$$

Substituting for the optimal quality choices from (D6) in the conditions in (D24) and (D26) yields the following inequality constraints that should hold for the bounds of the quality choices:

$$\bar{G}_1 \leq \frac{3}{2}\frac{\underline{F}}{\bar{F}_1}\underline{G},\ \sigma = 1 \tag{D27}$$

$$\bar{G}_1 \geq \frac{3}{2}\underline{F},\ \sigma = 0 \tag{D28}$$

Combining the conditions in (D18) and in (D27) gives the interval in which the maximal quality constraint of B_2 should lay in order when $\sigma = 1$ the conditions in (15) and (17) to be sufficient for having a solution with exogenously set two entrants of each type so that the mixed-quality combinations have no positive market shares and the market is covered. The interval is given by the following expression:

$$\left(2-\frac{\underline{F}}{\bar{F}_1}\right)\underline{G} \leq \bar{G}_1 \leq \frac{3}{2}\frac{\underline{F}}{\bar{F}_1}\underline{G},\ \sigma = 1 \tag{D29}$$

which is feasible only if the differentiation between the goods of type A is limited as required by the following condition:

$$\underline{F} > \frac{4}{5}\bar{F}_1 \tag{D30}$$

The analogous condition for $\sigma = 0$ is given by combining the conditions in (D19) and (D28):

$$2\underline{G} - \underline{F} \leq \bar{G}_1 \leq \frac{3}{2}\underline{F},\ \sigma = 0 \tag{D31}$$

which is feasible only when the lower bounds on the qualities of the two types satisfy the following condition:

$$\underline{F} > \frac{4}{5}\underline{G} \tag{D32}$$

The conditions in (D29) and (D30) are stricter than the conditions in (D31) and (D32).

To see that the inequality relations in (23) is sufficient for (D29) and (D30) to be satisfied, note that when $\bar{F}_1 > \frac{6}{5}\underline{G}$ as ensured by (23), the following inequality must hold:

$$\frac{5}{4} > \frac{3}{2}\frac{\underline{G}}{\bar{F}_1} \tag{D33}$$

Hence, having $\frac{3}{2}\frac{\underline{F}}{\bar{F}_1}\underline{G} > \bar{F}_1$ (ensured by (23) together with (D33)) implies that both the constraint from above in (D29) and the inequality in (D30) hold.

Furthermore, when (D30) is satisfied, the following inequality must also hold:

$$\frac{6}{5} > 2 - \frac{\underline{F}}{\bar{F}_1} \tag{D34}$$

Hence, having $\bar{G}_1 > \frac{6}{5}\underline{G}$ which is ensured by (23) together with (D34) implies also that the constraint from below in (D29) holds.

Since the condition in (23) ensures the validity of the conditions in (D29)–(D32) for having covered market with excluded mixed-quality combinations at both $\sigma = 0$ and $\sigma = 1$, we have all the assumptions for the subgame equilibrium solutions in (D2) and (D6) to be fulfilled. That is, the condition in (23) is sufficient for having the solution established in proposition 4. Q.E.D.

Since, the constraints in (19) imply having ranking 2 which is a mirror image of ranking 1, the result in (24) could be derived directly from (23) by trading the respective bounds of the qualities of type A and B.

E. Proof of Proposition 5

The single-good conditions on consumer-taste distribution in (15) and (17) are only sufficient but not necessary condition for the solution established in proposition 4.

Indeed, let's assume the less strict condition in (D4) instead of the one in (17) for having positive market share of A_2B_2. If we skip then the condition in (D22) so that $\underline{\theta}$ is limited by the stricter condition for covered market in (D21) instead of being constrained by the restriction in (15), we will finish with having the condition in (25) at which still two entrants of each type could make the same optimal pricing and quality choices. However, since (D22) is not required to hold, we do not need to restrict from above $\bar{G}_1$ and $\bar{F}_1$ in (D29) and (D30). Hence, the condition in (23) could be replaced by the following less restrictive inequality relations between the bounds of the range of the quality choices[8]:

$$\bar{F}_1 > \bar{G}_1 > 2\underline{G} > 2\underline{F} \tag{E1}$$

Correspondingly, the condition in (24) could be replaced by the following more relaxed inequality relations:

$$\bar{G}_1 > \bar{F}_1 > 2\underline{F} > 2\underline{G} \tag{E2}$$

The pairwise comparison of the conditions in (15), (17) and (23) versus the conditions in (25) and (E1) shows that there is a trade-off between the strictness of the conditions on the consumer taste and the strictness of the conditions on the quality bounds for having an efficient foreclosure of the mixed-quality combinations. To be consistent with the existing literature on single-good markets for vertically differentiated products, in proposition 4 we gave preference to the conditions that restrict more the quality-choice bounds than to

[8]Note that the inequality in (E1) is stricter than the inequality in (D18).

the ones that limit the consumer-taste distribution.

In proposition 5, however, we assume the condition in (25) because the inequalities in (23) are too restrictive and do not comply with the requirements for having an efficient foreclosure of the sales of an eventual third entrant in the market. The very requirements are derived below to show that the conditions in (26) or (27) are sufficient to ensure only two entrants of a type at equilibrium without violating the condition in (E1). We consider only the cases of perfect complements and non-perfect substitutes, $\sigma = 0$ and $\sigma = 1$, since the case of perfect substitutes $\sigma \to \infty$ was proven to be inconsistent with the conditions for having the mixed-quality combinations excluded from the market.

Suppose we have a third entrant of type A. Let's denote it by A_3. Combination A_3B_1 is the highest-ranked combination that could be formed with A_3. There are two possibilities. A_3B_1 could be of better quality than A_2B_2 or of worse.

For A_3B_1 to be of better quality than A_2B_2, the following condition must hold:

$$\chi_{31} > \chi_{22} \tag{E3}$$

When $\sigma = 1$ i.e. $\rho \to 0$ and the quality accumulation function gives the combination quality as a product of the qualities of the goods that form it, the inequality in (E3) takes the following form:

$$f(A_3)\, g(B_1) > f(A_2)\, g(B_2)\,,\ \sigma = 1 \tag{E4}$$

When $\sigma = 0$ i.e. $\rho \to -\infty$ and the quality accumulation function gives the combination quality as equal to the smaller of the qualities of the goods that form it, the inequality in (E1) takes the following form:

$$\min\left[f\left(A_3\right),g\left(B_1\right)\right] > \min\left[f\left(A_2\right),g\left(B_2\right)\right],\ \sigma = 0 \tag{E5}$$

which is inconsistent with the condition in (18) for any $f\left(A_3\right) < f\left(A_2\right)$. That is, we cannot have A_3B_1 of better quality than A_2B_2 when the qualities of the two goods are perfect complements.

Given that the conditions in (25), (E1) and (E4) are satisfied, the market share of A_3B_1 is given by the following expression:

$$D_{31} = \theta_{11/31} - \theta_{31/22} = \frac{p_1^A - p_3^A}{\chi_{11} - \chi_{31}} - \frac{p_3^A + p_1^B - p_2^A - p_2^B}{\chi_{31} - \chi_{22}} \tag{E6}$$

which after substituting for the optimal prices from (E2) takes the form:

$$D_{31}^{*} = \frac{\left[\left(\chi_{11} + 2\chi_{31} - 3\chi_{22}\right)\bar{\theta} - \left(4\chi_{11} - 2\chi_{31} - 2\chi_{22}\right)\underline{\theta} - 5p_3^A\right]\left(\chi_{11} - \chi_{22}\right)}{5\left(\chi_{11} - \chi_{31}\right)\left(\chi_{31} - \chi_{22}\right)} \tag{E7}$$

The expression in (E7) is negative as long as the following inequality holds:

$$\underline{\theta} > \frac{\left(\chi_{11} + 2\chi_{31} - 3\chi_{22}\right)}{4\chi_{11} - 2\left(\chi_{31} + \chi_{22}\right)}\bar{\theta} \tag{E8}$$

The condition in (D21) is stricter than the one in (E8) given the inequality below:

$$\chi_{31} < \frac{2\chi_{11}^2 - \chi_{11}\chi_{22} + \chi_{22}^2}{2\left(2\chi_{11} - \chi_{22}\right)} \tag{E9}$$

which is fulfilled whenever χ_{31} satisfies the following stricter inequality:

$$\chi_{31} < \frac{\chi_{11}}{2} \tag{E10}$$

When $\sigma = 1$ i.e. $\rho \to 0$ and the quality accumulation function gives the combination quality as a product of the qualities of the goods that form it, the inequality in (E10) takes the following form:

$$f(A_3)\,g(B_1) < \frac{f(A_1)\,g(B_1)}{2},\ \sigma = 1 \tag{E11}$$

which holds for any $f(A_1)$ that satisfies the following inequality:

$$f(A_1) > 2f(A_3) \tag{E12}$$

Combining the conditions in (E4) and (E12) yields the following general condition for efficient foreclosure of A_3B_1 when it is better than A_2B_2:

$$f(A_1) > 2f(A_3) > 2\frac{g(B_2)}{g(B_1)}f(A_2) \tag{E13}$$

Note that if we have a third entrant of type B, the same logic would define the condition for efficient foreclosure of A_1B_3 when it is better than A_2B_2 To derive the corresponding condition we should just trade $f(A_i)$ and $g\left(B_j\right)$ for $i = j$:

$$g(B_1) > 2g(B_3) > 2\frac{f(A_2)}{f(A_1)}g(B_2) \tag{E14}$$

Alternatively, when the condition in (E4) does not hold so that A_3B_1 is of worse quality than A_2B_2, for efficient foreclosure of the sales of A_3, it is sufficient to have the following inequality satisfied:

$$\underline{\theta} > \theta_{22/31} \tag{E15}$$

which after substituting for the optimal prices from (E2) takes the form:

$$\underline{\theta} > \frac{(\chi_{11} - \chi_{22})\,\bar{\theta}}{4\chi_{11} + \chi_{22} - 5\chi_{31}} \tag{E16}$$

The condition in (D21) is stricter than the one in (E16) as long as the following inequality holds:

$$\chi_{31} < \frac{2\chi_{11} + \chi_{22}}{4} \tag{E17}$$

When $\sigma = 1$ i.e. $\rho \to 0$ and the quality accumulation function gives the combination quality as a product of the qualities of the goods that form it, the inequality in (E17) takes the following form:

$$f(A_3) < \frac{f(A_1)}{2} + \frac{f(A_2)\,g(B_2)}{4g(B_1)},\ \sigma = 1 \tag{E18}$$

When $\sigma = 0$ i.e. $\rho \to -\infty$ and the quality accumulation function gives the combination quality as equal to the smaller of the qualities of the goods that form it, the inequality in (E15) takes the following form:

$$f(A_3) < \frac{f(A_1)}{2} + \frac{g(B_2)}{4},\ \sigma = 0 \tag{E19}$$

Both inequalities in (E18) and (E19) hold for any $g(B_1)$ that satisfies the following inequality:

$$f(A_1) > 2f(A_3) \tag{E20}$$

Note that if we have a third entrant of type B, the same logic would define the condition for efficient foreclosure of B_3 when A_1B_3 is of worse quality than A_2B_2. To derive the corresponding condition we should just swap $f(A_i)$ with $g\left(B_j\right)$ in (E19):

$$g(B_1) > 2g(B_3) \tag{E21}$$

The comparison between the inequality constraints in (E13) and (E20) implies that the condition in (E20) is stricter and therefore sufficient for having the sales of A_3 efficiently foreclosed. Similarly, the condition in (E21) is sufficient for having the sales of B_3 efficiently foreclosed.

Both the conditions in (E20) and in (E21), however, are satisfied as long as the maximal quality constraints satisfy whichever of the following inequality relations:

$$\bar{F}_1 > \bar{G}_1 > 2\bar{G}_3 > 2\bar{F}_3 \tag{E22}$$

$$\bar{G}_1 > \bar{F}_1 > 2\bar{F}_3 > 2\bar{G}_3 \tag{E23}$$

Hence, both the conditions in (26) and in (27) ensure that at most two firms of a type will enter the market at equilibrium. Q.E.D.

The inequality relations in (26) combine the conditions in (E1) and (E22) in the same way as the conditions in (E2) and (E23) are combined in the inequality relations in (27). Therefore, the inequality relations in (26) and (27) also ensure efficient foreclosure of the mixed-quality combinations and covered market at equilibrium. The choice of the relations between the maximal quality constraints $\bar{G}_3$ and $\bar{F}_3$ in (E22) and (E23) is not occasional. Indeed, if there is a third entrant, the optimal quality choice of the second-ranked entrant of its type will be set so that it is not possible to be exceeded by the quality choice of the third entrant. For example, if there is a third entrant of type A, the optimal choice for A_2 will be slightly above the maximal quality constraint of A_3:

$$f^*(A_2) = \bar{F}_3 + \varepsilon, \varepsilon > 0, \varepsilon \to 0 \tag{E24}$$

Similarly, if there is a third entrant of type B, the optimal choice for B_2 will be slightly above the maximal quality constraint of B_3:

$$g^*(B_2) = \bar{G}_3 + \varepsilon, \varepsilon > 0, \varepsilon \to 0 \tag{E25}$$

Therefore, for the condition in (18) to be satisfied in case both A_3 and B_3 enter the market at the first stage, $\bar{G}_3$ should exceed $\bar{F}_3$. Alternatively, $\bar{F}_3$ should exceed $\bar{G}_3$ for the condition in (19) to be satisfied under the same circumstances.

3 The Role of Consumer Taste Variegation and Quality Differentiation for the Entry Deterrence Effect of Bundling in a Market for Systems

In this paper we explore three special cases of a market for vertically differentiated systems to reveal the role of consumer taste variegation and system quality differences for the entry deterrence effect of bundling in such a market. The results imply that pure bundling and zero pricing cannot be effective entry-deterrence devices either against a superior-quality entrant, or against an inferior-quality entrant, if the variegation of consumer tastes is sufficient to ensure positive market shares for more than two systems at equilibrium. An additional condition for any potential entrant to have a positive market share is for the bottom-quality systems to be more differentiated than the top-quality systems. Accordingly, an inferior-quality entry could be effectively deterred, only if there is a restricted variegation of consumer tastes as well as less differentiated bottom-quality systems. In the latter case, the effect of bundling on both producer and consumer surpluses would be strictly negative.

Keywords: product tying, vertical differentiation, market foreclosure, system goods, social welfare, anti-trust policy

JEL classification: L11, L13, L15

3.1 Introduction

An important question in the competition policy debate is whether and when the decision of a firm with significant market power in one market to enter another competitive market should be legally restricted. Two main arguments are generally put forward why such behavior might not be socially acceptable. On the one hand, the firm could cross-subsidize its production in the competitive market by using the abnormal profit it makes in the monopoly market to cover its loss from undercutting the prices of its competitors (Faulhaber (1975), Berg and Weisman (1991), Boldon et al. (2000), Eckert and West (2003), Chen and Rey (2013)). On the other hand, if the products traded on the two markets are complements, the firm could make the purchase of its good in the less competitive (aka "core") market conditional on the purchase of its good in the more competitive (aka "adjacent") market (Whinston (1990), Nalebuff (2004), Carlton and Waldman (2002), Kováč (2007)). In both cases, the outcome could be a leverage of the market power of the firm in the core market to the adjacent market. The compelling question that economists try to answer is at what conditions the leverage outcome would occur at equilibrium.

In the current paper, we explore a model of a market for vertically differentiated systems consisting of two goods. Consumers buy systems but the competition between firms is good-specific and therefore takes place in two (core and adjacent) sub-markets, respectively. The main objective is to examine the importance of the consumer taste variability and quality differentiation of systems for the occurrence of entry-deterrence (i.e. monopoly) outcome in the sub-markets at equilibrium.

In the existing literature the focus is set on the product bundling strategies that the multi-product firm could implement to strengthen the price competition with a (potential) rival. The rival is forced to decrease its price, which lowers its profit and ultimately turns it into a loss. The result could then be a market exit (entry deterrence) of the entrant at equilibrium. In his seminal paper, Whinston (1990) describes two bundling strategies that could effectively deter the entry of a potential competitor. First, the multi-product firm could commit to offering its products only in a bundle (pure bundling strategy) which impedes its sales in the competitive market, leaving it with no other choice than to charge low prices and thus be a tougher competitor in case of an entry. This strategy makes sense only in a dynamic (two-period) market setting. Second, when consumers differ in their valuation for the monopoly-market good, the multi-product firm could effectively deter entry without precommitment, but by credibly threatening to sell its core-market good both as a separate good and in an underpriced bundle with its adjacent-market good. The offer of a bundle, together with one of the two bundled goods sold also as a separate good, is considered by Nalebuff (2003) to be a special case of mixed bundling where the good that is available separately is called a ”tying good”, and the good that is offered only in a bundle is called a ”tied good”. Kováč (2007) compares the two bundling arrangements in the setting of the standard static (one-period) model of vertical product differentiation of Shaked and Sutton (1982), especially adjusted to represent a market for system goods. He shows that the mixed bundling strategy with a tying good in the core market and a tied good in the adjacent market is dominant for the multi-product incumbent firm. Furthermore, it has the potential to effectively deter an entry of a potential inferior-quality rival in the core market without precommitment even if it faces a superior-quality rival in the adjacent market.

The results of both Whinston (1990) and Kováč (2007) imply that a mixed-bundling strategy is the optimal choice of the multi-product firm whether or not it results in effective entry-deterrence. Indeed, mixed bundling is a profit-maximizing strategy, not because of its exclusionary effect but because it allows for second-degree price discrimination between the consumers who value the tying good highly and lowly, respectively. Furthermore, in the setting of Kováč (2007) the duopoly outcome in the case of no bundling follows from the finiteness property[1] of the vertically differentiated markets whereas in Whinston (1990) it is a result from the assumed increasing returns to scale in the adjacent market. Therefore, Kovac's (2007) condition for effective entry-deterrence imposes additional requirements not only on the relations between the quality choices of the firms in both markets but also on the range within which consumer tastes could vary. For having the market only covered by the systems in which the elements of the multi-product firm take part, that is all consumers buying only these systems, their tastes need to vary in a certain narrow range analogous to the one defined by Shaked and Sutton (1982) for a covered-market duopoly.[2] The consumer taste variation requirement of Shaked and Sutton (1982), however, is derived for the case of a single-product vertical differentiation. Accordingly, in order to be able to adopt it Kovac (2007) makes the assumption that if bundling is not implemented by the multi-product firm (non-bundling strategy), consumers would buy the two goods forming a system at two independent single-product markets. For brevity, from now on we will refer to this assumption by calling it a "non-bundling independence assumption".

In this paper, we relax the non-bundling independence assumption of Kováč (2007)

[1] In contrast to the free-entry outcome in markets with monopolistically competitive structure (Chamberlin (1933)), when the entry barriers tend to zero on a vertically differentiated market, the number of firms with positive market share remains finite at equilibrium. (Shaked and Sutton (1983))

[2] Shaked and Sutton (1982) show that given uniform distribution of the consumer identification variable (income or taste) with support whose right endpoint is at most four times but not less than twice as large as its left endpoint, this is a necessary and sufficient condition for having exactly two firms with positive market share at equilibrium in a vertically differentiated market.

and explore a static model of a vertically differentiated market where consumers choose systems but not independent goods. As in Kováč (2007) we allow for a potential inferior-quality entrant in the core sub-market and a potential superior-quality entrant in the adjacent sub-market. For the particular market setting we show that the non-bundling equilibrium outcome established by Kováč (2007) under the non-bundling independence assumption could occur also in a vertically differentiated market for systems under certain conditions on consumer taste variability and system quality differentiation, which we derive explicitly in this paper.

Another key novelty in our approach is that we allow consumers who buy the bundled system to upgrade it with the good of the superior-quality entrant in the adjacent sub-market. This seemingly contradicts the requirement imposed by Whinston (1990) for pure bundling to have an exclusionary outcome. Namely, the multi-product firm should precommit to pure bundling. However, note that precommitment to pure bundling implies only that consumers find it technically impossible or prohibitively expensive to unbundle the multi-product firm's system, but not that they cannot combine it with other system elements. We refer to the latter assumption by calling it "assumption for irreversibility of bundling". Both Choi and Stefanadis (2001), as well as Carlton and Waldman (2002), apply it in the context of the US v. Microsoft antitrust case[3] by assuming that consumers are not allowed to use more than one browser application on the same operating system. However, Carlton and Waldman (2002) acknowledge that the integration of Internet Explorer as an embedded application of Windows 95 did not satisfy the assumption for irreversibility of bundling since consumers were still able to install and use Netscape

[3]In the late 1990s, the US Department of Justice had charges against Microsoft for violating the anti-trust law by tying its browser Internet Explorer to its operating system Windows. For detailed description and analysis of the US v. Microsoft antitrust case, see Economides (2001). For analysis of the close anti-trust cases against Microsoft in Europe, see Economides and Lianos (2009).

Navigator on Windows 95. Therefore, we find it reasonable to relax the irreversibility assumption and study the conditions for the exclusionary effect of pure bundling when a system upgrade is possible.

We propose three special settings of a market for vertically differentiated systems where a multi-product firm could implement pure bundling that allows for a system upgrade. The results bring out consumer taste variability and quality differentiation as decisive determinants of whether there could be an exclusionary outcome at equilibrium with or without the implementation of bundling.

In setting 1, we consider a simple case where the multi-product firm faces no threat of entry in one of the sub-markets and a potential superior-quality entrant in the other (adjacent) sub-market. The situation resembles the one modeled by Whinston (1990). The only difference is that instead of offering the monopoly good as a tying good that could be combined with the entrant's good, the multi-product firm makes the upgrade of the bundled system with the higher-quality good of its potential rival in the adjacent sub-market possible. The results imply that at equilibrium all the consumers that buy the bundled system choose to upgrade it. The multi-product firm and the potential entrant have identical profits at equilibrium with or without bundling. The market share of the multi-product firm in the core sub-market and that of the entrant in the adjacent sub-market stay the same. The only change that pure bundling causes is in the market share of the multi-product firm in the adjacent sub-market. It is zero at the non-bundling equilibrium but strictly positive at the bundling equilibrium. Therefore, if the multi-product firm implements bundling in such a market setting, this must be driven by an impetus to save its sales in the adjacent sub-market rather than by entry-deterrence or strategic foreclosure incentives.

In setting 2, we study the case suggested by Kováč (2007) where the multi-product firm faces an inferior-quality entrant in the core sub-market and a superior-quality entrant in the adjacent sub-market. However, we relax the non-bundling independence assumption, which allows us to establish a new non-bundling equilibrium where all the four possible systems are demanded by consumers. We derive certain conditions which presume sufficiently wide variability of consumer tastes and small differentiation of the higher-quality systems for the non-bundling equilibrium to exist. Except for having more systems with a positive market share at equilibrium, the non-bundling equilibrium results also imply a zero corner solution for the price of the multi-product firm's inferior-quality product sold in the adjacent market. The reason is that the goods of a type sold in the same sub-market are complements in prices, while the goods of different types that form systems are substitutes in prices. This allows the multi-product firm to maximize the surplus extracted from consumers in the core market by pricing its inferior-quality good at zero in the adjacent market. However, in the particular setting at which we show zero pricing to be optimal, it does not lead to effective entry-deterrence of any of the two potential rivals at equilibrium. If seen in the context of the US v. Microsoft antitrust case, the latter result complies with Davis and Murphy (2000) who argue that since the demand for the browser of Microsoft is ”lower” (i.e. more price sensitive) than the demand for its operating system, it is reasonable to expect Internet Explorer to be offered cheaper and even for free when this is feasible. The zero-pricing outcome is particularly consistent with the observation that “...many if not most markets with network externalities are characterized by the presence of two distinct sides whose ultimate benefit stems from interacting through a common platform” (Rochet and Tirole (2003), p.990).[4] For instance,

[4]We thank Volker Nocke for pointing out to us this possible interpretation of the results of our model in the context of the literature on the multi-sided markets.

software application developers might prefer to offer their products for free to the end users in exchange for sponsorship from the operating system (computing platform)[5] developers who are thus able to obtain a larger portion of the consumer surplus in the platform market.

In addition, our results for the bundling equilibrium solution suggest that pure bundling cannot be an effective entry-deterrence device in setting 2 (i.e. with a wide spread of consumer tastes and small differentiation of the higher-quality systems). In compliance with Choi and Stefanadis (2001) and Carlton and Waldman (2002), we show that pure bundling lowers the market shares and the profits of both the superior-quality and inferior-quality entrants. Yet, they are strictly positive because there are consumers with low enough tastes so that only the providers of the inferior-quality good have an interest in serving them.

In setting 3, again the multi-product firm faces an inferior-quality entrant in the core sub-market and a superior-quality entrant in the adjacent sub-market. However, the conditions on consumer taste variability and system quality differentiation differ from these in setting 2 so that not all systems have demand at a non-bundling equilibrium and not all potential entrants save their positive market shares at a pure-bundling equilibrium. The results resemble the equilibrium outcomes in Kováč (2007). At a non-tying equilibrium all firms make positive sales so that there is duopoly in both sub-markets but consumers buy only the best and the worst possible combinations. At pure-bundling equilibrium the inferior-quality single-good entrant is efficiently excluded from the core sub-market. The comparison of the results with those from the solution of setting 2 imply that consumer taste diversification and quality differentiation of systems are the main

[5]For a detailed discussion on the economic role of operating systems in computer-based industries, see Bresnahan (2001), Evans et al. (2005).

determinants of whether pure bundling is an effective entry-deterrence device against an inferior-quality supplier. Less differentiated systems, together with the systems of lowest quality in markets with less variegated consumer tastes, are the ones that remain unsold at a pure-bundling equilibrium. Hence, having pure bundling implemented is only a necessary but not sufficient condition for effective entry deterrence.

We also provide an analysis of the social welfare effect of pure bundling in setting 3. It indicates a strictly negative social welfare effect of pure bundling. When consumer tastes do not vary within a wide range, pure bundling lowers the market share of the upgraded bundled system, which leads to a decrease in both producer surplus and consumer surplus. The result is stronger than in the welfare analyses based on the models of Carlton and Waldman (2002) and Kováč (2007) which imply uncertain welfare effects of tying.

The paper is organized as follows. In Section 3.2 we present the supply side (3.2.1) and the demand side (3.2.2) of the model. In Section 3.3 the solution of the model for the three different market settings is established. Finally, we summarize the main implications in section 3.4.

3.2 The Model

We consider a market for systems consisting of two types of goods, type A and type B, respectively. The type A goods are sold in sub-market A, whereas the type B goods are sold in sub-market B.

3.2.1 Supply side

Accordingly, we assume the following potential entrants in the market:

1. a (multi-product) firm M which reckons upon entering the both sub-markets.
2. two single-product firms, firm A and firm B, potential entrants in sub-market A and sub-market B, respectively.

The strategic interaction between firms is modeled as a three-stage game.

Stage 1: Firms make simultaneous decisions whether to enter the market. Firm M chooses also whether to enter both sub-markets, or only one.

Stage 2: Firm M observes the actual entries in the two sub-markets at the first stage and decides to offer its goods in a bundle only or to sell them separately.

Stage 3: After observing Firm M's decision at the second stage, firms compete simultaneously on prices.

As is common in bundling models, the qualities of the products offered by the firms are assumed to be exogenously determined (Whinston (1990), Carlton and Waldman (2002)). So, we do not need to define explicit functional form to describe the relationship between good qualities and the qualities of the systems they form.

There are four systems that might be formed from the goods which the three firms could offer. Without loss of generality, we assume that the quality of the type-A good are decisive for the quality of the system so that the systems based on the better type A good (A_1B_1, A_1B_2) are of strictly higher quality than the systems based on the worse type A good (A_2B_1, A_2B_2). Accordingly, in the case of duopoly in both markets, sub-market A will be less competitive than sub-market B. So, to conform with the commonly agreed definitions, we will also call sub-market A a core sub-market and sub-market B an adjacent sub-market.

In addition, as in Kováč (2007), on the one hand, we assume that in the core-market firm A offers a good, denoted by A2, which is of strictly worse quality than the good of firm M, denoted by A1. On the other hand, in the adjacent market firm B is assumed to offer a good, denoted by B1, which is of strictly better quality than the good of firm M, denoted by B2. The letter in the notations of goods represents their type, while the number – their rank if ranked in decreasing order of their qualities. Accordingly, the four possible systems are denoted by A_iB_j, $i = 1,2;\ \ j = 1,2$; where i represents the quality rank of good Ai and j – the quality rank of good Bj. The corresponding ranking of the systems in decreasing order of their qualities is given in table 1 below:

Table 1: Ranking of the possible systems in decreasing order of their qualities

rank	**system**
1	A_1B_1
2	A_1B_2
3	A_2B_1
4	A_2B_2

The system A_1B_1 formed by the best type A good, A1, and the best good of type B, B1, is trivially also the best out of the four systems and therefore has a quality of rank 1. Similarly, system A_2B_2 formed by the worst type A good, A2, and the worst type B good, B2, is trivially also the worst out of the four systems and therefore has a quality rank of 4.

Accordingly, as it was assumed above, system A_1B_2 based on the better type A good, A1, is of strictly higher quality ($\chi_{12} > \chi_{21}$) than the system A_2B_1 which is based on the worse type A good, A2. Therefore, in table 1 the former system is of lower rank 2 than the rank 3 of the latter system.

We make another standard assumption for the models of vertical product differentiation. Namely, that firms do not incur entry costs. That is, as long as a firm can make positive sales, it enters the market (Shaked and Sutton (1982, 1983), Kováč (2007)). This assumption might look too restrictive to be attributable to markets like the ones for software systems where product development usually requires significant initial investment. At the same time, however, it is quite common practice nowadays for the development of operating systems or software applications to be financially and organizationally supported by non-profit organizations[6] based on free public license agreements with independent open-source developers[7]. Thus, the suppliers of these software products incur zero entry costs.

For simplicity, production cost is also assumed to be zero which might be argued to reflect the large economies of scale characterizing for example software production again. Indeed, it should not come as a surprise to anyone who has ever downloaded an application to a computer or smart phone that the marginal cost of the multiplication of a copy of an installation program is negligibly small and tends to zero as the number of copies increases. So, the profits of the entrants in our model coincide with their total revenues. Hence, the pricing subgame equilibrium is given by the solutions of the

[6] In 2006 Netscape open-sourced their browser code, and entrusted it to the newly formed non-profit Mozilla Foundation. The support for Netscape Navigator was ended and Mozilla Firefox was officially released as its free open-source successor. For more details see the Mozilla Public License uploaded at https://www.mozilla.org/MPL/NPL/1.1/ Similarly, a great deal of the Linux distributions are supported by the Free Software Foundation (FSF). The complete list is available at http://www.gnu.org/distros/free-distros.html

[7] For comprehensive discussion of the literature studying the incentives of developers and profit-oriented software suppliers to take part in open-source projects, see Rossi and Bonaccorsi (2006)

revenue-maximization problems of these of the three firms that decide to enter the market at the first stage. The revenue-maximization problems of the three firms could respectively be set as follows:

$$\begin{cases} \underset{p_{A1},p_{B2}}{Max}\ R_M = p_{A1}D_{A1} + p_{B2}D_{B2} = \\ = p_{A1}\sum_{j=1}^{2} D_{1j} + p_{B2}\sum_{i=1}^{2} D_{i2}, & \text{if A1 and B2 are sold separately} \\ \underset{p_M}{Max} R_M = p_M D_{12}, & \text{if A1 and B2 are sold in a bundle only} \end{cases} \tag{1}$$

$$\underset{p_{B1}}{Max} R_B = p_{B1}D_{B1} = p_{B1}\sum_{i=1}^{2} D_{i1} \tag{2}$$

$$\underset{p_{A2}}{Max} R_A = p_{A2}D_{A2} = p_{A2}\sum_{j=1}^{2} D_{2j} \tag{3}$$

where:

R_M, R_B, R_A - the total revenues of firms M, B and A, respectively

p_{A1}, p_{A2}, p_{B1}, p_{B2}, p_M - the prices of goods A1, A2, B1, B2, and bundle A1B2, respectively; $p_{Ai} \geq 0, p_{Bj} \geq 0, p_M \geq 0, i = 1,2; j = 1,2$

D_{A1}, D_{A2}, D_{B1}, D_{B2} - the demands for goods A1, A2, B1, B2, respectively

D_{ij} - demand for system consisting of good Ai and good Bj, $i = 1,2$, $j = 1,2$

The market shares of the systems are determined by consumers whose preferences and identification characteristics are described in subsection 3.2.2 below.

3.2.2 Demand side

As is common for the existing models of vertical product differentiation, we assume that the demand side of the market is represented by a continuum of consumers. Particularly, we adopt the approach of Mussa and Rosen (1978) and assume that consumers are characterized by their taste for quality θ which is a random variable following continuous uniform distribution with support given by the interval $[\underline{\theta}, \overline{\theta}]$, $\underline{\theta} > 0$.

Consumers do not gain from buying just a type A or B product. They need to have at least a system consisting of a pair of goods of each type to be able to gain a positive benefit from any of these goods. Therefore, consumers either buy a system or do not buy in the market at all.

The utility function, in compliance with Mussa and Rosen (1978), is designed to measure the individual surplus of any given consumer with taste θ from the purchase of one of the four systems A_iB_j:

$$U_{ij} = \chi_{ij} \cdot \theta - p_{ij}, i = 1,2; j = 1,2 \tag{4}$$

where:

U_{ij} - reduced-form utility function of consumer with taste θ from purchasing system A_iB_j

χ_{ij} – quality of system A_iB_j

p_{ij} – price of system A_iB_j; if good Ai and good Bj are sold separately, the system price is given as a pure sum of the goods' prices, $p_{ij} = p_{Ai} + p_{Bj}$

Consumer valuation of a system is given by the product of consumer taste and the quality of the system. This assumption ensures the main distinct property of the markets with vertical product differentiation. Namely, consumer valuation increases in taste but ranking of systems is consensually agreed by all consumers and remains the same across tastes (Mussa and Rosen (1978), Shaked and Sutton (1982, 1983)). To preclude negative values of the utility function in (4), we make another standard assumption that consumers have a free outside option with quality χ_0 which they could choose instead of buying the systems available on the market. The quality of the free outside option is accordingly chosen to be strictly smaller than the quality of the worst system A_2B_2, i.e. $\chi_0 < \chi_{22}$.

Distinct from the existing bundling models (Whinston (1990), Carlton and Waldman (2002), Kováč (2007)), we also relax the assumption for irreversibility of bundling, allowing consumers to combine a type A good with more than one type B good. Thus, we enable our model to better represent the market situation in the US v. Microsoft antitrust case where consumers could install and use more than one browser on their operating systems. Without loss of generality, the valuation of the upgraded system is assumed to be given by the valuation of the highest-quality system that could be formed by its goods. That is, there are no synergy effects from the upgrade. For example, if a consumer decides to upgrade firm M's system A_1B_2 with the firm B's good B1, the quality of the resulting system $A_1B_2B_1$ which we denote by χ_{121} is given by the quality of χ_{11} of the best system A_1B_1. In turn, mixed-bundling is not a feasible bundling option for firm M. Therefore, in the game defined in the previous subsection, firm M chooses only between pure bundling and no bundling. For simplicity, from now on we can skip the unnecessary attribute "pure" and will simply talk about bundling.

Note that theoretically firm M should be able to charge its adjacent-submarket good at a negative price. On the one hand, this would strengthen its resistance to the competition of the higher-quality entrant. On the other hand, firm M can cover the resulting losses by the extra consumer surplus it could extract through the sales of its core-submarket good to higher-taste consumers. However, negative pricing is hardly implementable when consumers can purchase and use more than one type B good in a system combination with a type A good. It is difficult for firm M to charge in practice B2 a negative price because this implies that firm M should in fact pay not only the consumers who buy its core-submarket good but also the one who combines its adjacent-submarket good with the good of the inferior-quality entrant. However, this would give all consumers an incentive to buy B2 and be paid whether they need it or not, because firm M cannot distinguish between consumers who use it alone and those who use also B1. Carlton and Waldman (2002) show that this pricing strategy which they call ”virtual tying” could have the same entry-deterrence outcome as pure bundling. In such a case however, negative pricing is easy to be proven and prosecuted as illegal practice. Therefore, when defining the producer side in the previous subsection, we precluded negative pricing and do not consider it as a feasible price choice in the non-bundling case.

Because consumers have, at most, unit consumption of a good and their tastes are uniformly distributed within the market range $\left(\underline{\theta} \leq \theta \leq \overline{\theta}\right)$, the bounds of the market shares of the product pairs are marked by the taste parameters of the so-called “marginal consumers”. A marginal consumer is defined as the consumer whose taste $\theta_{ij/i'j'}$, $i \neq i' \wedge j \neq j'$, makes it indifferent between given distinct pair of systems A_iB_j and $A_{i'}B_{j'}$.

The expression for the marginal consumer's taste variable could be directly derived from (4):

$$U_{ij} = U_{i'j'} \Rightarrow \theta_{ij/i'j'} = \frac{p_{ij} - p_{i'j'}}{\chi_{ij} - \chi_{i'j'}} = \frac{p_{ij} - p_{i'j'}}{d_{ij/i'j'}} \tag{5}$$

where:

$\theta_{ij/i'j'}$ – marginal taste at which consumers are indifferent between systems A_iB_j and $A_{i'}B_{j'}$, $i \neq i' \wedge j \neq j'$

$d_{ij/i'j'}$ – the difference in qualities between systems A_iB_j and $A_{i'}B_{j'}$, $d_{ij/i'j'} = \chi_{ij} - \chi_{i'j'}$

Given that all the three firms enter the market and all the four systems in table 1 have positive market shares, the latter can be expressed as follows:

$$D_{11} = \begin{cases} \overline{\theta} - \theta_{11/12}, & \text{if A1 and B2 are sold separately or in a bundle alone} \\ \overline{\theta} - \theta_{11/21}, & \text{if A1 and B2 are only sold in an upgraded bundle} \end{cases} \tag{6}$$

$$D_{12} = \begin{cases} \theta_{11/12} - \theta_{12/21}, & \text{if A1 and B2 are sold separately or in a bundle alone} \\ \overline{\theta} - \theta_{11/21}, & \text{if A1 and B2 are only sold in an upgraded bundle} \end{cases} \tag{7}$$

$$D_{21} = \begin{cases} \theta_{12/21} - \theta_{21/22}, & \text{if A1 and B2 are sold separately or in a bundle alone} \\ \theta_{11/21} - \theta_{21/22}, & \text{if A1 and B2 are only sold in an upgraded bundle} \end{cases} \tag{8}$$

$$D_{22} = \begin{cases} \theta_{21/22} - \underline{\theta}, & \text{if } \theta_{22/0} \leq \underline{\theta} \\ \theta_{21/22} - \theta_{22/0}, & \text{if } \theta_{22/0} > \underline{\theta} \end{cases} \tag{9}$$

where:

$\theta_{22/0}$ – marginal taste at which consumers are indifferent between system A_2B_2 and the outside option with quality χ_0, $\theta_{22/0} = \frac{p_{A2}+p_{B2}}{d_{22/0}}$

$d_{22/0}$ – quality difference between A_2B_2 and the outside option, $d_{22/0} = \chi_{22} - \chi_0$

If only the best and the worst systems, A_1B_1 and A_2B_2, have positive market shares, the latter are given by the following expressions:

$$D_{11} = \overline{\theta} - \theta_{11/22} \tag{10}$$

$$D_{22} = \theta_{11/22} - \underline{\theta} \tag{11}$$

Analogously, for the case when firm M is a monopolist in sub-market A, if both the systems that could be formed have positive market shares, we can express them as follows:

$$D_{11} = \overline{\theta} - \theta_{11/12} \tag{12}$$

$$D_{12} = \begin{cases} \theta_{11/12} - \underline{\theta}, & \text{if } \theta_{12/0} \leq \underline{\theta} \text{ for A1 and B2 sold separately} \\ \theta_{11/12} - \theta_{12/0}, & \text{if } \theta_{12/0} > \underline{\theta} \text{ for A1 and B2 sold separately} \\ \overline{\theta} - \underline{\theta}, & \text{if } \theta_{12/0} \leq \underline{\theta} \text{ for A1 and B2 sold in a bundle only} \\ \overline{\theta} - \theta_{12/0}, & \text{if } \theta_{12/0} > \underline{\theta} \text{ for A1 and B2 sold in a bundle only} \end{cases} \tag{13}$$

In what follows we solve the model for the following three particular market settings:

Setting 1: Firm A is excluded from the set of players. The endpoints of the support interval of the consumer taste uniform distribution and the system quality differences are assumed to satisfy the following conditions:

$$\underline{\theta} < \frac{1}{2}\overline{\theta} \tag{14}$$

$$d_{12/0} \leq \frac{4}{9}d_{11/0} \text{ i.e. } \chi_{12} \leq \frac{4}{9}\chi_{11} \tag{15}$$

In subsection 3.3.1 we claim and then in appendix section F we show that at the above conditions of setting 1 firm M has no incentive to sell its good B2 in the adjacent sub-market at a non-bundling equilibrium but strictly prefers to accommodate the superior quality entrant, firm B, at a bundling equilibrium.

Setting 2: The set of players includes all three firms. The endpoints of the support interval of the consumer taste uniform distribution and the system quality differences are assumed to satisfy the following conditions:

$$\frac{\overline{\theta}}{4} < \underline{\theta} < \frac{4d_{11/12}d_{12/21}}{5d_{11/12}d_{12/21} + 2d_{21/22}\left(2d_{11/12} + 3d_{12/21}\right)}\overline{\theta} \tag{16}$$

$$d_{22/0} > 3d_{12/21} > 2d_{21/22} > 3d_{11/12} > \frac{18d_{12/21}d_{21/22}}{11d_{12/21} - 4d_{21/22}} \tag{17}$$

In subsection 3.3.2 we claim and then in appendix section G we show that the above conditions of setting 2 are sufficient for having all systems with positive demands at a non-bundling equilibrium and all three firms with positive market shares at a bundling equilibrium.

Setting 3: The set of players includes all three firms. The endpoints of the support interval of the consumer taste uniform distribution and the system quality differences are assumed to satisfy the following conditions:

$$\frac{\left(4d_{22/0}+5d_{11/22}\right)}{8d_{22/0}+9d_{11/22}}\overline{\theta}<\underline{\theta}<\frac{6d_{11/22}}{8d_{22/0}+9d_{11/22}}\overline{\theta} \tag{18}$$

$$\frac{d_{11/22}}{4}>d_{22/0}>\frac{d_{11/22}}{6}>\frac{5d_{12/22}}{12} \tag{19}$$

In subsection 3.3.3 we claim and then in appendix section H we show that the above conditions of setting 3 are sufficient for having only the two most differentiated systems, A_1B_1 and A_2B_2, with positive demands at a non-bundling equilibrium, and the entry of firm A effectively deterred at a bundling equilibrium.

The inequality conditions in (16) could hold together only if the condition in (17) is satisfied. Similarly, the joint validity of the two inequalities in (18) is possible only when (19) holds true. Also, the right-hand side of (16) is smaller while the left-hand side of (19) is larger than $\frac{1}{2}\overline{\theta}$ so that the feasible intervals of $\underline{\theta}$ at settings 2 and 3 have no intersection. Accordingly, setting 2 represents a market that has more variegated consumer tastes than the market represented by setting 3. Furthermore, at setting 2, product differentiation decreases in system quality, making the competition between the top-ranked systems stronger, while at setting 3 it is the other way around. Quality differentiation increases in system quality, making the competition between the bottom-ranked systems stronger. Hence, even though we do not need to introduce an explicitly defined quality accumulation function for all the settings, the inequality condition in (17) implicitly specifies the following particular requirements on how the quality accumulation function should behave in setting 2:

$$\chi_{ij} = f\left(A_i, B_j\right) \text{ s.t. } \frac{\partial f\left(A_i, B_j\right)}{\partial A_i} > 0,\ \frac{\partial f\left(A_i, B_j\right)}{\partial B_j} > 0, \frac{\partial f\left(A_i, B_j\right)}{\partial B_j \partial A_i} < 0 \tag{20}$$

where:

A_i - exogenously determined quality of good Ai, $i = 1,\ 2; A_1 > A_2$

B_j - exogenously determined quality of good Bj, $j = 1,\ 2; B_1 > B_2$

$f\left(A_i, B_j\right)$ - implicit functional relationship between the quality of a system and the qualities of the goods of which it consists.

Similarly, the inequality condition in (19) implicitly specifies the following particular requirements on how the quality accumulation function should behave in setting 3:

$$\chi_{ij} = f\left(A_i, B_j\right) \text{ s.t. } \frac{\partial f\left(A_i, B_j\right)}{\partial A_i} > 0,\ \frac{\partial f\left(A_i, B_j\right)}{\partial B_j} > 0, \frac{\partial f\left(A_i, B_j\right)}{\partial B_j \partial A_i} > 0 \tag{21}$$

The equilibrium solutions for settings 1, 2, and 3 are represented in appendix sections F, G and H, respectively. The solution for each setting starts with the derivation of the pricing-stage subgame equilibrium which determines the equilibrium market shares of the entrants in each setting. The results allow us to define the optimal decisions of the firms at the entry stage by backward induction and deduce the optimal market structure at equilibrium. Finally, we apply the concept of perfect subgame equilibrium of Selten (1975) to justify the strict dominance of a pure bundling over a non-bundling strategy for the multi-product firm at each of the three settings. The results are established and explained in the next section.

3.3 Equilibrium Solutions

3.3.1 Setting 1

Setting 1 provides a scenario where on the one hand, if firm M is not allowed to apply bundling strategies, it is strictly better off of entering the market as a single-product firm operating alone in the core sub-market. On the other hand, if it is allowed to use bundling, firm M prefers to accommodate firm B by not preventing consumers from upgrading its bundled system A_1B_2 with the entrant's superior-quality good B1. The respective equilibria are established below in proposition 1 and proposition 2, respectively.

Proposition 1. Let the market be characterized by the conditions of setting 1 in (14) and (15), so that firm M is the only entrant in sub-market A, faces an entry threat in sub-market B and exogenous restriction on the use of bundling. Then, there is a unique subgame-perfect equilibrium of this restricted non-bundling game. At this equilibrium firm M enters only the core sub-market A while firm B enters and operates alone in the adjacent market B. The optimal strategies of the two firms at the pricing stage is accordingly given by the symmetric set of prices in (F16).[8] The corresponding outcome is that only system A_1B_1 has positive demand and the market is not covered.

Proof: see section F of the appendix.

In the restricted game where firm M is not allowed to choose bundling at the second stage, there are two possibilities for the multi-product firm. To enter both sub-markets and charge its goods prices, the sum of which is sufficiently low for the market to be exactly covered and both systems A_1B_1 and A_1B_2 to have positive demands. Or, alternatively, to enter the core sub-market only and to charge its good A1 at a price at which consumers buy it only in combination with the superior-quality good B1. In the latter case, the market would not be covered provided that consumer tastes are sufficiently variegated to satisfy condition (14). Still, as long as the superior-quality good is sufficiently differentiated from good B2 so that the condition in (15) holds, firm M could extract more consumer

[8] The expressions derived in the appendix have separate numeration for each section of the appendix which is marked by the letter of the corresponding section, e.g. (F16) is derived in section F of the appendix.

surplus by selling A1 solely in a system with B1. Therefore, the entry of firm M as a single-product firm in the core sub-market only, strictly dominates the other option in the non-bundling game.

Proposition 2. Let again the conditions of setting 1 in (14) and (15) hold, so that firm M is the only entrant in sub-market A, faces an entry threat in sub-market B, but is now also allowed to bundle its goods. Then, there is a unique subgame-perfect equilibrium of the (unrestricted) game. At this equilibrium firm M enters the both sub-markets and offers its goods only in a bundle but since consumers are allowed to upgrade the bundled system A_1B_2 with good B1, firm B enters the adjacent sub-market as well. The optimal strategies of the two firms are given by the symmetric price set in (F39). The corresponding equilibrium outcome implies that the bundling strategy is only weakly dominating the non-bundling one.

Proof: see section F of the appendix.

The result in proposition 2 should not come as a surprise after noticing that consumers value the upgraded system $A_1B_2B_1$ and the best-quality system A_1B_1 the same. Therefore, the same consumers who buy A_1B_1 at the non-bundling equilibrium established in proposition 1 are the ones that buy $A_1B_2B_1$ at the bundling equilibrium in proposition 2. Accordingly, firm M charges them at the same price for the bundled system as for A1 when sold alone. So, the payoffs of the two firms are identical at both equilibria. Bundling simply makes the purchase of good A1 conditional on the purchase of good B2 which drives consumers who would otherwise buy only A_1B_1 to get it together with A2. Thus firm M has positive sales in both sub-markets and therefore at equilibrium enters with two goods.

The weak dominance of the bundling strategy in the subgame-perfect equilibrium outcome established in proposition 2 depends primarily on the assumptions for the variegation of consumer tastes and quality differentiation in (14) and (15), respectively. When the latter conditions are not satisfied, it could be optimal for firm M to choose a pricing strategy at which the market would be covered and its system A_1B_2 would be demanded even without bundling applied (see (F21) and (F42) in the appendix).

3.3.2 Setting 2

Setting 2 is introduced to show that if bundling is precluded, zero-pricing could be an optimal choice of firm M without causing effective exclusion of any of its rivals in the two sub-markets. On the other hand, if allowed, firm M would strictly prefer to offer its goods only in a bundled system, but this would not have an effect on the entry decision of its potential entrants. The respective equilibria are established below in proposition 3 and 4, respectively

Proposition 3. Let the market be characterized by the conditions of setting 2 in (16) and (17), so that firm M faces potential competition by firms A and B in the two respective sub-markets, but the use of product bundling is precluded. Then there is a unique covered-market non-bundling subgame equilibrium at which all firms make a market entry and all the four systems in table 1 have positive sales, even though it is optimal for firm M to charge good B2 a zero price. The equilibrium is characterized by the prices in (G13) – (G15).

Proof: see section G of the appendix.

The key implication of the result in proposition 3 is that even though good B2 is charged zero price, all firms still enter the two sub-markets so that none of them is monopolized. The outcome could be explained by the same reasoning provided in the previous subsection as to why firm M would prefer to enter as a single-product firm in the core sub-market if it is prevented from using product bundling at the conditions of setting 1. Again, the lower price firm M charges B2, the higher the price at which it could sell A1. Therefore, by charging the lowest possible (zero) price for B2, firm M actually maximizes the consumer surplus it could extract through selling A1 as a part of the best-quality systems.

A reasonable question that occurs is why, at setting 2, firm M would choose to sell B2 even at zero price. The answer is hidden behind the observation that distinct from setting 1, at setting 2 firm M is not a monopolist in sub-market A, but is instead assumed to face the threat of entry of firm A. As a result, even if the market power of the firms in sub-market A is exogenously set to be larger than in sub-market B, this would hold true only if we have a duopoly in each of the two sub-markets. Accordingly, if firm M decides to enter only the submarket A as at setting 1, it will leave firm B operating alone in sub-market B. At setting 2, however, because there is a duopoly in sub-market A, the latter will be characterized by stronger competition and therefore, it will become an adjacent market, while sub-market B will become a core market.[9] So, firm M could only save its position as a quality leader in the core market by entering both sub-markets, which in fact also saves the positive sales and profit of firm A. In turn, all four systems have positive demands and the market is covered.

Proposition 4. Let the market be characterized by the conditions of setting 2 in (16) and (17) and firm M is allowed to offer its goods in a bundle in response to the potential competition by firm A and firm B in the two respective sub-markets. Then there is a unique bundling subgame equilibrium at which the entry of firm A is not efficiently deterred even though the worst-ranked system A_2B_2 is excluded from the set of systems demanded at equilibrium. The equilibrium is characterized by the prices in (G41) – (G43).

Proof: see section G of the appendix.

Even if allowed, consumers would still have no incentive to combine the bundled system with the inferior good in market A because it does not provide them with any additional utility, in contrast to the case when A_1B_2 is upgraded by B1. So, consumers have no reason to buy A2 in addition to A_1B_2. As a result, bundling reduces down to three the number of systems with positive demands at equilibrium. However, bundling cannot prevent consumers from purchasing the good of firm A in a system with the good of firm B. At the conditions of setting 2, consumer tastes are variegated enough and A_2B_1 is sufficiently differentiated from A_1B_2 for the former to have positive market share.

Firm M gains higher profit at the equilibrium in proposition 4 which implies that, if

[9] For explicit solution of the case of a single-product monopolist in one of the sub-markets and a potential duopoly in the other, see appendix section B of the second chapter of this thesis.

allowed at the conditions of setting 2, it would strictly prefer to offer its goods in a bundle and not separately. Note, however, that, here, distinct from the bundling equilibrium established in proposition 2, because firm M faces competition in both sub-markets, its payoff given bundling is strictly less than what it would gain if it entered as a single-product firm in sub-market A only. In the latter case, the equilibrium outcome is the same as at the equilibrium established in proposition 2. The only difference is in the competitive interaction between the firms that leads to this outcome.

Finally, note that compared to the outcome at the equilibrium established in proposition 3, at the bundling-equilibrium in proposition 4, firm M increases the market shares of its goods A1 and B2. However, this is at the expense of the reduced profits and market shares of firms A and B. That is, at the conditions of setting 2, pure bundling has the potential to restrict the consumer networks of firm A and firm B which in turn could reduce the qualities of their goods in the presence of positive network externalities.

3.3.3 Setting 3

Setting 3 reveals the decisive importance of consumer taste variegation, and the differentiation of the systems for the effective entry-deterrence outcome of bundling. On the one hand, we show that when consumer tastes do not vary in a wide range, there cannot be a solution of the restricted non-bundling game at which all the four systems have positive market shares. On the other hand, we demonstrate that when the lower-quality systems are more differentiated from the best system than from the worst, the entrants' system A_2B_1 cannot have a positive market share at equilibrium with or without bundling allowed. Thus, it appears as one of the two necessary conditions for the effective entry-deterrence of firm A. The second necessary condition is the use of bundling by firm M, which, as was already discussed in the previous subsection, makes system A_2B_2 unavailable, but has no direct effect on the demand for A_2B_1.

The non-bundling equilibrium is established in proposition 5 below.

Proposition 5. When conditions (18) and (19) are satisfied, a unique non-bundling subgame equilibrium exists, at which all firms enter the two markets but only the two most differentiated systems in table 1 have positive sales. The equilibrium is characterized by the prices in (H6) – (H9).

Proof: see section H of the appendix.

The left-hand side inequality condition of setting 3 in (18) restricts the variegation of consumer tastes. The lowest taste $\underline{\theta}$ is required to be less than two times smaller than the highest taste $\overline{\theta}$. As a result, the market demand is concentrated over a limited number of systems at equilibrium. Specifically, we cannot have all the four systems sold in positive amounts as at the equilibrium outcome of setting 2. In addition, the inequality conditions in (19) imply that when compared to the best system A_1B_1, the worst system A_2B_2 is more differentiated from the outside option and less differentiated from A_1B_2 and A_2B_1. This makes firm M prefer to sell B2 as a part of system A_2B_2 alone at a covered market. Furthermore, the latter outcome is achievable at a positive price of B2. However, the market-covering price of B2 also undercuts system A_2B_1, when the variegation of consumer tastes is not too restricted, which is ensured by the right-hand side inequality condition of setting 3 in (18). As a result, the equilibrium established in proposition 5 leads to an outcome that cannot occur at setting 2. Only the two most differentiated systems have positive demands. Still all three firms have an incentive to enter the market because they make positive profits at equilibrium.

The bundling solution is established in proposition 6 below.

Proposition 6. When conditions (18) and (19) are satisfied, there is a unique bundling subgame equilibrium, at which the entry of firm A is efficiently deterred. The corresponding equilibrium outcome coincides with the bundling subgame equilibrium outcome at setting 1. Accordingly, the equilibrium prices of firm M and firm B are given by the symmetric set of prices in (F39) while firm M does not enter the market. This is also the subgame-perfect equilibrium solution of the model at setting 3.

Proof: see section H of the appendix.

Like the bundling equilibrium outcome at setting 2, the outcome from the bundling equilibrium established in proposition 6 prevents firm A from selling its good in a system with the good of firm M. The variegation of consumer tastes, however, is restricted by the left-hand side inequality condition of setting 3 in (18). Therefore, the other system in which firm A could sell its good, A_2B_2, also has no demand. As a result, firm A would be effectively deterred in the case that firm M chooses to bundle its goods. The bundling strategy is optimal for firm M because it leads to the equilibrium outcome described in proposition 2. In particular, firm M is a monopolist in submarket A which allows it to maximize the consumer surplus it extracts by only selling its goods in a bundled system upgraded by B1.

As a final step of our analysis we measure the social welfare effect of the exclusion of firm A at the bundling subgame equilibrium established in proposition 5, relative to the non-bundling subgame equilibrium in proposition 6. The results are formally stated in proposition 7 below.

Proposition 7. When conditions (18) – (19) are satisfied, allowing for bundling has strictly negative social welfare at equilibrium. Both producer surplus and consumer surplus are lower at the bundling equilibrium established in proposition 6, relative to the non-bundling equilibrium in proposition 5.

Proof: see section H of the appendix.

Bundling affects social welfare in two ways. On the one hand, by excluding good A2 from the market, it leads to an increase in the average quality of the systems available in the market. However, at optimal prices, the systems available in the market are more expensive. Therefore, consumers with lower taste prefer to switch to the outside option which leads to decrease in the consumer surplus.[10] The inequality conditions of setting 3

[10]Note that the dominance of the negative effect would be stronger in the presence of positive network externalities as in the setup suggested by Carlton and Waldman (2002).

in (19) imply that the systems containing A2 are not much differentiated from the systems containing A1. Therefore, the negative effect of the market exclusion of A2 dominates the positive effect. Additionally, the market share of the bundled system also decreases, which reduces the producer surplus. Therefore, at setting 3 the social effect of bundling is negative.

3.4 Conclusion

In the presented paper we examine how important the quality differentiation of systems and the variability of consumer tastes are for the entry-deterrence effect of bundling implemented by a multi-product firm. We propose the solution of three particular market settings for vertically differentiated systems.

First, we demonstrate the importance of quality differentiation for the exclusionary effect of bundling in a simple market setting where a multi-product firm is monopolist in the (core) sub-market for one of the system elements, and faces a superior-quality entry threat in the other (adjacent) sub-market. Since consumers value systems that include the entrant's good more highly, the multi-product firm gains more from selling its monopoly good only as part of such a system. This allows it to extract more consumer surplus than if it sells its bundled system alone. However, if the multi-product firm does not bundle, it is suboptimal for it to sell in the adjacent sub-market. Thus, in this particular setting, the multi-product firm faces the dilemma of whether to sell its goods only bundled in a system or to turn itself into a single-product firm operating solely in the core sub-market. In both cases, the multi-product firm would have the same profit and market share in the monopoly sub-market.

Second, we analyze a market setting in which the multi-product firm faces an entry threat from a potential inferior-quality rival in the core sub-market, and from a potential superior-quality rival in the adjacent sub-market. In addition, for this particular setting we assume that the lower-quality systems are more differentiated than the higher-quality systems and consumer tastes are sufficiently variegated for showing that at these assumptions all three firms could have positive market share both at a non-bundled and bundled equilibrium. As in the first setting, consumers value the system formed by the goods of the multi-product firm and the superior-quality entrant most highly. Unlike the first setting, however, the multi-product firm cannot afford to exit the adjacent sub-market, because then the superior-quality entrant will have more market power to extract a larger share of the consumer surplus. Therefore, at the non-bundling equilibrium established in the paper, the multi-product firm chooses to stay, though only symbolically, in the adjacent sub-market by charging its good at a zero price. That is, it makes no profit directly from its sales in the adjacent sub-market but at the same time this allows it to preserve its dominant share of the extracted consumer surplus. Given bundling, the system formed by the goods of the multi-product firm and the good of the inferior entrant will be more expensive and of lower quality than the bundled system alone. Accordingly, at a pure-bundling equilibrium the core-submarket entrant sells its good only combined in a system with the good of the entrant in the adjacent submarket.

Third, again we consider a multi-product firm facing a potential inferior-quality entrant in the core sub-market, and potential superior-quality entrant in the adjacent sub-market. Distinct from the second setting, however, in this third setting we assume that the lower-quality systems are not more differentiated than the higher-quality systems and consumer tastes are not variegated enough to ensure demand for the goods of all the three firms at a pure-bundling equilibrium. As a result, we show that the adjacent-submarket good of the multi-product firm cannot be sold as an element of more than one system either at a non-bundling or at a pure-bundling equilibrium. At the non-bundling equilibrium, it could be sold only as a positively priced element of the system it forms with the good of the inferior-quality entrant. At the pure-bundling equilibrium, the multi-product firm's core-submarket good could be sold only as an element of the bundled system. In turn, this will lead to effective exclusion of the inferior-quality rival. The latter case brings higher profit to the multi-product firm but has a negative effect on both producer and consumer surplus, resulting in decreased social welfare.

In summary, we could infer from our results that the variability of consumer tastes and the pattern of quality differentiation are decisive for the entry-deterrence effect of the bundling or pricing strategies of a multi-product firm in a market for vertically differentiated systems. Therefore, these market characteristics should not be neglected when analyzing the competition in vertically differentiated markets for systems. The more differentiated the low-ranked systems' qualities and the further variegated the consumer tastes, the less likely it is that pure bundling could effectively exclude an inferior-quality entrant from the core sub-market. Furthermore, the multi-product firm strictly gains from accommodating a superior-quality rival in the adjacent sub-market.

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Appendix (Mathematical Proofs)

F. Solution for Setting 1 - Proof of Propositions 1 and 2

In this section, we derive the solution of the game for setting 1.

We start with an analysis of the (reduced) non-bundling game equilibrium and the proof of proposition 1.

First, we need to check what the optimal pricing strategies of firms M and B would be if they expected the market to be covered (i.e. $\theta_{12/0} \leq \underline{\theta}$) and both systems A_1B_1 and A_1B_2 (i.e. $\underline{\theta} < \theta_{11/12} < \overline{\theta}$), which could be formed by their goods to have positive demands. The equilibrium prices are given by the solutions of the revenue-maximization problems of the two firms derived by substituting with the corresponding expressions for D_{11} and D_{12} from (12) and (13) in (1) and (2), respectively:[11]

$$\underset{p_{A1},p_{B2}}{Max}\ R_M^{\text{S1NoBc12}} = p_{A1}\left(\overline{\theta} - \underline{\theta}\right) + p_{B2}\left(\frac{p_{B1} - p_{B2}}{d_{11/12}} - \underline{\theta}\right) \tag{F1}$$

$$\underset{p_{B1}}{Max} R_B^{\text{S1NoBc12}} = p_{B1}\left(\overline{\theta} - \frac{p_{B1} - p_{B2}}{d_{11/12}}\right) \tag{F2}$$

The optimal solutions for the above pair of problems is given by the solution of the following system of equations (resp. first-order optimality conditions):

$$\begin{aligned}
\frac{\partial R_M^{\text{S1NoBc12}}}{\partial p_{A1}} &= \overline{\theta} - \underline{\theta} = 0 \\
\frac{\partial R_M^{\text{S1NoBc12}}}{\partial p_{B2}} &= \frac{p_{B1} - 2p_{B2}}{d_{11/12}} - \underline{\theta} = 0 \\
\frac{\partial R_B^{\text{S1NoBc12}}}{\partial p_{B1}} &= \overline{\theta} - \frac{2p_{B1} - p_{B2}}{d_{11/12}} = 0
\end{aligned} \tag{F3}$$

[11]The superscript S1NoBc12 stands for setting 1 (S1), non-bundling (NoB) equilibrium, covered (c) market with both the systems of quality ranks 1 and 2 (12) having positive demands.

Note, however, that the first equation in (F3) can never be satisfied when the condition of setting 1 in (14) holds. Therefore, we have a corner solution for p_{A1} given by the highest value of the price of good A1 at which the market would still be covered by system A_1B_2, i.e. $\theta_{12/0} \leq \underline{\theta}$:

$$\begin{aligned}
p_{A1}^{\text{S1NoBc12}} &= d_{12/0}\underline{\theta} - p_{B2}^{\text{S1NBc12}} \\
p_{B2}^{\text{S1NoBc12}} &= \frac{1}{3}(\overline{\theta} - 2\underline{\theta})d_{11/12} \Rightarrow p_{A1}^{\text{S1NBc12}} = \frac{\left(2d_{11/12} + 3d_{12/0}\right)\underline{\theta} - d_{11/12}\overline{\theta}}{3} \\
p_{B1}^{\text{S1NoBc12}} &= \frac{1}{3}(2\overline{\theta} - \underline{\theta})d_{11/12}
\end{aligned} \tag{F4}$$

The market shares of the two systems A_1B_1 and A_1B_2 at the optimal prices in (F4) are as follows:

$$\begin{aligned}
D_{11}^{S1NoBc12} &= \overline{\theta} - \theta_{11/12}^{S1NoBc12} = \frac{2\overline{\theta} - \underline{\theta}}{3} \\
D_{12}^{S1NoBc12} &= \theta_{11/12}^{S1NoBc12} - \underline{\theta} = \frac{\overline{\theta} - 2\underline{\theta}}{3}
\end{aligned} \tag{F5}$$

Trivially, both expressions above are positive when the condition of setting 1 in (14) holds. The value of the revenue of firm M at the optimal prices in (F4) is given by the following expression:

$$R_M^{\text{S1NoBc12}*} = \frac{1}{9}\left(9(\overline{\theta} - \underline{\theta})\underline{\theta}d_{12/0} - \left(2\overline{\theta}^2 - 5\overline{\theta}\underline{\theta} + 2\underline{\theta}^2\right)d_{11/12}\right) \tag{F6}$$

The condition of setting 1 in (14) is not sufficient for the price of good A1 in (F4) to satisfy the non-negativity requirement. The following condition must also hold true:

$$\underline{\theta} \geq \frac{\overline{\theta}d_{11/12}}{3d_{12/0} + 2d_{11/12}} \tag{F7}$$

Otherwise, good A1 will have a zero optimal price and the optimal revenue of firm M will be given by the following expression:

$$R_M^{\text{S1NoBc12}**} = \frac{1}{9}(\overline{\theta} - 2\underline{\theta})^2 d_{11/12} \tag{F8}$$

Second, we need to check what the optimal pricing strategies of firms M and B would be if they expected the market to be non-covered (i.e. $\theta_{12/0} > \underline{\theta}$) and both systems A_1B_1 and A_1B_2 (i.e. $\theta_{12/0} < \theta_{11/12} < \overline{\theta}$), which could be formed by their goods to have positive demands. The equilibrium prices are again given by the solutions of the revenue-maximization problems of the two firms derived by substituting with the corresponding expressions for D_{11} and D_{12} from (12) and (13) in (1) and (2), respectively:[12]

$$\underset{p_M}{Max} R_M^{\text{S1NoBn12}} = p_{A1}\left(\overline{\theta} - \frac{p_{A1}+p_{B2}}{d_{12/0}}\right) + p_{B2}\left(\frac{p_{B1}-p_{B2}}{d_{11/12}} - \frac{p_{A1}+p_{B2}}{d_{12/0}}\right) \tag{F9}$$

$$\underset{p_{B1}}{Max} R_B^{\text{S1NoBn12}} = p_{B1}\left(\overline{\theta} - \frac{p_{B1}-p_{B2}}{d_{11/12}}\right) \tag{F10}$$

The optimal solutions for the above pair of problems is given by the solution of the following system of equations (resp. first-order optimality conditions):

$$\begin{aligned}
\frac{\partial R_M^{\text{S1NoBn12}}}{\partial p_{A1}} &= \overline{\theta} - \frac{2p_{A1}+p_{B2}}{d_{12/0}} = 0 \\
\frac{\partial R_M^{\text{S1NoBn12}}}{\partial p_{B2}} &= \frac{p_{B1}-2p_{B2}}{d_{11/12}} - \frac{p_{A1}+2p_{B2}}{d_{12/0}} = 0 \\
\frac{\partial R_B^{\text{S1NoBn12}}}{\partial p_{B1}} &= \overline{\theta} - \frac{2p_{B1}-p_{B2}}{d_{11/12}} = 0
\end{aligned} \tag{F11}$$

[12]The superscript S1NoBn12 stands for setting 1 (S1), non-bundling (NoB) equilibrium, non-covered (n) market with both the systems of quality ranks 1 and 2 (12) having positive demands.

The set of optimal prices is then given by the following expressions:

$$\begin{aligned}
p_{A1}^{\text{S1NoBn12}} &= \frac{\overline{\theta} d_{12/0}}{2} \\
p_{B2}^{\text{S1NoBn12}} &= 0 \\
p_{B1}^{\text{S1NoBn12}} &= \frac{\overline{\theta} d_{11/12}}{2}
\end{aligned} \tag{F12}$$

Note that this time the optimal expression for the price of good B2 does not satisfy the non-negativity condition. The optimal price of good B2 is therefore given by the zero-price corner solution. However, at the optimal prices in (F12) the initially assumed expectations of the firms for both systems A_1B_1 and A_1B_2 having positive demands are not met. Particularly, there is no demand for system A_1B_2 because the two marginal taste variables whose difference gives the demand for A_1B_2 have identical values:

$$D_{12}^{\text{S1NoBn12}} = \theta_{11/12} - \theta_{12/0} = \frac{\overline{\theta}}{2} - \frac{\overline{\theta}}{2} = 0 \tag{F13}$$

Hence, there is not such a non-bundling equilibrium at which the market is not covered and the both systems A_1B_1 and A_1B_2 have positive demands.

Accordingly, if firm M still expects the market not to be covered at a non-bundling equilibrium, its revenue-maximization problem should reflect the result in (F13). That is, it should represent firm M's revenue function when firm M is a single-product firm operating in the core sub-market only:[13]

$$\underset{p_M}{Max} R_M^{\text{S1NoBn1}} = p_{A1} \left(\overline{\theta} - \frac{p_{A1} + p_{B1}}{d_{11/0}} \right) \tag{F14}$$

[13]The superscript S1NoBn1 stands for setting 1 (S1), non-bundling (NoB) equilibrium, non-covered (n) market with only the system of quality rank 1 (1) having a positive demand.

$$\underset{p_{B1}}{Max} R_B^{\text{S1NoBn1}} = p_{B1}\left(\overline{\theta} - \frac{p_{B1} - p_{B1}}{d_{11/0}}\right) \tag{F15}$$

Thus, firm M is in an identical position as firm B. Therefore, the corresponding solution for prices is symmetric:

$$p_{A1}^{\text{S1NoBn1}*} = p_{B1}^{\text{S1NoBn1}*} = \frac{\bar{\theta}}{3} d_{11/0} \tag{F16}$$

The market share of system A_1B_1 is as follows:

$$D_{11}^{\text{S1NoBn1}} = \overline{\theta} - \theta_{11/0}^{\text{S1NoBn1}} = \frac{\overline{\theta}}{3} \tag{F17}$$

The optimal profits are identical:

$$R_M^{\text{S1NoBn1}*} = R_{B1}^{\text{S1NoBn1}*} = \frac{\bar{\theta}^2}{9} d_{11/0} \tag{F18}$$

Third, we compare the revenue expression in (F18) with that of firm M at the covered-market non-bundling subgame equilibrium in (F6) by subtracting the latter from the former:

$$R_M^{\text{S1NoBn1}*} - R_M^{\text{S1NoBc12}*} = \frac{1}{9}\left(\overline{\theta}^2 d_{11/0} - 9\underline{\theta}(\overline{\theta} - \underline{\theta}) d_{12/0} + \left(2\overline{\theta}^2 - 5\overline{\theta}\underline{\theta} + 2\underline{\theta}^2\right) d_{11/12}\right) \tag{F19}$$

The expression in (F19) is strictly decreasing in $\underline{\theta}$ for any value of it satisfying the condition of setting 1 in (14):

$$\frac{\partial\left(R_M^{\text{S1NoBn1}*}-R_M^{\text{S1NoBc12}*}\right)}{\partial\underline{\theta}}=\frac{2\underline{\theta}\left(9d_{12/0}+2d_{11/12}\right)-\overline{\theta}\left(9d_{12/0}+5d_{11/12}\right)}{9}<0$$
$$\text{for }\ \underline{\theta}<\frac{\overline{\theta}}{2}<\frac{\left(9d_{12/0}+5d_{11/12}\right)}{2\left(9d_{12/0}+2d_{11/12}\right)}\overline{\theta} \tag{F20}$$

At $\underline{\theta}=\frac{\overline{\theta}}{2}$, however, the revenue difference in (F19) is strictly positive when the conditions of setting 1 in (14) and (15) hold:

$$R_M^{\text{S1NoBn1}*}-R_M^{\text{S1NoBc12}*}=\frac{1}{4}\overline{\theta}^2\left(4d_{11/0}-9d_{12/0}\right)>0$$
$$\text{for }\ \underline{\theta}=\frac{\overline{\theta}}{2}\ \text{ and }\ d_{12/0}<\frac{4d_{11/0}}{9} \tag{F21}$$

Analogously, we could also compare the revenue expression in (F18) with that of firm M at the covered-market non-bundling equilibrium in (F8) for the case when (F7) is not satisfied and good A1 is charged a zero price:

$$R_M^{\text{S1NoBn1}*}-R_M^{\text{S1NoBc12}**}=\frac{1}{9}\left(\overline{\theta}^2d_{11/0}-(\overline{\theta}-2\underline{\theta})^2d_{11/12}\right) \tag{F22}$$

Note that the difference in the brackets is strictly positive when the condition of setting 1 in (14) is satisfied. So, in the non-bundling subgame firm M is strictly better off to enter as a single-product firm in the core-submarket only and set price for A1 at which only system A_1B_1 has positive demand in a non-covered market. This completes the proof of proposition 1.

Next, we continue with the proof of proposition 2.

We derive below the equilibrium solution of the pure-bundling subgame that would follow after firm M chooses to enter with both its good, A1 and B2, offered in a bundle only.

First, we need to check what the optimal pricing strategies of firms M and B would be if they expected the market to be covered (i.e. $\theta_{12/0} \leq \underline{\theta}$) and both systems $A_1B_2B_1$ and A_1B_2 (i.e. $\underline{\theta} < \theta_{11/12} < \overline{\theta}$), which could be formed by their goods to have positive demands. The equilibrium prices are again given by the solutions of the revenue-maximization problems of the two firms derived by substituting with the corresponding expressions for D_{11} and D_{12} from (12) and (13) in (1) and (2), respectively:[14]

$$\underset{p_M}{Max} R_M^{\text{S1Bc12}} = p_M \left(\overline{\theta} - \underline{\theta}\right) \tag{F23}$$

$$\underset{p_{B1}}{Max} R_B^{\text{S1Bc12}} = p_{B1} \left(\overline{\theta} - \frac{p_{B1}}{d_{11/12}}\right) \tag{F24}$$

The optimal solutions for the above pair of problems is given by the solution of the following system of equations (resp. first-order optimality conditions):

$$\begin{aligned} \frac{\partial R_M^{\text{S1Bc12}}}{\partial p_M} &= \overline{\theta} - \underline{\theta} = 0 \\ \frac{\partial R_B^{\text{S1Bc12}}}{\partial p_{B1}} &= \overline{\theta} - \frac{2p_{B1}}{d_{11/12}} = 0 \end{aligned} \tag{F25}$$

As in the solution for the non-bundling covered-market equilibrium, the first equation in (F25) can never be satisfied when the condition of setting 1 in (14) holds. Therefore, we have a corner solution for p_M given by the highest value of the price of the bundled system A_1B_2 at which the market would still be covered by system A_1B_2, i.e. $\theta_{12/0} \leq \underline{\theta}$:

$$\begin{aligned} p_M^{\text{S1Bc12}} &= \underline{\theta} d_{12/0} \\ p_{B1}^{\text{S1Bc12}} &= \frac{1}{3}\overline{\theta} d_{11/12} \end{aligned} \tag{F26}$$

The market shares of the two systems $A_1B_2B_1$ and A_1B_2 at the optimal prices in (F26) are

[14] The superscript S1Bc12 stands for setting 1 (S1), bundling (B) equilibrium, covered (c) market with both the systems of quality ranks 1 and 2 (12) having positive demands.

as follows:

$$D_{121}^{S1Bc12} = \overline{\theta} - \theta_{11/12}^{S1Bc12} = \frac{2\overline{\theta}}{3}$$
$$D_{12}^{S1Bc12} = \theta_{11/12}^{S1Bc12} - \underline{\theta} = \frac{\overline{\theta} - 3\underline{\theta}}{3} \tag{F27}$$

The value of the revenue of firm M at the optimal prices in (F26) is given by the following expression:

$$R_M^{\text{S1Bc12}*} = (\overline{\theta} - \underline{\theta})\underline{\theta} d_{12/0} \tag{F28}$$

Note that the condition of setting 1 in (14) is necessary but not sufficient for the demand of the bundled system alone to be positive. The following stricter condition must also hold:

$$\underline{\theta} < \frac{1}{3}\overline{\theta} \tag{F29}$$

Otherwise, a solution would not exist with an equilibrium outcome at which the market is covered and both systems, $A_1B_2B_1$ and A_1B_2, having positive demands. Instead only $A_1B_2B_1$ would be demanded and the optimal prices would be symmetric:

$$p_M^{\text{S1Bc1}} = p_{B1}^{\text{S1Bc1}} = \frac{\underline{\theta} d_{11/0}}{2} \tag{F30}$$

The corresponding expression for the revenue of firm M is given as follows:

$$R_M^{\text{S1Bc1}*} = \frac{1}{2}(\overline{\theta} - \underline{\theta})\underline{\theta} d_{11/0} \tag{F31}$$

which is strictly smaller than the revenue in (F28) when the condition of setting 1 in (15) holds.

Second, we need to check what the optimal pricing strategies of firms M and B would be if they expected the market to be non-covered (i.e. $\theta_{12/0} > \underline{\theta}$) and both systems $A_1B_2B_1$ and A_1B_2 (i.e. $\underline{\theta} < \theta_{11/12} < \overline{\theta}$), which could be formed by their goods to have positive demands. The equilibrium prices are again given by the solutions of the revenue-maximization problems of the two firms derived by substituting with the corresponding expressions for D_{11} and D_{12} from (12) and (13) in (1) and (2), respectively:[15]

$$\underset{p_M}{Max} R_M^{\text{S1Bn12}} = p_M \left(\overline{\theta} - \frac{p_M}{d_{12/0}} \right) \tag{F32}$$

$$\underset{p_{B1}}{Max} R_B^{\text{S1Bn12}} = p_{B1} \left(\overline{\theta} - \frac{p_{B1}}{d_{11/12}} \right) \tag{F33}$$

The optimal solutions for the above pair of problems is given by the solution of the following system of equations (resp. first-order optimality conditions):

$$\begin{aligned} \frac{\partial R_M^{\text{S1Bn12}}}{\partial p_M} &= \overline{\theta} - \frac{2p_M}{d_{12/0}} = 0 \\ \frac{\partial R_B^{\text{S1Bn12}}}{\partial p_{B1}} &= \overline{\theta} - \frac{2p_{B1}}{d_{11/12}} = 0 \end{aligned} \tag{F34}$$

Hence, we derive the following optimal prices:

$$\begin{aligned} p_M^{\text{S1Bn12}} &= \frac{\overline{\theta} d_{12/0}}{2} \\ p_{B1}^{\text{S1Bn12}} &= \frac{\overline{\theta} d_{11/12}}{2} \end{aligned} \tag{F35}$$

Again, the corresponding solution implies zero demand for the bundle alone. Firm M,

[15]The superscript S1Bn12 stands for setting 1 (S1), bundling (B) equilibrium, non-covered (n) market with both the systems of quality ranks 1 and 2 (12) having positive demands.

however, does not exit market B.

$$D_{121}^{S1Bn12} = \theta_{11/12}^{S1Bn12} - \theta_{12/0}^{S1Bn12} = \frac{\overline{\theta}}{2} - \frac{\overline{\theta}}{2} = 0 \tag{F36}$$

Consumers are not given an option to purchase A1 without B2. Therefore, whoever buys the best system A_1B_1 receives B2 together with it. The corresponding equilibrium prices are given by the solution of the following revenue-maximization problems:[16]

$$\underset{p_M}{Max} R_M^{\mathrm{S1Bn1}} = p_M \left(\overline{\theta} - \theta_{11/0}\right) = p_M \left(\overline{\theta} - \frac{p_M + p_{B1}}{d_{11/0}}\right) \tag{F37}$$

$$\underset{p_{B1}}{Max} R_{B1}^{\mathrm{S1Bn1}} = p_{B1} \left(\overline{\theta} - \theta_{11/0}\right) = p_{B1} \left(\overline{\theta} - \frac{p_M + p_{B1}}{d_{11/0}}\right) \tag{F38}$$

As in the solution for the non-bundling covered-market equilibrium, firm M is in identical position as firm B. Therefore, the corresponding solution for prices is symmetric:

$$p_M^{\mathrm{S1Bn1*}} = p_{B1}^{\mathrm{S1Bn1*}} = \frac{\overline{\theta}}{3} d_{11/0} \tag{F39}$$

The market share of the upgraded system $A_1B_2B_1$ is as follows:

$$D_{11}^{\mathrm{S1Bn1}} = \overline{\theta} - \theta_{11/0}^{S1Bn1} = \frac{\overline{\theta}}{3} \tag{F40}$$

The optimal profits are identical:

$$R_M^{\mathrm{S1Bn1*}} = R_{B1}^{\mathrm{S1Bn1*}} = \frac{\bar{\theta}^2}{9} d_{11/0} \tag{F41}$$

Third, we compare the revenue expression in (F41) with that of firm M at the

[16]The superscript S1Bn1 stands for setting 1 (S1), bundling (B) equilibrium, non-covered (n) market with only the (upgraded) system of quality rank 1 (1) having positive demand.

covered-market bundling subgame equilibrium in (F28) by subtracting the latter from the former:

$$R_M^{\text{S1Bn1}*} - R_M^{\text{S1Bc12}*} = \frac{\overline{\theta}^2 d_{11/0}}{9} - \underline{\theta}(\overline{\theta} - \underline{\theta}) d_{12/0} \tag{F42}$$

The revenue difference in (F42) is strictly positive when the conditions of setting 1 in (14) and (15) hold. Since the revenue of firm M in (F31) is strictly exceeded by $R_M^{\text{S1Bc12}*}$ in (F28), we could conclude that the two alternative covered-market pricing strategies of firm M are strictly dominated by its non-covered market strategy in (F39). Hence, firm M is strictly better off from setting a price of its bundled system at which only the consumers who prefer the upgraded system $A_1B_2B_1$ buy it and in turn the market is not covered.

Finally, note that allowing for bundling does not change the optimal payoffs of the two firms given by (F18) and (F41), respectively:

$$\begin{aligned} R_M^{\text{S1Bn1}*} &= R_M^{\text{S1NoBn1}*} = \frac{\bar{\theta}^2}{9} d_{11/0} \\ R_B^{\text{S1Bn1}*} &= R_B^{\text{S1NoBn1}*} = \frac{\bar{\theta}^2}{9} d_{11/0} \end{aligned} \tag{F43}$$

Hence, the bundling strategy only weakly dominates the no bundling strategy of firm M. The only difference between the subgame-perfect equilibria of the (reduced) non-bundling game and the bundling game is that consumers are not given an option to buy A1 alone without B2 so that the latter has positive demand:

$$D_{B2}^{\text{S1Bn1}*} = D_{11}^{\text{S1Bn1}} > 0 = D_{B2}^{\text{S1NoBn1}*} \tag{F44}$$

which completes the proof of proposition 2.

G. Solution for Setting 2 - Proof of Propositions 3 and 4

In this section, we derive the solution of the game for setting 2.

We start with analysis of the (reduced) non-bundling game equilibrium and the proof of proposition 3.

First, we need to check what would be the optimal pricing strategies of the three firms if they are all expected to enter the market, and for their systems to have positive market shares but still not to cover the market. The equilibrium prices are given by the solutions of the revenue-maximization problems of the two firms, derived by substituting with the corresponding expressions for D_{11}, D_{12}, D_{21} and D_{22} from (6) to (9) in (1), (2) and (3), respectively:[17]

$$\begin{aligned}\underset{p_{A1},p_{B2}}{Max}\ R_M^{\text{S2NoBn1234}} = p_{A1}&\left(\overline{\theta}-\frac{(p_{A1}+p_{B2})-(p_{A2}+p_{B1})}{d_{12/21}}\right)+\\ +p_{B2}&\left(\frac{(p_{B1}-p_{B2})}{d_{11/12}}-\frac{(p_{A1}+p_{B2})-(p_{A2}+p_{B1})}{d_{12/21}}\right)+\\ +p_{B2}&\left(\frac{(p_{B1}-p_{B2})}{d_{21/22}}-\frac{(p_{A2}+p_{B2})}{d_{22/0}}\right)\end{aligned} \tag{G1}$$

$$\begin{aligned}\underset{p_{B1}}{Max} R_B^{\text{S2NoBn1234}} =p_{B1}&\left(\overline{\theta}-\frac{(p_{B1}-p_{B2})}{d_{11/12}}\right)+\\ +p_{B1}&\left(\frac{(p_{A1}+p_{B2})-(p_{A2}+p_{B1})}{d_{12/21}}-\frac{(p_{B1}-p_{B2})}{d_{21/22}}\right)\end{aligned} \tag{G2}$$

$$\underset{p_{A2}}{Max} R_A^{\text{S2NoBn1234}} = p_{A2}\left(\frac{(p_{A1}+p_{B2})-(p_{A2}+p_{B1})}{d_{12/21}}-\frac{(p_{A2}+p_{B2})}{d_{22/0}}\right) \tag{G3}$$

[17]The superscript S2NoBn1234 stands for setting 2 (S2), non-bundling (NoB) equilibrium, non-covered (n) market with all the four systems of quality ranks 1, 2, 3 and 4 (1234) having positive demands.

The optimal solution of the above problems is given by the solution of the following system of equations (resp. first-order optimality conditions):

$$\begin{aligned}
\frac{\partial R_M^{\text{S2NoBn1234}}}{\partial p_{A1}} =& \overline{\theta} - \frac{(2p_{A1}+p_{B2})-(p_{A2}+p_{B1})}{d_{12/21}} = 0 \\
\frac{\partial R_M^{\text{S2NoBn1234}}}{\partial p_{B2}} =& \frac{(p_{B1}-2p_{B2})}{d_{11/12}} - \frac{(p_{A1}+2p_{B2})-(p_{A2}+p_{B1})}{d_{12/21}} + \\
& + \frac{(p_{B1}-2p_{B2})}{d_{21/22}} - \frac{(p_{A2}+2p_{B2})}{d_{22/0}} = 0 \\
\frac{\partial R_B^{\text{S2NoBn1234}}}{\partial p_{B1}} =& \overline{\theta} - \frac{(2p_{B1}-p_{B2})}{d_{11/12}} + \\
& + \frac{(p_{A1}+p_{B2})-(p_{A2}+2p_{B1})}{d_{12/21}} - \frac{(2p_{B1}-p_{B2})}{d_{21/22}} = 0 \\
\frac{\partial R_A^{\text{S2NoBn1234}}}{\partial p_{A2}} =& \frac{(p_{A1}+p_{B2})-(2p_{A2}+p_{B1})}{d_{12/21}} - \frac{(2p_{A2}+p_{B2})}{d_{22/0}} = 0
\end{aligned} \tag{G4}$$

Since the resulting expressions for the optimal prices that come from the solution of (G4) are very large we do not state them explicitly here but instead focus only on showing that the price of B2 does not satisfy the non-negative constraint:

$$p_{B2}^{\text{S2NoBn1234}*} = -\frac{X}{Z} \tag{G5}$$

where X and Z stand for:

$$\begin{aligned}
X = & 2\overline{\theta} d_{22/0} d_{11/12} d_{21/22} \left\{ d_{22/0} \left[d_{12/21} d_{21/22} + d_{11/12} \left(d_{12/21} + 2d_{21/22} \right) \right] + \right. \\
& \left. + d_{12/21} \left[2d_{12/21} d_{21/22} + d_{11/12} \left(2d_{12/21} + 3d_{21/22} \right) \right] \right\}
\end{aligned}$$

$$\begin{aligned}
Z = & \left\{ 3d_{11/12} d_{12/21} d_{21/22} \left[4d_{12/21} d_{21/22} + d_{11/12} \left(4d_{12/21} + 3d_{21/22} \right) \right] + \right. \\
& + d_{22/0}^2 \left(d_{11/12} + d_{21/22} \right) \left[9d_{12/21} d_{21/22} + d_{11/12} \left(9d_{12/21} + 8d_{21/22} \right) \right] + \\
& + 2d_{22/0} \left[6d_{12/21}^2 d_{21/22}^2 + d_{11/12} d_{12/21} d_{21/22} \left(12d_{12/21} + 11d_{21/22} \right) + \right. \\
& \left. \left. + d_{11/12}^2 \left(6d_{12/21}^2 + 11d_{12/21} d_{21/22} + 4d_{21/22}^2 \right) \right] \right\}
\end{aligned}$$

Note that both the expressions for X and for Z are strictly positive, and since their ratio enters with a negative sign in (G5), the latter does not satisfy the non-negativity condition. Therefore, we must have a corner solution $p_{B2}^{\text{S2NoBn1234}*} = 0$ at the pricing subgame equilibrium. Then, we could re-write the first and last first-order conditions in (G4) as follows:

$$\begin{aligned}
\frac{\partial R_M^{\text{S2NoBn1234}}}{\partial p_{A1}} &= \overline{\theta} - 2\theta_{12/21} - \frac{p_{A2}+p_{B1}}{d_{12/21}} = 0 \\
\frac{\partial R_A^{\text{S2NoBn1234}}}{\partial p_{A2}} &= \theta_{12/21} - 2\theta_{22/0} - \frac{p_{A2}}{d_{12/21}} = 0
\end{aligned} \tag{G6}$$

Accordingly, the equations in (G6) imply that, at the pricing subgame equilibrium, the marginal taste variables must be characterized by the following relationship:

$$\overline{\theta} > 2\theta_{12/21} > 4\theta_{22/0} \tag{G7}$$

Hence, when (16) holds we have that $\underline{\theta} > \theta_{22/0}$ i.e. the assumption for the existence of non-covered market equilibrium is not satisfied.

Second, we need to check what would be the optimal pricing strategies of the three firms if they all are again expected to enter the market, their systems to have positive market shares but now also the market needs to be covered. The equilibrium prices are given by the solutions of the revenue-maximization problems of the two firms derived by substituting with the corresponding expressions for D_{11}, D_{12}, D_{21} and D_{22} from (6) to (9) in (1), (2) and (3), respectively:[18]

$$\begin{aligned} \underset{p_{A1},p_{B2}}{Max}\ R_M^{\text{S2NoBc1234}} = p_{A1}&\left(\overline{\theta} - \frac{(p_{A1}+p_{B2})-(p_{A2}+p_{B1})}{d_{12/21}}\right)+ \\ +p_{B2}&\left(\frac{(p_{B1}-p_{B2})}{d_{11/12}} - \frac{(p_{A1}+p_{B2})-(p_{A2}+p_{B1})}{d_{12/21}}\right)+ \\ +p_{B2}&\left(\frac{(p_{B1}-p_{B2})}{d_{21/22}} - \underline{\theta}\right) \end{aligned} \tag{G8}$$

$$\begin{aligned} \underset{p_{B1}}{Max} R_B^{\text{S2NoBc1234}} =& p_{B1}\left(\overline{\theta} - \frac{(p_{B1}-p_{B2})}{d_{11/12}}\right)+ \\ &+p_{B1}\left(\frac{(p_{A1}+p_{B2})-(p_{A2}+p_{B1})}{d_{12/21}} - \frac{(p_{B1}-p_{B2})}{d_{21/22}}\right) \end{aligned} \tag{G9}$$

$$\underset{p_{A2}}{Max} R_A^{\text{S2NoBc1234}} = p_{A2}\left(\frac{(p_{A1}+p_{B2})-(p_{A2}+p_{B1})}{d_{12/21}} - \underline{\theta}\right) \tag{G10}$$

The optimal solution of the above problems is given by the solution of the following

[18]The superscript S2NoBc1234 stands for setting 2 (S2), non-bundling (NoB) equilibrium, covered (c) market with all the four systems of quality ranks 1, 2, 3 and 4 (1234) having positive demands.

system of equations (resp. first-order optimality conditions):

$$\begin{aligned}
\frac{\partial R_M^{\text{S2NoBc1234}}}{\partial p_{A1}} =& \overline{\theta} - \frac{(2p_{A1}+p_{B2})-(p_{A2}+p_{B1})}{d_{12/21}} = 0 \\
\frac{\partial R_M^{\text{S2NoBc1234}}}{\partial p_{B2}} =& \frac{(p_{B1}-2p_{B2})}{d_{11/12}} - \frac{(p_{A1}+2p_{B2})-(p_{A2}+p_{B1})}{d_{12/21}} + \\
& + \frac{(p_{B1}-2p_{B2})}{d_{21/22}} - \underline{\theta} = 0 \\
\frac{\partial R_B^{\text{S2NoBc1234}}}{\partial p_{B1}} =& \overline{\theta} - \frac{(2p_{B1}-p_{B2})}{d_{11/12}} + \\
& + \frac{(p_{A1}+p_{B2})-(p_{A2}+2p_{B1})}{d_{12/21}} - \frac{(2p_{B1}-p_{B2})}{d_{21/22}} = 0 \\
\frac{\partial R_A^{\text{S2NoBc1234}}}{\partial p_{A2}} =& \frac{(p_{A1}+p_{B2})-(2p_{A2}+p_{B1})}{d_{12/21}} - \underline{\theta} = 0
\end{aligned} \tag{G11}$$

The solution of the equations above results in a set of optimal pricing expressions which are too long to be explicitly stated. Therefore, here we only present the expression for the price of B2:

$$p_{B2}^{\text{S2NoBc1234}} = -\frac{d_{11/12}d_{21/22}\left((2\overline{\theta}+5\underline{\theta})d_{12/21}d_{21/22}+d_{11/12}\left((2\overline{\theta}+5\underline{\theta})d_{12/21}+4(\overline{\theta}+\underline{\theta})d_{21/22}\right)\right)}{\left(d_{11/12}+d_{21/22}\right)\left(9d_{12/21}d_{21/22}+d_{11/12}\left(9d_{12/21}+8d_{21/22}\right)\right)} < 0 \tag{G12}$$

which trivially does not satisfy the non-negative constraint. Therefore, we must have a corner solution $p_{B2}^{\text{S2NoBn1234}*} = 0$ at the pricing subgame equilibrium. Then, the rest of the optimal prices are given by the following expressions:

$$p_{A1}^{\text{S2NoBc1234}} = \frac{d_{1221}\left(2(2\overline{\theta}-\underline{\theta})d_{12/21}d_{21/22}+d_{11/12}\left((4\overline{\theta}-2\underline{\theta})d_{12/21}+(4\overline{\theta}-\underline{\theta})d_{21/22}\right)\right)}{6d_{12/21}d_{21/22}+d_{11/12}\left(6d_{12/21}+4d_{21/22}\right)} \tag{G13}$$

$$p_{B1}^{\text{S2NoBc1234}} = \frac{(4\overline{\theta}+\underline{\theta})d_{11/12}d_{12/21}d_{21/22}}{6d_{12/21}d_{21/22}+d_{11/12}\left(6d_{12/21}+4d_{21/22}\right)} \tag{G14}$$

$$p_{A2}^{\mathrm{S2NoBc1234}} = \frac{d_{12/21}\left(2(\overline{\theta}-2\underline{\theta})d_{12/21}d_{21/22}+d_{11/12}\left(2(\overline{\theta}-2\underline{\theta})d_{12/21}-3\underline{\theta}d_{21/22}\right)\right)}{6d_{12/21}d_{21/22}+d_{11/12}\left(6d_{12/21}+4d_{21/22}\right)} \tag{G15}$$

Note that both the expressions for $p_{A1}^{\mathrm{S2NoBc1234}}$ and $p_{B1}^{\mathrm{S2NoBc1234}}$ in (G13) and (G14), respectively, are strictly positive for any $\underline{\theta} < \overline{\theta}$, which holds by definition. The expression for $p_{A2}^{\mathrm{S2NoBc1234}}$ in (G15) is also strictly positive when the following inequality condition holds:

$$\underline{\theta} < \frac{2d_{12/21}\left(d_{11/12}+d_{21/22}\right)}{4d_{12/21}d_{21/22}+d_{11/12}\left(4d_{12/21}+3d_{21/22}\right)}\overline{\theta} \tag{G16}$$

which is less strict than the condition on $\underline{\theta}$ from above in (16) for any $d_{21/22} > \frac{d_{11/12}}{2}$. Accordingly, the validity of the latter inequality is ensured by the condition in (17). After substituting for $p_{B2}^{\mathrm{S2NoBn1234}*} = 0$ and $p_{A2}^{\mathrm{S2NoBc1234}}$ from (G15) into the general expression for the marginal taste variable $\theta_{22/0}$ given in the description of (9), we derive $\theta_{22/0}$ below in terms of the system quality differences and the endpoints of the consumer taste interval:

$$\theta_{22/0}^{\mathrm{S2NoBc1234}} = \frac{d_{12/21}\left[2(\overline{\theta}-2\underline{\theta})d_{12/21}d_{21/22}+d_{11/12}\left(2(\overline{\theta}-2\underline{\theta})d_{12/21}-3\underline{\theta}d_{21/22}\right)\right]}{2\left(3d_{12/21}d_{21/22}+d_{11/12}\left(3d_{12/21}+2d_{21/22}\right)\right)d_{22/0}} \tag{G17}$$

which is smaller than $\underline{\theta}$ i.e. implies a covered market at the above prices if the following condition holds:

$$\underline{\theta} > \frac{2d_{12/21}^2\left(d_{11/12}+d_{21/22}\right)}{\left[4d_{12/21}d_{21/22}+d_{11/12}\left(4d_{12/21}+3d_{21/22}\right)\right)+d_{22/0}\left(6d_{12/21}d_{21/22}+d_{11/12}\left(6d_{12/21}+4d_{21/22}\right)\right]d_{12/21}}\overline{\theta} \tag{G18}$$

Accordingly, the condition in (G18) is less strict than the condition on $\underline{\theta}$ from below in (16) for any $d_{22/0} > \frac{10}{9} d_{12/21}$ given that $d_{21/22} > \frac{3}{2} d_{11/12}$. Both inequalities are ensured by the condition in (17). After substituting for the optimal prices from (G13) – (G15) in the covered-market bundling version of the demand expressions (6) – (9), they take the form:

$$D_{11}^{\mathrm{S2NoBc1234}} = \frac{(2\overline{\theta}-\underline{\theta})d_{12/21}d_{21/22}+2\overline{\theta}d_{11/12}\left(3d_{12/21}+2d_{21/22}\right)}{6d_{12/21}d_{21/22}+2d_{11/12}\left(3d_{12/21}+2d_{21/22}\right)} \tag{G19}$$

$$D_{12}^{\mathrm{S2NoBc1234}} = \frac{(2\overline{\theta}-\underline{\theta})d_{12/21}d_{21/22}-d_{11/12}\left(2(\overline{\theta}+\underline{\theta})d_{12/21}+d_{21/22}\underline{\theta}\right)}{6d_{12/21}d_{21/22}+2d_{11/12}\left(3d_{12/21}+2d_{21/22}\right)} \tag{G20}$$

$$D_{21}^{\mathrm{S2NoBc1234}} = \frac{2(\overline{\theta}+\underline{\theta})d_{12/21}d_{21/22}+d_{11/12}\left((-2O+Q)d_{12/21}+Qd_{21/22}\right)}{6d_{12/21}d_{21/22}+d_{11/12}\left(6d_{12/21}+4d_{21/22}\right)} \tag{G21}$$

$$D_{22}^{\mathrm{S2NoBc1234}} = \frac{d_{1112}\left((4\overline{\theta}-5\underline{\theta})d_{1221}-4\underline{\theta}d_{2122}\right)-6\underline{\theta}d_{1221}d_{2122}}{6d_{1221}d_{2122}+2d_{1112}\left(3d_{1221}+2d_{2122}\right)} \tag{G22}$$

It is trivial that the demand expression for system A_1B_1 in (G19) is always strictly positive.

The condition for the positivity of (G20) sets a constraint from above on the spread of consumer tastes:

$$\underline{\theta} < \frac{2d_{12/21}\left(d_{21/22}-d_{11/12}\right)}{d_{12/21}d_{21/22}+d_{11/12}\left(2d_{12/21}+d_{21/22}\right)}\overline{\theta} \tag{G23}$$

The right-hand side is positive for any $d_{21/22} > d_{11/12}$ and less strict than the condition on $\underline{\theta}$ from above in (16) when $d_{21/22} > \frac{3}{2}d_{11/12}$. The validity of both inequalities is ensured by the condition in (17).

The demand expression for A_2B_1 in (G21) is positive when the following condition holds:

$$\underline{\theta} > \frac{2d_{12/21}\left(d_{11/12} - d_{21/22}\right)}{d_{12/21}d_{21/22} + d_{11/12}\left(2d_{12/21} + d_{21/22}\right)}\overline{\theta} \tag{G24}$$

Note, however, that the right-hand side of (G24) is equal to the right-hand side of (G23), that is it is strictly negative for any $d_{21/22} > d_{11/12}$ which is ensured by the condition in (17). Because $\underline{\theta}$ is positive by definition, the condition in (G24) holds true and A_2B_1 has a positive demand.

Finally, the positivity of (G22) is ensured by the very condition on $\underline{\theta}$ from above in (16).

Hence, the initial assumption that all the four systems have positive demands holds at the equilibrium given by the set of prices (G13) – (G15) when the conditions in (16) and (17) are satisfied. It could be shown (see section H of the appendix), that the latter conditions have no intersection with the corresponding conditions for having an equilibrium solution of the non-bundling game, where some of the systems do not have positive demand. Since the three firms make positive sales, they all have an incentive to enter the market at the first stage. Thus, the subgame-perfect equilibrium of the non-bundling game established in proposition 3 is unique which completes its proof.

The proof of proposition 4 follows the same steps but applied to the extended game allowing for bundling at the second stage.

We need to check what would be the optimal pricing strategies of the three firms if they are all expected to enter the market, firm M to offer its good only in a bundle. Note that consumers have no incentive to buy the bundled system upgraded by the inferior-quality good A2. Therefore, system A_2B_2 cannot have positive demand at equilibrium. Still, we assume that the other three systems have positive market shares.

First, we check for the existence of equilibrium at which the market is not covered.

If we assume that the bundled system is sold not only upgraded by A1, but also alone, the equilibrium prices are given by the solutions of the revenue-maximization problems of the two firms derived by substituting with the corresponding expressions for D_{11}, D_{12} and D_{21} from (6) to (9) in (1), (2) and (3), respectively:[19]

$$\underset{p_M}{Max} R_M^{\text{S2Bn123}} = p_M \left[\overline{\theta} - \frac{p_M - (p_{A2} + p_{B1})}{d_{12/21}} \right] \tag{G25}$$

$$\underset{p_{B1}}{Max} R_B^{\text{S2Bn123}} = p_{B1} \left[\left(\overline{\theta} - \frac{p_{B1}}{d_{11/12}} \right) + \frac{p_M - (p_{A2} + p_{B1})}{d_{12/21}} - \frac{(p_{A2} + p_{B1})}{d_{21/0}} \right] \tag{G26}$$

$$\underset{p_{A2}}{Max} R_A^{\text{S2Bn123}} = p_{A2} \left[\frac{p_M - (p_{A2} + p_{B1})}{d_{12/21}} - \frac{(p_{A2} + p_{B1})}{d_{21/0}} \right] \tag{G27}$$

[19]The superscript S2Bn123 stands for setting 2 (S2), bundling (B) equilibrium, non-covered (n) market with only the systems of quality ranks 1, 2, 3 (123) having positive demands.

The optimal solution of the above problems is given by the solution of the following system of equations (resp. first-order optimality conditions):

$$\begin{aligned}
\frac{\partial R_M^{\mathrm{S2Bn123}}}{\partial p_M} &= \overline{\theta} - \frac{2p_M - (p_{A2} + p_{B1})}{d_{12/21}} = 0 \\
\frac{\partial R_B^{\mathrm{S2Bn123}}}{\partial p_{B1}} &= \overline{\theta} - \frac{2p_{B1}}{d_{11/12}} + \frac{p_M - (p_{A2} + 2p_{B1})}{d_{12/21}} - \frac{p_{A2} + 2p_{B1}}{d_{21/0}} = 0 \\
\frac{\partial R_A^{\mathrm{S2Bn123}}}{\partial p_{A2}} &= \frac{p_M - (2p_{A2} + p_{B1})}{d_{12/21}} - \frac{(2p_{A2} + p_{B1})}{d_{21/0}} = 0
\end{aligned} \tag{G28}$$

The corresponding set of optimal prices is given by the following expressions:

$$p_M^{\mathrm{S2Bn123}} = \frac{d_{12/21}\left(d_{21/0} + d_{12/21}\right)\left(3d_{11/12}d_{12/21} + 4d_{21/0}\left(d_{11/12} + d_{12/21}\right)\right)}{6d_{11/12}d_{12/21}^2 + 2d_{21/0}d_{12/21}\left(5d_{11/12} + 4d_{12/21}\right) + d_{21/0}^2\left(4d_{11/12} + 6d_{12/21}\right)}\overline{\theta} \tag{G29}$$

$$p_{B1}^{\mathrm{S2Bn123}} = \frac{d_{210}d_{1112}d_{1221}\left(4d_{210} + 5d_{1221}\right)}{6d_{1112}d_{1221}^2 + 2d_{210}d_{1221}\left(5d_{1112} + 4d_{1221}\right) + d_{210}^2\left(4d_{1112} + 6d_{1221}\right)}\overline{\theta} \tag{G30}$$

$$p_{A2}^{\mathrm{S2Bn123}} = \frac{d_{210}\left(2d_{210} - d_{1112}\right)d_{1221}^2}{6d_{1112}d_{1221}^2 + 2d_{210}d_{1221}\left(5d_{1112} + 4d_{1221}\right) + d_{210}^2\left(4d_{1112} + 6d_{1221}\right)}\overline{\theta} \tag{G31}$$

After substituting for the optimal prices from (G29), (G30) and (G31) in (5), we could derive the equilibrium expressions for the marginal taste variables, $\theta_{11/12}$ and $\theta_{12/21}$, the difference of which is given as follows:

$$\theta_{11/12}^{\mathrm{S2Bn123}} - \theta_{12/21}^{\mathrm{S2Bn123}} = \frac{d_{12/21}\left(2d_{21/0}^2 - 3d_{11/12}d_{12/21} + d_{21/0}\left(d_{12/21} - 3d_{11/12}\right)\right)}{6d_{11/12}d_{12/21}^2 + 2d_{21/0}d_{12/21}\left(5d_{11/12} + 4d_{12/21}\right) + d_{21/0}^2\left(4d_{11/12} + 6d_{12/21}\right)}\overline{\theta} \tag{G32}$$

which is strictly positive for $d_{11/12} < \frac{d_{21/0}\left(2d_{21/0}+d_{12/21}\right)}{3\left(d_{21/0}+d_{12/21}\right)}$ i.e. when $d_{21/0} > 3d_{11/12}$. Therefore, when the condition in (17) is satisfied, the assumption that the bundled system is sold not only upgraded by A1, but also alone, holds true at equilibrium.

Analogously, we could also derive the equilibrium expression for $\theta_{21/0}$ as shown below:

$$\theta_{21/0} = \frac{d_{12/21}\left(2d_{11/12}d_{12/21} + d_{21/0}\left(2d_{11/12} + d_{12/21}\right)\right)}{3d_{11/12}d_{12/21}^2 + d_{21/0}^2\left(2d_{11/12} + 3d_{12/21}\right) + d_{21/0}d_{12/21}\left(5d_{11/12} + 4d_{12/21}\right)}\overline{\theta} \tag{G33}$$

which is smaller than $\underline{\theta}$, i.e. a non-covered market equilibrium does not exist if the following condition holds:

$$\underline{\theta} > \frac{d_{12/21}\left(2d_{11/12}d_{12/21} + d_{21/0}\left(2d_{11/12} + d_{12/21}\right)\right)}{3d_{11/12}d_{12/21}^2 + d_{21/0}^2\left(2d_{11/12} + 3d_{12/21}\right) + d_{21/0}d_{12/21}\left(5d_{11/12} + 4d_{12/21}\right)}\overline{\theta} \tag{G34}$$

The right-hand side of the inequality in (G18), however, is strictly smaller than the condition from below on $\underline{\theta}$ in (16), when the following inequality conditions jointly hold:

$$d_{21/0} > \frac{5d_{12/21}}{2} \tag{G35}$$

$$d_{11/12} > -\frac{3d_{21/0}^2 d_{12/21}}{2d_{21/0}^2 - 3d_{21/0}d_{12/21} - 5d_{12/21}^2} \tag{G36}$$

The condition in (G35) is satisfied when (17) holds true. Furthermore, the right-hand side of (G36) is strictly negative when the condition in (G35) is satisfied. Hence, since $d_{11/12}$ is positive by definition, when (17) holds, there cannot be non-covered market equilibrium of the bundling game.

Second, we derive the covered-market equilibrium of the bundling game at the conditions of setting 2.

If we assume that the bundled system is sold not only upgraded by A1, but also alone, the equilibrium prices are given by the solutions of the revenue-maximization problems of the two firms, derived by substituting with the corresponding expressions for D_{11}, D_{12} and D_{21} from (6) to (9) in (1), (2) and (3), respectively:[20]

$$\underset{p_M}{Max} R_M^{\text{S2Bc123}} = p_M \left[\overline{\theta} - \frac{p_M - (p_{A2} + p_{B1})}{d_{12/21}} \right] \tag{G37}$$

$$\underset{p_{B1}}{Max} R_B^{\text{S2Bc123}} = p_{B1} \left[\left(\overline{\theta} - \frac{p_{B1}}{d_{11/12}} \right) + \frac{p_M - (p_{A2} + p_{B1})}{d_{12/21}} - \underline{\theta} \right] \tag{G38}$$

$$\underset{p_{A2}}{Max} R_A^{\text{S2Bc123}} = p_{A2} \left[\frac{p_M - (p_{A2} + p_{B1})}{d_{12/21}} - \underline{\theta} \right] \tag{G39}$$

The optimal solution of the above problems is given by the solution of the following system of equations (resp. first-order optimality conditions):

$$\begin{aligned} \frac{\partial R_M^{\text{S2Bc123}}}{\partial p_M} &= \overline{\theta} - \frac{2p_M - (p_{A2} + p_{B1})}{d_{12/21}} = 0 \\ \frac{\partial R_B^{\text{S2Bc123}}}{\partial p_{B1}} &= \overline{\theta} - \frac{2p_{B1}}{d_{11/12}} + \frac{p_M - (p_{A2} + 2p_{B1})}{d_{12/21}} - \underline{\theta} = 0 \\ \frac{\partial R_A^{\text{S2Bc123}}}{\partial p_{A2}} &= \frac{p_M - (2p_{A2} + p_{B1})}{d_{12/21}} - \underline{\theta} = 0 \end{aligned} \tag{G40}$$

[20] The superscript S2Bc123 stands for setting 2 (S2), bundling (B) equilibrium, covered (c) market with only the systems of quality ranks 1, 2, 3 (123) having positive demands.

The corresponding set of optimal prices is given by the following expressions:

$$p_M^{\text{S2Bc123}} = \frac{(2\overline{\theta} - \underline{\theta})d_{12/21}\left(d_{11/12} + d_{12/21}\right)}{2d_{11/12} + 3d_{12/21}} \tag{G41}$$

$$p_{B1}^{\text{S2Bc123}} = \frac{(2\overline{\theta} - \underline{\theta})d_{11/12}d_{12/21}}{2d_{11/12} + 3d_{12/21}} \tag{G42}$$

$$p_{A2}^{\text{S2Bc123}} = \frac{d_{12/21}\left((\overline{\theta} - 2\underline{\theta})d_{12/21} - \underline{\theta}d_{11/12}\right)}{2d_{11/12} + 3d_{12/21}} \tag{G43}$$

It is trivial that the expressions in (G41) and (G42) are always strictly positive. Accordingly, the expression in (G43) is also strictly positive when the following condition holds true:

$$\underline{\theta} < \frac{d_{12/21}}{d_{11/12} + 2d_{12/21}}\overline{\theta} \tag{G44}$$

The right-hand side of (G44), however, is strictly larger than the condition from above on $\underline{\theta}$ in (16) for any $d_{21/22} > d_{11/12}$ which is ensured by (17). Hence, when the latter holds true, p_{A2}^{S2Bc123} is positive.

After substituting for the optimal prices from (G41) – (G43) in the covered-market bundling version of the demand expressions (6) – (9), they take the following forms:

$$D_{11}^{\text{S2Bc123}} = \frac{2\overline{\theta}d_{11/12} + (\overline{\theta} + \underline{\theta})d_{12/21}}{2d_{11/12} + 3d_{12/21}} \tag{G45}$$

$$D_{12}^{\text{S2Bc123}} = \frac{(\overline{\theta} - 2\underline{\theta})d_{12/21} - \underline{\theta}d_{11/12}}{2d_{11/12} + 3d_{12/21}} \quad \text{(G46)}$$

$$D_{21}^{\text{S2Bc123}} = \frac{(\overline{\theta} - 2\underline{\theta})d_{12/21} - \underline{\theta}d_{11/12}}{2d_{11/12} + 3d_{12/21}} \quad \text{(G47)}$$

$$D_{22}^{\text{S2Bc123}} = 0 \quad \text{(G48)}$$

It is trivial that on the one hand, the demand expression for system A_1B_1 in (G45) is always strictly positive. On the other hand, the demand expressions for the other two systems, A_1B_2 and A_2B_1, are identical and their positivity is ensured by the condition in (17) in the same way as the positivity of p_{A2}^{S2Bc123}. Hence, the initial assumption that the bundle system is sold not only upgraded by A1, but also alone, holds at equilibrium.

Next, if we substitute for the optimal prices from (G41) – (G43) in the expression for $\theta_{21/0}$ we get the following result:

$$\theta_{21/0}^{\text{S2Bc123}} = \frac{d_{12/21}\left(2(\overline{\theta} - \underline{\theta})d_{11/12} + (\overline{\theta} - 2\underline{\theta})d_{12/21}\right)}{d_{21/0}\left(2d_{11/12} + 3d_{12/21}\right)} \quad \text{(G49)}$$

which is smaller than $\underline{\theta}$, i.e. the assumption for a covered market will hold at equilibrium, as long as the following condition is satisfied:

$$\underline{\theta} > \frac{\overline{\theta}d_{12/21}\left(2d_{11/12} + d_{12/21}\right)}{2d_{12/21}\left(d_{11/12} + d_{12/21}\right) + d_{21/0}\left(2d_{11/12} + 3d_{12/21}\right)} \quad \text{(G50)}$$

The right-hand side of the inequality in (G50), however, is strictly smaller than the condition from below on $\underline{\theta}$ in (16). Hence, there is a bundling subgame equilibrium with all three firms entering the market, firm M choosing to sell its goods in a bundle, and all three firms having positive market shares and profits at the corresponding set of optimal prices in (G41) – (G43). The very profits could be derived by substituting for the optimal prices in the revenue expressions (G25) – (G27), respectively. The resulting expression for the profit of firm M is given below:

$$R_M^{\mathrm{S2Bc123}*} = \frac{(-2\overline{\theta}+\underline{\theta})^2 d_{12/21}\left(d_{11/12}+d_{12/21}\right)^2}{\left(2d_{11/12}+3d_{12/21}\right)^2} \tag{G51}$$

Similarly, we could derive the expression for the optimal revenue of firm M at the non-bundling equilibrium established by proposition 3:

$$R_M^{\mathrm{S2NoBc1234}*} = \frac{d_{12/21}\left(2(2\overline{\theta}-\underline{\theta})d_{12/21}d_{21/22}+d_{11/12}\left((4\overline{\theta}-2\overline{\theta})d_{12/21}+(4\overline{\theta}-\underline{\theta})d_{21/22}\right)\right)^2}{4\left(3d_{12/21}d_{21/22}+d_{11/12}\left(3d_{12/21}+2d_{21/22}\right)\right)^2} \tag{G52}$$

The difference between the firm M's optimal revenue expressions in (G51) and (G52) is given by the expression:

$$R_M^{\mathrm{S2Bc123}*} - R_M^{\mathrm{S2NoBc1234}*} = W \cdot Q \tag{G53}$$

where:

$$W = \frac{d_{11/12}d_{12/21}\left[12(2\overline{\theta}-\underline{\theta})d_{12/21}^2 d_{21/22}+d_{11/12}d_{12/21}\left(12(2\overline{\theta}-\underline{\theta})d_{12/21}+(40\overline{\theta}-17\underline{\theta})d_{21/22}\right)\right]}{4\left(2d_{11/12}+3d_{12/21}\right)^2\left(3d_{12/21}d_{21/22}+d_{11/12}\left(3d_{12/21}+2d_{21/22}\right)\right)^2}+$$
$$+\frac{2d_{11/12}^2 d_{11/12}d_{12/21}\left(5(2\overline{\theta}-\underline{\theta})d_{12/21}+(8\overline{\theta}-3\underline{\theta})d_{21/22}\right)}{4\left(2d_{11/12}+3d_{12/21}\right)^2\left(3d_{12/21}d_{21/22}+d_{11/12}\left(3d_{12/21}+2d_{21/22}\right)\right)^2}$$

$$Q = \left[-3\underline{\theta}d_{12/21}d_{21/22}+d_{11/12}\left((4\overline{\theta}-2\underline{\theta})d_{12/21}-2\underline{\theta}d_{21/22}\right)\right]$$

It is trivial that W is always strictly positive while the condition for the positivity of Q is given by the following inequality:

$$\underline{\theta} < \frac{4\overline{\theta}d_{11/12}d_{12/21}}{3d_{12/21}d_{21/22} + 2d_{11/12}\left(d_{12/21} + d_{21/22}\right)} \tag{G54}$$

which is obviously larger than the condition from above on $\underline{\theta}$ in (16). Hence, when the latter holds, firm M will be strictly better off from bundling.

To summarize, when the conditions in (16) and (17) hold true, there is a unique subgame perfect equilibrium characterized by all three firms entering the market, firm M offering its goods in a bundle only and the optimal prices given in (G41) – (G43), which completes the proof of proposition 4.

H. Solution for Setting 3 - Proof of Propositions 5, 6 and 7

In this section, we derive the solution of the game for setting 3.

We start with analysis of the (reduced) non-bundling game equilibrium and the proof of proposition 5.

First, note that since in setting 3 we have the same three firms as in setting 2, the profit-maximization problems are given by (G1) – (G3) for the non-bundling game when firms expect all the four systems to have positive demands but market not to be covered. Accordingly, the inequality relationship between the marginal taste variables in (G7) must hold again at equilibrium. However, the left-hand side of the inequality condition in (18) implies that in setting 3, $\underline{\theta}$ is less than twice smaller than $\overline{\theta}$. Hence, the following inequality holds true:

$$\underline{\theta} > \theta_{12/21} > \theta_{22/0} \tag{H1}$$

which implies non-existence of a non-covered market equilibrium with all four systems having positive demands. Similarly, it could be shown that the inequality condition in (18) is sufficient for the non-existence of non-covered market equilibrium with any number of systems.

We could also show how the solution for the covered-market equilibrium derived from (G11) implies that the inequality in (H1) must also hold when the market is expected to be covered. Therefore, in setting 3 a covered-market equilibrium at which all the four systems have positive demands does not exist. Furthermore, among all the other potential covered-market equilibria at which firm M enters as a multi-product firm in both sub-markets without bundling (S3NoBc12, S3NoBc14, S3NoBc123, S3NoBc123), that which allows for the biggest differentiation of the systems' qualities and therefore yields the highest profit for firm M is established in proposition 5 (S3NoBc14). For the sake of brevity, it is the only equilibrium we derive below.

Let us suppose that the covered-market equilibrium established in proposition 5 exists. Namely, all firms enter, but only A_1B_1 and A_2B_2 have positive demands, and the market is covered. Then, the equilibrium prices are given by the solutions of the revenue-maximization problems of the three firms, derived by substituting with the corresponding expressions for D_{11} and D_{22} from (6) and (9) as well as with D_{12}=$D_{21} = 0$ in (1), (2) and (3):[21]

$$\begin{aligned}\underset{p_{A1},p_{B2}}{Max}\ R_M^{\text{S3NoBc14}} =& p_{A1}\left(\overline{\theta} - \frac{(p_{A1}+p_{B1})-(p_{A2}+p_{B2})}{d_{11/22}}\right)+ \\ &+p_{B2}\left(\frac{(p_{A1}+p_{B1})-(p_{A2}+p_{B2})}{d_{11/22}} - \underline{\theta}\right)\end{aligned} \tag{H2}$$

$$\underset{p_{B1}}{Max} R_B^{\text{S3NoBc14}} = p_{B1}\left(\overline{\theta} - \frac{(p_{A1}+p_{B1})-(p_{A2}+p_{B2})}{d_{11/22}}\right) \tag{H3}$$

$$\underset{p_{A2}}{Max} R_A^{\text{S3NoBc14}} = p_{A2}\left(\frac{(p_{A1}+p_{B1})-(p_{A2}+p_{B2})}{d_{11/22}} - \underline{\theta}\right) \tag{H4}$$

[21] The superscript S3NoBc14 stands for setting 3 (S3), non-bundling (NoB) equilibrium, covered (c) market with only systems of quality ranks 1 and 4 (14) having positive demands.

The optimal solution of the above problems is given by the solution of the following system of equations (resp. first-order optimality conditions):

$$
\begin{aligned}
\frac{\partial R_M^{\text{S3NoBc14}}}{\partial p_{A1}} &= \overline{\theta} - \frac{(2p_{A1}+p_{B1})-(p_{A2}+2p_{B2})}{d_{11/22}} = 0 \\
\frac{\partial R_M^{\text{S3NoBc14}}}{\partial p_{B2}} &= \frac{(2p_{A1}+p_{B1})-(p_{A2}+2p_{B2})}{d_{11/22}} - \underline{\theta} = 0 \\
\frac{\partial R_B^{\text{S3NoBc14}}}{\partial p_{B1}} &= \overline{\theta} - \frac{(p_{A1}+2p_{B1})-(p_{A2}+p_{B2})}{d_{11/22}} = 0 \\
\frac{\partial R_A^{\text{S2NoBc14}}}{\partial p_{A2}} &= \frac{(p_{A1}+p_{B1})-(2p_{A2}+p_{B2})}{d_{11/22}} - \underline{\theta} = 0
\end{aligned}
\tag{H5}
$$

Note that the first two equations in (H5) are inconsistent. Therefore, we should have a corner solution for the price of B2, i.e. it obtains the highest value at which the market would still be covered. The corresponding set of optimal prices is given as follows:

$$p_{A1}^{\text{S3NoBc14}} = \frac{1}{2}\underline{\theta}\left(2d_{22/0}+d_{11/22}\right) \tag{H6}$$

$$p_{A2}^{\text{S3NoBc14}} = \frac{1}{4}(2\overline{\theta}-3\underline{\theta})d_{11/22} \tag{H7}$$

$$p_{B1}^{\text{S3NoBc14}} = \frac{1}{4}(2\overline{\theta}-\underline{\theta})d_{11/22} \tag{H8}$$

$$p_{B2}^{\text{S3NoBc14}} = \underline{\theta}d_{22/0} + \frac{1}{4}(-2\overline{\theta}+3\underline{\theta})d_{11/22} \tag{H9}$$

The condition for the price expression in (H9) to be positive is:

$$\underline{\theta} > \frac{2d_{11/22}}{4d_{22/0}+3d_{11/22}}\overline{\theta} \tag{H10}$$

which is less strict than the right-hand side inequality condition in (18) when $d_{22/0} > \frac{1}{4}\left(\sqrt{7}-2\right)d_{11/22}$. The validity of the latter inequality is ensured by (19).

The set of optimal prices (H6)–(H9) implies that A_1B_2 is cheaper than A_2B_1 as long as the right-hand inequality condition in (18) is satisfied:

$$\left(p_{A1}^{\text{S3NoBc14}}+p_{B2}^{\text{S3NoBc14}}\right)-\left(p_{A2}^{\text{S3NoBc14}}+p_{B1}^{\text{S3NoBc14}}\right)=\frac{8\underline{\theta}d_{22/0}-3(2\overline{\theta}-3\underline{\theta})d_{11/22}}{4}<0 \tag{H11}$$

Since A_1B_2 is of higher rank that A_2B_1, however, the latter cannot have a positive market share when charged a higher price.

Then, it remains only to show that A_1B_2 has no demand at the prices (H6)–(H9), when A_2B_1 is excluded from the market. The corresponding expression for its market share is given as follows:

$$\begin{aligned} D_{12}^{\text{S3NoBc14}} &= \theta_{11/12}^{\text{S3NoBc}}-\theta_{12/22}^{\text{S3NoBc}}= \\ &= \frac{\left[(2\overline{\theta}-5\underline{\theta})d_{11/12}+4(\overline{\theta}-\underline{\theta})d_{12/22}\right]d_{11/22}-4\underline{\theta}d_{22/0}\left(d_{11/12}+d_{12/22}\right)}{4d_{11/12}d_{12/22}} \end{aligned} \tag{H12}$$

which is non-positive when the following inequality holds:

$$\underline{\theta}>\frac{2\left(d_{11/22}+d_{12/22}\right)}{4d_{22/0}+5d_{11/22}-d_{12/22}}\overline{\theta} \tag{H13}$$

The inequality in (H13) is less strict than the left-hand side inequality condition in (18) as long as $d_{12/22}<\frac{103}{237}d_{11/22}$ which is ensured by (19).

Hence, only the most differentiated systems A_1B_1 and A_2B_2 have positive market share when conditions (18)–(19) are satisfied:

$$D_{11}^{\text{S3NoBc14}}=\frac{(2\overline{\theta}-\underline{\theta})}{4} \tag{H14}$$

$$D_{22}^{\text{S3NoBc14}}=\frac{(2\overline{\theta}-3\underline{\theta})}{4} \tag{H15}$$

This result completes the proof of proposition 5.

The proof of proposition 6 follows directly from the observation that the left-hand side of the inequality condition in (18) is inconsistent with the condition for a positive price of good A2 in (G44). The corresponding set of optimal prices is given by the following expressions:

$$p_M^{\text{S3Bc14}} = \frac{\left((3\overline{\theta} - \underline{\theta})d_{11/12} + 2\overline{\theta} d_{12/21}\right)}{3d_{11/12} + 4d_{12/21}} \tag{H16}$$

$$p_{B1}^{\text{S3Bc14}} = \frac{\left((3\overline{\theta} - 2\underline{\theta})d_{11/12} d_{12/21}\right)}{3d_{11/12} + 4d_{12/21}} \tag{H17}$$

$$p_{A2}^{\text{S3Bc14}} = 0 \tag{H18}$$

When the condition in (G44) is not satisfied, however, both expressions for the market shares of A_1B_2 and A_2B_1 are non-positive:

$$D_{12}^{\text{S3Bc14}} = \frac{(\overline{\theta} - 2\underline{\theta})d_{12/21} - \underline{\theta} d_{11/12}}{3d_{11/12} + 4d_{12/21}} < 0 \tag{H19}$$

$$D_{21}^{\text{S3Bc14}} = \frac{2(\overline{\theta} - 2\underline{\theta})d_{12/21} - 2\underline{\theta} d_{11/12}}{3d_{11/12} + 4d_{12/21}} < 0 \tag{H20}$$

Hence, at the conditions in (18) and (19) a bundling subgame equilibrium does not exist at which A_2B_1 has positive market share. Therefore, firm 2 has no incentive to enter the market which leads to the subgame equilibrium outcome at setting 1 under the conditions of setting 3.

To derive the optimal bundling strategy we compare the profits of firm M in case of exclusionary bundling and in case of non-bundling. The optimal expressions for profits are given as follows:

$$R_M^{\text{S3Bn1}*} = R_M^{\text{S1Bn1}*} = \frac{\bar{\theta}^2}{9} d_{11/0} \tag{H21}$$

$$R_M^{\text{S3NoBc14}*} = (\overline{\theta} - \underline{\theta})\underline{\theta} d_{22/0} - \frac{\left(4\overline{\theta}^2 - 16\overline{\theta}\underline{\theta} + 11\underline{\theta}^2\right)}{16} d_{11/22} \tag{H22}$$

The difference between (H21) and (H22) is given by the expression:

$$R_M^{\text{S3Bn1}*} - R_M^{\text{S3NoBc14}*} = \frac{1}{9}\overline{\theta}^2 d_{11/0} - (\overline{\theta} - \underline{\theta})\underline{\theta} d_{22/0} + \frac{1}{16}\left(4\overline{\theta}^2 - 16\overline{\theta}\underline{\theta} + 11\underline{\theta}^2\right) d_{11/22} \tag{H23}$$

which is strictly positive when the following condition holds:

$$d_{22/0} > -\frac{\left(52\overline{\theta}^2 - 144\overline{\theta}\underline{\theta} + 99\underline{\theta}^2\right)}{16\left(\overline{\theta}^2 - 9\overline{\theta}\underline{\theta} + 9\underline{\theta}^2\right)} d_{11/22} \tag{H24}$$

The right-hand side of (H24) is strictly decreasing in $\underline{\theta}$ when the condition of setting 3 in (18) holds. Hence, the inequality in (H24) is satisfied as long as $d_{22/0}$ satisfies the following condition:

$$d_{22/0} > \frac{4d_{22/0}d_{11/22}\left(52d_{22/0} + 9d_{11/22}\right)}{-64d_{22/0}^2 + 288d_{22/0}d_{11/22} + 81d_{11/22}^2} \tag{H25}$$

The inequality in (H25) is derived by substituting $\underline{\theta}$ in (H24) by its lower bound in (18). The condition in (H25) is satisfied for any $d_{22/0} < \frac{1}{8}\left(5 + \sqrt{70}\right) d_{11/22}$ which is ensured by (19). Therefore, firm M finds it optimal to apply bundling and efficiently deter the entry of firm A in the primary market. This completes the proof of proposition 6.

Finally, to prove proposition 7, we measure the change in the social welfare, the producer surplus and the consumer surplus in case of bundling relative to the case of no bundling at setting 3 of the model.

The social welfare in the two subgames is given as a sum of the integrals of the consumer valuations of the systems they buy based on their tastes:

$$SW^{\text{S3Bn1}} = \int_{\underline{\theta}}^{\theta_{11/0}} \chi_0 \theta d\theta + \int_{\theta_{11/0}}^{\bar{\theta}} \chi_{11} \theta d\theta = \int_{\underline{\theta}}^{\frac{3}{4}\bar{\theta}} \chi_0 \theta d\theta + \int_{\frac{3}{4}\bar{\theta}}^{\bar{\theta}} \chi_{11} \theta d\theta \tag{H26}$$

$$SW^{\text{S3NoBc14}} = \int_{\underline{\theta}}^{\theta_{11/22}} \chi_{22} \theta d\theta + \int_{\theta_{11/22}}^{\bar{\theta}} \chi_{11} \theta d\theta = \int_{\underline{\theta}}^{\frac{2\bar{\theta}+\underline{\theta}}{4}} \chi_{22} \theta d\theta + \int_{\frac{2\bar{\theta}+\underline{\theta}}{4}}^{\bar{\theta}} \chi_{11} \theta d\theta \tag{H27}$$

In explicit form the expressions in (H27) and (H26) look as follows:

$$SW^{\text{S3Bn1}*} = \frac{\left[5\bar{\theta}^2 \chi_{11} + \left(4\bar{\theta}^2 - 9\underline{\theta}^2\right) \chi_0\right]}{18} \tag{H28}$$

$$SW^{\text{S3NoBc14}*} = \frac{\left[\left(12\bar{\theta}^2 - 4\bar{\theta}\underline{\theta} - \underline{\theta}^2\right) \chi_{11} + \left(4\bar{\theta}^2 + 4\bar{\theta}\underline{\theta} - 15\underline{\theta}^2\right) \chi_{22}\right]}{32} \tag{H29}$$

The difference between (H28) and (H29) gives a measure of the change in social welfare from applying bundling at setting 3:

$$\begin{aligned} & SW^{\text{S3Bn1}*} - SW^{\text{S3NoBc14}*} = \\ & = -\frac{\left(2\bar{\theta} - 3\underline{\theta}\right)\left[\left(14\bar{\theta} + 3\underline{\theta}\right)\chi_{11} + 9\left(2\bar{\theta} + 5\underline{\theta}\right)\chi_{22} - 16\left(2\bar{\theta} + 3\underline{\theta}\right)\chi_0\right]}{288} = \\ & = -\frac{1}{288}(2\overline{\theta} - 3\underline{\theta})\left(16(2\overline{\theta} + 3\underline{\theta})d_{22/0} + (14\overline{\theta} + 3\underline{\theta})d_{11/22}\right) \end{aligned} \tag{H30}$$

The expression in (H30) is strictly negative for any $\underline{\theta}$ that satisfies the condition of setting 3 in (18).

Producer surplus in each subgame equilibrium is given by the sum of the corresponding profits made by the firms in the two sub-markets:

$$PS^{\mathrm{S3Bn1}*} = R_M^{\mathrm{S1Bn1}*} + R_{B1}^{\mathrm{S1Bn1}*} = \frac{1}{9}\overline{\theta}^2 d_{11/0} + \frac{1}{9}\overline{\theta}^2 d_{11/0} = \frac{2}{9}\overline{\theta}^2 \left(d_{11/22} + d_{22/0}\right) \quad \text{(H31)}$$

$$\begin{aligned} PS^{\mathrm{S3NoBc14}*} &= R_M^{\mathrm{S3NoBc14}*} + R_{B1}^{\mathrm{S3NoBc14}*} + R_{A2}^{\mathrm{S3NoBc14}*} = \\ &= (\overline{\theta} - \underline{\theta})\underline{\theta} d_{22/0} - \frac{1}{16}\left(4\overline{\theta}^2 - 16\overline{\theta}\underline{\theta} + 11\underline{\theta}^2\right) d_{11/22} + \frac{(2\overline{\theta} - \underline{\theta})^2 d_{11/22}}{16} + \\ &\qquad + \frac{(2\overline{\theta} - 3\underline{\theta})^2 d_{11/22}}{16} = \\ &= (\overline{\theta} - \underline{\theta})\underline{\theta} d_{22/0} + \frac{1}{16}\left(4\overline{\theta}^2 - \underline{\theta}^2\right) d_{11/22} \end{aligned} \quad \text{(H32)}$$

The difference between the expressions in (H31) and (H32) gives a measure of the change in the producer surplus due to bundling at setting 3:

$$PS^{\mathrm{S3Bn1}*} - PS^{\mathrm{S3NoBc14}*} = -\frac{1}{144}(2\overline{\theta} - 3\underline{\theta})\left(16(3\underline{\theta} - \overline{\theta})d_{22/0} + (2\overline{\theta} + 3\underline{\theta})d_{11/22}\right) \quad \text{(H33)}$$

which is strictly negative when $\underline{\theta}$ satisfies the condition of setting 3 in (18).

Finally, subtracting the right-hand side of (H33) from the right-hand side of (H30) yields the following measure of the effect of bundling on the consumer surplus:

$$CS^{\mathrm{S3Bn1}*} - CS^{\mathrm{S3NoBc14}*} = -\frac{1}{288}(2\overline{\theta} - 3\underline{\theta})\left(16(4\overline{\theta} - 3\underline{\theta})d_{22/0} + (10\overline{\theta} - 3\underline{\theta})d_{11/22}\right) \quad \text{(H34)}$$

The expression in (H34) is strictly negative for any $\underline{\theta}$ which satisfies the condition of setting 3 in (18).

The expressions in (H30), (H33) and (H34) imply that bundling at setting 3 leads to reduction in both producer and consumer surpluses and thus has strictly negative social welfare effect. This result completes the proof of proposition 7.