

Title: Relational Approach to Universal Algebra

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Abstract: We give some descriptions of certain algebraic properties using relations and relational structures. In the first part, we focus on Neumann's lattice of interpretability types of varieties. First, we prove a characterization of varieties defined by linear identities, and we prove that some conditions cannot be characterized by linear identities. Next, we provide a partial result on Taylor's modularity conjecture, and we discuss several related problems. Namely, we show that the interpretability join of two idempotent varieties that are not congruence modular is not congruence modular either, and the analogue for idempotent varieties with a cube term. In the second part, we give a relational description of higher commutator operators, which were introduced by Bulatov, in varieties with a Mal'cev term. Furthermore, we use this result to prove that for every algebra with a Mal'cev term there exists a largest clone containing the Mal'cev operation and having the same congruence lattice and the same higher commutator operators as the original algebra, and to describe explicit (though infinite) set of identities describing supernilpotence of a fixed degree in any congruence permutable variety.

Keywords: linear varieties, clone, interpretability, Mal'cev condition, Mal'cev algebra, commutator, supernilpotence