

Title: The realization problem for von Neumann regular rings

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Abstract: With every unital ring R , one can associate the abelian monoid $\mathcal{V}(R)$ of isomorphism classes of finitely generated projective right R -modules. Said monoid is a conical monoid with order-unit. Moreover, for von Neumann regular rings, it satisfies the Riesz refinement property. In the thesis, we deal with the question, under what conditions an abelian conical refinement monoid with order-unit can be realized as $\mathcal{V}(R)$ for some unital von Neumann regular ring or algebra, with emphasis on countable monoids. Two generalizations of the construction of $\mathcal{V}(R)$ to the context of nonunital rings are presented and their interrelation is analyzed. To that end, necessary properties of rings with local units and modules over such rings are developed. Further, the construction of Leavitt path algebras over quivers is presented, as well as the construction of a monoid associated with a quiver that is isomorphic to $\mathcal{V}(R)$ of the Leavitt path algebra over the same quiver. These methods are then used to realize directed unions of finitely generated free abelian monoids as $\mathcal{V}(R)$ of algebras over any given field. A method of constructing abelian conical refinement monoids that are not realizable as $\mathcal{V}(R)$ of regular algebras over any uncountable field is also presented. The thesis is concluded by computation of the monoid $\mathcal{V}(R)$ of an algebra R over a countable field, constructed by Chuang and Lee.

Keywords: von Neumann regular ring, refinement monoid, Leavitt path algebra, nonunital Morita equivalence