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MASTER'S THESIS

Understanding co-jumps in financial markets

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Declaration of Authorship

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Prague, May 13, 2016

Signature

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Abstract

This thesis focuses on impact of jumps and simultaneous jumps (co-jumps) in asset prices on future volatility. Our main contribution to the empirical literature lies in the use of panel Heterogeneous Autoregressive (HAR) model that allows us to obtain average effect of jumps for both the portfolio of 29 U.S. stocks and 8 individual market sectors our stocks belong to. On top of that we investigate the effect of sign for both jumps and co-jumps. The estimation results indicate that the impact of jumps on future volatility is positive whereas for co-jumps it is negative. We also document tendency of downward jumps and co-jumps to be followed by increase in volatility and that upward jumps and co-jumps are followed by decrease in volatility. Finally, results for individual sectors reveal that estimated effects vary across industries - for cyclical sectors volatility is in general more sensitive to negative jumps and less sensitive to positive jumps than for defensive sectors.

Keywords Realized variance, Jumps, Co-jumps, HAR,
Asymmetric effect

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Abstrakt

Tato diplomová práce se zabývá dopadem skoků a simultánních skoků v cenách aktiv na budoucí volatilitu. Hlavním přínosem této práce je využití heterogenního autoregresivního (HAR) modelu pro panelová data, který nám umožňuje odhadnout průměrný efekt skoků jak pro celé portfolio obsahující 29 akcií, tak pro jednotlivé tržní sektory. Navíc analyzujeme, zda je efekt rozdílný pro pozitivní a negativní (simultánní) skoky. Výsledky aplikovaných modelů dokazují, že vliv skoků na budoucí volatilitu je kladný, zatímco vliv simultánních skoků je záporný. Také naznačují, že negativní skoky mají tendenci vést k vyšší budoucí volatilitě, zatímco v důsledku pozitivních skoků volatilita klesá. Nakonec výsledky naší analýzy dokazují, že odhadované dopady na volatilitu se liší pro jednotlivé sektory - cyklické sektory jsou obecně citlivější vůči negativním skokům a méně citlivé vůči pozitivním skokům než proticyklická odvětví.

Klíčová slova

Realizovaná variance, Skoky, Simultánní skoky, HAR, Asymetrický efekt

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Acronyms

BNS	(Jump test proposed by) Barndorff-Nielsen & Shephard
BPV	Bipower Variation
GICS	Global Industry Classification Standard
HAR	Heterogeneous Autoregressive (model)
HAR-RV	Heterogeneous Autoregressive Model for Realized Variance
NYSE	New York Stock Exchange
OLS	Ordinary Least Squares
QV	Quadratic Variation
RS	Realized Semivariance
RV	Realized Variance
RVol	Realized Volatility
TPQ	Tri-power Quarticity

Master's Thesis Proposal

Author	Bc. Richard Thoma
Supervisor	prof. Jozef Baruník, Ph.D.
Proposed topic	Understanding co-jumps in financial markets

Motivation There is a vast literature assuming that the modelling of asset prices as a continuous time diffusion process can be improved by allowing for price discontinuities - jumps. Consequently various tests have been developed in order to detect such a sudden and large discontinuities that are driven primarily by firm-specific and macroeconomic news announcements and liquidity shocks.

Recently, literature studying simultaneous jumps across many assets (co-jumps) attracted attention of researchers as their detection can reveal unique information. In contrast to univariate jumps co-jumps across many assets can represent systemic risk and thus substantially reduce the gain from diversification.

The main objective of this thesis is to investigate the effect of co-jumps in a panel of stocks on their future volatilities and correlations. Thus far, two studies (Caporin et al., 2014 and Clements Liao, 2013) analyzing these dynamics using HAR models has been conducted, but their results are contradictory. Furthermore, I would like to inspect whether the frequency of co-jump arrival changed after the beginning of crisis.

Hypotheses

Hypothesis #1: Co-jumps have a positive effect on future volatility of stocks.

Hypothesis #2: Co-jumps have a positive effect on future correlations of stocks.

Hypothesis #3: Co-jumps in a panel of stocks are more frequent during recession (after 2007).

Methodology In my thesis high-frequency data covering 21 most liquid U.S. stocks which represent seven main market sectors will be used.

Firstly, we must identify co-jumps. Possible solution would be to apply one of univariate jump tests to all stocks and then to use the intersection to detect co-jumps by so-called coexceedance rule. However, as our dataset will consist of a large panel of high-frequency returns it will be more efficient to use the test proposed by Bollerslev et al. (2008) which utilizes the cross-covariance structure in the returns to identify non-diversifiable jumps. Once the co-jumps are identified, we can observe their distribution over time and argue whether third hypothesis is correct or not.

To find out whether co-jumps have an impact on future volatilities and correlations of stocks the HAR (heterogeneous autoregressive) model proposed by Corsi (2009) will be applied. HAR-RV model is a simple tool for volatility forecasting, an AR-type model in the realized volatility with the feature of considering volatilities realized over different time horizons. We will modify this HAR-RV model by including the co-jump indicator function in the same way the HAR-RV-J is derived.

Expected Contribution Understanding co-jumps and their impact on stock correlations and volatilities is of utmost importance for asset allocation and risk management. Among the consequences of a positive effect of co-jumps on volatility and correlations of stocks would be a more volatile portfolio with lower diversification potential. Furthermore higher occurrence of co-jumps during crisis would deteriorate the diversification potential of portfolio even more.

In comparison with previous studies I would like to estimate whether the effect of co-jumps on volatility and correlation is the same for pre-crisis and post-crisis period. Furthermore I will take into account one of the important characteristics of volatility - the leverage effect. As Patton Sheppard (2015) found out, days dominated by negative jumps lead to significantly higher future volatility, days with positive jumps lead to lower future volatility and the response to negative jumps is larger in magnitude. I expect that the same holds for positive and negative co-jumps.

Outline

1. Introduction
2. Theory of realized measures of volatility
3. Jump and co-jump tests
4. HAR models
5. Description of data

6. Results
7. Conclusion

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Chapter 1

Introduction

One of the most important topics in financial econometrics is modeling and forecasting volatility of asset returns. Volatility serves as a proxy for risk and is frequently used for derivatives pricing, asset allocation or risk management. Originally the economic literature focused on volatility forecasting using parametric models (ARCH/GARCH family or stochastic volatility models) has been dominant, however the importance of non-parametric modeling has been growing since high frequency data became wide-spread available and the ex-post non-parametric measure of volatility called realized variance was proposed by Andersen *et al.* (2001). Then Barndorff-Nielsen (2004) introduced bipower variation, an ex-post non-parametric measure of volatility that consistently estimates continuous variation only and a simple Hausman-type test was constructed to identify jumps (price discontinuities driven mainly by exogenous events) based on the difference between two measures of variance - realized variance which contains both continuous and jump variation and jump-robust bipower variance. Subsequently, many univariate jump tests were developed (Andersen *et al.* (2007b), Lee & Mykland (2008), Andersen *et al.* (2010b) or Corsi *et al.* (2010)) and the jump-related literature became a popular field of research.

The literature on co-jumps, simultaneous discontinuities in prices of many assets, is much scarcer. One of the key topics is the co-jumping behavior of two or more asset prices and the relationship of macroeconomic news announcements and co-jumps. Chatrath *et al.* (2014) focused on currency co-jumps, Lahaye *et al.* (2011) on co-jumps between stock index futures, bond futures, and exchange rates. Dungey *et al.* (2009) examined co-jumps across the term structure of the US Treasury bond market and Dungey & Hvozdnyk (2012) studied the joint behavior of spot and futures prices in the US Treasury bond market. Strong co-jumping behavior of the U.S. and Asian (Chinese and Korean) stock markets during the recent financial crisis was found by Li & Zhang (2013) and Kim & Ryu (2015).

The main objective of this thesis is to investigate whether jumps and simultaneous jumps across many stocks have an impact on future volatility. There is a vast literature studying the impact of idiosyncratic jumps on volatility forecasting. Many previous studies (Andersen *et al.* (2007a), Giot & Laurent (2007) or Busch *et al.* (2011)) found that decomposition of total variation into the continuous and jump component leads to better forecasting performance, but the impact of univariate jumps on future volatilities was found mostly insignificant. However, Corsi *et al.* (2010) used their own measure of continuous component (threshold bipower variation) and found significant and positive effect of daily jump component on future volatility. Finally, Patton & Sheppard (2015) showed that the impact of jump component depends on its sign - negative jumps lead to significantly higher future volatility, positive jumps lead to significantly lower future volatility and the response to negative jumps is larger in magnitude. All in all, there is no clear consensus on how jumps affect future volatility and in the case of co-jumps the situation is even less clear as two studies analyzing the predictive power of common jumps on future volatility in a panel of stocks (Clements & Liao (2013) and Caporin *et al.* (2014)) reached different conclusions.

To find out the true effects, we will employ the jump and co-jump tests presented in Chapter 3 and for volatility forecasting we will use a simple and efficient tool - the HAR model proposed by Corsi (2009). Our first contribution to the current jump/co-jump literature lies in employment of panel HAR model that enables us to estimate an average effect of jumps and co-jumps on volatility in a panel of stocks. All the previous studies investigated the effect of significant jumps and co-jumps for individual stocks only. Secondly, we also investigate the average effect across industries as our stocks can be classified into 8 market sectors according to the Global Industry Classification Standard (GICS). Finally, we examine if there is a difference between the impact of positive and negative co-jumps on future volatility in the same way as documented by Patton & Sheppard (2015) for univariate jumps.

Our results show that on average volatility increases following the jump. However, the effect is not the same across different industries. The information technology, financial and energy sectors are rather insensitive to upward jumps, but following the downward jump their volatility drastically increases. Consequently the three above-mentioned sectors drive the positive impact of jumps on future volatility the most. The effect of co-jumps is found negative and we conclude that the sign of the common jump matters as in the case of idiosyncratic jumps.

The rest of this thesis is organized as follows. Chapter 2 provides a summary of the theory of realized measures of variance and introduces two important concepts - realized variance and bipower variation. Chapter 3 presents an overview of tests that are designed

to identify jumps and common jumps. In Chapter 4 the HAR model and its extensions are introduced. Our dataset is described within Chapter 5. In Chapter 6 the results are presented. Last chapter concludes.

Chapter 2

Realized measures of volatility

The structure of this chapter follows mainly Andersen *et al.* (2010a). First we define the price process evolving in continuous time under the condition of no arbitrage and frictionless markets and show how the returns can be decomposed into an expected return and innovation component. Then we define the quadratic variation and its estimator, realized variance. Finally, we introduce the jump-robust counterpart to realized variance - bipower variation.

2.1 Continuous-time no-arbitrage price process

Assume a univariate risky logarithmic price process p_t defined on a complete probability space (Ω, \mathcal{F}, P) which evolves in continuous time over the interval $[0, T]$, where T is a (finite) integer and $\{\mathcal{F}_t\}_{t \in [0, T]} \subseteq \mathcal{F}$ denotes associated natural filtration, all the historical information about the asset price process and other relevant state variables known at time t , which is assumed to satisfy the usual conditions of right continuity and P -completeness.

Then the continuously compounded return over the time interval $[t-h, t]$ is defined as

$$r_{t,h} = p_t - p_{t-h}, \quad 0 \leq h \leq t \leq T. \quad (2.1)$$

Consequently the cumulative return over the $[0, t]$ time interval can be derived as

$$r_t \equiv r_{t,t} = p_t - p_0, \quad 0 \leq t \leq T. \quad (2.2)$$

Finally the following relation between the period-by-period and cumulative returns must hold

$$r_{t,h} = r_t - r_{t-h}, \quad 0 \leq h \leq t \leq T. \quad (2.3)$$

Following Andersen *et al.* (2010a) we assume that the asset price process remains almost surely strictly positive and finite in order to have p_t and r_t well defined over $[0, T]$.

Furthermore, the number of jump points in the return process r_t is countable over the $[0, T]$ time interval and price and return processes are squared integrable. Lastly, we define $r_{t-} \equiv \lim_{\tau \rightarrow t, \tau < t} r_\tau$ and $r_{t+} \equiv \lim_{\tau \rightarrow t, \tau > t} r_\tau$ and uniquely determine the process as the right-continuous left-limit (*càdlàg*) if $r_t = r_{t+}$ and as the left-continuous right-limit (*càglàd*) if $r_t = r_{t-}$ for all t in $[0, t]$. Thenceforth we can work with *càdlàg* version of the return process without loss of generality. Jumps in cumulative price and return processes are then represented as

$$\Delta r_t \equiv r_t - r_{t-}, \quad 0 \leq t \leq T \quad (2.4)$$

which implies that for continuity points $\Delta r_t = 0$. As number of jumps is at most countably infinite, for arbitrarily chosen t in $[0, T]$ $P[\Delta r_t \neq 0] = 0$. To rule out jump processes that explode (= with infinite number of jumps over some discrete interval) we define regular (pure) jump process which is characterized by finite number of jumps.

2.2 Return decomposition

Under the standard assumptions of frictionless setting, no arbitrage and finite instantaneous mean the asset price process will belong to the class of special semimartingales. The defining characteristic of a special semimartingale is that the finite variation process in the decomposition can be taken to be predictable. Informally speaking, a predictable process is one for which the value at time t is known just before t (Back 1991, p. 375). That allows us to perform following decomposition as shown by Protter (1992).

Proposition 1. Any arbitrage-free logarithmic price process subject to the regularity conditions outlined above may be uniquely represented as

$$r_t \equiv r_{t,t} = p_t - p_0 = \mu_t + M_t = \mu_t + M_t^C + M_t^J \quad (2.5)$$

where μ_t is a predictable and finite-variation process, M_t is a local martingale that may be further decomposed into M_t^C , a continuous sample path, infinite-variation local martingale component, and M_t^J , a compensated jump martingale. We may normalize the initial conditions such that all components may be assumed to have initial conditions normalized such that $\mu_0 \equiv M_0 \equiv M_0^C \equiv M_0^J \equiv 0$, which implies that $r_t \equiv p_t$ (Andersen *et al.* 2010a, p. 71).

Thus we can uniquely decompose the instantaneous return into an expected return component and an (martingale) innovation where martingale can be defined as a process for which the best prediction of the next realization is the current value of the process given the historical information about past values.

Moreover, the predictable finite-variation component might include jumps as well. The following proposition (Andersen *et al.* 2003, p. 583) implies that whenever there is a predictable jump in price (which means that we know the time and size of the jump before the jump occurs), there is a simultaneous jump in the martingale component to offset it. Otherwise there would be an arbitrage opportunity.

Proposition 2. The predictable jumps are associated with genuine jump risk, in the sense that if $\mu_t^J \neq 0$, then

$$P[\text{sgn}(\mu_t^J) = -\text{sgn}(\mu_t^J + M_t^J)] > 0 \quad (2.6)$$

where $\text{sgn}(x) \equiv 1$ for $x \geq 0$ and $\text{sgn}(x) \equiv -1$ for $x < 0$.

Generally the martingale component is the dominant one. As stated by Andersen *et al.* (2010a, p. 72), the continuous component, μ_t^C , which is of finite variation, must be locally an order of magnitude smaller than the corresponding contribution from the continuous component of the innovation term, M_t^C . If an asset earning a positive expected return over the risk-free rate didn't have innovations an order of magnitude larger than the expected return over infinitesimal intervals, a sustained long position (infinitely, many periods over any interval) in the risky asset would tend to be perfectly diversified due to a law of large numbers, as the martingale part is uncorrelated. Thus, the risk-return relation would become unbalanced.

As for the jump component, the predictable jumps must be offset by large jump innovation risk. Furthermore, predictable jumps must be associated with release of new information at a predetermined time, but in case of any uncertainty about the exact timing the jump might not be predictable anymore. Therefore in consensus with the eco-

economic literature we consider the predictable jump component negligible and we will not take it into account in further analysis.

2.3 Quadratic variation

This section will be devoted to the introduction of the quadratic variation. The martingale component, which is dominant in the return decomposition, is of primary interest now. However we cannot observe it unless we have access to continuous data. Unfortunately continuous data are not available and the data at very high frequency that might be perceived close to being continuous cannot be used due to microstructure effects (i.e. bid-ask bounce). Therefore we will focus on variation measures in the discrete setting (over a discrete time interval).

Definition 1. Let X_t denote any (special) semimartingale. The unique quadratic variation process, $[X, X]_t, t \in [0, T]$, associated with X_t is formally defined as

$$[X, X]_t \equiv X_t^2 - 2 \int_0^t X_{s-} dX_s, \quad 0 < t \leq T, \quad (2.7)$$

where the stochastic integral of the adapted *càglàd* process, X_{s-} , with respect to the *càdlàg* semimartingale, X_s , is well-defined (Andersen *et al.* 2010a, p. 75).

Then we can see that quadratic variation is an increasing stochastic process and jumps in the sample path of the quadratic variation process occur simultaneously with the jumps in the underlying semimartingale process.

Definition 2. The Notional Volatility over $[t - h, t], 0 < h \leq t \leq T$, is

$$v_{t,h}^2 \equiv [M, M]_t - [M, M]_{t-h} = [M^C, M^C]_t - [M^C, M^C]_{t-h} + \sum_{t-h < s \leq t} \Delta M_s^2. \quad (2.8)$$

$$v_{t,h}^2 \equiv [r, r]_t - [r, r]_{t-h} = [M^C, M^C]_t - [M^C, M^C]_{t-h} + \sum_{t-h < s \leq t} \Delta r_s^2. \quad (2.9)$$

Following Andersen *et al.* (2010a, pp. 75-76) we defined the notional volatility as an ex-post return variability over a fixed time interval which equals (the increment to) the quadratic variation for the return series. We can approximate it arbitrarily well by the

sum of high frequency squared returns using the measure called realized variance that will be defined in the next section.

The major stream of economic literature focused on asset and derivatives pricing assumes that sample paths are continuous and the corresponding diffusion processes are given in the form of stochastic differential equations. Following proposition demonstrates that there is a relation between the integral representation of a price process and the representation based on stochastic differential equations.

Proposition 3. For any univariate, square-integrable, continuous sample path, logarithmic price process, which is not locally riskless, there exists a representation such that for all $0 \leq t \leq T$, a.s.(P),

$$r_{t,h} = \mu_{t,h} + M_{t,h} = \int_{t-h}^t \mu_s ds + \int_{t-h}^t \sigma_s dW_s, \quad (2.10)$$

where μ_s is an integrable, predictable, and finite-variation stochastic process, σ_s is a strictly positive *càdlàg* stochastic process satisfying

$$P \left[\int_{t-h}^t \sigma_s^2 ds < \infty \right] = 1, \quad (2.11)$$

and W_s is a standard Brownian motion (Andersen *et al.* 2010a, p. 79).

Therefore without any loss of generality the price process defined above can be described using stochastic differential equation as follows

$$dp_t = \mu_t dt + \sigma_t dW_t, \quad 0 \leq t \leq T, \quad (2.12)$$

where μ_t is predictable and of finite variation, σ_t is square integrable and strictly positive *càdlàg* and W_t is a standard Brownian motion. The quadratic variation of this process over the time interval $[t, h]$ equals

$$QV_{t,h} = \int_{t-h}^t \sigma_s^2 ds \quad (2.13)$$

Finally we introduce jumps into the previous model. In the following chapter we will assume that price processes evolve as described by this standard jump-diffusion model

$$dp_t = \mu_t dt + \sigma_t dW_t + J_t dq_t, \quad (2.14)$$

where μ_t refers to the drift term which is continuous and of finite variation and σ_t is strictly positive local volatility process, W_t is a standard Brownian motion and $J_t dq_t$ refers to the pure jump component, where q_t is Poisson process with finite intensity λ_t and J_t is the size of the corresponding jumps.

Then the variability of the squared returns will be represented by quadratic variation as follows

$$QV_{t,h} = \int_{t-h}^t \sigma_s^2 ds + \sum_{(t-h) < s \leq t} J_s^2. \quad (2.15)$$

As we can see in this case the quadratic variation (QV) consists of two components. The first one is called integrated variance (IV) and represents the contribution of a continuous sample path variation. The second one represents jump variation. Therefore in the absence of jumps QV and IV are both equal to $\int_{t-h}^t \sigma_s^2 ds$.

2.4 Realized variance

As we stated before the return quadratic variation can be approximated arbitrarily well by the sum of squared returns observed at high frequency. It is worth notice that using the sum of squared returns to measure volatility is not a recently developed idea. However, few decades ago daily data were considered as high frequency data and for example Poterba & Summers (1986) or French *et al.* (1987) use monthly sample variances computed from daily returns. An increasing availability of high frequency data is one of the reasons why the use of non-parametric measures of volatility has become so popular in the last decade.

The formal definition of so-called realized variance follows (Andersen *et al.* 2010a, p. 109).

Definition 3. The Realized variance over $[t-h, t]$, $0 < h \leq t \leq T$, is defined by

$$RV_{t,h} = \sum_{i=1}^m r_{t-h+h(\frac{i}{m})}^2, \quad (2.16)$$

where m is the number of observations with the corresponding sampling frequency

$1/m$. As we can see the realized volatility is just the second sample moment of the return process over a fixed interval of length h .

In the economic literature realized variance and realized volatility are two terms that are often used interchangeably. In this thesis realized volatility will denote the squared root of realized variance, i.e. $RVol_{t,h} = \sqrt{RV_{t,h}}$.

The following two propositions (Andersen *et al.* 2010a, pp. 110-111) show that the ex-post realized variance is an unbiased estimator of expected ex-ante variance $\zeta_{t,h}^2$ and that realized variance converges in probability to quadratic variation as $m \rightarrow \infty$ (this holds for all semimartingales as noted by Barndorff-Nielsen & Shephard (2002)).

Proposition 4. If the return process is square-integrable and $\mu_t \equiv 0$, then for any value of $m \geq 1$ and $h > 0$,

$$\zeta_{t,h}^2 = E[v_{t,h}^2 | \mathcal{F}_{t-h}] = E[M_{t,h}^2 | \mathcal{F}_{t-h}] = E[RV_{t,h} | \mathcal{F}_{t-h}]. \quad (2.17)$$

Proposition 5. The realized variance provides a consistent nonparametric measure of the notional volatility,

$$\text{plim}_{m \rightarrow \infty} RV_{t,h} = v_{t,h}^2 \equiv QV_{t,h}, \quad 0 < h \leq t \leq T, \quad (2.18)$$

where the convergence is uniform in probability.

Thus in the jump-diffusion framework we introduced in Equation 2.14

$$\text{plim}_{m \rightarrow \infty} RV_{t,h} = QV_{t,h} = \int_{t-h}^t \sigma_s^2 ds + \sum_{(t-h) < s \leq t} J_s^2. \quad (2.19)$$

Even though the proposition 5 suggests to use the data at the highest frequency available (i.e. tick-by-tick data), we will not use them in our thesis. The reason behind is the effect of microstructure noise (mainly bid-ask bounce) that would contaminate our data. We will thus follow the tradition and use 5-minute returns in the empirical part.

The valuable extension of realized variance are so-called realized semivariances introduced by Barndorff-Nielsen *et al.* (2010) that capture the variation only due to positive or negative returns.

Definition 4. The upside (respectively downside) semivariance over $[t-h, t]$, $0 < h \leq t \leq T$ is defined as follows

$$RS_{t,h}^+ = \sum_{i=1}^m r_{t-h+h(\frac{i}{m})}^2 I\{r_{t-h+h(\frac{i}{m})} > 0\} \quad (2.20)$$

$$RS_{t,h}^- = \sum_{i=1}^m r_{t-h+h(\frac{i}{m})}^2 I\{r_{t-h+h(\frac{i}{m})} < 0\}, \quad (2.21)$$

where the sum of two semivariances equals to realized variance. Barndorff-Nielsen *et al.* (2010) also show the limiting behavior of realized semivariances - both of them converge to one half of the continuous sample-path variance plus the sum of squared jumps with a positive/negative sign:

$$\text{plim}_{m \rightarrow \infty} RS_{t,h}^+ = \frac{1}{2} \int_{t-h}^t \sigma_s^2 ds + \sum_{(t-h) < s \leq t} J_s^2 I\{J_s > 0\} \quad (2.22)$$

and

$$\text{plim}_{m \rightarrow \infty} RS_{t,h}^- = \frac{1}{2} \int_{t-h}^t \sigma_s^2 ds + \sum_{(t-h) < s \leq t} J_s^2 I\{J_s < 0\}. \quad (2.23)$$

Finally, Patton & Sheppard (2015) use realized semivariances to construct a signed jump variation measure in order to investigate the impact of the sign of the jump on volatility.

Definition 5. Signed jump variation over $[t-h, t]$, $0 < h \leq t \leq T$ is defined as the difference between positive and negative realized semivariances as follows:

$$\begin{aligned} \Delta J_{t,h}^2 &\equiv RS_{t,h}^+ - RS_{t,h}^- = \\ &= \sum_{i=1}^m r_{t-h+h(\frac{i}{m})}^2 I\{r_{t-h+h(\frac{i}{m})} > 0\} - \sum_{i=1}^m r_{t-h+h(\frac{i}{m})}^2 I\{r_{t-h+h(\frac{i}{m})} < 0\}. \end{aligned} \quad (2.24)$$

This way we eliminate the common continuous variation term and thus signed jump variation converges to the difference between the sums of squared jumps with positive and negative sign.

$$\text{plim}_{m \rightarrow \infty} \Delta J_{t,h}^2 = \sum_{(t-h) < s \leq t} J_s^2 I\{J_s > 0\} - \sum_{(t-h) < s \leq t} J_s^2 I\{J_s < 0\}. \quad (2.25)$$

Therefore signed jump variation will be positive when a day is dominated by an upward jump, negative in the opposite case and close to zero if a day is dominated by neither positive nor negative jumps.

2.5 Bipower variation

In the next chapter we will propose several tests that might be employed to identify jumps and co-jumps. The easiest approach to detecting jumps in prices is to compare two estimators of quadratic variation - one of them must measure both integrated and jump variation (realized variance is the obvious choice), the second one must be robust to jumps. Very simple estimator of integrated variance called bipower variation was proposed by Barndorff-Nielsen (2004). The last section of this chapter will be dedicated to this concept. First we will provide formal definition of bipower variation and highlight its most important property, then we will suggest an enhanced version that is more robust to microstructure noise and finally we will propose two alternative estimators of integrated variance.

Definition 6. The realized bipower variation over $[t - h, t]$, $0 < h \leq t \leq T$, is defined as

$$BPV_{t,h} = \mu_1^{-2} \frac{m}{m-1} \sum_{i=2}^m \left| r_{t-h+h\frac{(i-1)}{m}} \right| \left| r_{t-h+h\frac{i}{m}} \right|, \quad (2.26)$$

where $\mu_1 = \sqrt{2/\pi}$.

The main idea behind this measure is following - unless there are two consecutive jumps, the product of two adjacent returns has only a minor effect on bipower variation. Moreover, as the sampling frequency increases, the probability of observing jumps in two adjacent returns goes to zero. The jump return is paired with the continuous one, thus the jump process has negligible impact on probability limit of bipower variation. To put it more formally, Barndorff-Nielsen (2004) showed that bipower variation is a consistent estimator of the integrated variance.

$$\text{plim}_{m \rightarrow \infty} BPV_{t,h} = IV_{t,h} = \int_{t-h}^t \sigma_s^2 ds \quad (2.27)$$

As bipower variation is sensitive to microstructure noise, Huang & Tauchen (2005) and Andersen *et al.* (2007a) advocate using staggered (skip-one) returns instead of adjacent ones to make it more robust against market microstructure effects.

Definition 7. The staggered realized bipower variation over $[t-h, t]$, $0 < h \leq t \leq T$, is defined as

$$BPV_{t,h} = \mu_1^{-2} \frac{m}{m-2} \sum_{i=3}^m \left| r_{t-h+h\frac{(i-2)}{m}} \right| \left| r_{t-h+h\frac{i}{m}} \right|, \quad (2.28)$$

where $\mu_1 = \sqrt{2/\pi}$.

The staggered realized bipower variation can be further modified into more general skip- q bipower variation estimator.

Definition 8. The skip- q realized bipower variation over $[t-h, t]$, $0 < h \leq t \leq T$, is defined as

$$BPV_{t,h} = \mu_1^{-2} \frac{m}{m-1-q} \sum_{i=2+q}^m \left| r_{t-h+h\frac{(i-1-q)}{m}} \right| \left| r_{t-h+h\frac{i}{m}} \right|, \quad (2.29)$$

where $\mu_1 = \sqrt{2/\pi}$.

Thus for $q = 0$ we obtain the usual BPV from Equation 2.27, for $q = 1$ the skip-one version from Equation 2.28 and so on. Patton & Sheppard (2015) suggest to use an average of multiple skip- q bipower variation estimators and note that it is a compromise between locality (skip-0) and robustness to microstructure noise (skip- q with high value of q).

Even though the bipower variance is a consistent estimator of integrated variance, in finite sample there might be an upward bias in the presence of jumps that will lead to the underestimation of the jump component. Shrinking the interval by using higher sampling frequency is not a solution - in that case the noise would dominate the series. Therefore an alternative solution is to use another measure of integrated variance with better theoretical properties, for example threshold bipower variation proposed by Corsi *et al.* (2010) or MedRV introduced by Andersen *et al.* (2012).

Chapter 3

Jump and co-jump tests

In this thesis we assume that the asset's logarithmic price process p_t is a semi-martingale to rule out arbitrage opportunities and that p_t within the active part of the trading day evolves as a standard jump-diffusion process represented by Equation 2.14. Given $m + 1$ equidistant price observations for day t , the i th intraday return can be defined as $r_{t,i} = p_{t-1+i/m} - p_{t-1+(i-1)/m}$, $j = 1, 2, \dots, m$. Then we can construct realized variance and bipower variation for day t . The former is a consistent estimator of the corresponding quadratic variation and the latter consistently estimates the integrated variance.

$$RV_t = \sum_{i=1}^m r_{t,i}^2, \quad \text{plim}_{m \rightarrow \infty} RV_t = QV_t = \int_{t-1}^t \sigma_s^2 ds + \sum_{(t-1) < s \leq t} J_s^2. \quad (3.1)$$

$$BPV_t = \mu_1^{-2} \frac{m}{m-1} \sum_{i=2}^m |r_{t,i}| |r_{t,i-1}|, \quad \text{plim}_{m \rightarrow \infty} BPV_t = IV_t = \int_{t-1}^t \sigma_s^2 ds \quad (3.2)$$

where $\mu_1^{-2} = \pi/2$.

Based on these simple measures, we will now develop the simple test for jump detection. Then we will explain how we can identify simultaneous jumps across many assets.

3.1 Jump tests

We can estimate the total variation coming from jumps by $RV_t - BPV_t$ or using relative jump measure $RJ_t = \frac{RV_t - BPV_t}{RV_t}$ introduced by Huang & Tauchen (2005). Barndorff-

Nielsen (2004) showed that the difference between the realized variance and bipower variation consistently estimates the jump variation:

$$\text{plim}_{m \rightarrow \infty}(RV_t - BPV_t) = \sum_{(t-1) < s \leq t} J_s^2. \quad (3.3)$$

However, we are interested only in significant jumps and the difference between two measures can hardly be interpreted as a size of the jump. Firstly, it can assume negative values. Secondly, low positive values might be just the result of the sampling variation. Thus we would attribute part of the continuous price movements to the jump process.

Therefore the jump test based on the difference between the two measures was proposed by Barndorff-Nielsen & Shephard (2006). We will present the version of this test introduced by Andersen *et al.* (2011). Under the regularity conditions and assumption of no within-day jumps, the test statistic Z_t is asymptotically standard normally distributed and can be used to test if there is at least one jump at day t .

$$Z_t = \frac{\frac{RV_t - BPV_t}{RV_t}}{\sqrt{\left(\left(\frac{\pi}{2}\right)^2 + \pi - 5\right) \frac{1}{m} \max\left(1, \frac{TPQ_t}{BPV_t^2}\right)}}, \quad (3.4)$$

where TPQ_t (tri-power quarticity) = $m\mu_{4/3}^{-3} \left(\frac{m}{m-2-2q}\right) \sum_{i=3+2q}^m |r_{t,i-2-2q}|^{\frac{4}{3}} |r_{t,i-1-q}|^{\frac{4}{3}} |r_{t,i}|^{\frac{4}{3}}$. As we can see the tri-power quarticity depends on skip parameter q introduced in previous chapter together with bipower variation.

Andersen *et al.* (2007a) proposed a simple method how to separate the jump variation and the continuous sample path variation so that their sum would equal to the realized variation. The jump variation reads as

$$J_t = I\{Z_t > \Phi_\alpha\} \cdot (RV_t - BPV_t) \quad (3.5)$$

and the continuous sample path variation as

$$C_t = I\{Z_t \leq \Phi_\alpha\} \cdot RV_t + I\{Z_t > \Phi_\alpha\} \cdot BPV_t, \quad (3.6)$$

where $I\{\cdot\}$ is an indicator function and Φ_α represents an appropriate critical value from the standard normal distribution. We will use these measures later in the HAR-RV-CJ model.

The BNS test we introduced will detect only the daily jumps, it carries no information about the exact intraday timing and size of the jump. Therefore Andersen *et al.* (2010b) proposed the sequential version of BNS test. In other words, they recommend to first use the BNS test to identify jump days and then to select the largest squared returns as intraday jumps. Afterwards the squared returns that belong to the previously identified jumps are replaced by the average of the remaining squared returns and the BNS test is applied once more. The procedure is repeated until the jump test is insignificant for all days.

There are many modifications of the BNS test, i.e. Corsi *et al.* (2010) replaced bipower variation by threshold bipower variation. Finally, Andersen *et al.* (2007b) and Lee & Mykland (2008) developed their own tests that are able to detect intraday jumps.

3.2 Co-jump tests

There are two ways how to identify simultaneous jumps across many assets. The simple approach is to use the so-called co-exceedance rule, defined by Gilder *et al.* (2014, p. 446) as

$$\sum_{j=1}^N I\{Jump_{t,i,j} > 0\} \begin{cases} \geq 2 & \text{Cojump,} \\ \leq 1 & \text{No Cojump,} \end{cases} \quad (3.7)$$

where $I\{Jump_{t,i,j} > 0\}$ is an indicator function taking the value 1 when a jump is detected in asset j during intraday interval i on day t . Thus the co-jump is identified if at least two out of N assets jump simultaneously.

The alternative solution is to employ a bivariate co-jump test (see i.e. Jacod & Todorov (2009)) or multivariate co-jump test that can be applied to detect simultaneous jump in a large portfolio of assets. We will focus on the test proposed by Bollerslev *et al.* (2008).

Bollerslev *et al.* (2008) argue that an association between significant jumps in individual stocks (detected by univariate test) and the jump in the market portfolio might be very weak as univariate test (together with co-exceedance rule) might not be able to identify modest-sized co-jumps due to the large amount of idiosyncratic noise in individual returns.

Therefore they suggest to use the cross products of intraday returns in a large panel of N stocks. Let mcp denote the mean cross-product specified as follows:

$$mcp_{t,i} = \frac{2}{N(N-1)} \sum_{j=1}^{N-1} \sum_{l=j+1}^N r_{j,t,i} r_{l,t,i}, \quad i = 1, 2, \dots, m. \quad (3.8)$$

In the absence of co-jumps the product of returns should be small - even the influence of a single idiosyncratic jump will be dampened by other (continuous) returns. However, the statistic should be very sensitive to the presence of co-jumps as the cross-product of two large jumps with the same sign will inflate its value. Thus the mcp -statistic measures the co-movement of the stocks.

The studentized test statistic assumes form:

$$z_{mcp,t,i} = \frac{mcp_{t,i} - \overline{mcp}_t}{s_{mcp,t}}, \quad i = 1, 2, \dots, m, \quad (3.9)$$

where

$$\overline{mcp}_t = \frac{1}{m} mcp_t = \frac{1}{m} \sum_{i=1}^m mcp_{t,i} \quad (3.10)$$

and

$$s_{mcp,t} = \sqrt{\frac{1}{m-1} \sum_{i=1}^m (mcp_{t,i} - \overline{mcp}_t)^2}. \quad (3.11)$$

To find critical values of z_{mcp} statistic we need its distribution under the null hypothesis of no jumps. As the distribution of the test statistic cannot be approximated by any standard distribution, we have to bootstrap it.

When applying the test, we assume that the location and scale of the test statistic are approximately constant throughout the day. The presumption can be challenged by the known U-shaped intraday volatility pattern, but Bollerslev *et al.* (2008, p. 241) claim that this does not affect their main findings. Furthermore, a possible remedy would be to apply the intraday volatility corrector as Gilder *et al.* (2014) suggest.

The large cross-section of returns is crucial for the implementation of the new mcp test in order to diversify away the effect of idiosyncratic jumps. The extension of this test for small N is provided by Gnabo *et al.* (2014).

Furthermore, a new test for co-jump detection was proposed by Caporin *et al.* (2014).

It is based on the difference between two types of smoothed power variations and detects the presence of simultaneous jumps among at least M stocks in the portfolio comprising of N stocks, where $M \leq N$.

Chapter 4

HAR models

In the last chapter dedicated to theory we will focus on Heterogeneous Autoregressive (HAR) model proposed by Corsi (2009). As the key objective of this thesis is to find out whether jumps and simultaneous jumps have an effect on future volatilities, we need a model that is able to forecast volatility using high frequency data. HAR model is a simple tool for volatility forecasting, but in terms of performance it is comparable to much more complicated models. First, we will derive the simple HAR-RV model, then we will present its extensions that account for the presence of jumps.

The inspiration for HAR model comes from Heterogeneous Market Hypothesis presented by Müller *et al.* (1993). The difference between heterogeneous and homogeneous markets is reflected in the correlation of volatility and number of market participants. In the homogeneous market, the higher number of traders implies faster consensus and convergence to the true market value. Thus volatility is lower and the correlation between volatility and number of agents is negative. This contradicts the empirical findings - there is a positive correlation of volatility and market presence. Therefore the heterogeneous market hypothesis seems to be the more plausible one. In this model various agents have different time horizons and dealing frequencies and consequently they create volatility.

The logic behind the HAR model is that various agents have different investment horizons and thus they influence (and are influenced by) different types of volatility components. Corsi (2009) operates with three volatility components - daily, weekly and monthly. There are dealers, market makers and speculators who trade at daily or higher frequency. Then we can identify medium-term traders with weekly investment horizon and finally, there are long-term investors (i.e. insurance companies and pension funds) who trade at monthly or lower frequency.

As noted by Corsi (2009, p. 178), volatility over longer time intervals has a stronger

influence on volatility over shorter time intervals than vice-versa. This property is used to construct the HAR model, therefore we can call it an additive cascade model of different volatility components. In the next section we follow Corsi (2009) and derive the simple HAR-RV model.

4.1 HAR-RV

Let $\tilde{\sigma}_t^{(d)}$, $\tilde{\sigma}_t^{(w)}$ and $\tilde{\sigma}_t^{(m)}$ denote daily, weekly and monthly partial volatilities, where partial volatility is generated by specific market component. The return process is determined by the daily volatility component (i.e. the one with the highest frequency) as

$$r_t = \sigma_t^{(d)} \epsilon_t, \quad \epsilon_t \sim NID(0, 1) \quad (4.1)$$

In our model the unobserved partial volatility depends on the ex-post observed realized volatility experienced at the same time scale and on the expected value of the longer term partial volatility, thus incorporating the cascade structure in a following way:

$$\tilde{\sigma}_{t+1m}^{(m)} = c^{(m)} + \phi^{(m)} RVol_t^{(m)} + \tilde{\omega}_{t+1m}^{(m)}, \quad (4.2)$$

$$\tilde{\sigma}_{t+1w}^{(w)} = c^{(w)} + \phi^{(w)} RVol_t^{(w)} + \gamma^{(w)} E_t \left[\tilde{\sigma}_{t+1m}^{(m)} \right] + \tilde{\omega}_{t+1w}^{(w)}, \quad (4.3)$$

$$\tilde{\sigma}_{t+1d}^{(d)} = c^{(d)} + \phi^{(d)} RVol_t^{(d)} + \gamma^{(d)} E_t \left[\tilde{\sigma}_{t+1w}^{(w)} \right] + \tilde{\omega}_{t+1d}^{(d)}, \quad (4.4)$$

where $RVol_t^{(d)}$, $RVol_t^{(w)}$ and $RVol_t^{(m)}$ are the daily, weekly, and monthly observed realized volatilities and the innovations $\tilde{\omega}_{t+1d}^{(d)}$, $\tilde{\omega}_{t+1w}^{(w)}$ and $\tilde{\omega}_{t+1m}^{(m)}$ are contemporaneously and serially independent with zero mean and truncated left tail so that the partial volatilities are positive.

By recursive substitutions into the third equation and using the fact that $\tilde{\sigma}_t^{(d)} = \sigma_t^{(d)}$ we get

$$\sigma_{t+1d}^{(d)} = c + \beta^{(d)} RVol_t^{(d)} + \beta^{(w)} RVol_t^{(w)} + \beta^{(m)} RVol_t^{(m)} + \tilde{\omega}_{t+1d}^{(d)}, \quad (4.5)$$

which is a three-factor stochastic volatility model, where each factor represents the past realized volatility at different frequencies. Finally, we can express the latent volatility from the left hand side of the previous equation as

$$\sigma_{t+1d}^{(d)} = RVol_{t+1d}^{(d)} + \omega_{t+1d}^{(d)}, \quad (4.6)$$

where $\omega_{t+1d}^{(d)}$ denotes measurement and estimation errors of latent daily volatility. Conse-

quently, we obtain the HAR-RV model.

$$RVol_{t+1d}^{(d)} = c + \beta^{(d)} RVol_t^{(d)} + \beta^{(w)} RVol_t^{(w)} + \beta^{(m)} RVol_t^{(m)} + \omega_{t+1d}, \quad (4.7)$$

where $\omega_{t+1d} = \tilde{\omega}_{t+1d}^{(d)} - \omega_{t+1d}^{(d)}$. The weekly and monthly realized volatilities are defined as follows:

$$RVol_t^{(w)} = \frac{1}{5} \sum_{i=0}^4 RVol_{t-i} \quad (4.8)$$

$$RVol_t^{(m)} = \frac{1}{22} \sum_{i=0}^{21} RVol_{t-i} \quad (4.9)$$

In the same manner we can define HAR-RV model for realized variances instead of realized volatilities or its logarithmic version as

$$RV_{t+1}^{(d)} = c + \beta^{(d)} RV_t^{(d)} + \beta^{(w)} RV_t^{(w)} + \beta^{(m)} RV_t^{(m)} + \omega_{t+1}, \quad (4.10)$$

and

$$\log \left(RV_{t+1}^{(d)} \right) = c + \beta^{(d)} \log \left(RV_t^{(d)} \right) + \beta^{(w)} \log \left(RV_t^{(w)} \right) + \beta^{(m)} \log \left(RV_t^{(m)} \right) + \omega_{t+1}. \quad (4.11)$$

The HAR model can be easily estimated by OLS. However, the residuals often suffer from serial correlation and heteroskedasticity. Therefore Newey-West correction of standard errors should be employed to ensure the correct estimation of t-statistics.

Many studies use all the three measures of volatility - realized variance, realized volatility as well as logarithmic transformation of realized variance - when they employ HAR model. In comparison with realized variance, its logarithmic transformation has an advantage of positiveness, lower skewness and lower kurtosis, therefore we decided to use only the $\log(RV)$ measure even though it is not as useful in economic applications as predictions of volatility in levels. Therefore we will present HAR model extensions that are commonly used and for each extension we will specify the logarithmic version we will apply in the results section of our thesis.

4.2 HAR-RV-J/HAR-RV-CJ

A simple extension of HAR-RV model was proposed by Andersen *et al.* (2007a) to account for the presence of jumps. More specifically, they include the jump variation component

we derived in the previous chapter into the HAR-RV model as an additional variable.

$$RV_{t+1}^{(d)} = c + \beta^{(d)} RV_t^{(d)} + \beta^{(w)} RV_t^{(w)} + \beta^{(m)} RV_t^{(m)} + \beta_J^{(d)} J_t^{(d)} + \omega_{t+1}, \quad (4.12)$$

The HAR-RV-J model was further extended by decomposition of realized variance into the jump and continuous sample path variation components. First we have to construct the average weekly and monthly jump/continuous variation components:

$$\begin{aligned} J_t^{(w)} &= \frac{1}{5} \sum_{i=0}^4 J_{t-i}, & C_t^{(w)} &= \frac{1}{5} \sum_{i=0}^4 C_{t-i}, \\ J_t^{(m)} &= \frac{1}{22} \sum_{i=0}^{21} J_{t-i}, & C_t^{(m)} &= \frac{1}{22} \sum_{i=0}^{21} C_{t-i}, \end{aligned} \quad (4.13)$$

where J_t and C_t are defined as in Equations 3.5 and 3.6. The HAR-RV-CJ model then reads as

$$RV_{t+1}^{(d)} = c + \beta_C^{(d)} C_t^{(d)} + \beta_C^{(w)} C_t^{(w)} + \beta_C^{(m)} C_t^{(m)} + \beta_J^{(d)} J_t^{(d)} + \beta_J^{(w)} J_t^{(w)} + \beta_J^{(m)} J_t^{(m)} + \omega_{t+1} \quad (4.14)$$

If we want to build a logarithmic version of HAR-RV-J or HAR-CJ model we have to take into account that jump variation might be equal to zero. Therefore we have to add 1 to the jump component to make it non-negative. Logarithmic form is then defined as

$$\begin{aligned} \log \left(RV_{t+1}^{(d)} \right) &= c + \beta_C^{(d)} \log \left(BPV_t^{(d)} \right) + \beta_J^{(d)} \log \left(1 + J_t^{(d)} \right) \\ &+ \beta^{(w)} \log \left(RV_t^{(w)} \right) + \beta^{(m)} \log \left(RV_t^{(m)} \right) + \omega_{t+1}. \end{aligned} \quad (4.15)$$

We follow the logic behind HAR-RV-CJ model and replaced the daily RV component with the estimator of continuous variation, bipower variation. The HAR-RV-CJ will not be employed in this thesis, instead we will derive logarithmic version of HAR-RV-J model where the jump variation, J_t , is replaced by the binary time series with value of 1 if $J_t > 0$ and 0 otherwise. Thus we can estimate the effect of the size of the jump as well as the effect of the presence of the jump. Our HAR-RV-Jump then reads as

$$\begin{aligned} \log \left(RV_{t+1}^{(d)} \right) &= c + \beta_C^{(d)} \log \left(BPV_t^{(d)} \right) + \beta_{Jump}^{(d)} Jump_t^{(d)} \\ &+ \beta^{(w)} \log \left(RV_t^{(w)} \right) + \beta^{(m)} \log \left(RV_t^{(m)} \right) + \omega_{t+1}. \end{aligned} \quad (4.16)$$

4.3 HAR models with asymmetric volatility

Here we present various extensions of a simple HAR model introduced by Patton & Sheppard (2015) in order to capture the asymmetric impact of signed returns and jumps on volatility. Firstly, the effect of positive and negative returns on future volatility can be investigated by decomposing the realized variance in the HAR model into realized semivariances as follows

$$RV_{t+1}^{(d)} = c + \beta^{+(d)} RS_t^{+(d)} + \beta^{-(d)} RS_t^{-(d)} + \beta^{(w)} RV_t^{(w)} + \beta^{(m)} RV_t^{(m)} + \omega_{t+1} \quad (4.17)$$

and

$$\begin{aligned} \log \left(RV_{t+1}^{(d)} \right) &= c + \beta^{+(d)} \log \left(RS_t^{+(d)} \right) + \beta^{-(d)} \log \left(RS_t^{-(d)} \right) \\ &+ \beta^{(w)} \log \left(RV_t^{(w)} \right) + \beta^{(m)} \log \left(RV_t^{(m)} \right) + \omega_{t+1} \end{aligned} \quad (4.18)$$

if we want to employ logarithmic transformation. The weekly and monthly components can be naturally decomposed in the same way as daily realized variance. The impact of the sign of a jump can be estimated using the signed jump variation which is defined as $\Delta J_t^2 \equiv RS_t^+ - RS_t^-$. The HAR-RV-J-Sign model then reads as

$$RV_{t+1}^{(d)} = c + \beta_C^{(d)} BPV_t^{(d)} + \beta_{\Delta J}^{(d)} \Delta J_t^2 + \beta^{(w)} RV_t^{(w)} + \beta^{(m)} RV_t^{(m)} + \omega_{t+1} \quad (4.19)$$

In order to derive the logarithmic form we follow Patton & Sheppard (2015) and replace ΔJ_t^2 with percentage jump variation, $\% \Delta J_t^2 = \log(1 + \Delta J_t^2 / RV_t)$, so that the final model is defined as

$$\begin{aligned} \log \left(RV_{t+1}^{(d)} \right) &= c + \beta_C^{(d)} \log \left(BPV_t^{(d)} \right) + \beta_{\Delta J}^{(d)} \% \Delta J_t^2 \\ &+ \beta^{(w)} \log \left(RV_t^{(w)} \right) + \beta^{(m)} \log \left(RV_t^{(m)} \right) + \omega_{t+1} \end{aligned} \quad (4.20)$$

Here we suppose that positive news will lead to lower volatility and negative news will lead to higher volatility. But what if the response of volatility to good and bad news is asymmetric (the effect of bad news has greater impact than the effect of good news)? One option would be to use a difference between realized semivariances and one-half of bipower variation, but we follow Patton & Sheppard (2015) and construct two signed jump measures using an indicator function which has a value of 1 if corresponding realized semivariance is higher and 0 otherwise. To put it more formally:

$$\begin{aligned} \Delta J_t^{2+} &= (RS_t^+ - RS_t^-) I\{(RS_t^+ - RS_t^-) > 0\} \\ \Delta J_t^{2-} &= (RS_t^+ - RS_t^-) I\{(RS_t^+ - RS_t^-) < 0\} \end{aligned} \quad (4.21)$$

where the first term is positive signed jump variation and the second one is negative signed jump variation. Consequently, we derive the percentage-based measures in the following way:

$$\begin{aligned}\% \Delta J_t^{2+} &= \log(1 + \Delta J_t^2 / RV_t) I\{\Delta J_t^2 > 0\} \\ \% \Delta J_t^{2-} &= \log(1 + \Delta J_t^2 / RV_t) I\{\Delta J_t^2 < 0\}\end{aligned}\quad (4.22)$$

and the logarithmic HAR model with the decomposition of signed jump variation is defined as

$$\begin{aligned}\log(RV_{t+1}^{(d)}) &= c + \beta_C^{(d)} \log(BPV_t^{(d)}) + \beta_{\Delta J+}^{(d)} \% \Delta J_t^{2+(d)} + \\ &+ \beta_{\Delta J-}^{(d)} \% \Delta J_t^{2-(d)} + \beta^{(w)} \log(RV_t^{(w)}) + \beta^{(m)} \log(RV_t^{(m)}) + \omega_{t+1}\end{aligned}\quad (4.23)$$

4.4 HAR-RV-Cojump

Inspired by models introduced in Clements & Liao (2013) and Caporin *et al.* (2014) we can construct an additional variable that represents co-jumps like the following one:

$$RV_{t+1}^{(d)} = c + \beta^{(d)} RV_t^{(d)} + \beta^{(w)} RV_t^{(w)} + \beta^{(m)} RV_t^{(m)} + \beta_{CJ}^{(d)} CJ_t^{(d)} + \omega_{t+1}, \quad (4.24)$$

where CJ_t is an indicator function taking the value 1 when a co-jump is detected across panel of stocks on day t . We transform the model into the logarithmic form and replace the daily realized variance with continuous component as in HAR-RV-J model. Logarithmic HAR-RV-Cojump then reads as

$$\begin{aligned}\log(RV_{t+1}^{(d)}) &= c + \beta_C^{(d)} \log(BPV_t^{(d)}) + \beta_{CJ}^{(d)} CJ_t^{(d)} \\ &+ \beta^{(w)} \log(RV_t^{(w)}) + \beta^{(m)} \log(RV_t^{(m)}) + \omega_{t+1}.\end{aligned}\quad (4.25)$$

As an example we also present HAR model where realized variances are replaced by realized correlations:

$$RCorr_{t+1}^{(d)} = c + \beta^{(d)} RCorr_t^{(d)} + \beta^{(w)} RCorr_t^{(w)} + \beta^{(m)} RCorr_t^{(m)} + \beta_{CJ}^{(d)} CJ_t^{(d)} + \omega_{t+1}, \quad (4.26)$$

where realized correlation of assets A and B is defined as

$$RCorr_{t,A,B} = \frac{RCov_{t,A,B}}{\sqrt{RV_{t,A}} \sqrt{RV_{t,B}}}\quad (4.27)$$

and realized covariance as

$$RCov_{t,A,B} = \sum_{i=1}^M r_{a,t,i} r_{b,t,i}. \quad (4.28)$$

4.5 Panel HAR

All the HAR models we presented in this chapter can be employed to predict volatility of a single asset. But what if we have a panel of assets at our disposal? Then we can either estimate all the models separately for every single asset or we can estimate the average effect across the portfolio using panel HAR model proposed by Patton & Sheppard (2015) which is defined as

$$RV_{j,t+1}^{(d)} = c_j + \beta^{(d)} RV_{j,t}^{(d)} + \beta^{(w)} RV_{j,t}^{(w)} + \beta^{(m)} RV_{j,t}^{(m)} + \omega_{j,t+1}, \quad j = 1, 2, \dots, N, \quad (4.29)$$

where N is number of assets and c_j is a fixed effect that enables every asset to have different level of long-run volatility. We can eliminate c_j by demeaning and then the model is estimated by OLS.

In this thesis we heavily rely on the panel HAR modifications of the models we have introduced before as our data set contains a portfolio of stocks. We use it to estimate an average effect across the portfolio as well as for analysis of individual market sectors. But before we discuss the results, we should acquaint ourselves with the data.

Chapter 5

Data

Previous chapters were dedicated to the theory behind realized measures of volatility, introduction of jump and co-jump tests and last but not least to the overview of HAR models that will be used in the empirical part of this thesis in order to find out the impact of jumps and common jumps on future volatility. Before the results are presented and discussed we should describe our data set which consists of 29 U.S. stocks traded at New York Stock Exchange (NYSE) and spans from August 2004 to December 2015.

Nowadays high frequency tick-by-tick data are available and contain enormous number of observations. Even though more observations might give us more information, we cannot use the data at the highest frequency possible due to microstructure noise effects and 5-minute returns will suit our purpose much better. The 5-minute logarithmic returns were constructed from the original 1-minute prices we had at our disposal. Our data were already pre-processed, therefore we didn't have to perform a challenging task of cleaning and filtering raw tick-by-tick data. Yet we will make a few remarks about high frequency data handling.

5.1 Handling high frequency data

Due to technological advancement in data storing and computing power the high frequency data are collected at a very fast rate every time a new information arrives. On one hand the more information we have, the better we can understand the markets. On the other hand the management of high frequency data is a complex task that involves a lot of choices that may have an impact on the results one obtains. Therefore let's look at the challenges anyone who works with high frequency data has to face.

First of all, raw data contain erroneous records. Decimal, transposition and other typing errors might be caused by human reporters, especially if the trading intensity is high. Trades might be cancelled or delayed. The original tick-by-tick data are also asynchronous, highly liquid assets are traded at higher frequency than their less liquid counterparts and the tick frequency also differs within the same security as there are intraday seasonal patterns such as well-known U-shape that is typical particularly for high-volume equities. Finally data might be structurally different across sample periods.

Sometimes we can be sure that the outlier is present and we can safely remove it without changing statistical properties of the true data, but in some cases it might be difficult to assess whether the observation is really an outlier or not. If our data-cleaning algorithm is too strict we can lose the information that is hidden within the data. In the opposite case the data might not be usable. To sum it up, there is no single “one size fits all” filter. As Falkenberry (2002, p. 3) suggests, the primary objective in developing a set of tick filters is to manage the over-scrub/under-scrub tradeoff in such a fashion as to produce a time series that removes false outliers in the trader’s base unit of analysis that can support historical backtesting without removing real-time properties of the data.

Fortunately our data are already filtered and synchronized by the same time stamp and we do not have to face the challenges listed above. Therefore we only leave a reference to the influential papers in this field written by Brownlees & Gallo (2006) and Falkenberry (2002) for those who want to learn more about high frequency data cleaning and management.

5.2 Construction of realized measures

The data set used in this thesis consists of one-minute price observations for 29 U.S. stocks with the sample period from August 19, 2004 to December 31, 2015 which is equivalent to the total of 2863 days. Therefore our sample covers the recent financial crisis as well as pre-crisis and post-crisis periods. Only prices corresponding to the trading hours (from Monday through Friday 9:30 a.m. to 4:00 p.m.) of the New York Stock Exchange (NYSE) were used and observations outside this interval were eliminated.

The trading hours of NYSE correspond to 390 intraday one-minute price observations. However, for many days some observations were missing. If the number of missing observations was low, the corresponding days were retained in our sample. However, if there were less than 300 intraday price observations for a given day, it was dropped from

the sample as illiquid. Otherwise the realized measures could have been undervalued or overvalued. This way 30 days (mostly U.S. holidays) were dropped.

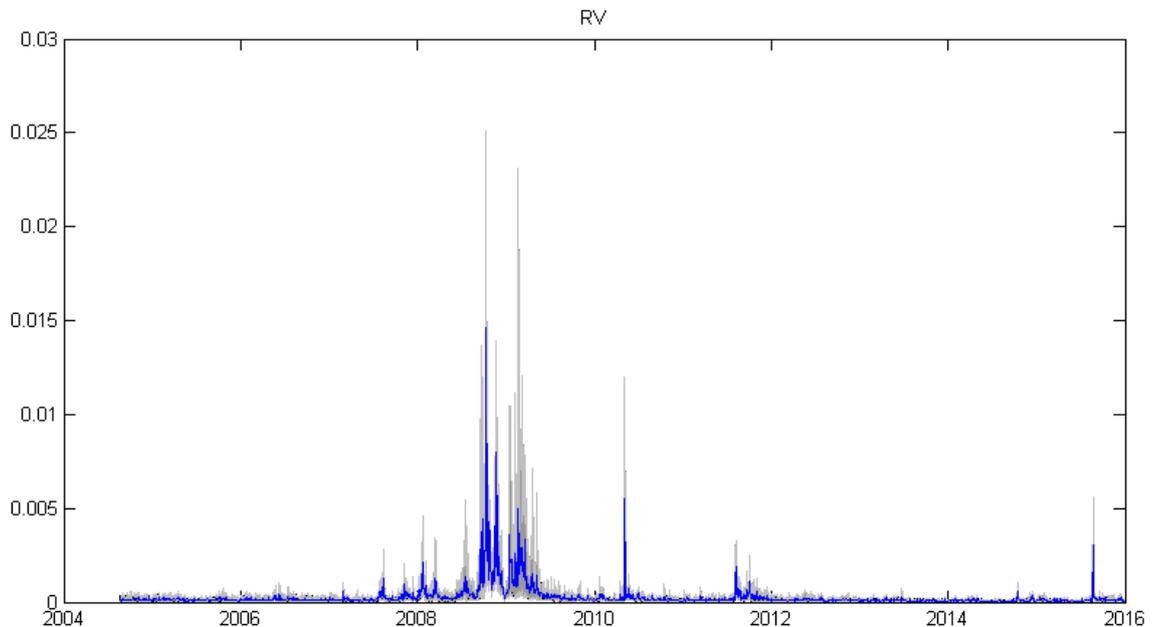
Then we used every fifth intraday price observation in order to build the series of 5-minute prices, thus we obtained maximum of 79 price observations a day. Those were used to construct 5-minute logarithmic returns as $r_{t,i} = \log(P_{t,i}/P_{t,i-1})$. Consequently the log-returns we obtained were employed to compute realized variance for all 29 stocks as $RV_t = \sum_{i=1}^m r_{t,i}^2$, where m equals to 78. For some days RV was extremely high ($RV > 0.1$), therefore we treated corresponding 7 days as outliers and dropped them from our sample. Our final sample thus consists of 2826 days.

Furthermore we followed Patton & Sheppard (2015) and decomposed realized variance into positive and negative semivariances as $RS_t^+ = \sum_{i=1}^m r_{t,i}^2 I_{[r_{t,i}>0]}$ and $RS_t^- = \sum_{i=1}^m r_{t,i}^2 I_{[r_{t,i}<0]}$. Finally, as we want to identify univariate jumps using BNS test we computed bipower variation which serves as a measure of integrated variation. We use the staggered (skip-one) version described in Equation 2.28 in order to make the estimator robust to microstructure noise:

$$BPV_t = \mu_1^{-2} \left(\frac{m-2}{m-2} \right) \sum_{i=3}^m |r_{t,i}| |r_{t,i-2}|$$

where $\mu_1 = \sqrt{2/\pi}$.

Figure 5.1: Average Realized Variance



Source: Author's calculations.

Figure 5.1 depicts the average value of realized variance across the portfolio of 29 stocks

(blue line) as well as the area between 10 % and 90 % quantiles (gray-shaded region). We can clearly see the period of heightened volatility which can be attributed to the financial crisis of 2008-2009 or a spike representing the flash crash of May 6, 2010.

5.3 Sectors

The 29 stocks we have at hand belong to 8 sectors classified according to the Global Industry Classification Standard (GICS). GICS was developed by Morgan Stanley Capital International (MSCI) and Standard & Poor's in 1999 and is regularly updated since then. It has become a global classification standard used by all groups of market participants and consists of 10 sectors, 24 industry groups, 67 industries and 156 sub-industries. Now let us have a closer look at the eight sectors to which our stocks belong and their characteristics.

The Information Technology Sector encompasses companies involved in software development, information technology services as well as manufacturing and distribution of hardware, semiconductors and corresponding equipment. The IT is currently the largest sector of the S&P 500 index with a share of 21% and the same holds for our sample which contains seven IT stocks - Apple, Cisco Systems, IBM, Intel, Microsoft, Oracle and QUALCOMM.

The Consumer Discretionary Sector contains businesses that sell nonessential goods and services such as retailers, media, automobile manufacturers or consumer services and consumer durables companies that represent 13% of the S&P 500 index. Amazon, Walt Disney, Home Depot and McDonald's are its representatives in our sample.

The Financials Sector comprises companies engaged in banking, insurance and real estate industries. It is the second largest sector of the S&P 500 index with a share of 16% and four financial stocks - Bank of America, Citigroup, JPMorgan Chase and Wells Fargo - belong to our sample.

The contribution of telecommunications services providers to the S&P 500 index is rather negligible (less than 3%). However, there are three representatives of **the Telecommunication Services Sector** in our sample - Comcast, AT&T and Verizon.

The Energy Sector comprises companies that are involved in exploration, production, marketing, refining, storage or transportation of oil, gas, coal and other consumable fuels. The energy producers and suppliers contribute to the S&P 500 index by only 7% and in our sample they are represented by three companies - Chevron, Schlumberger and ExxonMobil.

The Industrials Sector consists of three industry groups - manufacture and distribution of capital goods, provision of commercial services and supplies and transportation. Its market capitalization constitutes 10% of the S&P 500 index, but General Electric is the only representative of this sector within our sample.

The Health Care Sector contains health care equipment and services providers as well as pharmaceutical and biotechnology companies that represent 14% of the S&P 500 index. Three health care stocks belong to our sample - Johnson & Johnson, Merck and Pfizer.

Finally, **the Consumer Staples Sector** encompasses businesses that sell essential products such as food, beverage, tobacco or non-durable household goods. Its share of the S&P 500 index equals to 10% and four companies - Coca-Cola, Pepsi, Procter & Gamble and Wal-Mart - represent the sector in our sample.

As we can see our sample contains both cyclical sectors such as the information technology, consumer discretionary or financial sector that are more sensitive to the business cycle as well as defensive sectors such as the health care or consumer staples.

Table 5.1: Sectors and corresponding stocks

Sector	Stocks
Information Technology	Apple (AAPL), Cisco Systems (CSCO), International Business Machines Corporation (IBM), Intel Corporation (INTC), Microsoft Corporation (MSFT), Oracle Corporation (ORCL), QUALCOMM (QCOM)
Consumer Discretionary	Amazon.com (AMZN), Walt Disney Company (DIS), Home Depot Inc (HD), McDonald's Corporation (MCD)
Financials	Bank of America Corporation (BAC), Citigroup (C), JPMorgan Chase & Co. (JPM), Wells Fargo & Company (WFC)
Telecommunication Services	Comcast Corporation (CMCSA), AT&T(T), Verizon Communications (VZ)
Energy	Chevron Corporation (CVX), Schlumberger Limited (SLB), ExxonMobil Corporation (XOM)
Industrials	General Electric Company (GE)
Health Care	Johnson & Johnson (JNJ), Merck & Co. (MRK), Pfizer (PFE)
Consumer Staples	Coca-Cola Company (KO), PepsiCo (PEP), Procter & Gamble Co. (PG), Wal-Mart Stores (WMT)

Table 5.1 shows the list of sectors and corresponding stocks. With the exception of the industrial sector we have at least three representative stocks for seven remaining sectors. Therefore we can create subsamples that consist only of stocks from one sector to find out if our hypotheses hold for all of them or if there are some differences among individual sectors.

The descriptive statistics of daily returns for all stocks in our portfolio are reported in Table 5.2. All the stocks have excess kurtosis as expected, but those from financial sector seem to have the heaviest tails. The financial stocks are also the most volatile ones as hinted by high standard deviation of returns. Figure 5.2 which is a box plot showing logarithmic realized variance of all 29 stocks supports this finding. Particularly

the outlying observations (red + signs) of Bank of America and Citigroup stocks represent days with extremely high realized variance.

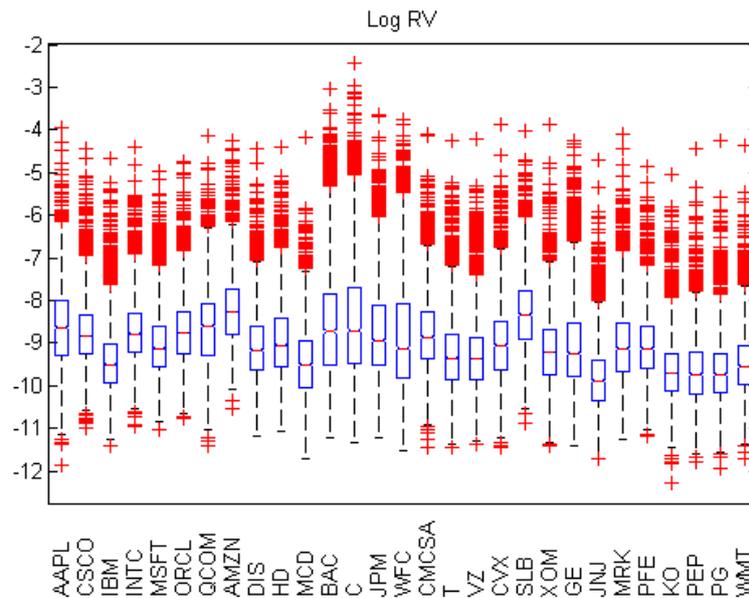
On the contrary, the consumer staples stocks are on average the least volatile which corresponds with their defensive nature.

Table 5.2: Descriptive Statistics for Daily Returns

Information Technology								
	AAPL	CSCO	IBM	INTC	MSFT	ORCL	QCOM	
Mean	-0.0005	-0.0000	0.0006	0.0001	0.0002	0.0004	0.0000	
Std.dev.	0.0178	0.0140	0.0107	0.0146	0.0129	0.0141	0.0151	
Skewness	-0.2688	-0.2294	-0.2131	0.1130	0.0880	-0.1633	-0.3233	
Kurtosis	6.9618	7.6712	7.4316	7.0654	8.7337	7.1269	6.8503	
Minimum	-0.1223	-0.0922	-0.0677	-0.0907	-0.0755	-0.0975	-0.1195	
Maximum	0.1123	0.0802	0.0636	0.0880	0.1102	0.0774	0.0925	
Consumer Discretionary				Financials				
	AMZN	DIS	HD	MCD	BAC	C	JPM	WFC
Mean	0.0011	0.0006	0.0003	0.0003	-0.0016	-0.0027	0.0001	-0.0001
Std.dev.	0.0197	0.0136	0.0148	0.0106	0.0276	0.0286	0.0214	0.0225
Skewness	0.3119	0.4358	0.6869	0.3307	-0.4877	-2.1095	0.3763	0.2681
Kurtosis	8.2795	11.3497	10.1314	10.4240	21.3447	32.1312	17.3702	22.1464
Minimum	-0.1313	-0.0909	-0.0763	-0.0799	-0.2509	-0.3468	-0.1804	-0.2081
Maximum	0.1388	0.1185	0.1127	0.1035	0.2014	0.1992	0.1618	0.1933
Telecommunication Services			Energy			Industrials		
	CMCSA	T	VZ	CVX	SLB	XOM	GE	
Mean	0.0003	-0.0000	-0.0001	0.0002	0.0000	0.0005	-0.0003	
Std.dev.	0.0161	0.0114	0.0112	0.0137	0.0187	0.0128	0.0150	
Skewness	0.6079	0.5480	0.5394	0.0623	-0.4068	0.0044	-0.2745	
Kurtosis	24.8503	14.6429	12.4582	16.1456	9.7968	15.2499	15.7594	
Minimum	-0.1416	-0.0629	-0.0760	-0.1296	-0.1552	-0.1261	-0.1178	
Maximum	0.2325	0.1242	0.1118	0.1460	0.1253	0.1189	0.1166	
Health Care			Consumer Staples					
	JNJ	MRK	PFE	KO	PEP	PG	WMT	
Mean	0.0002	-0.0000	-0.0003	0.0001	0.0004	0.0005	-0.0000	
Std.dev.	0.0088	0.0135	0.0118	0.0095	0.0091	0.0089	0.0099	
Skewness	0.7189	-0.1708	0.0889	0.0376	0.1694	-0.0579	-0.0969	
Kurtosis	21.7714	11.1575	6.8228	11.5861	10.9142	10.2867	13.9058	
Minimum	-0.0803	-0.1092	-0.0696	-0.0717	-0.0657	-0.0660	-0.1053	
Maximum	0.1158	0.0919	0.0714	0.0795	0.0878	0.0776	0.0762	

Source: Author's calculations.

Figure 5.2: Box plot of log RV for 29 stocks



Source: Author's calculations.

5.4 Jump identification

We have already calculated RV and BPV , now we can use them to detect jumps using the BNS test. Therefore we computed Z_t test statistic as in Equation 3.4 and identified jumps that are statistically significant at 5% level. On average 11.6% of days were labeled as jump days and the last column of Table 5.3 shows jump frequency for all 29 stocks. The least jumpy sector is definitely the energy sector, whereas consumer staples stocks display the highest occurrence of jumps. We should keep in mind that the highest occurrence of jumps in the consumer staples sector does not necessarily mean that corresponding stocks display also the highest jump variation, J_t , which is computed as in Equation 3.5.

The fourth column of Table 5.3 shows relative contribution of jumps to total variation using the relative jump measure $RJ_t = \frac{RV_t - BPV_t}{RV_t}$. We conclude that jumps account for approximately 4.6% of the total return variation.

One of our objectives is to investigate if the impact of a jump on future volatility depends on the sign of the jump. Therefore we have to identify if the jump was upward or downward one. However, as we have already stated the BNS test is not able to detect the intraday timing of jumps, thus we cannot assess if the jump was positive or negative.

Table 5.3: Frequency of Jumps

Company	Ticker	Sector	RJ measure	Jump frequency
Apple	AAPL	Information Technology	4.51%	9.84%
Cisco Systems	CSCO	Information Technology	2.98%	10.86%
IBM	IBM	Information Technology	3.66%	11.57%
Intel	INTC	Information Technology	3.05%	10.26%
Microsoft	MSFT	Information Technology	4.10%	12.77%
Oracle	ORCL	Information Technology	4.17%	12.70%
QUALCOMM	QCOM	Information Technology	4.90%	11.71%
Amazon	AMZN	Consumer Discretionary	3.99%	10.47%
Walt Disney	DIS	Consumer Discretionary	5.71%	11.39%
Home Depot	HD	Consumer Discretionary	5.11%	11.57%
McDonald's	MCD	Consumer Discretionary	4.52%	12.46%
Bank of America	BAC	Financials	4.56%	12.07%
Citigroup	C	Financials	6.61%	12.53%
JPMorgan Chase	JPM	Financials	3.41%	10.72%
Wells Fargo	WFC	Financials	4.37%	12.07%
Comcast	CMCSA	Telecommunication Services	5.31%	11.93%
AT&T	T	Telecommunication Services	4.30%	13.02%
Verizon	VZ	Telecommunication Services	4.08%	11.47%
Chevron	CVX	Energy	3.92%	8.78%
Schlumberger	SLB	Energy	3.15%	8.42%
ExxonMobil	XOM	Energy	2.64%	7.78%
General Electric	GE	Industrials	2.96%	10.08%
Johnson & Johnson	JNJ	Health Care	9.26%	14.58%
Merck	MRK	Health Care	6.98%	12.77%
Pfizer	PFE	Health Care	4.80%	11.57%
Coca-Cola	KO	Consumer Staples	6.08%	13.94%
Pepsi	PEP	Consumer Staples	8.07%	13.91%
Procter & Gamble	PG	Consumer Staples	5.87%	12.70%
Wal-Mart	WMT	Consumer Staples	4.39%	12.31%
Portfolio			4.61%	11.60%

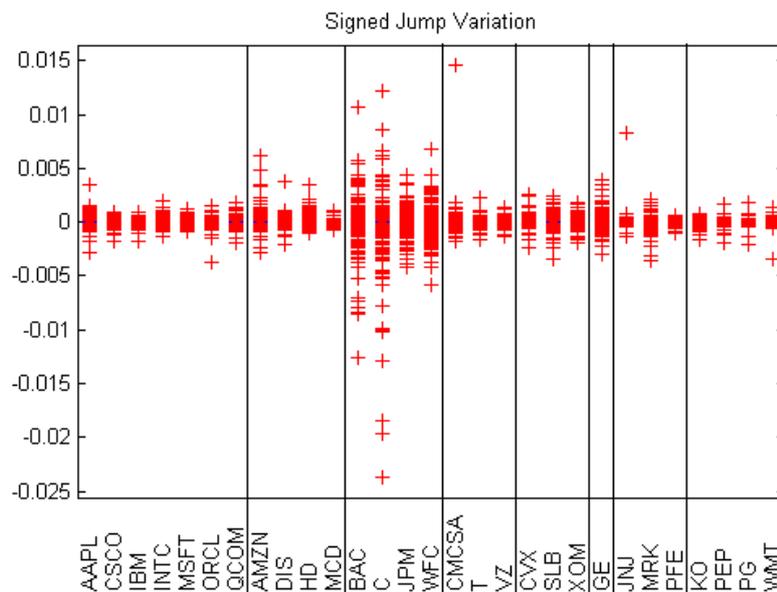
Source: Author's calculations.

We could have used either the sequential version of BNS test or one of the intraday jump tests, instead we opted for signed jump variation proposed by Patton & Sheppard (2015).

The signed jump variation is defined as $\Delta J_t^2 \equiv RS_t^+ - RS_t^-$ and is positive if a day is dominated by positive jumps and negative if a day is dominated by negative jumps. In order to examine whether the effect of good and bad news is the same in absolute value we decompose signed jump variation into ΔJ_t^{2+} and ΔJ_t^{2-} as in Equation 4.21.

Finally, Figure 5.3 depicts a box plot of signed jump variation for our panel of stocks and confirms that financial stocks are the most volatile. We can see that most values are close to zero as the graph shows no boxes that represent values between 25th and 75th percentiles, only outliers.

Figure 5.3: Box plot of Signed Jump Variation for 29 stocks



Source: Author's calculations.

5.5 Co-jump identification

Last but not least we have to detect common jumps within sectors and across the panel of 29 stocks. We cannot employ the test proposed by Bollerslev *et al.* (2008) in order to detect sector co-jumps as it requires large cross-section of returns to function properly and our sectors consist of only 3-7 companies. Therefore common jumps on the sector

level are identified using a simple co-exceedance rule - if at least two stocks from one sector jump simultaneously sector co-jump is detected.

However, we can use the above-mentioned test to detect common jumps in our portfolio of 29 stocks. In comparison with the univariate jump test which is based on the difference between realized variance and jump-robust bipower variation this co-jump test depends on the cross-products between returns. As we noted earlier some price observations are missing, but now we need exactly 78 log returns each day to properly construct z_{mcp} statistic defined in Equation 3.9. Therefore the one-minute data are in this case adjusted so that missing price observations are replaced by prevailing prices and then the 5-min logarithmic returns are computed.

The test statistic does not follow any standard distribution, therefore we have to simulate its distribution under the condition of no jumps. Firstly we obtain the covariance matrix of original intraday returns and use it to bootstrap diffusion process with zero drift and given covariance matrix. This way we obtain 220 428 (78 realizations along 2826 days) x 29 log returns and compute the test statistic. The process is repeated 1000 times as Bollerslev *et al.* (2008) did and in this way 220 428 000 simulated values of z_{mcp} statistic are obtained. Then we can easily set the significance level and get critical value. As Bollerslev *et al.* (2008) showed the critical values depend on the number of intraday returns whereas they are rather insensitive to the number of stocks and covariance matrix of their returns. For 99.9% quantile we obtained critical value of 5.88 which is in accordance with the simulated values from the original paper. In this case we found total amount of 867 jumps which corresponds to 30.68% of intraday 5-min intervals and 0.39% of days. We are particularly interested in the most significant co-jumps that might be difficult to be diversified away and are likely driven by macroeconomic news like Federal Funds Target Rate announcements. Therefore we focused also on higher quantiles. The frequency of common jump occurrence for various significance levels can be observed in Table 5.4.

One of our hypotheses is that the impact of “good” and “bad” jumps is not the same. The test is designed to detect co-jumps if the prices of majority of stocks move in the same direction, because the mean cross-product of two stocks that move in the opposite direction is negative and lowers z_{mcp} statistic. Therefore we divided common jumps into two groups based on the prevailing sign of returns within given interval. For significance levels lower than 0.1% at least 18 stocks move in one direction and 11 or less move in the opposite way. Therefore with respect to the nature of test we can be fairly sure that the distribution of co-jumps into two groups reflects the sign of common jump. The frequency of positive and negative common jumps can be observed again in Tables 5.5 and 5.6.

Table 5.4: Cojumps - descriptive statistics

Significance level	No. of co-jumps	% of co-jumps (intraday)	% of co-jumps (daily)
0.1%	867	0.39%	30.68%
0.01%	308	0.14%	10.90%
0.001%	131	0.06%	4.64%
0.0001%	63	0.03%	2.23%

Table 5.5: Positive cojumps - descriptive statistics

Significance level	No. of co-jumps	% of co-jumps (intraday)	% of co-jumps (daily)
0.01%	153	0.07%	5.41%
0.001%	68	0.03%	2.41%
0.0001%	38	0.02%	1.34%

Table 5.6: Negative cojumps - descriptive statistics

Significance level	No. of co-jumps	% of co-jumps (intraday)	% of co-jumps (daily)
0.01%	155	0.07%	5.48%
0.001%	63	0.03%	2.23%
0.0001%	25	0.01%	0.88%

Source: Author's calculations.

Chapter 6

Results

In the previous chapter we introduced our data set, constructed realized measures of variance and identified jumps and common jumps. We will follow up with the discussion of results. But first we will summarize what our primary objectives and hypotheses are.

First of all, we believe that jumps have a positive effect on future volatility even though many empirical studies (Andersen *et al.* (2007a), Giot & Laurent (2007) or Busch *et al.* (2011)) found the effect to be insignificant or even negative. However, Corsi *et al.* (2010) argued that jumps should have an impact on volatility for two reasons. Firstly, they note that periods of high volatility are usually initiated by an unexpected movement of asset prices. Secondly, the occurrence of a jump might increase the uncertainty and consequently lead to higher volatility. They applied threshold bipower variation as a measure of continuous variation and found that the impact of jumps on volatility for the S&P 500 index futures, six individual stocks and US Treasury Bond futures is positive and predominantly significant.

The conclusions drawn from the paper written by Patton & Sheppard (2015) support our hypothesis as they report that the impact of a jump depends on its sign - negative jumps lead to higher future volatility and positive jumps lead to lower future volatility. This could explain insignificant jump variation coefficients in most studies if the effects of upward and downward jumps just cancel each other. However, Patton & Sheppard (2015) also showed that response to negative jumps is larger in magnitude. These findings correspond with the leverage effect hypothesis proposed by Black (1976) according to which a decline in the value of the stock increases financial leverage, which in turn makes the stock riskier and more volatile. Thus the stronger impact of downward jumps might drive the positive effect of jumps on volatility.

Secondly, we will investigate if the common jumps help to predict future volatility in

the same way as idiosyncratic jumps do. Whereas Clements & Liao (2013) used test proposed by Bollerslev *et al.* (2008) and found this effect to be negative and significant, Caporin *et al.* (2014) employed their own novel test and reported positive, but insignificant results. Furthermore, we will examine if the impact of co-jump depends on its sign.

Last but not least we will estimate our models for seven sectors that contain at least three representative stocks. Therefore, we will examine whether our results are driven by all the sectors or if our hypotheses hold for only some of them.

As we stated before we apply only logarithmic forms of HAR models due to their favorable properties. We employ HAR models for individual stocks and panel HAR models to obtain average effects for sectors and for the panel of stocks. The results of panel HAR models are presented in corresponding sections whereas the summaries for individual stocks can be found in Appendix. For each model we report the coefficients, robust t-statistics (in parentheses) and R^2 .

The results are presented in the following order. We begin with simple HAR-RV model in order to establish a set of reference results. Then we add jump variation into our modeling framework and investigate the effect of jumps on volatility. The decomposition of realized variance into semivariances is performed in Section 6.3 and the impact of signed returns and signed jumps is estimated. Finally, Section 6.4 is dedicated to simultaneous jumps and their impact on future volatility.

6.1 Volatility modeling using HAR-RV

First of all, we need a baseline model that will provide us with a set of reference results. Therefore we employ standard HAR-RV model and present the results for the portfolio and market sectors in Table 6.1. In the last row we can see the average result for the panel of stocks. The daily realized variance seems to have the largest predictive power whereas the monthly component is clearly the least important and all the coefficients are positive and highly significant.

The results for individual sectors reveal additional information. For market sectors that are clearly cyclical (Information Technology, Consumer Discretionary, Financials) the coefficient of the daily component is higher (and much more significant) than the coefficient of the weekly component. But for the rest of industries $\beta^{(w)}$ is higher than $\beta^{(d)}$. The summary of estimation for 29 stocks is provided in Table A.2 and confirms our findings with only few exceptions like IBM for which the weekly component is more important than the daily one.

Table 6.1: Panel HAR-RV Estimation Results

$$\log(RV_{j,t+1}^{(d)}) = c_j + \beta^{(d)} \log(RV_{j,t}^{(d)}) + \beta^{(w)} \log(RV_{j,t}^{(w)}) + \beta^{(m)} \log(RV_{j,t}^{(m)}) + \omega_{j,t+1}$$

	$\beta^{(d)}$		$\beta^{(w)}$		$\beta^{(m)}$		R^2
Information Technology	0.370	(33.650)	0.332	(19.944)	0.242	(20.782)	0.665
Consumer Discretionary	0.350	(24.821)	0.339	(16.169)	0.261	(15.986)	0.682
Financials	0.448	(29.656)	0.308	(16.125)	0.220	(14.465)	0.843
Telecommunication Services	0.335	(19.906)	0.374	(16.431)	0.242	(14.164)	0.695
Energy	0.388	(23.032)	0.407	(19.990)	0.155	(10.190)	0.730
Health Care	0.340	(19.255)	0.368	(15.431)	0.226	(11.799)	0.627
Consumer Staples	0.318	(21.104)	0.380	(18.517)	0.242	(14.832)	0.641
Portfolio	0.368	(65.617)	0.354	(46.501)	0.233	(40.055)	0.721

The table presents estimated parameters of HAR-RV model for the panel of 29 stocks and 7 market sectors. Robust t-statistics based on Newey-West correction (with lag length of 44) are reported in parentheses.

Source: Author's calculations.

The HAR-RV model explains 72.1% of total variation. As we will see in the following sections only slight improvements of R^2 will be obtained even though the extended models will provide us with valuable information. The value of R^2 for sector-related estimations fluctuates between 0.627 (health care sector) and 0.843 (financial sector).

6.2 The impact of jumps

From now on we will focus on the decomposition of the daily RV component. First of all, we will decompose it into bipower variation (continuous component) and jump variation.

Table 6.2 confirms our hypothesis that jumps have a positive effect on future volatility as the jump variation coefficient $\beta_j^{(d)}$ is statistically significant at the 95% confidence level for the panel of stocks and the health care sector and statistically significant at the 99% confidence level for the information technology sector. As a robustness check we estimated the HAR-RV-J model with realized volatility (square root of realized variance) instead of its logarithmic transformation as the key variable. The coefficient of the health care sector is no longer significant, but the impact of jumps on volatility in financial sector becomes statistically significant as documented in Table A.1 (the coefficients of HAR models in standard deviation form are in general less significant, but in line with results of HAR models in logarithmic form).

Then we estimate an alternative specification where the jump variation is replaced by the jump dummy variable. Thus we will not investigate if the impact of the size of

Table 6.2: The Impact of Jumps on Future Volatility (Panel HAR-RV-J)

$$\log(RV_{j,t+1}^{(d)}) = c_j + \beta_C^{(d)} \log(BPV_{j,t}^{(d)}) + \beta_J^{(d)} \log(1 + J_{j,t}^{(d)}) + \beta^{(w)} \log(RV_{j,t}^{(w)}) + \beta^{(m)} \log(RV_{j,t}^{(m)}) + \omega_{j,t+1}$$

	$\beta_C^{(d)}$		$\beta_J^{(d)}$		$\beta^{(w)}$		$\beta^{(m)}$		R ²
Information Technology	0.359	(33.504)	117.046	(3.379)	0.342	(20.573)	0.240	(20.346)	0.665
Consumer Discretionary	0.352	(25.292)	25.079	(0.944)	0.339	(16.128)	0.258	(15.767)	0.684
Financials	0.441	(30.654)	36.743	(1.483)	0.311	(16.305)	0.223	(14.680)	0.844
Telecommunication Services	0.328	(19.406)	3.989	(0.105)	0.383	(16.602)	0.239	(13.920)	0.695
Energy	0.384	(22.710)	10.126	(0.225)	0.412	(19.813)	0.152	(9.931)	0.731
Health Care	0.348	(19.209)	111.880	(2.288)	0.364	(15.314)	0.221	(11.476)	0.629
Consumer Staples	0.323	(21.812)	-61.491	(-0.828)	0.379	(18.890)	0.239	(14.768)	0.643
Portfolio	0.365	(66.224)	43.913	(2.393)	0.358	(47.119)	0.231	(39.716)	0.722

The table presents estimated parameters of Panel HAR-RV-J model for the panel of 29 stocks and 7 market sectors. Robust t-statistics based on Newey-West correction (with lag length of 44) are reported in parentheses.

Source: Author's calculations.

Table 6.3: The Impact of Jumps on Future Volatility (Panel HAR-RV-Jump)

$$\log(RV_{j,t+1}^{(d)}) = c_j + \beta_C^{(d)} \log(BPV_{j,t}^{(d)}) + \beta_{Jump}^{(d)} Jump_{j,t}^{(d)} + \beta^{(w)} \log(RV_{j,t}^{(w)}) + \beta^{(m)} \log(RV_{j,t}^{(m)}) + \omega_{j,t+1}$$

	$\beta_C^{(d)}$		$\beta_{Jump}^{(d)}$		$\beta^{(w)}$		$\beta^{(m)}$		R ²
Information Technology	0.378	(34.409)	0.103	(8.772)	0.325	(19.561)	0.240	(20.715)	0.666
Consumer Discretionary	0.363	(25.699)	0.062	(4.033)	0.328	(15.640)	0.258	(15.882)	0.684
Financials	0.459	(31.095)	0.106	(6.684)	0.297	(15.559)	0.221	(14.491)	0.844
Telecommunication Services	0.343	(19.837)	0.076	(4.985)	0.368	(15.963)	0.240	(14.120)	0.696
Energy	0.398	(23.721)	0.095	(5.403)	0.398	(19.584)	0.153	(10.123)	0.732
Health Care	0.362	(19.397)	0.066	(3.943)	0.352	(14.752)	0.221	(11.698)	0.630
Consumer Staples	0.335	(22.130)	0.061	(4.204)	0.366	(17.950)	0.239	(14.908)	0.643
Portfolio	0.380	(67.499)	0.083	(14.787)	0.344	(45.154)	0.231	(39.927)	0.723

The table presents estimated parameters of Panel HAR-RV-Jump model for the panel of 29 stocks and 7 market sectors. Robust t-statistics based on Newey-West correction (with lag length of 44) are reported in parentheses.

Source: Author's calculations.

the jump matters, but rather if the mere presence of the jump has an effect on future volatility. The results that are presented in Table 6.3 clearly show that volatility increases following the jump in the price of a stock. The coefficient $\beta_{Jump}^{(d)}$ is positive and highly significant for all the sectors and the information technology and financial sectors are the most sensitive to jumps.

Tables A.3 and A.4 contain the results of our estimations for 29 stocks. The results from the first table are a bit ambiguous as there are seven stocks for which the jump variation coefficient is statistically significant at the 95% confidence level, but only for five of them the coefficient is positive. The second table, however, reveals positive and significant effect for majority of stocks.

To sum it up, our results are in accordance with the findings of Corsi *et al.* (2010) for the S&P 500 index futures and six individual stocks. We have found that the average effect of jumps on volatility is positive across the portfolio with the technology sector being the most sensitive to jumps in stock prices.

6.3 The impact of signed returns and jumps

Now we will follow Patton & Sheppard (2015) in order to find out the impact of signed jumps. Our aim is to provide an explanation of the positive effect of jumps on volatility.

Table 6.4: The Impact of Positive and Negative Semivariances on Future Volatility (Panel HAR-RS)

$$\log(RV_{j,t+1}^{(d)}) = c_j + \beta^{+(d)} \log(RS_{j,t}^{+(d)}) + \beta^{-(d)} \log(RS_{j,t}^{-(d)}) + \beta^{(w)} \log(RV_{j,t}^{(w)}) + \beta^{(m)} \log(RV_{j,t}^{(m)}) + \omega_{j,t+1}$$

	$\beta^{+(d)}$		$\beta^{-(d)}$		$\beta^{(w)}$		$\beta^{(m)}$		R^2
Information Technology	0.118	(13.377)	0.245	(24.766)	0.339	(20.445)	0.240	(20.543)	0.666
Consumer Discretionary	0.108	(9.662)	0.244	(20.816)	0.339	(16.069)	0.259	(15.941)	0.684
Financials	0.150	(12.357)	0.297	(20.354)	0.311	(16.162)	0.218	(14.309)	0.844
Telecommunication Services	0.121	(9.878)	0.214	(14.261)	0.377	(16.484)	0.240	(14.192)	0.696
Energy	0.130	(10.540)	0.248	(16.385)	0.417	(20.551)	0.154	(10.166)	0.731
Health Care	0.119	(8.933)	0.228	(14.848)	0.365	(15.150)	0.222	(11.639)	0.628
Consumer Staples	0.116	(9.811)	0.207	(16.111)	0.377	(18.552)	0.240	(14.815)	0.642
Portfolio	0.123	(28.636)	0.244	(49.380)	0.357	(46.770)	0.231	(39.789)	0.722

The table presents estimated parameters of Panel HAR-RS model for the panel of 29 stocks and 7 market sectors. Robust t-statistics based on Newey-West correction (with lag length of 44) are reported in parentheses.

Source: Author's calculations.

We expect the effect to be driven by downward jumps that lead to higher volatility. Furthermore, we hypothesize that upward jumps lead to lower volatility and that the response to downward jumps is larger in magnitude.

Therefore we will first decompose the realized variance into semivariances, then we will use the semivariances to obtain signed jump variation measure and finally we will decompose the signed jump variation into positive and negative signed jump components in the same way Patton & Sheppard (2015) did with their portfolio of stocks. On top of that we will again perform sectoral analysis which will be our unique contribution to their findings.

Tables 6.4 and A.5 show the results of HAR-RS estimations. All the coefficients of the daily semivariances are positive and highly significant, but the impact of negative realized semivariance is clearly larger than the impact of its positive counterpart for the portfolio as well as for all the sectors and individual stocks. The outcome of panel HAR-RS model is also in line with the results obtained by Patton & Sheppard (2015).

Table 6.5: The Impact of Signed Jumps on Future Volatility
(Panel HAR-RV-J-Sign)

$$\log(RV_{j,t+1}^{(d)}) = c_j + \beta_C^{(d)} \log(BPV_{j,t}^{(d)}) + \beta_{\Delta J}^{(d)} \Delta J_{j,t}^{2(d)} + \beta^{(w)} \log(RV_{j,t}^{(w)}) + \beta^{(m)} \log(RV_{j,t}^{(m)}) + \omega_{j,t+1}$$

	$\beta_C^{(d)}$	$\beta_{\Delta J}^{(d)}$	$\beta^{(w)}$	$\beta^{(m)}$	R^2
Information Technology	0.352 (33.172)	-0.145 (-9.298)	0.349 (21.138)	0.240 (20.373)	0.667
Consumer Discretionary	0.348 (25.123)	-0.142 (-7.567)	0.345 (16.490)	0.257 (15.727)	0.685
Financials	0.438 (30.642)	-0.169 (-7.909)	0.316 (16.452)	0.221 (14.248)	0.845
Telecommunication Services	0.324 (19.609)	-0.106 (-4.774)	0.386 (17.122)	0.238 (14.030)	0.696
Energy	0.375 (22.634)	-0.141 (-6.213)	0.421 (20.698)	0.152 (10.003)	0.732
Health Care	0.348 (19.030)	-0.109 (-4.924)	0.366 (15.263)	0.220 (11.667)	0.630
Consumer Staples	0.321 (22.020)	-0.098 (-4.898)	0.379 (19.261)	0.240 (14.904)	0.644
Portfolio	0.361 (66.015)	-0.134 (-17.989)	0.363 (47.945)	0.230 (39.538)	0.723

The table presents estimated parameters of Panel HAR-RV-J-Sign model for the panel of 29 stocks and 7 market sectors. Robust t-statistics based on Newey-West correction (with lag length of 44) are reported in parentheses.

Source: Author's calculations.

In Table 6.5 we present the results of HAR model in which the daily realized variance is decomposed into bipower variation and signed jump variation. The coefficient of signed jump variation, $\beta_{\Delta J}^{(d)}$, is unanimously negative and highly significant for all the sectors and naturally also for the portfolio. The same holds for most of the stocks as Table A.6 suggests. Therefore we claim that days dominated by negative jumps ($\Delta J_t^{2(d)} < 0$) lead to higher future volatility and days dominated by positive jumps ($\Delta J_t^{2(d)} > 0$) lead to lower

future volatility which is again in accordance with the conclusions of Patton & Sheppard (2015).

As we stated before when discussing the results of panel HAR-RV model, for cyclical sectors (Information Technology, Consumer Discretionary, Financials) the daily realized variance has higher predictive power than its weekly average whereas for the rest of the sectors the opposite is true. Consequently, we can expect the coefficient of daily signed jump variation, $\beta_{\Delta J}^{(d)}$, to be higher and more significant for cyclical sectors as well. Table 6.5 proves our expectations to be correct with one exception - the energy sector exhibits the same behavior as three cyclical sectors.

Finally, we decompose the signed jump variation in order to prove existence of the asymmetric response of volatility to positive and negative jumps. Table 6.6 shows that the response of volatility to positive and negative jumps is indeed asymmetric as the decrease in volatility following the positive jump is smaller than the increase in volatility after the negative jump.

Table 6.6: The Impact of Positive and Negative Signed Jumps on Future Volatility (Panel HAR-RV-J-Sign)

$$\log(RV_{j,t+1}^{(d)}) = c_j + \beta_C^{(d)} \log(BPV_{j,t}^{(d)}) + \beta_{\Delta J+}^{(d)} \% \Delta J_{j,t}^{2+(d)} + \beta_{\Delta J-}^{(d)} \% \Delta J_{j,t}^{2-(d)} + \beta^{(w)} \log(RV_{j,t}^{(w)}) + \beta^{(m)} \log(RV_{j,t}^{(m)}) + \omega_{j,t+1}$$

	$\beta_C^{(d)}$	$\beta_{\Delta J+}^{(d)}$	$\beta_{\Delta J-}^{(d)}$	$\beta^{(w)}$	$\beta^{(m)}$	R ²
Information Technology	0.350 (32.998)	-0.016 (-0.429)	-0.228 (-8.326)	0.350 (21.204)	0.241 (20.437)	0.667
Consumer Discretionary	0.348 (25.057)	-0.160 (-3.832)	-0.130 (-3.935)	0.345 (16.466)	0.257 (15.726)	0.685
Financials	0.436 (30.543)	-0.002 (-0.045)	-0.267 (-7.621)	0.317 (16.568)	0.221 (14.290)	0.845
Telecom. Services	0.324 (19.585)	-0.119 (-2.371)	-0.098 (-2.632)	0.386 (17.104)	0.238 (13.950)	0.696
Energy	0.375 (22.694)	0.002 (0.041)	-0.230 (-5.796)	0.421 (20.809)	0.152 (9.996)	0.733
Health Care	0.348 (18.997)	-0.111 (-2.138)	-0.107 (-2.992)	0.366 (15.273)	0.220 (11.623)	0.630
Consumer Staples	0.322 (22.090)	-0.130 (-2.787)	-0.078 (-2.303)	0.379 (19.230)	0.239 (14.838)	0.644
Portfolio	0.360 (65.876)	-0.064 (-3.752)	-0.177 (-13.817)	0.363 (48.051)	0.231 (39.604)	0.723

The table presents estimated parameters of Panel HAR-RV-J-Sign model with decomposed signed jump variation for the panel of 29 stocks and 7 market sectors. Robust t-statistics based on Newey-West correction (with lag length of 44) are reported in parentheses.

Source: Author's calculations.

However, at the sector level we can observe that for 4 sectors (Consumer Discretionary, Telecommunication Services, Health Care and Consumer Staples) the impact of negative signed jump variation is not larger in magnitude than the impact of positive signed jump variation and for some sectors it is even higher. On the contrary, 3 remaining sectors (Information Technology, Financials and Energy) exhibit the dramatic increase in volatility following the downward jump, but their volatility seems to be insensitive to upward

jumps. This is a novel finding about the volatility of stock returns which underlines the importance of taking sector-specific characteristics into account.

Finally, Table A.7 contains the results for individual stocks. We can notice that the $\beta_{\Delta J_+}^{(d)}$ is surprisingly positive for Apple and Bank of America, the volatility of two stocks should therefore increase following an upward jump. However, for the majority of stocks the coefficient is negative.

6.4 The impact of co-jumps

We have already found that the effect of jumps on future volatility is positive and that this effect might be driven particularly by negative jumps in technology, financial and energy sectors. Now we will investigate if simultaneous jumps have some predictive power for volatility and if the same holds when we take into account idiosyncratic jumps.

6.4.1 The impact of co-jumps within sectors

First, we focus on simultaneous jumps of two or more stocks in one sector. Estimated parameters of panel HAR-RV-Cojump model are reported in Table 6.7. For information technology, financial and energy sectors the impact of sector common jumps on future volatility is positive. For the rest of the sectors the binary variable representing co-jumps seems to have no forecasting power for volatility. The only exception is the consumer staples sector - the $\beta_{C,J}^{(d)}$ coefficient is negative and significant at the 90% confidence level.

Particularly energy sector seems to be sensitive to common jumps. Whereas for both technology and financial sectors the co-jump coefficient decreased by one half, for energy it slightly increased (even though corresponding t-statistic decreased).

A possible explanation for negative and significant $\beta_{C,J}^{(d)}$ coefficient in the last row of Table 6.7 might be related to the results of HAR-RV-J-Sign model that are presented in Table 6.6. For consumer staples sector the impact of positive signed jump variation is larger in magnitude than the impact of negative signed jump variation, thus the decrease in volatility following the upward jump is larger than increase in volatility following the downward jump. Therefore, the negative effect of sector co-jumps on volatility in this industry might be driven by upward co-jumps.

As a robustness check we added jump indicator variable into panel HAR-RV-Cojump model, because the effect of common jumps reported in Table 6.7 might be based only on

Table 6.7: The Impact of Co-jumps on Future Volatility (Panel HAR-RV-Cojump[a])

$$\log(RV_{j,t+1}^{(d)}) = c_j + \beta_C^{(d)} \log(BPV_{j,t}^{(d)}) + \beta_{CJ}^{(d)} CJ_{j,t}^{(d)} + \beta^{(w)} \log(RV_{j,t}^{(w)}) + \beta^{(m)} \log(RV_{j,t}^{(m)}) + \omega_{j,t+1}$$

	$\beta_C^{(d)}$	$\beta_{CJ}^{(d)}$	$\beta^{(w)}$	$\beta^{(m)}$	R^2
Information Technology	0.364 (33.307)	0.041 (4.653)	0.340 (20.455)	0.239 (20.424)	0.665
Consumer Discretionary	0.352 (25.229)	0.005 (0.269)	0.340 (16.183)	0.257 (15.760)	0.684
Financials	0.444 (30.661)	0.054 (2.839)	0.312 (16.171)	0.221 (14.353)	0.844
Telecommunication Services	0.329 (19.618)	0.009 (0.455)	0.382 (16.765)	0.239 (13.957)	0.695
Energy	0.389 (22.974)	0.106 (3.734)	0.408 (20.090)	0.152 (10.080)	0.731
Health Care	0.350 (19.045)	-0.018 (-0.916)	0.366 (15.223)	0.218 (11.522)	0.629
Consumer Staples	0.321 (21.755)	-0.029 (-1.877)	0.379 (19.011)	0.240 (14.940)	0.643

The table presents estimated parameters of Panel HAR-RV-Cojump model for 7 market sectors. Co-jumps are detected if at least two stocks from given sector jump simultaneously. Robust t-statistics based on Newey-West correction (with lag length of 44) are reported in parentheses.

Source: Author's calculations.

Table 6.8: The Impact of Co-jumps on Future Volatility (Panel HAR-RV-Cojump[b])

$$\log(RV_{j,t+1}^{(d)}) = c_j + \beta_C^{(d)} \log(BPV_{j,t}^{(d)}) + \beta_{Jump}^{(d)} Jump_{j,t}^{(d)} + \beta_{CJ}^{(d)} CJ_{j,t}^{(d)} + \beta^{(w)} \log(RV_{j,t}^{(w)}) + \beta^{(m)} \log(RV_{j,t}^{(m)}) + \omega_{j,t+1}$$

	$\beta_C^{(d)}$	$\beta_{Jump}^{(d)}$	$\beta_{CJ}^{(d)}$	$\beta^{(w)}$	$\beta^{(m)}$	R^2
Information Technology	0.379 (34.361)	0.096 (7.845)	0.017 (1.833)	0.325 (19.581)	0.240 (20.738)	0.666
Consumer Discretionary	0.363 (25.653)	0.069 (4.149)	-0.024 (-1.255)	0.328 (15.602)	0.259 (15.924)	0.684
Financials	0.459 (31.097)	0.104 (5.996)	0.005 (0.263)	0.297 (15.575)	0.221 (14.502)	0.844
Telecommunication Services	0.342 (19.861)	0.087 (5.322)	-0.039 (-1.765)	0.369 (15.977)	0.240 (14.115)	0.696
Energy	0.398 (23.730)	0.078 (4.013)	0.058 (1.830)	0.398 (19.656)	0.153 (10.156)	0.732
Health Care	0.363 (19.319)	0.083 (4.471)	-0.066 (-2.973)	0.351 (14.695)	0.221 (11.746)	0.630
Consumer Staples	0.335 (22.089)	0.084 (5.284)	-0.065 (-3.830)	0.364 (17.866)	0.241 (15.102)	0.644

The table presents estimated parameters of Panel HAR-RV-Cojump model with jump dummy for 7 market sectors. Co-jumps are detected if at least two stocks from given sector jump simultaneously. Robust t-statistics based on Newey-West correction (with lag length of 44) are reported in parentheses.

Source: Author's calculations.

the predictive power of idiosyncratic jumps. The co-jump coefficient in Table 6.8 remains positive and significant (but only at the 90% confidence level) for information technology and energy sectors. However, for telecommunication services and two defensive sectors, health care and consumer staples, the simultaneous jumps lead to lower future volatility. The results thus indicate that the average effect of sector co-jumps might not be positive as for univariate jumps, but rather negative.

6.4.2 The impact of co-jumps within portfolio

In the previous section we concentrated on sector co-jumps, now we will focus on common jumps across portfolio. Detection of portfolio co-jumps using the co-exceedance rule would not be the optimal solution. However, we can use the test based on cross-products of returns proposed by Bollerslev *et al.* (2008) to detect co-jumps in the large portfolio of assets. This co-jump test is designed to diversify away idiosyncratic jumps and identify non-diversifiable (often modest-sized) intraday common movements across the panel. As stated in the data section, we identify co-jumps at the 0.1%, 0.01%, 0.001% and 0.0001% significance levels. Table 6.9 contains the results of panel HAR-RV-Cojump model. For all the significance levels at which co-jumps were detected common jumps have negative and highly significant effect on future volatility.

Table 6.9: The Impact of Co-jumps on Future Volatility (Panel HAR-RV-Cojump[c])

$$\log(RV_{j,t+1}^{(d)}) = c_j + \beta_C^{(d)} \log(BPV_{j,t}^{(d)}) + \beta_{CJ}^{(d)} C_{j,t}^{(d)} + \beta^{(w)} \log(RV_{j,t}^{(w)}) + \beta^{(m)} \log(RV_{j,t}^{(m)}) + \omega_{j,t+1}$$

Significance level	$\beta_C^{(d)}$	$\beta_{CJ}^{(d)}$	$\beta^{(w)}$	$\beta^{(m)}$	R ²
0.1%	0.366 (66.167)	-0.048 (-13.882)	0.360 (47.332)	0.228 (39.019)	0.723
0.01%	0.366 (66.396)	-0.053 (-9.984)	0.358 (47.124)	0.230 (39.586)	0.722
0.001%	0.366 (66.283)	-0.065 (-8.428)	0.358 (47.031)	0.230 (39.398)	0.722
0.0001%	0.366 (66.211)	-0.060 (-5.354)	0.359 (47.158)	0.230 (39.428)	0.722

The table presents estimated parameters of Panel HAR-RV-Cojump model for the panel of 29 stocks. Co-jumps are detected using BLT test based on cross-products of returns. Robust t-statistics based on Newey-West correction (with lag length of 44) are reported in parentheses.

Source: Author's calculations.

These findings are in line with results of Clements & Liao (2013) who use the same co-jump test. We attempted to add the co-jump indicator variable with time index $t + 1$ into the model to find out how the volatility behaves on the days with co-jumps. The volatility is higher, particularly for downward co-jumps. But as our results suggest the next day volatility drops.

In order to better understand the impact of co-jumps on volatility we also decided to replace the co-jump indicator function by its positive and negative counterpart. Table 6.10 shows that the effect of positive and negative co-jumps is not the same. More specifically, positive co-jumps lead to significantly lower future volatility and the impact of negative co-jumps is not clear - for the most significant co-jumps it is even positive as we assumed. The effect of sign is thus in general the same as for idiosyncratic jumps.

Table 6.10: The Impact of Positive and Negative Co-jumps on Future Volatility
(Panel HAR-RV-Cojump[d])

$$\log(RV_{j,t+1}^{(d)}) = c_j + \beta_C^{(d)} \log(BPV_{j,t}^{(d)}) + \beta_{CJ}^{+(d)} CJ_{j,t}^{+(d)} + \beta_{CJ}^{- (d)} CJ_{j,t}^{- (d)} + \beta^{(w)} \log(RV_{j,t}^{(w)}) + \beta^{(m)} \log(RV_{j,t}^{(m)}) + \omega_{j,t+1}$$

Significance level	$\beta_C^{(d)}$	$\beta_{CJ}^{+(d)}$	$\beta_{CJ}^{- (d)}$	$\beta^{(w)}$	$\beta^{(m)}$	R ²
0.01%	0.365 (66.312)	-0.088 (-12.088)	-0.017 (-2.440)	0.359 (47.195)	0.229 (39.334)	0.722
0.001%	0.366 (66.279)	-0.094 (-9.200)	-0.034 (-3.003)	0.358 (47.117)	0.230 (39.329)	0.722
0.0001%	0.365 (66.280)	-0.121 (-8.801)	0.032 (1.825)	0.359 (47.209)	0.230 (39.382)	0.722

The table presents estimated parameters of Panel HAR-RV-Cojump model with positive and negative co-jumps for the panel of 29 stocks. Co-jumps are detected using BLT test depending on cross-products of returns. Robust t-statistics based on Newey-West correction (with lag length of 44) are reported in parentheses.

Source: Author's calculations.

Whereas the jump-related literature features well-established tests proposed by Barndorff-Nielsen & Shephard (2006) or Lee & Mykland (2008), the co-jump tests are scarce and not proven enough - there is no clear consensus on which test is the appropriate one. Using the test based on cross-products of returns leads to identification of negative and significant effect, but Caporin *et al.* (2014) found positive (albeit insignificant) effect of common jumps on volatility using their novel test. Thus the effect of co-jumps might critically depend on the test used. However, we are confident that taking into account the difference between positive and negative common jumps would reveal additional information regardless of the test used. All in all, co-jumps are still an interesting area for future research.

Chapter 7

Conclusion

In this work we focus on jumps and the role they play in volatility modeling. Jumps are price discontinuities driven primarily by firm-specific and macroeconomic news announcements and liquidity shocks, therefore they should affect the riskiness of assets and hence volatility. We aim to enrich the stream of relevant literature by several novel findings - we take an interest in the effect of jumps and common jumps on future volatility with special focus on the impact of signed jumps and the effect at the market sector and portfolio level.

The first part of this thesis concentrates on theory. We present quadratic variation and standard jump-diffusion model in which the price process can be decomposed into continuous and jump component. The ex-post non-parametric measure of quadratic variation, realized variance (RV), is introduced as well as bipower variation (BPV) which is robust to jumps and consistently estimates the continuous path variation. Consequently, we can detect jumps in asset prices using simple Hausman-type test proposed by Barndorff-Nielsen & Shephard (2006) that is based on the difference between RV and BPV . To reveal the impact of the sign of a jump on volatility we construct a measure called signed jump variation as well as its positive and negative components that allow us to estimate the response of volatility to positive and negative jumps. Then attention is given to detection of common jumps which is handled in two ways. Sector co-jumps are identified if at least two stocks from on sector jump simultaneously. Portfolio co-jumps are handled using the test based on cross-products of returns developed by Bollerslev *et al.* (2008).

For volatility modeling we employ HAR model proposed by Corsi (2009), in which future volatility depends on volatilities realized over different time horizons. Various extensions of HAR model are presented including the panel HAR which enables us to

estimate the average effect of jumps and common jumps on future volatility across the panel of stocks and different market sectors.

In the second part of this work we describe the data and present the results. Our data set, which spans from August 2004 to December 2015, consists of 29 U.S. stocks traded at New York Stock Exchange (NYSE) that can be classified into 8 market sectors according to the Global Industry Classification Standard (GICS). Therefore we are able to investigate if the impact of jumps and common jumps on volatility is the same for all sectors or not, which is our unique contribution to the relevant literature. Furthermore, we assume that equity markets are more sensitive to bad news than to good news. Therefore our main hypothesis is that volatility increases following the jump and that the effect is driven by downward jumps that lead to higher volatility whereas upward jumps lead to lower volatility, but their impact is smaller in magnitude. Moreover, we suppose that the same might hold for common jumps.

The results confirm most of our hypotheses. Firstly, the consensus about the effect of jumps on future volatility has not been reached in the empirical literature, but we do find it positive regardless of whether we investigate the effect of the size of a jump or the presence of a jump. However, the latter effect is stronger and significant across all market sectors.

Secondly, Patton & Sheppard (2015) found that the decrease in volatility following the positive jump is smaller than the increase in volatility after the negative jump for a panel of stocks. We derive the same results and on top of that we perform sectoral analysis. Surprisingly, for four sectors (Consumer Discretionary, Telecommunication Services, Health Care and Consumer Staples) upward jumps lead to lower future volatility and downward jumps to higher future volatility and the effect of upward jumps is larger in magnitude or roughly the same. The impact of upward jumps for three remaining sectors (Information Technology, Financials and Energy) is close to zero, they are however very sensitive to downward jumps that are followed by large increase in volatility.

Finally, estimation of HAR model identifying the impact of sector co-jumps on volatility indicates that the effect is rather positive. Inclusion of the jump indicator variable, however, reveals that the effect might be predominantly driven by idiosyncratic jumps and only for energy and information technology it is truly positive whereas for telecommunication services, health care and consumer staples sectors the impact of sector common jumps on future volatility is negative. We also show that co-jumps identified across the panel of stocks lead to lower volatility and thus confirm the results of Clements & Liao (2013). Moreover, we examine if there is a difference between impact of positive and negative co-jumps. We find out that the effect of upward co-jumps is consistently negative

whereas the effect of downward co-jumps is less clear - for the most significant common jumps it becomes positive.

To sum it up, the effect of jumps on volatility was found positive, but for co-jumps it is rather negative. Taking into account the sign of the jump or common jump helps to predict future volatility, because in general positive jumps lead to lower volatility and negative jumps to higher volatility. Finally, results for individual sectors reveal that estimated effects vary across industries. More specifically, for cyclical sectors volatility is in general more sensitive to negative jumps and less sensitive to positive jumps than for defensive sectors.

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Appendix A

Supplementary results

Table A.1: The Impact of Jumps on Future Volatility (Panel HAR-RVol-J)

$$\sqrt{RV_{j,t+1}^{(d)}} = c_j + \beta_C^{(d)} \sqrt{BPV_{j,t}^{(d)}} + \beta_J^{(d)} \sqrt{J_{j,t}^{(d)}} + \beta^{(w)} \sqrt{RV_{j,t}^{(w)}} + \beta^{(m)} \sqrt{RV_{j,t}^{(m)}} + \omega_{j,t+1}$$

	$\beta_C^{(d)}$	$\beta_J^{(d)}$	$\beta^{(w)}$	$\beta^{(m)}$	R ²
Information Technology	0.339 (14.973)	0.037 (2.436)	0.399 (8.653)	0.204 (8.946)	0.650
Consumer Discretionary	0.369 (12.556)	-0.019 (-0.750)	0.390 (9.393)	0.196 (8.243)	0.691
Financials	0.526 (15.477)	0.098 (2.110)	0.263 (7.912)	0.181 (7.521)	0.797
Telecommunication Services	0.362 (11.409)	-0.021 (-0.566)	0.388 (8.164)	0.209 (8.222)	0.710
Energy	0.395 (12.452)	-0.005 (-0.150)	0.433 (8.904)	0.124 (6.282)	0.723
Health Care	0.341 (11.153)	0.048 (0.875)	0.333 (9.674)	0.257 (9.510)	0.568
Consumer Staples	0.324 (11.005)	-0.032 (-0.539)	0.402 (12.134)	0.224 (9.650)	0.629
Portfolio	0.422 (22.966)	0.035 (1.857)	0.346 (17.455)	0.191 (15.064)	0.729

The table presents estimated parameters of Panel HAR-RVol-J model for the panel of 29 stocks and 7 market sectors. Robust t-statistics based on Newey-West correction (with lag length of 44) are reported in parentheses.

Source: Author's calculations.

Table A.2: HAR-RV Estimation Results

$$\log(RV_{t+1}^{(d)}) = c + \beta^{(d)} \log(RV_t^{(d)}) + \beta^{(w)} \log(RV_t^{(w)}) + \beta^{(m)} \log(RV_t^{(m)}) + \omega_{t+1}$$

Ticker	c		$\beta^{(d)}$		$\beta^{(w)}$		$\beta^{(m)}$		R ²
AAPL	-0.552	(-4.070)	0.439	(16.380)	0.231	(5.549)	0.265	(10.045)	0.631
CSCO	-0.464	(-3.018)	0.373	(12.897)	0.308	(6.934)	0.266	(7.798)	0.668
IBM	-0.545	(-2.915)	0.325	(10.946)	0.451	(11.217)	0.166	(6.069)	0.683
INTC	-0.474	(-3.302)	0.375	(14.777)	0.325	(9.162)	0.246	(9.089)	0.668
MSFT	-0.597	(-3.273)	0.329	(11.953)	0.395	(9.122)	0.210	(6.610)	0.633
ORCL	-0.478	(-3.206)	0.362	(12.838)	0.326	(8.837)	0.258	(8.651)	0.660
QCOM	-0.377	(-3.228)	0.350	(12.304)	0.339	(7.058)	0.267	(7.736)	0.713
AMZN	-0.519	(-3.266)	0.339	(13.065)	0.350	(8.444)	0.247	(8.108)	0.631
DIS	-0.486	(-2.916)	0.389	(11.540)	0.324	(7.153)	0.233	(7.229)	0.683
HD	-0.349	(-2.853)	0.345	(14.125)	0.353	(9.546)	0.263	(8.344)	0.732
MCD	-0.453	(-2.695)	0.328	(12.312)	0.321	(7.417)	0.304	(8.424)	0.672
BAC	-0.220	(-2.825)	0.462	(16.362)	0.289	(7.579)	0.223	(7.032)	0.839
C	-0.187	(-2.594)	0.434	(12.776)	0.321	(8.384)	0.223	(6.941)	0.853
JPM	-0.251	(-3.125)	0.466	(17.364)	0.294	(8.419)	0.211	(7.493)	0.818
WFC	-0.186	(-2.482)	0.430	(13.721)	0.329	(8.089)	0.221	(7.648)	0.858
CMCSA	-0.389	(-2.865)	0.316	(11.234)	0.374	(10.000)	0.266	(8.052)	0.706
T	-0.448	(-2.655)	0.341	(11.527)	0.380	(8.734)	0.231	(7.793)	0.700
VZ	-0.497	(-2.705)	0.349	(11.798)	0.368	(9.969)	0.230	(8.646)	0.677
CVX	-0.481	(-3.567)	0.387	(12.960)	0.426	(12.017)	0.133	(5.742)	0.726
SLB	-0.338	(-3.362)	0.348	(12.735)	0.406	(11.304)	0.206	(6.505)	0.746
XOM	-0.515	(-3.449)	0.425	(14.682)	0.383	(11.062)	0.136	(5.671)	0.720
GE	-0.311	(-2.750)	0.395	(14.102)	0.345	(9.947)	0.225	(8.050)	0.781
JNJ	-0.761	(-3.229)	0.350	(11.331)	0.373	(10.741)	0.200	(6.462)	0.601
MRK	-0.543	(-3.342)	0.326	(11.261)	0.375	(8.500)	0.239	(6.717)	0.643
PFE	-0.573	(-3.317)	0.344	(10.566)	0.352	(7.761)	0.240	(7.302)	0.632
KO	-0.573	(-2.557)	0.363	(12.040)	0.304	(7.450)	0.273	(9.256)	0.636
PEP	-0.551	(-2.748)	0.289	(9.556)	0.408	(10.488)	0.246	(8.618)	0.648
PG	-0.637	(-3.045)	0.334	(10.898)	0.399	(10.067)	0.201	(6.829)	0.637
WMT	-0.536	(-2.958)	0.288	(10.154)	0.405	(9.322)	0.250	(5.991)	0.645

The table presents estimated parameters of HAR-RV model for 29 stocks. Robust t-statistics based on Newey-West correction (with lag length of 44) are reported in parentheses.

Source: Author's calculations.

Table A.3: The Impact of Jumps on Future Volatility (HAR-RV-J)

$$\log(RV_{t+1}^{(d)}) = c + \beta_C^{(d)} \log(BPV_t^{(d)}) + \beta_J^{(d)} \log(1 + J_t^{(d)}) \\ + \beta^{(w)} \log(RV_t^{(w)}) + \beta^{(m)} \log(RV_t^{(m)}) + \omega_{t+1}$$

Ticker	c	$\beta_C^{(d)}$	$\beta_J^{(d)}$	$\beta^{(w)}$	$\beta^{(m)}$	R ²
AAPL	-0.570 (-4.240)	0.420 (15.846)	104.692 (2.234)	0.243 (5.758)	0.269 (9.863)	0.629
CSCO	-0.510 (-3.302)	0.360 (12.371)	365.062 (1.693)	0.318 (7.150)	0.263 (7.709)	0.668
IBM	-0.543 (-2.805)	0.319 (11.054)	-104.173 (-0.737)	0.458 (11.575)	0.164 (5.978)	0.683
INTC	-0.506 (-3.685)	0.367 (14.561)	279.961 (0.848)	0.331 (9.477)	0.242 (9.123)	0.669
MSFT	-0.626 (-3.258)	0.316 (11.537)	191.030 (0.932)	0.407 (8.958)	0.206 (6.310)	0.632
ORCL	-0.469 (-3.119)	0.346 (12.687)	31.443 (0.125)	0.339 (9.151)	0.259 (8.369)	0.659
QCOM	-0.373 (-3.163)	0.353 (12.744)	103.224 (1.952)	0.338 (6.988)	0.264 (7.582)	0.715
AMZN	-0.514 (-3.096)	0.336 (13.032)	26.037 (0.626)	0.354 (8.412)	0.246 (7.864)	0.632
DIS	-0.471 (-2.881)	0.400 (12.043)	32.541 (1.642)	0.317 (7.007)	0.229 (7.121)	0.686
HD	-0.328 (-2.495)	0.349 (14.445)	-41.639 (-0.870)	0.355 (9.560)	0.258 (8.307)	0.733
MCD	-0.469 (-3.014)	0.325 (12.697)	313.426 (1.116)	0.321 (7.784)	0.303 (8.524)	0.674
BAC	-0.211 (-2.860)	0.455 (16.551)	50.141 (0.797)	0.291 (7.696)	0.227 (7.177)	0.840
C	-0.147 (-2.143)	0.431 (14.166)	23.568 (0.963)	0.317 (8.316)	0.232 (7.186)	0.854
JPM	-0.297 (-3.763)	0.443 (17.034)	269.756 (4.773)	0.306 (8.686)	0.215 (7.726)	0.817
WFC	-0.172 (-2.180)	0.432 (14.129)	23.027 (0.338)	0.327 (8.154)	0.219 (7.576)	0.859
CMCSA	-0.379 (-2.570)	0.299 (10.620)	-4.613 (-0.116)	0.389 (9.960)	0.268 (8.032)	0.705
T	-0.459 (-2.687)	0.337 (11.477)	0.839 (0.009)	0.386 (8.782)	0.225 (7.652)	0.701
VZ	-0.507 (-2.749)	0.347 (11.645)	161.661 (1.533)	0.372 (10.283)	0.225 (8.346)	0.678
CVX	-0.484 (-3.417)	0.386 (13.250)	-59.034 (-2.912)	0.431 (12.429)	0.127 (5.424)	0.727
SLB	-0.340 (-3.128)	0.342 (11.953)	47.751 (0.691)	0.412 (10.807)	0.203 (6.426)	0.747
XOM	-0.547 (-3.757)	0.423 (14.621)	319.331 (3.274)	0.381 (10.991)	0.135 (5.654)	0.720
GE	-0.314 (-2.539)	0.392 (13.621)	-62.589 (-0.441)	0.351 (10.355)	0.221 (7.827)	0.781
JNJ	-0.765 (-3.356)	0.372 (12.023)	62.798 (2.534)	0.362 (10.745)	0.186 (5.838)	0.607
MRK	-0.541 (-3.824)	0.327 (11.445)	216.000 (1.867)	0.374 (8.679)	0.237 (6.684)	0.645
PFE	-0.565 (-3.023)	0.345 (9.968)	13.897 (0.169)	0.352 (7.429)	0.238 (7.389)	0.633
KO	-0.530 (-2.195)	0.355 (11.800)	-177.475 (-0.848)	0.322 (7.721)	0.265 (9.332)	0.636
PEP	-0.500 (-2.220)	0.293 (9.555)	-64.869 (-1.873)	0.410 (10.947)	0.244 (8.372)	0.649
PG	-0.593 (-2.560)	0.352 (11.345)	-197.344 (-2.323)	0.390 (10.070)	0.194 (6.754)	0.641
WMT	-0.595 (-3.911)	0.296 (11.388)	631.883 (2.086)	0.388 (9.195)	0.251 (6.070)	0.647

The table presents estimated parameters of HAR-RV-J model for 29 stocks. Robust t-statistics based on Newey-West correction (with lag length of 44) are reported in parentheses.

Source: Author's calculations.

Table A.4: The Impact of Jumps on Future Volatility (HAR-RV-Jump)

$$\log(RV_{t+1}^{(d)}) = c + \beta_C^{(d)} \log(BPV_t^{(d)}) + \beta_{Jump}^{(d)} Jump_t^{(d)} + \beta^{(w)} \log(RV_t^{(w)}) + \beta^{(m)} \log(RV_t^{(m)}) + \omega_{t+1}$$

Ticker	c	$\beta_C^{(d)}$	$\beta_{Jump}^{(d)}$	$\beta^{(w)}$	$\beta^{(m)}$	R ²
AAPL	-0.554 (-4.094)	0.442 (16.327)	0.162 (4.177)	0.229 (5.487)	0.265 (10.011)	0.631
CSCO	-0.470 (-3.068)	0.378 (12.906)	0.099 (3.480)	0.304 (6.706)	0.264 (7.790)	0.669
IBM	-0.546 (-2.935)	0.331 (11.071)	0.064 (2.054)	0.444 (11.017)	0.165 (6.054)	0.684
INTC	-0.484 (-3.378)	0.383 (15.067)	0.090 (3.313)	0.318 (9.061)	0.243 (9.145)	0.670
MSFT	-0.598 (-3.285)	0.335 (12.340)	0.097 (3.505)	0.390 (8.975)	0.208 (6.485)	0.633
ORCL	-0.474 (-3.206)	0.376 (13.690)	0.140 (5.066)	0.311 (8.574)	0.258 (8.720)	0.662
QCOM	-0.361 (-3.092)	0.362 (12.977)	0.051 (1.686)	0.332 (6.911)	0.264 (7.634)	0.715
AMZN	-0.508 (-3.196)	0.348 (13.463)	0.071 (2.159)	0.343 (8.244)	0.247 (8.010)	0.632
DIS	-0.469 (-2.872)	0.410 (12.191)	0.052 (1.834)	0.308 (6.773)	0.228 (7.163)	0.686
HD	-0.338 (-2.755)	0.360 (15.181)	0.061 (2.049)	0.341 (9.258)	0.260 (8.408)	0.734
MCD	-0.449 (-2.686)	0.338 (12.323)	0.064 (2.024)	0.313 (7.229)	0.301 (8.406)	0.674
BAC	-0.203 (-2.629)	0.471 (16.981)	0.097 (2.745)	0.279 (7.369)	0.224 (7.124)	0.840
C	-0.147 (-2.141)	0.445 (13.670)	0.075 (2.255)	0.310 (7.971)	0.227 (6.888)	0.854
JPM	-0.251 (-3.152)	0.470 (17.444)	0.166 (5.181)	0.289 (8.124)	0.211 (7.497)	0.818
WFC	-0.174 (-2.354)	0.448 (14.643)	0.089 (3.689)	0.311 (7.844)	0.219 (7.663)	0.859
CMCSA	-0.387 (-2.827)	0.318 (11.071)	0.102 (3.696)	0.371 (9.795)	0.266 (8.058)	0.706
T	-0.456 (-2.699)	0.347 (11.485)	0.047 (1.824)	0.376 (8.522)	0.227 (7.753)	0.701
VZ	-0.492 (-2.724)	0.362 (12.035)	0.079 (3.131)	0.357 (9.643)	0.227 (8.507)	0.679
CVX	-0.479 (-3.575)	0.403 (13.924)	0.124 (4.124)	0.412 (11.947)	0.131 (5.703)	0.728
SLB	-0.334 (-3.262)	0.352 (12.295)	0.053 (1.907)	0.404 (10.948)	0.204 (6.456)	0.747
XOM	-0.515 (-3.417)	0.433 (15.346)	0.100 (3.166)	0.374 (10.796)	0.135 (5.685)	0.721
GE	-0.314 (-2.741)	0.403 (14.261)	0.070 (2.280)	0.339 (9.825)	0.223 (7.966)	0.782
JNJ	-0.753 (-3.315)	0.385 (11.794)	0.053 (1.780)	0.349 (10.208)	0.187 (6.042)	0.607
MRK	-0.519 (-3.333)	0.345 (11.257)	0.084 (2.781)	0.361 (8.126)	0.235 (6.808)	0.645
PFE	-0.565 (-3.181)	0.356 (10.519)	0.060 (2.380)	0.342 (7.429)	0.239 (7.411)	0.634
KO	-0.566 (-2.488)	0.378 (12.195)	0.106 (3.470)	0.294 (7.021)	0.269 (9.368)	0.638
PEP	-0.525 (-2.549)	0.306 (10.085)	0.065 (2.451)	0.394 (10.491)	0.244 (8.542)	0.650
PG	-0.624 (-2.982)	0.351 (11.324)	0.012 (0.414)	0.385 (9.672)	0.197 (6.737)	0.640
WMT	-0.540 (-2.972)	0.306 (11.150)	0.056 (1.892)	0.387 (9.004)	0.249 (6.092)	0.647

The table presents estimated parameters of HAR-RV-Jump model for 29 stocks. Robust t-statistics based on Newey-West correction (with lag length of 44) are reported in parentheses.

Source: Author's calculations.

Table A.5: The Impact of Positive and Negative Semivariances on Future Volatility (HAR-RS)

$$\log \left(RV_{j,t+1}^{(d)} \right) = c_j + \beta^{+(d)} \log \left(RS_{j,t}^{+(d)} \right) + \beta^{-(d)} \log \left(RS_{j,t}^{-(d)} \right) + \beta^{(w)} \log \left(RV_{j,t}^{(w)} \right) + \beta^{(m)} \log \left(RV_{j,t}^{(m)} \right) + \omega_{j,t+1}$$

Ticker	c	$\beta^{+(d)}$	$\beta^{-(d)}$	$\beta^{(w)}$	$\beta^{(m)}$	R ²
AAPL	-0.271 (-2.015)	0.142 (5.240)	0.272 (10.055)	0.252 (5.996)	0.267 (9.856)	0.629
CSCO	-0.195 (-1.273)	0.150 (6.389)	0.220 (8.843)	0.309 (6.920)	0.268 (7.890)	0.668
IBM	-0.331 (-1.854)	0.075 (3.509)	0.239 (10.257)	0.463 (11.399)	0.163 (5.928)	0.685
INTC	-0.202 (-1.389)	0.105 (5.256)	0.274 (10.994)	0.327 (9.200)	0.240 (8.787)	0.672
MSFT	-0.362 (-1.980)	0.140 (6.448)	0.188 (7.570)	0.399 (9.092)	0.207 (6.452)	0.632
ORCL	-0.216 (-1.458)	0.133 (6.587)	0.230 (8.368)	0.325 (8.860)	0.257 (8.592)	0.661
QCOM	-0.129 (-1.166)	0.074 (3.158)	0.269 (10.603)	0.346 (7.323)	0.267 (7.772)	0.717
AMZN	-0.263 (-1.671)	0.081 (4.201)	0.262 (11.334)	0.341 (8.240)	0.252 (8.404)	0.635
DIS	-0.195 (-1.150)	0.118 (4.363)	0.270 (11.959)	0.333 (7.104)	0.226 (6.998)	0.686
HD	-0.102 (-0.810)	0.127 (6.168)	0.217 (8.950)	0.357 (9.590)	0.259 (8.237)	0.732
MCD	-0.214 (-1.291)	0.107 (4.852)	0.224 (9.926)	0.318 (7.344)	0.302 (8.462)	0.674
BAC	0.111 (1.346)	0.155 (6.423)	0.298 (10.394)	0.300 (7.750)	0.221 (6.936)	0.838
C	0.116 (1.478)	0.144 (5.266)	0.294 (8.874)	0.316 (8.194)	0.222 (6.916)	0.853
JPM	0.087 (1.025)	0.154 (6.834)	0.308 (11.093)	0.298 (8.577)	0.212 (7.504)	0.818
WFC	0.136 (1.733)	0.147 (6.363)	0.288 (10.660)	0.329 (8.011)	0.216 (7.414)	0.859
CMCSA	-0.154 (-1.095)	0.098 (5.494)	0.216 (7.807)	0.379 (9.992)	0.262 (8.017)	0.707
T	-0.204 (-1.175)	0.149 (6.748)	0.193 (7.264)	0.381 (8.841)	0.229 (7.770)	0.701
VZ	-0.254 (-1.352)	0.120 (5.155)	0.229 (9.800)	0.368 (9.865)	0.229 (8.741)	0.678
CVX	-0.221 (-1.557)	0.120 (5.476)	0.255 (9.449)	0.436 (12.405)	0.134 (5.776)	0.728
SLB	-0.105 (-0.991)	0.110 (5.171)	0.228 (9.161)	0.416 (11.295)	0.205 (6.476)	0.746
XOM	-0.218 (-1.397)	0.159 (7.805)	0.258 (9.837)	0.392 (11.696)	0.135 (5.619)	0.720
GE	-0.039 (-0.322)	0.123 (6.011)	0.272 (10.200)	0.348 (9.895)	0.221 (7.884)	0.781
JNJ	-0.499 (-2.121)	0.089 (4.115)	0.277 (10.231)	0.366 (10.331)	0.189 (6.217)	0.606
MRK	-0.291 (-1.772)	0.125 (5.839)	0.208 (7.853)	0.372 (8.360)	0.237 (6.701)	0.644
PFE	-0.320 (-1.799)	0.151 (6.015)	0.196 (8.039)	0.351 (7.663)	0.239 (7.288)	0.632
KO	-0.304 (-1.335)	0.163 (7.344)	0.204 (7.154)	0.302 (7.535)	0.272 (9.228)	0.637
PEP	-0.345 (-1.706)	0.103 (4.252)	0.191 (8.864)	0.405 (10.652)	0.243 (8.508)	0.649
PG	-0.384 (-1.828)	0.097 (4.591)	0.240 (8.879)	0.396 (9.993)	0.202 (6.962)	0.639
WMT	-0.328 (-1.766)	0.102 (4.013)	0.195 (7.995)	0.399 (9.292)	0.247 (5.985)	0.646

The table presents estimated parameters of HAR-RS model for 29 stocks. Robust t-statistics based on Newey-West correction (with lag length of 44) are reported in parentheses.

Source: Author's calculations.

Table A.6: The Impact of Signed Jump Variation on Future Volatility (HAR-RV-J-Sign)

$$\log(RV_{t+1}^{(d)}) = c + \beta_C^{(d)} \log(BPV_t^{(d)}) + \beta_{\Delta J}^{(d)} \% \Delta J_t^{2(d)} + \beta^{(w)} \log(RV_t^{(w)}) + \beta^{(m)} \log(RV_t^{(m)}) + \omega_{t+1}$$

Ticker	c	$\beta_C^{(d)}$	$\beta_{\Delta J}^{(d)}$	$\beta^{(w)}$	$\beta^{(m)}$	R ²
AAPL	-0.572 (-4.149)	0.406 (15.561)	-0.191 (-4.190)	0.257 (6.147)	0.269 (9.896)	0.632
CSCO	-0.484 (-3.142)	0.355 (12.127)	-0.082 (-2.080)	0.324 (7.403)	0.264 (7.811)	0.669
IBM	-0.560 (-3.134)	0.307 (10.869)	-0.184 (-5.258)	0.466 (12.060)	0.165 (6.039)	0.686
INTC	-0.494 (-3.503)	0.363 (14.663)	-0.166 (-4.332)	0.340 (9.664)	0.239 (8.794)	0.671
MSFT	-0.621 (-3.408)	0.313 (11.494)	-0.062 (-1.561)	0.411 (9.076)	0.205 (6.263)	0.632
ORCL	-0.470 (-3.214)	0.341 (12.804)	-0.098 (-2.425)	0.343 (9.378)	0.260 (8.386)	0.660
QCOM	-0.364 (-3.195)	0.340 (12.433)	-0.195 (-4.812)	0.352 (7.432)	0.264 (7.594)	0.718
AMZN	-0.506 (-3.215)	0.336 (13.223)	-0.204 (-5.662)	0.353 (8.671)	0.248 (8.016)	0.636
DIS	-0.476 (-2.908)	0.390 (11.685)	-0.147 (-3.873)	0.330 (7.136)	0.226 (6.979)	0.687
HD	-0.336 (-2.704)	0.344 (14.409)	-0.095 (-2.444)	0.359 (9.772)	0.258 (8.347)	0.734
MCD	-0.449 (-2.665)	0.324 (12.425)	-0.116 (-3.185)	0.327 (7.497)	0.300 (8.383)	0.675
BAC	-0.220 (-2.979)	0.452 (16.619)	-0.183 (-4.264)	0.295 (7.838)	0.224 (7.060)	0.841
C	-0.161 (-2.320)	0.430 (14.046)	-0.156 (-3.211)	0.322 (8.161)	0.228 (6.809)	0.855
JPM	-0.265 (-3.212)	0.438 (16.874)	-0.183 (-4.424)	0.318 (8.697)	0.212 (7.421)	0.818
WFC	-0.175 (-2.358)	0.431 (14.225)	-0.156 (-4.147)	0.331 (8.401)	0.216 (7.408)	0.860
CMCSA	-0.387 (-2.801)	0.296 (10.913)	-0.132 (-3.303)	0.393 (10.421)	0.266 (8.065)	0.706
T	-0.465 (-2.760)	0.335 (11.576)	-0.054 (-1.411)	0.387 (9.002)	0.226 (7.683)	0.701
VZ	-0.509 (-2.792)	0.343 (11.745)	-0.124 (-3.383)	0.376 (10.484)	0.225 (8.470)	0.679
CVX	-0.502 (-3.628)	0.374 (13.404)	-0.166 (-3.995)	0.439 (13.064)	0.130 (5.572)	0.729
SLB	-0.344 (-3.353)	0.336 (11.647)	-0.132 (-3.582)	0.420 (10.995)	0.202 (6.347)	0.748
XOM	-0.526 (-3.458)	0.411 (14.683)	-0.119 (-3.079)	0.395 (11.543)	0.135 (5.674)	0.721
GE	-0.343 (-3.016)	0.393 (13.796)	-0.173 (-4.481)	0.347 (10.412)	0.221 (7.913)	0.783
JNJ	-0.778 (-3.461)	0.375 (11.997)	-0.187 (-4.939)	0.359 (10.771)	0.185 (5.944)	0.610
MRK	-0.504 (-3.199)	0.330 (11.321)	-0.086 (-2.269)	0.377 (8.608)	0.235 (6.894)	0.645
PFE	-0.565 (-3.117)	0.344 (9.870)	-0.048 (-1.374)	0.354 (7.471)	0.238 (7.405)	0.634
KO	-0.557 (-2.436)	0.353 (12.010)	-0.058 (-1.366)	0.320 (7.997)	0.267 (9.413)	0.637
PEP	-0.527 (-2.507)	0.290 (9.586)	-0.093 (-2.666)	0.410 (11.241)	0.244 (8.508)	0.650
PG	-0.619 (-2.989)	0.347 (11.602)	-0.146 (-3.622)	0.388 (10.181)	0.199 (6.833)	0.642
WMT	-0.541 (-2.935)	0.297 (11.506)	-0.094 (-2.372)	0.394 (9.351)	0.251 (6.121)	0.647

The table presents estimated parameters of HAR-RV-J-Sign model for 29 stocks. Robust t-statistics based on Newey-West correction (with lag length of 44) are reported in parentheses.

Source: Author's calculations.

Table A.7: The Impact of Positive and Negative Signed Jump Variation on Future Volatility (HAR-RV-J-Sign)

$$\log(RV_{t+1}^{(d)}) = c + \beta_C^{(d)} \log(BPV_t^{(d)}) + \beta_{\Delta J_+}^{(d)} \% \Delta J_t^{2+(d)} + \beta_{\Delta J_-}^{(d)} \% \Delta J_t^{2-(d)} + \beta^{(w)} \log(RV_t^{(w)}) + \beta^{(m)} \log(RV_t^{(m)}) + \omega_{t+1}$$

Ticker	c	$\beta_C^{(d)}$	$\beta_{\Delta J_+}^{(d)}$	$\beta_{\Delta J_-}^{(d)}$	$\beta^{(w)}$	$\beta^{(m)}$	R ²
AAPL	-0.643 (-4.648)	0.397 (14.983)	0.342 (3.212)	-0.517 (-8.506)	0.271 (6.391)	0.267 (10.106)	0.637
CSCO	-0.510 (-3.233)	0.354 (12.110)	0.061 (0.686)	-0.179 (-2.609)	0.325 (7.442)	0.264 (7.798)	0.669
IBM	-0.560 (-3.175)	0.307 (10.866)	-0.180 (-2.233)	-0.186 (-2.998)	0.466 (12.098)	0.165 (6.039)	0.686
INTC	-0.486 (-3.500)	0.363 (14.633)	-0.221 (-2.435)	-0.132 (-2.020)	0.341 (9.695)	0.238 (8.744)	0.671
MSFT	-0.645 (-3.562)	0.312 (11.439)	0.053 (0.599)	-0.136 (-1.815)	0.410 (9.085)	0.207 (6.298)	0.633
ORCL	-0.482 (-3.283)	0.340 (12.753)	-0.025 (-0.271)	-0.146 (-2.038)	0.344 (9.421)	0.261 (8.389)	0.660
QCOM	-0.365 (-3.123)	0.340 (12.369)	-0.192 (-1.992)	-0.197 (-2.456)	0.352 (7.428)	0.264 (7.598)	0.718
AMZN	-0.515 (-3.274)	0.335 (13.233)	-0.147 (-1.862)	-0.247 (-3.677)	0.354 (8.716)	0.247 (7.965)	0.636
DIS	-0.465 (-2.779)	0.390 (11.684)	-0.197 (-2.284)	-0.118 (-1.716)	0.331 (7.164)	0.224 (6.951)	0.688
HD	-0.337 (-2.690)	0.344 (14.399)	-0.085 (-1.039)	-0.101 (-1.534)	0.359 (9.773)	0.258 (8.344)	0.734
MCD	-0.439 (-2.599)	0.325 (12.355)	-0.191 (-2.192)	-0.072 (-1.172)	0.326 (7.433)	0.300 (8.383)	0.675
BAC	-0.332 (-4.414)	0.445 (16.392)	0.271 (2.891)	-0.444 (-7.000)	0.298 (7.952)	0.225 (7.106)	0.843
C	-0.170 (-2.470)	0.429 (14.084)	-0.108 (-0.969)	-0.183 (-2.367)	0.322 (8.168)	0.228 (6.814)	0.855
JPM	-0.295 (-3.535)	0.436 (16.885)	-0.050 (-0.559)	-0.272 (-4.089)	0.319 (8.818)	0.211 (7.430)	0.818
WFC	-0.186 (-2.480)	0.430 (14.276)	-0.102 (-1.238)	-0.185 (-3.198)	0.331 (8.418)	0.217 (7.425)	0.860
CMCSA	-0.398 (-2.902)	0.295 (10.861)	-0.076 (-0.909)	-0.169 (-2.245)	0.393 (10.479)	0.266 (8.058)	0.706
T	-0.459 (-2.718)	0.335 (11.590)	-0.095 (-1.062)	-0.030 (-0.512)	0.388 (8.948)	0.225 (7.534)	0.701
VZ	-0.500 (-2.749)	0.344 (11.761)	-0.190 (-2.136)	-0.088 (-1.527)	0.375 (10.432)	0.224 (8.453)	0.680
CVX	-0.516 (-3.819)	0.375 (13.437)	-0.042 (-0.434)	-0.244 (-3.131)	0.440 (13.147)	0.129 (5.578)	0.729
SLB	-0.359 (-3.417)	0.337 (11.753)	-0.017 (-0.203)	-0.202 (-3.146)	0.420 (11.005)	0.201 (6.326)	0.748
XOM	-0.557 (-3.714)	0.410 (14.586)	0.068 (0.778)	-0.238 (-3.798)	0.396 (11.643)	0.135 (5.666)	0.722
GE	-0.389 (-3.508)	0.387 (13.474)	0.090 (1.039)	-0.342 (-4.790)	0.348 (10.398)	0.225 (8.035)	0.784
JNJ	-0.770 (-3.489)	0.375 (12.001)	-0.230 (-2.740)	-0.160 (-2.680)	0.360 (10.854)	0.184 (5.871)	0.610
MRK	-0.505 (-3.204)	0.330 (11.345)	-0.082 (-0.984)	-0.089 (-1.471)	0.377 (8.602)	0.235 (6.887)	0.645
PFE	-0.572 (-3.124)	0.343 (9.808)	-0.011 (-0.114)	-0.071 (-1.168)	0.354 (7.477)	0.239 (7.379)	0.634
KO	-0.564 (-2.453)	0.352 (12.008)	-0.011 (-0.118)	-0.086 (-1.260)	0.320 (8.016)	0.268 (9.362)	0.637
PEP	-0.507 (-2.377)	0.290 (9.598)	-0.207 (-2.284)	-0.020 (-0.305)	0.412 (11.057)	0.242 (8.324)	0.650
PG	-0.613 (-2.936)	0.347 (11.626)	-0.178 (-2.064)	-0.126 (-1.638)	0.388 (10.187)	0.199 (6.854)	0.642
WMT	-0.540 (-2.948)	0.297 (11.507)	-0.111 (-1.124)	-0.084 (-1.380)	0.394 (9.352)	0.250 (6.076)	0.647

The table presents estimated parameters of HAR-RV-J-Sign model with decomposed signed jump variation for 29 stocks. Robust t-statistics based on Newey-West correction (with lag length of 44) are reported in parentheses.

Source: Author's calculations.