We present two possible thermodynamical approaches towards a derivation of a model, proposed by Korteweg at the beginning of the 20th century, that is suitable to describe phase transitions liquid-vapor with non-sharp interfaces. The first approach (Dunn, Serrin (1985)) is based on classical rational continuum thermodynamics. The second approach (Heida, Málek (2010)) stems from the principles of classical nonequilibrium continuum thermodynamics. We compare both approaches in favor of the second one. The considered constitutive equation for the Cauchy stress is nonlinear. Nonlinearity and higher order derivatives of the density makes the analysis of relevant problems for the Navier-Stokes-Korteweg (NSK) fluid more difficult in comparison to problems concerning Navier-Stokes equations. Special attention is devoted to the appropriate choice of the boundary conditions. We also investigate the influence of compressibility on the stability of bubbles by comparing numerical simulations for compressible NSK fluid and its incompressible variant. Instabilities observed for a compressible NSK fluid are due to the pressure that has a different meaning for incompressible fluid.