Charles University in Prague

Faculty of Social Sciences
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MASTER'S THESIS

Robust Investment Portfolios

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Declaration of Authorship

The author hereby declares that he compiled this thesis independently, using only the listed resources and literature.

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Prague, July 31, 2014

Signature

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Abstract

This master's thesis pursues the construction of stable, robust and growth portfolios in active portfolio management. These portfolios provide limited downside risks, short-time drawdowns and substantial growth prospects. We argue that the construction of such portfolios is based on security selection as well as on portfolio theory (the Mean-Variance Model, MVM). The equity based portfolios were constructed and tested on real market data from the 1995-2014 period. The tested portfolios outperformed the S&P 500 out of and within the risk-reward ratio domain.

Robust portfolios build on the MVM but they are less sensitive to errors of parameters estimation. We experimented with several robust approaches and the results confirmed that the robust portfolios were less sensitive to outliers, less volatile and more stable (robust).

The bottom-up process of security selection seems time consuming and labor intensive. Therefore we proposed an alternative approach to select stocks with so-called "cluster analysis". Generally, the cluster analysis classifies similar objects (companies) into similar groups. Technical and fundamental parameters of companies provided needed classification parameters. The classification was run over companies from the German DAX index. The achieved results were surprisingly supportive and valuable.

We argue that the robustness of a portfolio is primarily driven by security selection, therefore we describe what matters in our opinion. The robustness of a portfolio can be measured by many measures. The personally selected measures are size, frequency of drawdowns, drawdown period and risk-return measures (such as Sharpe ratio). The selected measures were evaluated on the historical data. The experimental verification supported our assumptions that robust portfolios provide lower drawdowns and high risk-return measures.

JEL Classification	C1, C6, C8, G0, G1
Keywords	Portfolio construction, equity, MVM, cluster analysis, robustness, optimization, reward-to-variability ratios

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Abstrakt

Diplomová práce se zabývá konstrukcí stabilního, robustního a růstového portfolia v rámci aktivního portfolio managementu. Takováto portfolia poskytují omezení rizika poklesu hodnoty aktiv, krátkodobý propad hodnoty portfolia a přiměřené růstové perspektivy. Tvrdíme, že konstrukce takovýchto portfolií je závislá na výběru cenných papírů a teorii portfolia (MV model, MVM). Námi vytvořená a testovaná akciová portfolia byla založena na reálných tržních datech za období 1995-2014. Portfolia prokázala nadvýnos nad tržním indexem S&P 500 mimo doménu i v doméně rizikově upraveného zisku.

Robustní portfolia stavějí na MVM, ale jsou méně citlivá k chybám odhadu parametrů. Experimentovali jsme s několika robustními přístupy a výsledky potvrdily, že robustní portfolia jsou méně citlivá k vychýleným hodnotám, méně volatilní a více stabilní (robustní).

Výběr vhodných cenných papírů je časově a dovednostně náročný. Z tohoto důvodu jsme použili alternativu ke klasifikaci akcií využitím klasifikačního algoritmu tzv. "shlukové analýzy". Obecně shluková analýza nám přiřadí do podobných cílových skupin (clusterů) parametrově podobné objekty (společnosti). Technické a fundamentální parametry společností poskytly potřebné klasifikační parametry. Klasifikace probíhala nad akciovými společnostmi obsaženými v německém indexu DAX. Dosažené výsledky byly překvapivě hodnotné.

Tvrdíme, že robustnost portfolia je primárně ovlivněna výběrem cenných papírů, proto uvádíme důležité výběrové faktory, na kterých podle našeho názoru záleží. Robustnost portfolia je možné měřit mnoha mírami. Námi vybrané faktory jsou rozsah poklesů, frekvence propadů hodnoty, doba propadu a tzv. rizikové míry (např. Sharpeho poměr) na historických datech. Experimentální ověření podpořilo naše předpoklady, že robustní portfolia poskytují nižší propady a vyšší rizikově výnosové míry.

Klasifikace JEL	C1, C6, C8, G0, G1
Klíčová slova	Konstrukce portfolia, akcie, MVM, shluková analýza, robustnost, optimalizace, výnosově-rizikové míry

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Acronyms

ACF/PACF Auto Correlation Function/Parcial ACF

CA Cluster Analysis

CAL Capital Allocation Line

CAPM Capital Asset Pricing Model

CDF/PDF Cumulative/Probability Distribution Function

VaR/CVaR Value at Risk/Conditional VaR

DAX German 'Large Cap' equity index

DD Drawdown

DY Dividend Yield

ETF Exchange Traded Fund

EWMA Exponentially Weighed Moving Average

FHS Filtered Historical Simulation

(G)ARCH (Generalized) Autoregressive Conditional Heteroskedasticity

IR/M2 Information/Modigliani-Modigliani Ratio

MACD Moving Average Convergence/Divergence

MaxDD/MaxDDG/EMaxDD Maximum DD/MaxDD Geometric/Expected MaxDD

MCap Market Capitalization of a company

MV/MVM/MVP Mean-Variance/Model/Problem

MVO/MVEF Mean-Variance Optimization/Efficient Frontier

MPT Modern Portfolio Theory

PCA Principal Component Analysis

PE/PB Price to Earnings/Price to Book

RMV/RMVM Robust Mean-Variance/Model

ROE Return on Equity

SP500/SPX/SPY Symbol/Ticker/ETF of the Standard & Poor's 500

SR/SoR/TR Sharpe/Sortino/Treynor Ratio

TE Tracking Error

Vol Volatility

Notations

t	generic time
T	length of time series
n	number of securities
β	measure of the risk arising from general market movements
S	dimension of market parameters
$f : x \rightarrow y$	a function f mapping $x \in X$ into $y \in Y$
$\min\{x\}$	minimization function (inf)
$\max\{x\}$	maximization function (sup)
$a \approx b$	a is approximately equal b
$a \succeq b$	a is preferred to b
X	generic random variable
x	realized value of the X
Q_X	quantile of the X
$Cor\{X, Y\}$	correlation of the X and Y
$Cov\{X, Y\}$	covariance of the X and Y
$\mathbb{E}\{X\}$	expected value of the X
$Var\{X\}$	variance of the univariate the X
$Std\{X\}$	standard deviation of the X
$\rho_{xy}\{X, Y\}$	correlation coefficient of the X and Y
$\mu, \hat{\mu}$	expected value, estimation of μ
$\mathbf{\Sigma}, \hat{\mathbf{\Sigma}}$	covariance matrix, estimation of the matrix $\mathbf{\Sigma}$
$\theta, \hat{\theta}$	parameter, estimation of the parameter θ
$N(\mu, \sigma^2)$	univariate normal distribution with expected value μ and variance σ^2
$N(\mu, \mathbf{\Sigma})$	multivariate normal distribution with expected value μ and variance $\mathbf{\Sigma}$
$\mathbf{A}, \mathbf{A}^T$	matrix, transpose of the matrix $\mathbf{A}$
$\mathbf{Cor}$	correlation matrix
$\mathbf{\Sigma}$	covariance matrix
$\mathbf{1}_n, \mathbf{1}_n^T$	n-dimensional vector of ones, transpose of the vector $\mathbf{1}_n$
$\mathbf{x}, \mathbf{x}^T$	row vector, transpose of the vector $\mathbf{x}$
$\Re^N$	Euclidean N-dimensional vector space

Master's Thesis Proposal

Author	Ing. Zdeněk Konfršt, Ph.D.
Supervisor	prof. Ing. Oldřich Dědek, CSc.
Proposed topic	Robust Investment Portfolios

Topic characteristics Institutional investors such as investment banks, mutual funds, pension funds and hedge funds as well as retail investors construct various investment portfolios. The investment portfolios are constructed from base investable assets such as short-term securities, bonds, equities, property, currencies, commodities, and other alternative and structured instruments [Cip00, Ste99]. In portfolio management, there is a huge demand for robust long-term investment portfolios. These portfolios provide a significant downside risk, low volatility and also substantial asset growth.

Hypotheses At this early stage of my research, there are several questions, which could be potentially answered in the thesis. Firstly, I would like to show that robust portfolios are constructed from equities. Secondly, I confirm that robustness of the portfolios has been traceable during the periods of stock market downturns. Thirdly, I need to verify that the portfolios offer high reward-to-variability ratios.

Methodology To analyze the robust portfolios, we start with small size equity portfolios. The tracking index can be the US large cap index (the S&P 500) that has been selected for tracking purposes.

The tested portfolios were of three types: (i) long-only equity, (ii) long-short equity and (iii) long-short equity with leverage (due to the Kelly criterion [Tho97, Wil07]). The long equity side is constructed from subsets of carefully selected equities. The short side is achieved via short-selling equities or market indexes.

There are many derivatives available for hedging purposes such as options, warrants, futures and leveraged ETF (Exchange Traded Funds). We assume that we make do without them during our experiments.

The tested portfolios can be loaded from zero up to 100% leverage (200% total exposure, but still underbetting the Kelly criterion [Tho97]).

Outline

1. Introduction
2. Theoretical Background
3. Related Work
4. The Model
5. Empirical Verification
6. Conclusion

The thesis is structured as follows: Chapter 1 provides an introduction. Chapter 2 gives some hints and information regarding background. Chapter 3 covers the related work on robust portfolios. Chapter 4 gives an overview of our experimental model. Chapter 5 presents an empirical verification and experiments. Chapter 6 is related to discussion of results. Chapter 7 summarizes our findings and concludes the thesis.

Core bibliography

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Author | Supervisor

Chapter 1

Introduction

The investment portfolios are constructed from elementary investable assets [BD11] such as money-market securities, bonds, stocks, currencies, commodities, structured and alternative instruments [Cip00, Ste99]. To mitigate or reduce portfolio risks, derivative instruments [Hul08] are employed for hedging purposes and mitigation of portfolio volatility.[1] Since two relatively recent disaster events in capital markets, the IT bubble (in 2000) and the mortgage bubble (in 2008), were both followed by deep recessions, we have seen an increasing interest and demand for growth and robust long-term investment portfolios [EGBG07, CWS07]. These portfolios are constructed on unknown future characteristics of investment or speculative assets and their derivatives but they can be forecasted or estimated. These problems fit perfectly into the robust optimization domain that solves an optimization problem with uncertain parameters to achieve good objective function values for the realization of these parameters in given uncertainty set. From a more practical view, we see robust portfolios as the portfolios that ensure a stable growth of the invested principal with a low volatility and a strict downside protection.

The chapter is structured as follows: **Section 1.1** explains some general terms from the capital markets. **Section 1.2** describes research and development in the portfolio optimization. **Section 1.3** explains the classical mean-variance model. **Section 1.4** is devoted to the robust portfolio optimization. **Section 1.5** summarizes the chapter. **Section 1.6** is about the motivation and goals. Outline of the thesis is in **Section 1.7**.

1.1 General Terms

We clarify several terms of the capital markets, portfolio theory and management to use them later in the thesis. We assume that they are quite basic but they deserve your attention.

[1] Volatility is a measure for variation of price of a financial instrument over time.

Investing [Cip00] is a process of deploying capital in order to get appropriate return on the capital as well as return of the capital with a high probability. On the other hand, *betting* (speculating) is a process of deploying capital with quite uncertain outcomes. Both approaches are pooling securities to form portfolios to be managed.

Portfolios [Cip00] are pools of securities that are managed by portfolio managers. The portfolio managers are responsible for insightful, intelligent and low-risk deployment of capital and managing the capital pools. These pools are made of money-market securities, bonds, equities, mutual funds, derivatives and alternative assets.

Bond security (also bond) [Cip00] is a financial instrument of indebtedness of the bond issuer to the bond holders. The issuer owes a debt and is obliged to pay the holders interest (the coupon) and to repay the principal at the maturity date. Interest is usually payable at fixed intervals (semiannual, annual). Bonds are usually used by companies, municipalities, states and governments to finance a variety of projects and activities. Bonds are commonly referred to as fixed-income securities.

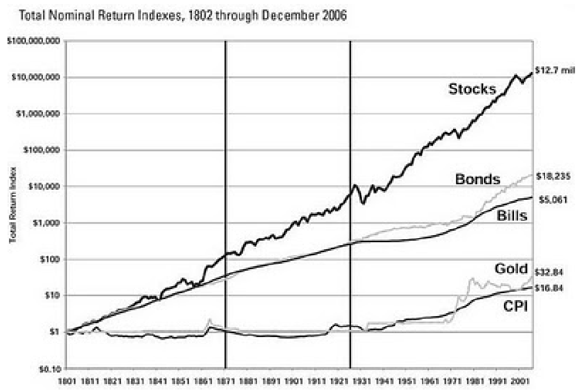

Figure 1.1: The superiority of equities. The return superiority of equities over other asset classes is described during the period between 1802 and 2006. Source: seekingalpha.com.

Equity security (also common stock, share, equity) [Cip00] is an asset class that enables an investor to be a (minority) owner of a company. Equities tend to be more risky but more profitable than bonds (see in Figure 1.1). Equities are highly correlated with an economic cycle, capital markets and negatively correlated to bonds.

Equity/bond mix is a mix of equities and bonds in an investment portfolio recommended by investment advisors. The risky part of the portfolio is formed of equities and the conservative part is of bonds. The more equities in the portfolio, the more the overall portfolio is risky. Generally, the 60/40 portfolio[2] (the ratio of

[2]Throughout the year 2013 we ran the experimental study that tested the performance of the equity/bond portfolio mix on the period of 10 and more years. The portfolio mix was done from

equities/bonds) is recommended to the general public. The tested 60/40 portfolio performance against the S&P 500 is depicted in Figure 1.2.

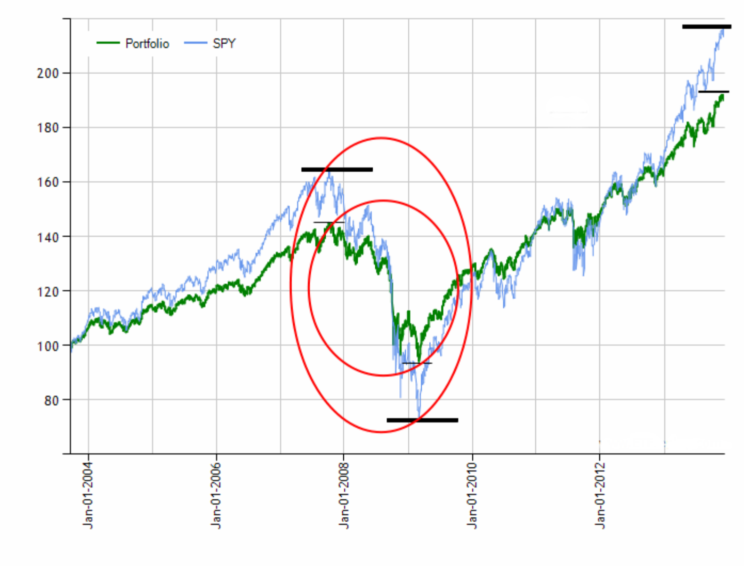

Figure 1.2: The 60/40 portfolio. The 60/40 portfolio performance against the S&P 500 (SPY) is depicted. The portfolio experienced 91.6% total return (vs. 115.5% of SPY), 6.6% compound annual growth rate (7.8%), 11.7% volatility (20.2%) and -35.08% max drawdown (-55.20%) on the period 22nd Sep 2003 - 10th Dec 2013. The equities were represented by the S&P 500 (SPY) and the bonds by iShares Core Total US Bond (4-5yr) (AGG). The strategy was 'buy and hold' with reinvestments of dividends and no rebalancing.

A *hedge fund* [Cip00] is a pooled investment vehicle administered by a professional management firm, and often structured as a limited partnership, limited liability company, or similar vehicle. Hedge funds invest in a diverse range of markets, use a wide variety of investment styles and financial instruments (including derivatives). The name "hedge fund" refers to the hedging techniques traditionally used by hedge funds, but hedge funds today do not necessarily hedge.

We have reviewed the terms such as investing vs. speculation, portfolio, bond,

0/100 to 100/0 ratios. The equities were represented by S&P 500 (SPY) and the bonds by iShares Core Total US Bond (4-5yr) (AGG). The 60/40 portfolio overweighted equities performed well on risk-adjusted basis against the S&P 500. We argued that a skilled portfolio manager would be able to construct an equity-cash portfolio with reward-to-variability ratios not significantly worse than the 60/40 portfolio. The bond part could be replaced by cash and equities in defensive sectors such as consumer staples, pharma or utilities. Our results confirmed the research and publications [Sie13] by Prof. Jeremy J. Siegel from the Wharton School of the University of Pennsylvania. To conclude, these were the reasons why we have focused on equities as a major asset class in the thesis.

equity, equity/bond mix and hedge fund. We turn our attention to State of the Art on the portfolio selection and optimization.

1.2 State of the Art

This part follows several sources [Lu08, FFK07, FKPF10] on the robust portfolio selection and optimization.

Portfolio selection problem is concerned with determining a portfolio such that the "return" and "risk" of the portfolio have a favorable trade-off. The first mathematical model of the portfolio selection problem was developed by Markowitz [Mar52] six decades ago, in which an optimal or efficient portfolio can be identified by solving a convex quadratic program (CQP). In his model, the "return" and "risk" of a portfolio are measured by the mean and variance of the random portfolio return, respectively, therefore is also known as the mean-variance model.

Despite the theoretical elegance and importance of the mean-variance model, it continues to encounter skepticism among the investment practitioners [FKPF10]. One of the main reasons is that the optimal portfolios determined by the mean-variance model are often sensitive to perturbations in the parameters of the problem (e.g., expected returns and the covariance matrix), and thus lead to large turnover ratios with periodic adjustments of the problem parameters. Various aspects of this phenomenon have also been extensively studied in the literature, for example, see [CZ93, Mic98].

As a recently emerging modeling tool, robust optimization can incorporate the perturbations in the parameters of the problems into the decision making process. Generally speaking, robust optimization aims to find solutions to given optimization problems with uncertain problem parameters that will achieve good objective values for all or most of realizations of the uncertain problem parameters. For details, see [FHZ10, FFK07, RS09, FKPF10]. Recently, robust optimization has been applied to model portfolio selection problems in order to alleviate the sensitivity of optimal portfolios to statistical errors in the estimates of problem parameters. Goldfarb and Iyengar [GI03] considered a factor model for the random portfolio returns, and proposed some statistical procedures for constructing uncertainty sets for the model parameters. For these uncertainty sets, they showed that the robust portfolio selection problems can be reformulated as second-order cone programs.

Alternatively, Tütüncü and Koenig [TK04] considered a box-type uncertainty structure for the mean and covariance matrix of the assets returns. For this uncertainty structure, they showed that the robust portfolio selection problems can be formulated and solved as smooth saddle-point problems that involve semidefinite constraints. In addition, for finite uncertainty sets, Ben-Tal et al. [BTMN00] studied the robust formulations of multi-stage portfolio selection problems. Also, El Ghaoui et al. [GOO03] considered the robust Value-at-Risk (VaR[3]) problems given the partial information on the distribution of the returns, and they showed that these problems can be cast as semidefinite programs. Zhu and Fukushima [ZF05] showed that the robust conditional Value-at-Risk (CVaR[4]) problems can be reformulated as linear programs or second-order cone programs for some simple uncertainty structures of the distributions of the returns. Recently, DeMiguel and Nogales [DN08] proposed a novel approach for portfolio selection by minimizing certain robust estimators of portfolio risk. In their method, robust estimation and portfolio optimization are performed by solving a single nonlinear program.

There are several very new articles that extend the domain. One of them [LT14] is a numerical study of a robust active portfolio selection model with downside risk and multiple weights constraints. The study tracks numerical performance of solutions with the classical MV tracking error model and the naive $1/n$ portfolio strategy from real China market and other markets.

1.3 The Classical Mean-Variance Model

The classical Mean-Variance Model (MVM, also as the Modern Portfolio Theory, MPT) [Wil07] suggests that the return on individual assets are represented by normal distribution with the analysis, with a certain mean and standard deviation over a specified period. Therefore the mean-variance framework models an asset's return as a normally distributed function (or more generally as an elliptically distributed random variable), defines risk as the standard deviation of return, and models a portfolio as a weighted combination of assets, so that the return of a portfolio is the weighted combination of the assets' returns. By combining different assets whose returns are not perfectly positively correlated, the MVM seeks to reduce the total variance of the portfolio return. The MVM also assumes that investors are rational and the markets are efficient. The theory was invented by Harry Markowitz in 1952 [Mar52]. The following subsections of the MVM are based on published works and accessible research [FKPF10, Hul08, Wil07].

1.3.1 The Fundamental Concept

The fundamental concept behind the MVM [Mar52] is that the assets in an investment portfolio should not be selected individually, each on its own merits. One needs

[3]VaR is a Value-at-Risk measure. VaR is a downside risk measure developed by JP Morgan, as a part of the Risk Metrics software, in 1994. VaR measures the predicted maximum loss at a specified probability level (95% or 99%) over a certain time period (10 days or 1 month). The measure has been frequently used in financial institution to track and report the market risk exposure of the trading portfolios [FKPF10].

[4]CVaR is a conditional VaR measure that elaborates on VaR issues. CVaR measures the expected amount of losses in the tail of the distribution of possible portfolio losses, beyond the portfolio VaR [FKPF10].

to consider how each asset changes in price relative to how every other asset in the portfolio changes in price.

Investing is a tradeoff between risk and expected return. In general, assets with higher expected returns are riskier. For a given amount of risk, the MVM describes how to select a portfolio with the highest possible expected return. Simply said, for a given expected return, the MVM explains how to select a portfolio with the lowest possible risk.

1.3.2 Risk And Expected Return

The MVM[5] assumes that investors are risk averse [Mar52], meaning that given two portfolios that offer the same expected return, investors will prefer the less risky one. Thus, an investor will take on increased risk only if compensated by higher expected returns. Conversely, an investor who wants higher expected returns must accept more risk. The exact trade-off will be the same for all investors, but different investors will evaluate the trade-off differently based on individual risk aversion characteristics. The implication is that a rational investor will not invest in a portfolio if a second portfolio exists with a more favorable risk-expected return profile. That is if for that level of risk an alternative portfolio exists that has better expected returns.

Equations of the MVM

This part on the MVM follows the related publications [Mar52, RS09]. Optimal portfolio asset allocation problems are quadratic programming problems (QP). Some of them can be formulated as convex QP that is minimizing a quadratic function subject to linear constraints.

Let n be the number of the available assets, and

$$X = \{\mathbf{x} \in \Re^n | \sum_i^n x_i = 1, x_i \geq 0, i = 1 \dots n\} \tag{1.1}$$

be a set of the feasible portfolios. Next, let μ be the estimated expected return vector of the given assets while matrix Σ is the covariance matrix of these returns. Then the mean-variance model can be formulated as follows:

1. **Maximize the expected return** subject to an upper limit on the variance,

$$\begin{aligned} &\max \mu^T \mathbf{x} \\ s.t. \quad &\mathbf{x}^T \mathbf{\Sigma} \mathbf{x} \leq \sigma \\ &\mathbf{x} \in X. \end{aligned} \tag{1.2}$$

[5] Note that the framework uses standard deviation of return as a proxy for risk, which is valid if asset returns are jointly normally distributed or otherwise elliptically distributed. There are several problems with this idea as it is explained later.

2. **Minimize the variance subject** to a lower limit on the expected return:

$$\begin{aligned} \min\ & \mathbf{x}^T \mathbf{\Sigma} \mathbf{x} \\ s.t.\quad & \mu^T \mathbf{x} \geq R \\ & \mathbf{x} \in X. \end{aligned} \tag{1.3}$$

3. **Maximize the risk-adjusted return**:

$$\begin{aligned} \max\ & \mu^T \mathbf{x} - \lambda \mathbf{x}^T \mathbf{\Sigma} \mathbf{x} \\ s.t.\quad & \mathbf{x} \in X, \end{aligned} \tag{1.4}$$

   where $\lambda \in \mathcal{R}$ denotes a risk-aversion parameter.

These three models are parameterized by the variance limit, the expected return limit and the risk-aversion parameter, respectively. The variance constraint is a nonlinear constrain, so the first formulation can not be classified as as a convex QP formulation, while the later two are convex QP formulations.

A published study of Black and Litterman [BL92] demonstrated that small changes in the expected returns may have a substantial impact in the portfolio composition. Large estimation errors in the expected returns influence significantly the optimal allocation. The mean-variance model seems to be less sensitive to inaccuracies in the estimate of the covariance matrix Σ than to estimation errors in the expected returns but insurance against uncertainty in these estimates is recommended.

These equations were elementary equations of the MVM to be used in the portfolio analysis and construction.

1.3.3 Diversification

An investor [Wil07] can reduce portfolio risk simply by holding combinations of instruments that are not perfectly positively correlated (correlation coefficient ρ_{xy},[6] $-1 \leq \rho_{xy} < 1$). In other words, investors can reduce their exposure to individual asset risk by holding a diversified portfolio of assets. Diversification may allow for the same portfolio expected return with reduced risk. These ideas [Wil07] have been started by Markowitz and then reinforced by other economists and mathematicians such as Andrew Brennan who have expressed ideas in the limitation of variance through portfolio theory.

[6] If all the asset pairs have correlations of $\rho_{xy} = 0$, they are perfectly uncorrelated. The portfolio's return variance is the sum over all assets of the square of the fraction held in the asset times the asset's return variance and the portfolio standard deviation is the square root of this sum.

1.3.4 The Mean Variance Efficient Frontier

As shown in Figure 1.3, every possible combination of the risky assets, without including any holdings of the risk-free asset, can be plotted in risk-expected return space, and the collection of all such possible portfolios defines a region in this space. The left boundary of this region is a hyperbola, and the upper edge of this region is the (mean-variance) efficient frontier (the EF, MVEF) in the absence of a risk-free asset (sometimes called 'the Markowitz bullet'). Combinations along this upper edge represent portfolios (including no holdings of the risk-free asset) for which there is lowest risk for a given level of expected return. Equivalently, a portfolio lying on the efficient frontier represents the combination offering the best possible expected return for given risk level.

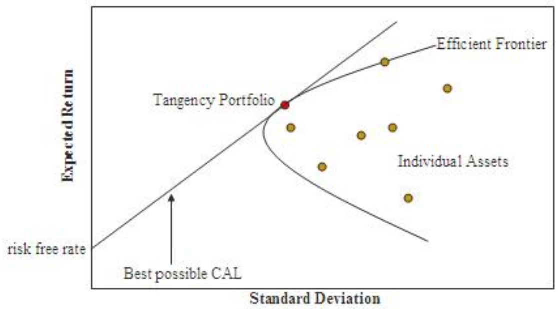

Figure 1.3: The Mean-Variance Efficient Frontier (the MVEF). The hyperbola is the efficient frontier if no risk-free asset is available. With a risk-free asset, the straight line is the efficient frontier. Source: [Mar52, Wil07].

1.3.5 The Capital Allocation Line

The risk-free asset [Wil07] is the (hypothetical) asset that pays a risk-free rate. In practice, short-term government securities (such as US treasury bills) are used as a risk-free asset, because they pay a fixed rate of interest and have exceptionally low default risk. The risk-free asset has zero variance in returns (hence is risk-free); it is also uncorrelated with any other asset (by definition, since its variance is zero). As a result, when it is combined with any other asset or portfolio of assets, the change in return is linearly related to the change in risk as the proportions in the combination vary.

When a risk-free asset is introduced, the half-line shown in the figure is the new efficient frontier. It is tangent to the hyperbola at the pure risky portfolio with the highest Sharpe ratio. Its horizontal intercept represents a portfolio with 100% of holdings in the risk-free asset; the tangency with the hyperbola represents a portfolio with no risk-free holdings and 100% of assets held in the portfolio occurring at the tangency point; points between those points are portfolios containing positive amounts of both the risky tangency portfolio and the risk-free asset; and points on the half-line beyond the tangency point are leveraged portfolios involving negative holdings of the risk-free asset (the latter has been sold short—in other words, the investor has borrowed at the risk-free rate) and an amount invested in the tangency portfolio equal to more than 100% of the investor's initial capital. This efficient half-line is called the capital allocation line (CAL), and its formula can be shown to be

$$r_p = r_f + \sigma_p \frac{r_r - r_f}{\sigma_r}, \tag{1.5}$$

where r_r is return of the sub-portfolio of risky assets at the tangency with the Markowitz bullet, r_f is return of the risk-free asset, σ_p, σ_r are respective return volatilities, and r_p is a return combination of risk free f and risky portfolios r.

The introduction of the risk-free asset as a possible component of the portfolio has improved the range of risk-expected return combinations available, because everywhere except at the tangency portfolio the half-line gives a higher expected return than the hyperbola does at every possible risk level. The fact that all points on the linear efficient locus can be achieved by a combination of holdings of the risk-free asset and the tangency portfolio is known as the one mutual fund theorem, where the mutual fund referred to is the tangency portfolio.

1.4 Robust Portfolio Optimization

Despite the theoretical support, the availability of computer programs and the elegance of the mean-variance model, there are several pitfalls [Wil07]. The optimal portfolios are not well diversified but concentrated, require large data for the accurate estimation of inputs and are very sensitive to changes in input parameters such as expected returns, variances and covariances. Portfolio managers demand to reduce the complexity of the framework and the sensitivity of a portfolio on input parameters. There are various approaches to resolve these issues, one of them is the Black-Litterman model optimization [BL92] and the other is the robust optimization. The robust framework models optimization problems with data uncertainty to receive a solution that is 'good' under all possible circumstances.

1.4.1 Robustness

There are several facts related to the robustness and the portfolio robustness [FHZ10]. A robust system is a responsive system that is insensitive to extreme input parameters irrespective how wildly they fluctuate. We propose a working definition of robustness of a portfolio as follows.

Robustness of a portfolio ρ is the ability of a financial trading system (portfolio) $\chi(\vartheta, \epsilon, \xi)$ to remain effective under different markets ϑ and changing market conditions ϵ, or the ability of a portfolio model to remain valid under various assumptions, input parameters and initial conditions ξ.

$$\rho : \forall\ \vartheta, \epsilon, \xi \quad \chi(\vartheta, \epsilon, \xi) \leq \delta_\rho, \tag{1.6}$$

where δ_ρ is a robust parameter that says the system is still robust. Sometimes inputs ϑ, ϵ and ξ can fluctuate excessively but $\chi(.) \leq \delta_\rho$ holds.

This feature ensures that a managed portfolio does not react excessively to changes and outliers of inputs. And therefore one can assume that *robust investment portfolios* provide a significant downside risk, low volatility and substantial wealth growth over time. Generally, in bull markets, these portfolios grow faster than tracking market indexes. During economic downturns, market crashes and recessions (bear markets) they are relatively stable or decreasing far less than the tracking benchmarks. This is an ideal and theoretical case, but the reality is not far different. There are some examples of such robust portfolios:

- bond and call option (on a stock or equity index)
- stock and put option (on an underlying stock or equity index)
- 60/40 equity-bond mix
- 130/30 strategy[7]
- general long/short strategy
- multi-strategy portfolios.[8]

All the above examples are robust strategies from simple to more complicated ones as they ensure robustness with the limited downside and the unlimited upside.

Generally, the robust portfolios repeatedly deliver *high reward-to-variability* ratios (such as Sharpe, Sortino or Treynor ratios [Wil07]). The reward-to-variability ratio is a measure of the excess return (or risk premium) per unit of deviation in a portfolio. Such characteristics of general asset portfolios are often appreciated in active management of mutual funds and hedge funds. See the performance statistics of the latter in Table 1.1.

[7]This is a long-short strategy that shorts 30% of a portfolio and the received cash from shorts is redeployed to the long side of the portfolio. The investable example is 'Credit Suisse 130/30 Large Cap Index'.

[8]A multi-strategy approach focuses on two and more investment strategies to gain a profit from an investment portfolio. These strategies are the mix of the following ones: convertible arbitrage, dedicated short, emerging markets, event driven, fixed income arbitrage, market neutral, global macro, long-short and managed futures. When exploited skillfully, the portfolio provides low volatility, diversification effects, high robustness and (uncorrelated) growth.

1.4.2 Robust MVM

Let us assume [RS09] the uncertain mean return vector μ and the uncertain covariance matrix $\mathbf{\Sigma}$ of the asset return belong to uncertain sets of the following form:

$$U_\mu = \{\mu : \mu^L \leq \mu \leq \mu^U\} \text{ and } U_{\mathbf{\Sigma}} = \{\mathbf{\Sigma} : \mathbf{\Sigma} \succeq 0, \mathbf{\Sigma}^L \leq \mathbf{\Sigma} \leq \mathbf{\Sigma}^U\}.$$

The end-points of the intervals may correspond to the extreme values of the corresponding statistics in historical data or in analyst estimates. Alternatively, an analyst may choose a confidence level and then generate estimates of returns and covariance parameters in the form of prediction intervals.

The first robust problem determines a feasible portfolio x such that its maximum risk-adjusted expected return, where both parameters vary in the given uncertainty sets, is the minimum ones among the feasible portfolios,

$$\max_{\mathbf{x}\in X} \Big\{ \min_{\mu\in U_\mu, \mathbf{\Sigma}\in U_{\mathbf{\Sigma}}} \quad \mu^T\mathbf{x} - \lambda\mathbf{x}^T\mathbf{\Sigma}\mathbf{x}\Big\} \tag{1.7}$$

and

Hedge funds, Indexes and Sharpe ratios

Strategies	Avg. Ret. [%]	Std. Dev. [%]	SR
Convertible Arbitrage	9.04	4.62	**1.09**
Dedicated Short Bias	-2.39	16.97	-0.38
Emerging Markets	9.25	16.00	0.33
Equity Market Neutral	10.01	2.88	**2.09**
Event Driven	11.77	5.54	**1.40**
Fixed Income Arbitrage	6.46	3.66	0.67
Global Macro	13.54	10.75	0.89
Long/Short Equity	12.09	10.05	0.81
Managed Futures	6.50	11.84	0.21
Multi-Strategy	9.57	4.29	**1.30**
D&J 30	9.18	14.60	0.35
Nasdaq	8.87	26.10	0.19
S&P 500	8.66	14.27	0.33

Table 1.1: Hedge funds, Indexes and Sharpe ratios. There are several statistics such as average return (Avg. Ret.), standard deviation (Std. Dev.) and Sharpe ratio (SR). These statistics are of hedge funds (HFs) with various investment strategies. The equity market indexes are for comparison. One can see that Sharpe ratios of HFs can be over 1.00. Source: cairn.info (May 2007).

$$\begin{aligned} \min \max_{\mathbf{\Sigma} \in U_{\mathbf{\Sigma}}} \quad & \mathbf{x}^T \mathbf{\Sigma} \mathbf{x} \\ s.t. \min_{\mu \in U_{\mu}} \quad & \mu^T \mathbf{x} \geq R, \\ & \mathbf{x} \in X. \end{aligned} \tag{1.8}$$

On the other hand, the latter robust problem looks for a feasible portfolio which guarantees the lower limit R on the expected return also in the worst case, i.e., for the worst realization of parameter μ in U_{μ}, and which minimizes in the worst realization of parameter $\mathbf{\Sigma}$ according to the uncertainty set $U_{\mathbf{\Sigma}}$.

Under certain simplifying assumptions, that is when $\mathbf{\Sigma}^U$ is a positive semidefinite matrix, these robust problems can be reduced to pure MVO problems. In such a special case, the best asset allocation can in fact be determined by first fixing the worst-case input data in the considered uncertainty sets, that is μ^L for the uncertain mean return vector μ and $\mathbf{\Sigma}^U$ for the uncertain covariance matrix $\mathbf{\Sigma}$ and then solving the resulting QP problems [RS09].

1.4.3 Robust Portfolios

Unobservable θ

Suppose θ and $\hat{\theta}$ represent the true and estimated input parameters in a portfolio selection model [FHZ10], respectively. For example, θ denotes the mean μ and the covariance matrix $\mathbf{\Sigma}$ in the mean-variance model, it represents the distribution of portfolio return. Typically, θ is unobservable but is believed to belong to a certain set $\mathcal{P}$ which is generated from the estimated parameter $\hat{\theta}$, i.e., $\theta \in \mathcal{P} = \mathcal{P}_{\hat{\theta}}$. We aim at constructing a portfolio so that the risk is as small as possible with respect to the worst-case scenario of the uncertain parameters in this set $\mathcal{P}$.

1.4.4 Portfolio With Known Moments

We consider a general portfolio optimization model [FHZ10] where the investor seeks to maximize the expectation of his utility $u(.)$.[9] The investor solves the following general stochastic mathematical program:

$$\max_{\mathbf{x} \in X} \mathbb{E}[u(\mathbf{r}^T \mathbf{x})], \tag{1.9}$$

where $X = \{\mathbf{x} \in \Re^n : \mathbf{1}_n^T \mathbf{x} = 1\}$. When the distribution of the portfolio return $\mathbf{r}$ is exactly known, problem (1.2) is a general one-stage stochastic optimization problem

[9] Utility function $u(.)$ is an economic function that measures usefulness of goods or services to a consumer. A function $u : X \rightarrow R$ is a utility function that represents preference $\succeq$ such as $\forall\, x, y \in X$ where $x \succeq y \leftrightarrow u(x) \geq u(y)$.

without recourse. Particularly, if $u(.)$ is a quadratic utility $u(\mathbf{r}) = c_2\mathbf{r}^2 + c_1\mathbf{r} + c_0$, the problem (1.2) does not depend on the actual distribution of $\mathbf{r}$, except its mean μ and covariance $\mathbf{\Sigma}$, and amounts to:

$$\max_{\mathbf{x}\in X} c_2\mathbf{x}^T(\mathbf{\Sigma} + \mu\mu^T)\mathbf{x} + c_1\mu^T\mathbf{x} + c_0. \tag{1.10}$$

This only applies to the case of quadratic utility functions, partially explaining why it is particularly favored in economics and finance models, especially for portfolio selection. Details of the general case, when $u(.)$ is not quadratic and the distribution $p(\mathbf{r})$ is partially known, are in [FHZ10].

1.4.5 Portfolio With Unknown Mean

We follow [FHZ10] to discuss the robust version of the mean-variance portfolio problem where uncertainty is present only in the expected return and $\mathbf{\Sigma}$ is known, so $\theta = \mu \in \mathcal{P}_{\hat{\mu}}$.

Box uncertainty on mean

The simples choice for the uncertain set μ is box,

$$\mathcal{P}_{\hat{\mu}} = \{\mu : |\mu_i - \hat{\mu}_i| \leq \delta_i, i = 1 \ldots n\}.$$

The δ_i parameters can be related to some confidence interval around the estimated expected return. The robust portfolio problem can be formulated as

$$\min_{x\in X} \left\{\mathbf{x}^T\mathbf{\Sigma}\mathbf{x} : \min_{\mu} \mu^T\mathbf{x} \geq \mu_0, |\mu_i - \hat{\mu}_i| \leq \delta_i, i = 1, \ldots, n\right\},$$

which can be further formulated as follows

$$\min_{\mathbf{x}\in X} \left\{\mathbf{x}^T\mathbf{\Sigma}\mathbf{x} : (\hat{\mu} - \mu_\delta)^T\mathbf{x} \geq \mu_0\right\}, \tag{1.11}$$

where $\mu_\delta = (\text{sign}(x_1)\delta_1, \ldots, \text{sign}(x_n)\delta_n)^T$. The term $\text{sign}(x)$ is the sign function equal to 1 if $x \geq 0$ and 0 otherwise.

The term $\hat{\mu} - \mu_\delta$ can be viewed as a shrinkage estimator[10] of the expectation of portfolio returns. In other words, constructing a robust portfolio for μ from $\hat{\mu}$ is equivalent to constructing a portfolio from $\hat{\mu} - \mu_\delta$. If the weight of asset i in the portfolio is negative, the expected return on this asset is increased, $\mu_i + \delta_i$ and vice versa.

[10] Shrinkage estimator improves the estimate that is made closer to the value supplied by the 'other information' than the raw estimate.

1.4.6 Portfolio With Unknown Mean And Covariance

We continue to follow the material [FHZ10]. There are some situations when the covariance matrix $\mathbf{\Sigma}$ is subject to estimation error. Then, $\theta = (\mu, \mathbf{\Sigma}) \in \mathcal{P}_{(\hat{\mu}, \hat{\mathbf{\Sigma}})}$. Therefore there are several methods for modeling uncertainty in the covariance matrix. Some are superimposed on top of factor models for returns [GI03] and others consider confidence intervals for the individual matrix entries [TK04]. For more details, see [FHZ10].

1.5 Summary

The Mean-Variance Model (MVM) is an elegant framework for portfolio optimization. The framework operates with the return and risk of investable assets and their asset weights to form an optimal portfolio. The returns and risks of these assets are derived from the means and variances of historical data series or various estimation models.

The MVM seeks to reduce the total variance of the portfolio returns and proposes an efficient frontier for the assets, where the best possible MV portfolios reside. Mutual correlations of the assets impact the portfolio formation and weights of the assets forming the portfolio. The MVM assumes that investors are rational and the capital markets are efficient. In our opinion this is a strong assumption that holds most of the time but does not hold all the time (bubbles, recessions).

Despite the elegance of the framework, the MVM suffers with the over-concentration and sensitivity issues. Therefore these issues of the model were studied extensively and several correcting techniques have been proposed. There have been two paths to overcome the disturbing issues. One is the mitigation of the issues and the other is robust portfolio optimization.

We reviewed several practical as well as theoretical cases of the possible robustness. The theoretical cases include:

1. Robust MVM
2. Portfolios with known moments
3. Portfolios with unknown mean
4. Portfolios with unknown mean and covariance.

Topics such as the MV framework, the mitigation of MVM issues, robust optimization, the classification of securities, robustness and robust portfolios are developed in next chapters.

1.6 Motivation

The goals of the thesis were motivated by problems encountered during an analysis, construction and optimization of risky asset portfolios. The robust investment portfolios are of a practical applicability but their fundamentals are of a theoretical as well as of a practical nature. There are four goals to be declared.

- *The first one is to test the mean-variance model (MVM) and investigate its disadvantages.*
- *The second one is to evaluate a robust optimization technique to overcome the disadvantages of the MVM.*
- *The third one is to provide and test an intuitive framework for classification or selection of risky assets for a robust portfolio.*
- *The fourth one is to give a practical allocation procedure and to verify that robust portfolios offer low and infrequent drawdowns and high reward-to-variability ratios.*

In the presented thesis there are no complete answers to many questions but they provide elementary building blocks and guidance to the solution.

1.7 Outline Of Thesis

The thesis is organized so that the reader can view the sections independently. The main directives of investigation and research are related to MV/RMV portfolios. One can read some relevant sources of portfolio analysis, optimization and management, of which there are plenty elsewhere [Cip00, FHZ10, Wil07].

The thesis is structured as follows: **Chapter 1** is a brief introduction to the thesis. **Chapter 2** provides the theory on the mean-to-variance portfolios. **Chapter 3** is about robust portfolios. **Chapter 4** gives some hints on equity selection and classification for the robust portfolios. **Chapter 5** covers the robustness, robust portfolios, drawdowns, risk-reward ratios of the robust portfolios and some empirical verifications. **Chapter 6** summarizes and discusses the results. **Chapter 7** concludes the thesis. There are five appendixes. **Appendix A** provides a description of two well-known equity indexes and a growth fund. **Appendix B** is about Value at Risk method (VaR) and Mean-Variance Model (MVM). **Appendix C** gives insight into the clustering algorithm, input data and other results. **Appendix D** is about details on Reward-To-Variability Ratios and additional experimental results. **Appendix E** informs about Data Sources among other things.

Chapter 2

MVM Portfolio

The chapter[1] provides some insight into portfolio analysis and construction but complete references [Cip00, Hul08] offer proper details.

Markowitz [Mar52] proposed and published a solution for portfolio selection problems. The basic idea was elegant, innovative and intuitive such as to allocate investment capital over a number of assets in order to maximize the 'return' and minimize the 'risk'. The solution was substantially researched and two main weaknesses appeared [FHZ10].

The chapter is structured as follows: **Section 2.1** gives some background about portfolios, investments and related work. **Section 2.2** specifies the MV model (MVM). **Section 2.3** is related to empirical verification and **Section 2.4** discusses results. **Section 2.5** concludes the chapter.

2.1 Background

This section reviews the research work related to portfolio selection and management. It explains essential ideas, facts and context of the topic. The section starts with portfolio selection problems researched by Markowitz.

Portfolio selection problems [RS09] were formulated for the first time by Markowitz [Mar52]. They consist of allocating capital over a number of available assets in order to maximize the 'return' on the investment while minimizing the 'risk' using mathematical techniques. In the proposed models, the return is measured by the expected value of the random portfolio return, while the risk is quantified by the variance of the portfolio (mean-variance models).

Despite the strong theoretical support [RS09], the availability of efficient computer codes to solve them and the elegance of the models, they present some practical pitfalls: the optimal portfolios are not well diversified; in fact they tend to concen-

[1] This chapter was rewritten and published at the conference [Kon13]. The requested revisions and suggestions were incorporated into the chapter.

trate on a small subset of the available securities and, above all, they are often very sensitive to changes in the input parameters.

Also, a critical weakness of mean-variance analysis [FHZ10] is the use of variance as a measure of risk. In some sense, risk is a subjective concept and different investors adopt diverse investment strategies in seeking to realize their investment objectives, and hence the exogenous characteristics of investors mean that probably no unique risk measure exists that can accommodate every investor's problem.

For example, one investor may be concerned about dramatic market fluctuation no matter whether this movement is upside or downside, whereas another investor may be more concerned with the downside movements, which usually imply severe loss consequences. In this case, the variance is obviously not sufficient to express or measure the investors' risk. On the other hand, the market may change in nature.

The introduction [FHZ10] of new derivatives and investment strategies may require the formulation of an alternative risk measure more appropriate for different investors. This is because the portfolio distribution with derivatives such as futures and options is skewed and heavy-tailed, which calls for a risk measure to respond to downside and upside deviations asymmetrically.

2.2 The Mean-Variance Model

Portfolio

Consider n risky assets [FHZ10] that are chosen by an investor in the financial market. Let $\mathbf{r} = (r_1, \ldots, r_n)^T \in \Re^n$ denote the uncertain returns of the n risky assets from the current time $t = 0$ to a fixed future time $t = T$. Let $\mathbf{x} = (x_1, \ldots, x_n)^T \in \Re^n$ denote the percentage of the available funds to be allocated in each of the n risky assets. A portfolio allocation model aims at finding the optimal (best) portfolio $\mathbf{x}$ to be constructed at $t = 0$, in order to maximize the portfolio's future return $\mathbf{r}^T\mathbf{x}$ from $t = 0$ to $t = T$. The definition of the portfolio is extended in the next subsection.

The Mean-Variance Problem

In this section we consider a one-period portfolio selection problem [FHZ10, Mar52]. Let the random vector $\mathbf{r} = (r_1, \ldots, r_n) \in \Re^n$ denote random returns of the n risky assets, and $\mathbf{x} = (x_1, \ldots, x_n)^T \in X, X = \{\mathbf{x} \in \Re^n : \mathbf{1}_n{}^T\mathbf{x} = 1\}$ denote the proportion of the portfolio to be invested in the n risky assets, where T means transposition and $\mathbf{1}_n$ denotes a vector of all ones. Suppose that $\mathbf{r}$ has a probability distribution $p(\mathbf{r})$ with mean vector μ and covariance matrix $\mathbf{\Sigma}$. Then the target of the investor is to choose an optimal portfolio $\mathbf{x}$ that rests on the mean-variance efficient frontier. In the Markowitz model [Mar52], the 'mean' of a portfolio is defined as the expected value of the portfolio return, $\mu^T \mathbf{x}$, and the 'risk' is defined as the variance of the portfolio return, $\mathbf{x}^T \mathbf{\Sigma} \mathbf{x}$.

Mathematically, minimizing the variance subject to target and budget constraints leads to a formulation like:

$$\min_{\mathbf{x}} \left\{ \mathbf{x}^T \mathbf{\Sigma} \mathbf{x} : \mu^T \mathbf{x} \geq \mu_0, \mathbf{1}_n{}^T \mathbf{x} = 1 \right\}, \tag{2.1}$$

where μ_0 is the minimum expected return. There are two implicit assumptions in this formulation: i) the first two moments of portfolio return exist and ii) the initial wealth is normalized to be 1 without loss of generality.

If the moment parameters are known, the analytical solution [FHZ10] to the above formulation is straightforward to apply, and the above problem can be solved numerically under various practical constraints, such as no-short-selling or position limits. However, the moment parameters are never known in practice and they have to be estimated from an unknown distribution with limited data. Typically, the procedure of minimizing the portfolio variance with a given expected return can be decomposed into three steps: (i) estimate the expected return and covariance, (ii) use the above optimization problem to create an efficient frontier, and (iii) select a point on the efficient frontier or select a mix of the risk-free assets and the optimal risky asset allocation according to the investor's risk tolerance. This procedure is clearly not optimal, and hence robust procedures for making a good use of portfolio theory are called for in the presence of parameter, model uncertainties or both.

2.3 Empirical Verification

The section gives an overview of the MV method, preliminary test cases and other experimental results with the MVM. These are true unobservable values (μ, $\mathbf{\Sigma}$), which are estimated as ($\hat{\mu}$, $\hat{\mathbf{\Sigma}}$)[2] from historical data series.[3] We assume that holds $\mu \approx \hat{\mu}$, $\mathbf{\Sigma} \approx \hat{\mathbf{\Sigma}}$.

2.3.1 The MVM Method

Let us review the procedure of minimizing the portfolio variance as a four step method.

[**Step 1**] Estimate the expected returns ($\hat{\mu}_s$) and covariance matrix ($\hat{\mathbf{\Sigma}}_\mathbf{s}$) over the period S as

[2] $\mathbf{\Sigma}$ includes σ, so we will operate only with $\mathbf{\Sigma}$.

[3] The process of parameters' estimation ($\mu, \mathbf{\Sigma}$) is also interesting one. If not stated otherwise, we calculated the estimates as simple averages from historical series over the stated periods (sample estimators). This is not the best estimation method but it is sufficient for our demonstration purposes. Better options are shrinkage estimators (i.e. James-Stein or Bayes-Stein shrinkage estimators) [FKPF10].

$$\hat{\mu}_s = \frac{1}{S}\sum_{s=1}^{S} r_s, \hat{\mathbf{\Sigma}}_s = \frac{1}{S}\sum_{s=1}^{S}(r_s - \hat{\mu}_s)(r_s - \hat{\mu}_s)^T. \tag{2.2}$$

[**Step 2**] Use the optimization problem to create an efficient frontier.

[**Step 3**] Select a point on the efficient frontier or select a mix of the risk-free assets and the optimal risky asset allocation.

[**Step 4**] Calculate weights of risky assets for the selected mix.

We have to find an estimation of the expected return, the standard deviation and the correlation of risky assets. Based on that, one calculates the covariance matrix of the returns and an efficient frontier. Finally, one optimizes the portfolio.

2.3.2 Data

The primary stock selection was narrowed to the U.S. traded stocks (the Dow 30, the S&P 500) due to long time series and data availability. Experimental data were retrieved from one financial information source [YA]. The estimations of sample means, variances and correlations of risky assets were calculated as equally-weighted from the historical time series (1.1.1995-2.1.2014). We assumed that these statistical parameters were 'close' and 'reasonable' approximations of true parameters of the constructed portfolio.

2.3.3 Test Case

We experimented with three US stocks[4] – Coca-Cola (KO), Procter&Gamble (PG) and IBM (IBM). If not stated otherwise, we kept this order of the stocks. The index S&P 500 (SPX) was a comparative benchmark which includes all three stocks. The main task of the setup is to searched for weights $\mathbf{x}_{op} = (x_{KO}, x_{PG}, x_{IBM})$ to get these portfolio parameters (μ_{op}, σ_{op}) of our portfolio.

[**Step 1**] In Table 2.1, there are the estimation of expected returns $(\hat{\mu})$ and standard deviations $(\hat{\sigma})$ for the stocks and the S&P 500 index.

In matrices $(\hat{\mathbf{Cor}}, \hat{\mathbf{\Sigma}})$, there are estimated correlations and calculated covariances between the three selected assets.

$$\text{Correlation} \qquad \hat{\mathbf{Cor}} = \begin{pmatrix} 1.0000 & 0.7705 & 0.8504 \\ 0.7705 & 1.0000 & 0.8502 \\ 0.8504 & 0.8502 & 1.0000 \end{pmatrix} \tag{2.3}$$

[4] These are global large capitalization companies, leaders in their industries with capable management and distributing regular dividends. Operating results of the companies are a sort of uncorrelated therefore they form a portfolio.

	$\hat{\mu}$	$\hat{\sigma}$
KO	0.0640	0.1840
PG	0.1001	0.1724
IBM	0.1260	0.2747
SPX	0.0600	0.2000

Table 2.1: ($\hat{\mu}$, $\hat{\sigma}$) pairs. There are expected returns ($\hat{\mu}$) and standard deviations ($\hat{\sigma}$) for the stocks and the S&P 500 index.

$$\text{Covariance} \qquad \hat{\mathbf{\Sigma}} = \begin{pmatrix} 0.0339 & 0.0244 & 0.0430 \\ 0.0244 & 0.0297 & 0.0403 \\ 0.0430 & 0.0403 & 0.0755 \end{pmatrix} \tag{2.4}$$

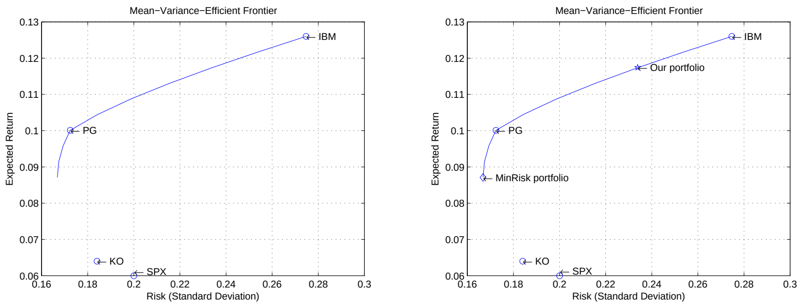

Figure 2.1: The MVEF with three risky assets and the S&P 500 index (left) and the S&P 500 index and two portfolios. Our and MinRisk portfolios are depicted (right).

[**Step 2**] Then we ran a portfolio optimization procedure to receive the efficient frontier of the risky assets. The result is depicted in Figure 2.1 (left).

[**Step 3**] We selected a point on the efficient frontier $(\mu_{op}, \sigma_{op}) = (0.1174, 0.2339)$ that reflected our risk aversion. The result is depicted in Figure 2.1 (right).

[**Step 4**] Finally, we calculated weights of the selected assets. The weights of the three risky assets were $\mathbf{x}_{op} = (0.0000, 0.3335, 0.6665)$. One could see that the MinRisk portfolio had parameters $(\mu_{mr}, \sigma_{mr}) = (0.0871, 0.1668)$ with weights $\mathbf{x}_{mr} = (0.0000, 0.3593, 0.6407)$. And that was the final step of the optimization.

2.3.4 Test Case (reiterated)

We ran the previous setup with the same steps but very recent data. Recent risk-return statistics are in Table 2.2. ($\tilde{\mu}$, $\tilde{\sigma}$) are estimated parameters of the true return and variance of the true parameters ($\mu \approx \tilde{\mu}$, $\mathbf{\Sigma} \approx \tilde{\mathbf{\Sigma}}$).

The rest of the data ($\tilde{\mathbf{Cor}}$, $\tilde{\mathbf{\Sigma}}$) could be found in Appendix D. We do not describe

	$\tilde{\mu}$	$\tilde{\sigma}$
KO	0.1036	0.0611
PG	0.1293	0.1164
IBM	0.0186	0.0375
SPX	0.1884	0.0946

Table 2.2: $(\tilde{\mu}, \tilde{\sigma})$ pairs. There are expected returns $(\tilde{\mu})$ and standard deviations $(\tilde{\sigma})$ for the stocks and the S&P 500 index.

procedural details again but we concentrate on results. And the results look quite differently. Let us compare Figures 2.2 (original left, reiterated right).

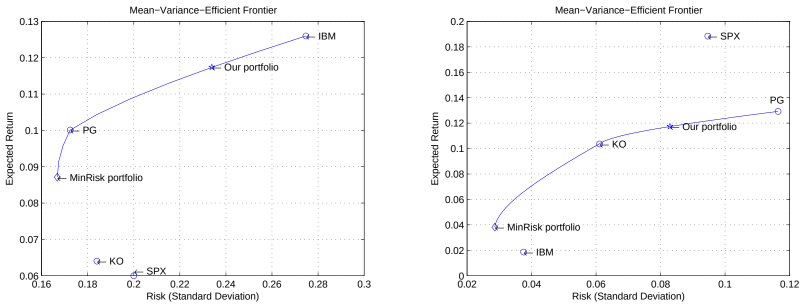

Figure 2.2: The MVEF with three risky assets, the S&P 500 index and two portfolios (original, left). Our and MinRisk portfolios are depicted (reiterated, right).

Required parameters of our portfolio were $(\tilde{\mu}_{op}, \tilde{\sigma}_{op}) = (0.1174, 0.0829)$. We calculated weights as previously. The weights of the three risky assets were $\tilde{\mathbf{x}}_{op} = (0.4662, 0.5388, 0.0000)$. The MinRisk portfolio had parameters $(\tilde{\mu}_{mr}, \tilde{\sigma}_{mr}) = (0.0382, 0.0286)$ with weights $\tilde{\mathbf{x}}_{mr} = (0.0000, 0.1770, 0.8230)$. One can assume that the expected return of the assets is higher, the expected risk is lower and the S&P 500 outperformed all three risky assets. We continued with the long term data from the previous Test Case as we assumed they were more reliable.

2.3.5 Experimental Setup

The tested portfolios were simple three-stock equity based portfolios.[5] The tracking index was the US large cap index (the S&P 500 [SPX]). The tested portfolios were of two types: (i) long-only equity and (ii) long-short equity.

From a practical perspective, the long equity side was constructed from subsets of carefully selected equities. The short side could be achieved via short-selling stocks, equity market indexes or using derivatives.[6] In our case, we shorted 1/4 of

[5]These are small portfolios, but one can understand them easily.

[6]These are options, warrants, swaps, futures, leveraged certificates, ETFs (Exchange Traded Funds) and others.

SPX against the overall portfolio value. We could imagine more aggressive shorting scenario, but we would not want to exploit *the ex-post* knowledge.

The tested portfolios could take up of 100% leverage (200% of the total gross long exposure, but this setup was still underbetting the Kelly criterion[7]. The primary modeling software was Matlab 2011a. Some experiments used examples from the optimization library by Attilio Meucci.[8]

2.3.6 Results

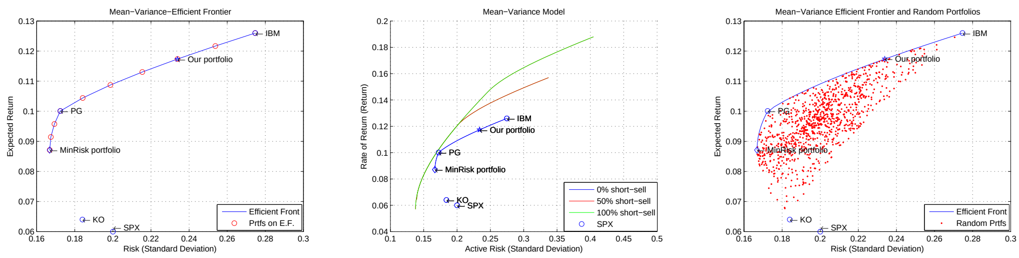

Figure 2.3: The MVEF I. The MV efficient frontier and 10 risky portfolios on the efficient frontier (left). Portfolios weights are in Table 2.3. Three types of portfolios with different levels of short-selling (0%, 50% and 100%) (middle). There are also depicted the MVEF and 1,000 randomly generated portfolios (right).

The first results with the MV model are presented in Figures 2.3 and in Table 2.3. In Figure 2.3 (left), the MVEF is depicted. It shows the efficient frontier with 10 risky portfolios, represented as small circles, on the efficient frontier along the S&P 500 index. The higher risk means the higher expected return and vice versa. In Table 2.3, one can find weights of these 10 risky portfolios.

#	1	2	3	4	5	6	7	8	9	10
x_{KO}	**0.36**	0.24	0.12	0.01	0.00	0.00	0.00	0.00	0.00	0.00
x_{PG}	**0.64**	0.76	0.88	0.99	0.83	0.67	0.50	0.33	0.17	0.00
x_{IBM}	0.00	0.00	0.00	0.00	0.17	0.33	0.50	0.67	0.83	**1.00**

Table 2.3: Portfolios weights and risks. Calculated portfolio weights are related to Figure 2.3. The lowest risk-return portfolio splits weights between KO (0.36) and PG (0.64) [left]. The highest risk portfolio concentrates in IBM stock (1.00) [right].

[7]The Kelly criterion [Wil07] is a criterion for maximizing expected growth of assets by investing a fixed fraction of your wealth. The criterion suggests use of leverage for growth of assets, when the leverage is lower than recommended, we are 'underbetting' the criterion.

[8]The library is available at `http://symmys.com`.

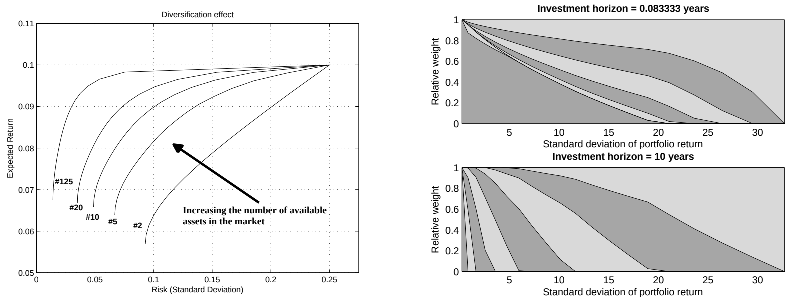

Figure 2.4: The MVEF II. Figure (left) displays the diversification effect of increasing the number of assets in the market (from 2 to 125 securities). The efficient frontier also depends on the investment horizon (right).

Figure displays the diversification effect of increasing the number of assets in the market (from 2 to 125 securities) 2.4(left). This reflects the intuitive thinking because the more asset available, the more possibilities to construct a better performing portfolio. The efficient frontier depends on the investment horizon and highlights the problems arising from an incorrect use of compounded returns instead of linear returns for this analysis in Figure 2.4 (right). The investment horizon is one month and 10 years.

2.4 Discussion

The presented research explains a simple procedure how to construct, optimize and balance the equity based portfolios. The current algorithmic setup provides far more functionality than was presented. E.g. splitting stocks into industry groups, restricting on local and global regions, other limiting and restrictive constrains might be declared on the algorithms.

We experimented with the simple optimization portfolio strategy (the MVM). The results were presented in the form of figures, tables and appropriate comments. The achieved results reflect the reality and counter-testing. These experiments have shown the relation between risk and return that reflects the financial markets. As mentioned previously, there are some minor flaws[9] in the model but anyway this simple model works quite consistently.

There was no discussion about the relevance to capital markets, allocation strategies, nor real-life portfolio construction. Some parts of applied statistics [Fel66] and econometrics [Bal11] might need more attention. The presented study was about understanding the portfolio theory, applied statistics and simulation software. The

[9] One of them is a directional concept of volatility. In some case, any increased volatility is a problem and in the other the increased volatility with falling markets is the serious issue.

experiments with extended stress-testing, re-allocations and re-optimizations were not completed.

In Appendix B we presented results from minor experiments with VaR (Value-at-Risk). The results are only a set of Figures and related comments. As dipping our toes into VaR, let us clarify that VaR (Value-at-Risk) is one of the risk measures and metrics used in banking and the financial markets. It helps to construct and guard portfolios of financial assets. For the sake of context, the mean-VaR model for portfolio selection [FHZ10] was proposed in 2000.

2.5 Summary

The chapter defined and explained the Mean-Variance Model (MVM), the Mean-Variance Problem (MVP), asset portfolio and the optimization as a minimization of the portfolio variance. The MVM works with estimates of expected returns of assets and their covariance matrix. The estimates are usually calculated as sample means from historical time series.

The experiments involved two test cases. The test cases optimized a portfolio of the same three risky assets. Due to calculation sample means from two time series with different periods, we received different expected returns and covariance matrix for the assets and then two different MV portfolios. We continued with the more conservative estimation and the portfolio. Therefore we can claim:

- The MVM provides a simple, elegant and intuitive framework for portfolio optimization. It includes portfolio leverage, short-selling, correlated and uncorrelated assets, various asset classes and so on.

- The MVM was tested with 3-stock portfolios against the capital market data for the 1995-2014 period. The tests showed the favorable diversification, optimization and performance effects of individual securities formed in the portfolio to the equity index. The stock portfolios showed the MV superiority over the benchmark index.

- The asset estimation of expected returns and related covariance matrix is quite vital to the MVM. Small inaccuracies in the estimation of asset parameters lead to absurd or unacceptable results. This is a disturbing feature of the MVM.

There are many sophisticated approaches to deal with these unpleasant issues of the MVM such as mitigation approaches and robust optimization.

Chapter 3

Robust Portfolio

The chapter provides some insight into robust estimates, robust portfolio methods, analysis and construction. We start with the Mean-Variance Model (MVM) and its issues. The issues have been researched and several adequate solutions proposed. The solutions lead to robust estimates in portfolio management.

Markowitz [Mar52] proposed and published a solution for portfolio selection problems, so called the MVM. The solution was substantially researched and two main weaknesses appeared [FKPF10]. The weaknesses were the sensitivity to estimation errors and small changes in the inputs. These weaknesses can be mitigated with four approaches to make the classical mean-variance approach more robust:

1. Improve the accuracy of inputs
2. Use constraints for the portfolio weights
3. Use portfolio resampling to calculate the portfolio weights
4. Apply the robust optimization framework to the portfolio allocation process.

The other direction to solve the weaknesses is robust optimization. Robust optimization has its roots in robust statistics and robust control engineering [FKPF10]. The robust portfolio optimization includes uncertainty in the optimization process. The uncertain parameters are assumed to vary in specified uncertainty sets that are defined on statistical and probability techniques.

Making the portfolio optimization process robust [FKPF10] with respect to uncertainty in the parameters is not very expensive in computational costs but it may result in a worse objective value. This feature can be corrected by using "smart" uncertainty sets for parameters that do not make the expected portfolio return very conservative.

There is a strong empirical evidence that the robust optimization reduces portfolio turnover, transaction costs, improves worst-case performance, limits drawdowns, leads to increased and stable returns in the long run [FKPF10, Meu10]. These are the features which are quite attractive, demanded and useful in the investment industry.

The chapter is structured as follows: **Section 3.1** provides some background about the topic. **Section 3.2** specifies the robust approaches and allocations. **Section 3.3** is devoted to experiments. **Section 3.4** discusses results. **Section 3.5** closes the chapter.

3.1 Background

This section reviews the topic related to robust estimates and optimization. As stated in the previous section, we know that the classical MVM has two weaknesses – the sensitivity to estimation errors and small changes in the inputs. The presented approaches either mitigate the weaknesses or provide direct robust solution.

While experimenting with the MVM, one needs to understand that there are three frontiers [FKPF10] – true (unobservable), estimated and actual frontiers. The true and estimated ones seem evident. The actual frontier takes portfolios on the efficient frontier and calculates their expected returns using the true expected returns. The estimated frontier is the most optimistic frontier of the three as the actual one is the most pessimistic. The optimization target is to get as close as possible to the true unobservable frontiers therefore the mitigation and robust approaches might be the required path.

The next part follows several published references [FKPF10, Meu09a, Meu09b, Meu10, Meu11]. Any details, derivations, technicalities and proofs of robust approaches are in the references above, therefore we give only a short overview of each model. There are four approaches – Bayesian, Black-Litterman, Robust and Robust Bayesian approaches[1] to be experimented with.

3.1.1 Bayesian Approach

The Bayesian approach limits parameters' sensitivity. Bayesian allocations are presented in terms of the predictive distribution of the market, as well as the classical-equivalent Bayesian allocation, which relies on Bayes-Stein shrinkage estimators of the market parameters, the estimator [FHZ10] is defined as,

$$\hat{\mu}_{BS} = (1-\nu)\hat{\mu} + \nu\hat{\mu}_g \mathbf{1}_n, \tag{3.1}$$

where ν is the weight given to the shrinkage target $\hat{\mu}_g$ and $1-\nu$ is the weight on

[1]Actually, there is at least one more – Michaud's resampling technique [FKPF10]. The rationale behind this approach consists in limiting the extreme sensitivity of the optimal allocation function to the market parameters by averaging several sample based allocations in different scenarios but we did not work with this one.

the sample mean. The target is the average excess return of the sample minimum-variance portfolio,

$$\hat{\mu}_g = \frac{\mathbf{1}_n^T \tilde{\mathbf{\Sigma}}^{-1} \hat{\mu}}{\mathbf{1}_n^T \tilde{\mathbf{\Sigma}}^{-1} \mathbf{1}_n} = \frac{\mathbf{1}_n^T \hat{\mathbf{\Sigma}}^{-1} \hat{\mu}}{\mathbf{1}_n^T \hat{\mathbf{\Sigma}}^{-1} \mathbf{1}_n} \tag{3.2}$$

and the weight is

$$\nu = \frac{n+2}{(n+2) + T(\hat{\mu} - \hat{\mu}_g \mathbf{1}_n)^T \tilde{\mathbf{\Sigma}}^{-1} (\hat{\mu} - \hat{\mu}_g \mathbf{1}_n)}, \tag{3.3}$$

where $\tilde{\mathbf{\Sigma}}$ is defined as $\frac{T}{T-n-2}\hat{\mathbf{\Sigma}}$. From a shrinkage point of view, combing $\hat{\mu}_{BS}$ with $\tilde{\mathbf{\Sigma}}$ gives an estimator of the optimal portfolio weights.

The Bayesian approach provides a mechanism that mixes the positive features of the prior allocation and the sample-based allocation: the estimate of the market is shrunk towards the investor's prior view in a self-adjusting way and the overall opportunity cost is reduced.

3.1.2 Black-Litterman Approach

The Black-Litterman (BL) approach controls the extreme sensitivity of the optimal allocation function to the unknown market parameters,

$$\hat{\mu}_{BL} = [(\tau\mathbf{\Sigma})^{-1} + \mathbf{P}^T \mathbf{\Omega}^{-1} \mathbf{P}]^{-1} [(\tau\mathbf{\Sigma})^{-1} \mathbf{\Pi} + \mathbf{P}^T \mathbf{\Omega}^{-1} \mathbf{q}], \tag{3.4}$$

where $\mathbf{\Sigma}$ is the $n \times n$ covariance matrix of returns, $\mathbf{\Pi}$ is $[\Pi_1, \ldots, \Pi_n]^T$ which is the vector of expected excess returns, computed from an equilibrium model such as CAPM, τ is a scalar that represents the confidence in the estimation of the market prior, $\mathbf{q}$ is a k-dimensional vector of k investor views, $\mathbf{P}$ is a $k \times n$ matrix of investor views, $\mathbf{\Omega}$ is a $k \times k$ matrix expressing the confidence in the investor's view.[2]

Like the Bayesian approach, the Black-Litterman methodology makes use of Bayes' rule. In this case the market is directly shrunk towards the investor's prior views, rather than indirectly through the market parameters. We present the theory in a general context, performing the computations explicitly in the case of normally distributed markets.

3.1.3 Robust Approach

Rather than trying to limit the sensitivity of the optimal allocation function, the robust approach aims at determining the "best" allocation in the presence of estimation risk. In other words, robust allocations minimize the opportunity cost over a reasonable set of potential markets. The conceptually intuitive robust approach, as is defined in Chapter 1 (Robust MVM), is hard to implement in the general case.

[2] The matrix $\mathbf{\Omega}$ is assumed to be diagonal, that is, investor views are assumed to be independent.

Therefore, the two-step mean-variance framework helps as follows: (i) under suitable assumptions for the investment constraints the optimal allocations solve a second-order cone programming problem: (ii) as a result, the optimal allocations can be efficiently determined numerically. The robust approach can also be blended with the Bayesian approach (**Robust Bayesian approach**). There are some disadvantages of the robust approach: (i) the markets are defined quite arbitrarily and (ii) the investor's prior view are not taken into the account.

3.2 Experiments

3.2.1 Data and Setup

The data series were artificially generated or retrieved from one financial source [YA]. The retrieved data were narrowed to the U.S. index (the S&P 500) and its industry components. The means, variances and correlations of the index were calculated from historical time series (sample data).

The primary calculation software was Matlab 2011a. The additional software sources were adjusted and rebuilt from the optimization examples created by Attilio Meucci[3] and the SeDuMi[4] library v.1.1 by Jos F. Sturm.

3.2.2 Results

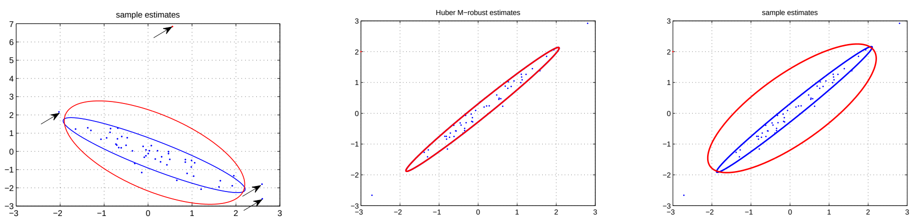

Figure 3.1: Robust estimate – The sample one is sensitive to outliers and the robust one is not. The arrows point to important outliers (left). There are Huber M-robust estimates (middle) and sample estimates with the robust set for comparison (right).

There are presented experimental results based on the ARPM and the SeDuMi libraries. We experimented with robust statistics (Huber M-robust estimates) but the experiments are not described due to the space limitations. We provide only three illustrative and self-descriptive diagrams in Figure 3.1.

[3]These are available at the resource for Advanced Risk and Portfolio Management (ARPM, `http://symmys.com`).

[4]The software for optimization over symmetric cones, available at `http://sedumi.ie.lehigh.edu/downloads`.

Bayesian allocation The sequence of diagrams shows the evaluations of the Bayesian allocation, which replaces the true, unknown market parameter in the optimal allocation policy with a Bayesian classical-equivalent (point) estimate in Figure 3.2.

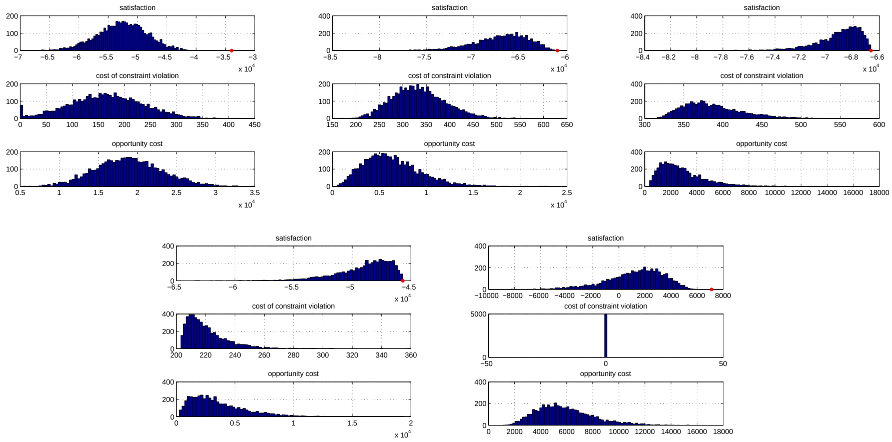

Figure 3.2: Bayesian allocation – Charts display the distribution of satisfaction, cost of constraint violation and opportunity cost for each value of the market stress-test parameters (in this case the correlation; time passes from top left to bottom right).

Black-Litterman approach The figures show the motivations for the Black-Litterman approach. The estimation risk is such that the true ("optimal") frontier estimated with naive estimators changes quite wildly for different time series realizations and corner solutions when inputing the views in Figure 3.3.

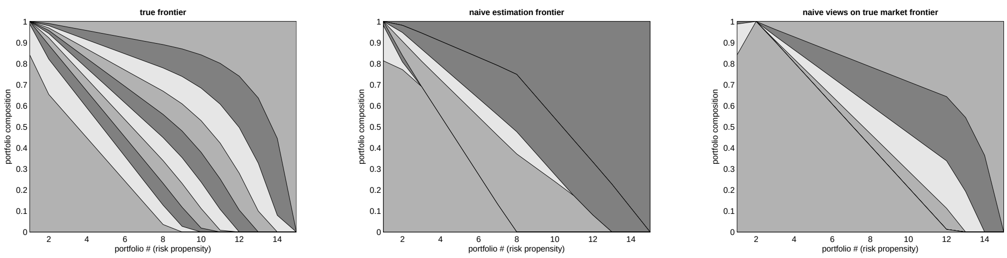

Figure 3.3: Robust motivation for BL – there are three scenarios such as true frontier (left), naive estimation frontier (middle) and naive views on true market frontier (right). Each of the figures is represented in the risk propensity/portfolio composition view.

Black-Litterman approach There has been shown a comparison among: (i) the true efficient frontier, (ii) the pseudo-efficient frontier stemming from the BL general equilibrium prior and (iii) the pseudo-efficient frontier stemming from the BL posterior (general equilibrium plus views). Results are depicted in Figure 3.4.

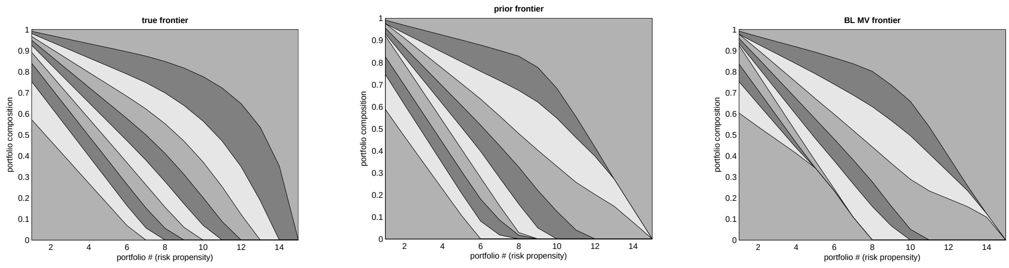

Figure 3.4: Robust BL and the frontiers. The comparison relates true frontier (left), prior frontier (middle) and BL MV frontier (right).

Black-Litterman approach This scenario compares the Black-Litterman approach to inputing views on the market with a brute force approach, which gives rise to corner solutions in Figure 3.5.

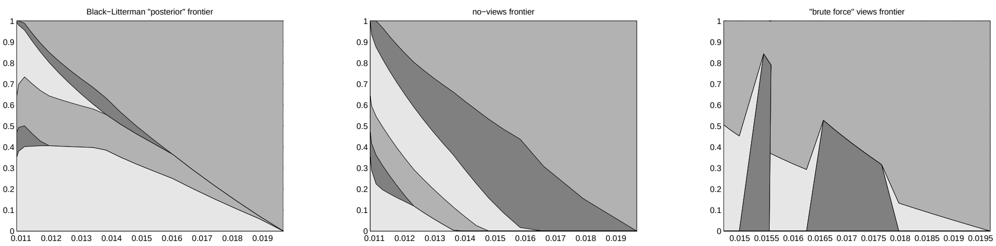

Figure 3.5: Robust BL and three cases. There are three cases to compare: BL "posterior" frontier (left), no views frontier (middle) and brute force views frontier (right).

Robust MV approach The diagram shows the robust mean-variance allocation by means of SOCP (a second-order cone program). The uncertainty region for the expected value is elliptical and no uncertainty is assumed on the covariance matrix in Figure 3.6 (left).

There are (i) the sample MV frontier, (ii) the Bayesian MV frontier and (iii) the robust Bayesian MV frontier while the experiment is repeated for the number $S = 20$ of simulations. The results are depicted in Figure 3.6 (right). The figure is displayed in the mean-variance coordinates.

Robust Bayesian MV approach The figure displays the robust Bayesian allocation frontier, which shrinks the sample estimate toward the practitioner's prior view by including elliptical uncertainty on both location and scatter parameters of a normal market. Three frontiers are computed for three different levels $(T_0^i, \nu_0^i, T^i)_{i=1}^3$ of shrinkage (confidence levels) in Figure 3.7.

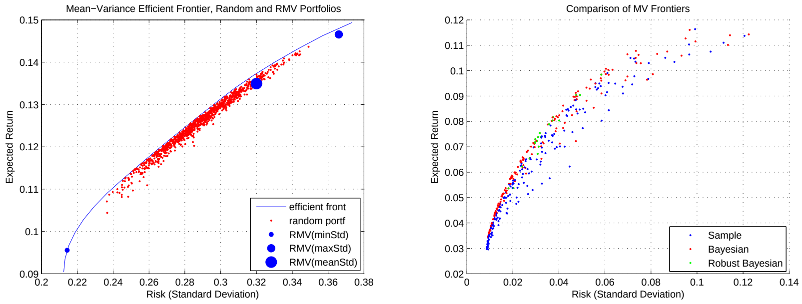

Figure 3.6: Robust MV (left) – There is a robust mean variance (RMV) allocation based on the random input data. The RMVM is controlled with the variance parameter. MV comparison (right) – There are depicted (i) the sample MV frontier, (ii) Bayesian MV frontier and (iii) Robust Bayesian MV frontier.

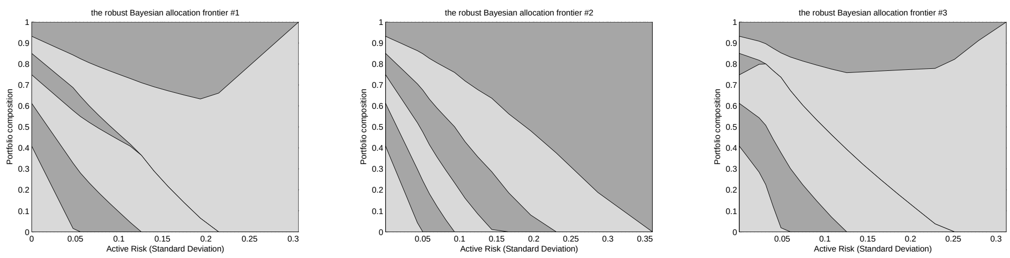

Figure 3.7: Robust Bayesian MV simulations – Three frontiers are computed for three different levels of shrinkage (confidence levels). The number of assets is 6.

This computation compares robust Bayesian allocation, which shrinks the sample estimate toward the practitioner's prior view by including elliptical uncertainty on both location and scatter parameters of a normal market. That was done with a simplistic sample-based allocation, which estimated the market parameters without processing inputs from the investor. The market was represented by industry sectors of the S&P 500. The results are in Figure 3.8.

3.3 Discussion

There are several points for the discussion. Generally, robust statistics (i.e. Huber M-means) resist outliers. M-means provide better estimations and close solutions.

All the approaches (Bayesian, Black-Litterman, Robust and Robust Bayesian) demonstrated their abilities to improve or completely avoid the weaknesses of the MVM. In Figure 3.6, the advantages of the Bayesian, Robust MV and Robust

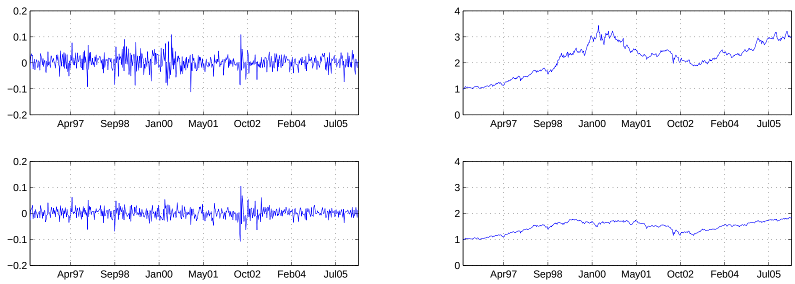

Figure 3.8: Robust Bayesian approach on the S&P 500 – sample and robust returns (top and bottom left). The sample and robust returns of the portfolio over time (top and bottom right) over the period. The robust samples are less volatile but in this particular example they appreciated less in value than the regular data sample.

Bayesian approaches were sufficiently demonstrated. The approaches headed towards a low variance and the best risk-reward ratio.

Nevertheless, robust optimization methods can decrease returns of the invested capital as shown in Figure 3.8. This needs to be dealt with accordingly.

Optimization software and rich statistics were new for us. Therefore we felt that we only possessed a superficial knowledge of the topic. The topic is rich, complex but applicable and stimulative.

3.4 Summary

The chapter has explained the weaknesses of the MVM and corrective approaches. The corrective approaches include robust statistics, mitigation of the MVM weaknesses, "robustification" of the MVM and the RMVM (Robust Mean-Variance Model).

There were four elemental approaches to deal with the MVM issues. These are Bayesian, Black-Litterman, Robust and Robust Bayesian approaches. The first two are mitigating ones and the last two are robust ones. The robust ones are determining the best allocation in the presence of estimation risk. There were described several experiments to demonstrate the ability of the approaches.

The portfolio management application that compared sample and robust returns of the S&P 500 industries demonstrated the decrease of the volatility and drawdowns. This result supports the empirical and theoretical evidence [Meu10] that the robust portfolio limits drawdowns, improves worst-case performance and provides stable returns in the long-term horizons.

Chapter 4

Equity Classification

In this chapter we describe our analysis of equities via explorative techniques such as cluster analysis (CA) or principal component analysis (PCA). The equities were from the DAX index. The explorative analysis worked with technical as well as fundamental parameters of the equities.

The notion of a "cluster" varies between methods and is one of the many decisions to make when choosing the appropriate algorithm for a particular problem. At first the terminology of a cluster seems obvious: a group of data objects. However, the clusters found by different algorithms vary significantly in their properties, and understanding these "cluster models" is key to understanding the differences between the various algorithms [Har75].

The remainder of the chapter[1] is structured as follows: **Section 4.1** gives details on cluster analysis and portfolio selection. **Section 4.2** gives a short overview of the financial data and introduces the data and some preliminary statistical tests. **Section 4.3** specifies the details of cluster analysis used in the investigation. **Section 4.4** and **Section 4.5** presents the obtained empirical analysis and results. Finally, **Section 4.6** summarizes the results and concludes the most important findings.

4.1 Background

4.1.1 Cluster Analysis And PCA

Cluster analysis or clustering is the task of assigning a set of objects into groups (called *clusters*) so that the objects in the same cluster are more similar (in some sense or another) to each other than to those in other clusters [Har75]. The intuitive and explanatory example of clustering is in Figure 4.1.

Clustering is a main task of explorative data mining [Har75], and a common technique for statistical data analysis used in many fields, including machine learn-

[1] This chapter was rewritten into a conference paper [Kon14].

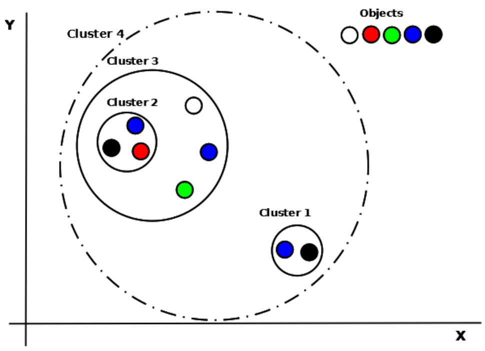

Figure 4.1: A cluster diagram. This is a simple example of cluster analysis of circular objects in a 2D space. There are 8 objects and 4 clusters available. Steps of the clustering process are also usually depicted by a dendrogram.

ing, pattern recognition, image analysis, information retrieval, econometrics [Bal11], finance [Bro08, Wil07] and bioinformatics.

Cluster analysis [Har75] itself is not one specific algorithm, but the general task to be solved. It can be achieved by various algorithms that differ significantly in their notion of what constitutes a cluster and how to efficiently find them. Popular notions of clusters include groups with low distances among the cluster members, dense areas of the data space, intervals or particular statistical distributions. Clustering can therefore be formulated as a multi-objective optimization problem. The appropriate clustering algorithm and parameter settings (including values such as the distance function to use, a density threshold or the number of expected clusters) depend on the individual data set and intended use of the results. Cluster analysis as such is not an automatic task, but an iterative process of knowledge discovery or interactive multi-objective optimization that involves trial and failure. It is often necessary to modify preprocessing and parameters until the result achieves the desired properties.

Principal Component Analysis (PCA) [Jol02] is a statistical procedure that uses orthogonal transformation to convert a set of observations of possibly correlated variables into a set of values of linearly uncorrelated variables called *principal components*. Usually, it has been used to decrease a dimensionality of the input data and to provide the order of the highest significant components in the explored set.

4.1.2 Portfolio Selection

A portfolio manager of a fund picks investment instruments into a portfolio. The instruments are selected with technical, fundamental and hybrid analytical approaches. Portfolio managers make use of quantitative heuristics, applied statistics and numerical computing in their decision making.

Therefore our elementary idea was to use cluster analysis in portfolio management. Due to several resource constraints, the task was a bit simplified. We focused on a small sized index of 'Large Cap' companies, which of the financial data were freely available and could be used in the exploration via cluster analysis. The results should have provided the classification of companies into clusters and revealed the similarity of companies in a cluster. The portfolio manager could use this approach as an analytical or screening tool of potential investments.

4.2 Data Description

The financial data were of German large companies in the DAX index. The DAX has been an industry indicator of the national economy in Germany. The DAX (also as the DAX 30) consists of 30 companies with the largest market capitalization and the national importance. The DAX floated around $7,074$ points at the time of data retrieval.[2] The weighting of companies is based on the free float market capitalization, i.e. on the freely tradable shares and not on the entire market value of a company.

Stated briefly, the DAX is a German 'Large Cap' index. The index consists of 30 blue chips companies such as SIEMENS AG, SAP AG, BASF SE, DAIMLER AG, BMW AG, ALLIANZ SE, DEUTSCHE TELECOM AG, E.ON AG, DEUTSCHE BANK AG and so on. A complete table of the companies and their industries are placed in Appendix C (Table C.1).

Descriptive statistics, technical and fundamental parameters of equities were employed. The technical parameters were Market Capitalization (MCap, Mio EUR), Beta (β), Volatility (Vol) and Momentum (Mom, Moment) in the last 250 days. The fundamental parameters were Price To Earnings (PE), Price to Book (PB), Dividend Yield (DY) and Return On Equity (ROE) (last two in percents).

These statistical variables were retrieved from financial data sources ([`DB`, `DI`]). Then the financial data were processed automatically, standardized[3] and inserted into three test files (`tech, fund, tech+fund`). The summary statistics of our financial data before scaling are placed in Table 4.1.

4.3 Methodology

Our motivation was to find out if cluster analysis on technical and fundamental data of companies would be helpful in the investment process. The empirical analysis was conducted via one descriptive statistics and cluster analysis. The basic idea was that

[2] This was in 21.8.2012 at 11:00 CET. Since then, the DAX has advanced to $9,644$ points in 25.7.2014 and crossed the $10,000$ level three times in 2014.

[3] It was with the embedded R function `scale()`.

the companies with the same descriptive statistics should create clusters together. The technical and fundamental data needed to be standardized before clustering.

There is a number of different clustering methods available. Ward's minimum variance method aims at finding compact, spherical clusters. The Ward Hierarchical Clustering method was run with the Manhattan distance (`manhattan, ward`). The largest data set was also run with another set of clustering parameters (`euclidian, complete`). The final experiments employed K-Means clustering method.

Our computation scripts were a modification of the original R script.[4] The scripts were redesigned to ensure required calculations, diagrams and charts. Data statistics and cluster analysis were calculated in the R software package version 2.15.1.

4.4 Cluster Analysis

The additional empirical results are also described in Appendix C. Here, we summarize the most important results.

The number of clusters was set to 6 due to the industry segregation of the DAX. The industry groups were set as: (1) Finance; (2) Services and Transport; (3) Stores, Telecom and Utilities; (4) Apparel, Cosmetics and Medical Equipment; (5) Autos and Machinery; and (6) Chemicals, Energy, Materials and Semiconductors.

Clustering of technical parameters is shown in Figure 4.2 (left). Figure shows a dendrogram which is a hierarchical graph of the clustering method. The clustering of fundamental parameters is described in Figure 4.2 (right). We clustered technical and fundamental parameters together in Figure 4.3 (left and right).

We assume that the best results are from the very last clustering. This is because the companies from related industries often finished in the same cluster. Two different clusterings (`euclidian, complete`) were utilized in the experiments. However, the clustering methods did not classify the companies as uniformly as expected, the classification process was very explorative in our opinion.

[4] `http://www.statmethods.net/advstats/cluster.html`

Technical and fundamental parameters

	MCap	β	Vol	Mom	PE	PB	DY	ROE
Min.:	4.56	0.36	20.35	0.62	5.42	0.27	0.00	−12.23
1st Q_X:	8.24	0.75	28.08	1.02	8.46	1.08	1.78	4.59
Median:	19.84	0.93	36.53	1.23	12.62	1.75	2.79	12.23
Mean:	25.92	0.97	35.71	1.18	13.39	1.79	3.29	11.25
3rd Q_X:	39.73	1.19	41.53	1.34	16.41	2.32	5.07	15.87
Max.:	68.22	1.65	66.63	1.53	27.61	4.89	7.39	31.22

Table 4.1: Technical and fundamental parameters of the DAX companies.

In all dendrograms one can see that COMMRZB and DBANK, FMC and FSE, BASF and BAYER are grouped together. These companies operate in the same or the related industry. In the last dendrogram one can detect groups such as (COMMRZB, DBANK), (FMC, FSE, HENKEL, BRSDRF), (EON, RWE, DTEL, METRO), (BMW, DAIMLER, VW, SIEMENS) and (BASF, BAYER). In our assumption, the resultant clusters of the companies are related to the economic cyclicality and fundamentals. There are some more examples of the smooth classification, see for example (DLUFT, THSSKRP) or (ALLIANZ, MUNCHRE).

It needs to be emphasized that the quality of classification improved notably after standardization of the input data. To achieve a better classification a larger set of parameters would have been required. It is presumed that something between $15-25$ weighted parameters for each company would be satisfactory.

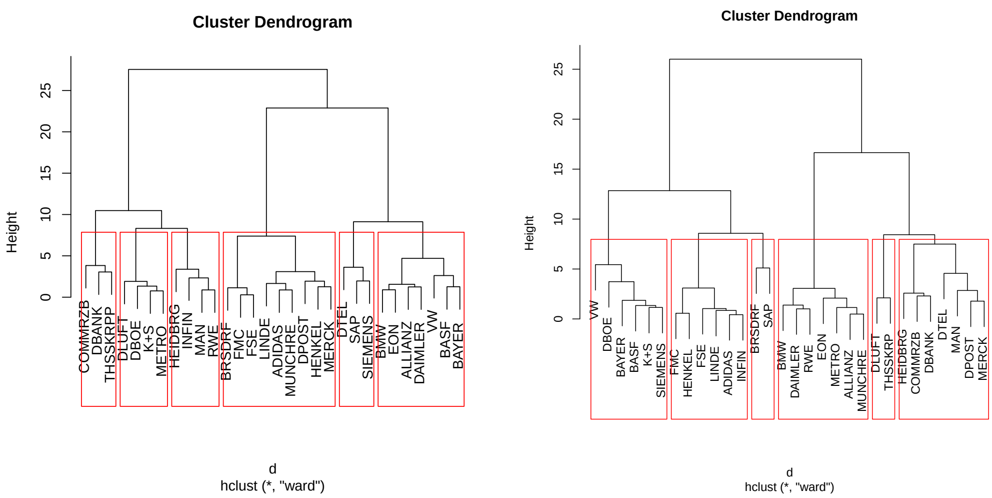

Figure 4.2: Dendrogram (left) for technical parameters with cluster numbering $[3,(4),6,(1),(5),2]$ and clustering parameters (`manhattan, ward`). Dendrogram (right) for fundamental parameters with cluster numbering $[(\mathbf{3}),(1),4,(2),6,5]$ and clustering parameters (`manhattan, ward`). Clusters in brackets are fundamentally strong and the best one is in bold.

Matrix visualizations of the parameters interaction are shown in Figures C.1 (technical) and (fundamental), C.2 (technical and fundamental). There are strong dependencies between Beta and Volatility and also some between PE/PB, PB/ROE, PE/DY, and PB/DY. These dependencies confirm scientific references regarding the relation between technical, fundamental statistics and growth/value stocks.

There are statistic means calculated in Tables 4.2, 4.3 and 4.4. These are statistics inside of each cluster after the clustering method finished. The enumeration of a cluster in the table relates to the bracket number in the matching dendrogram.

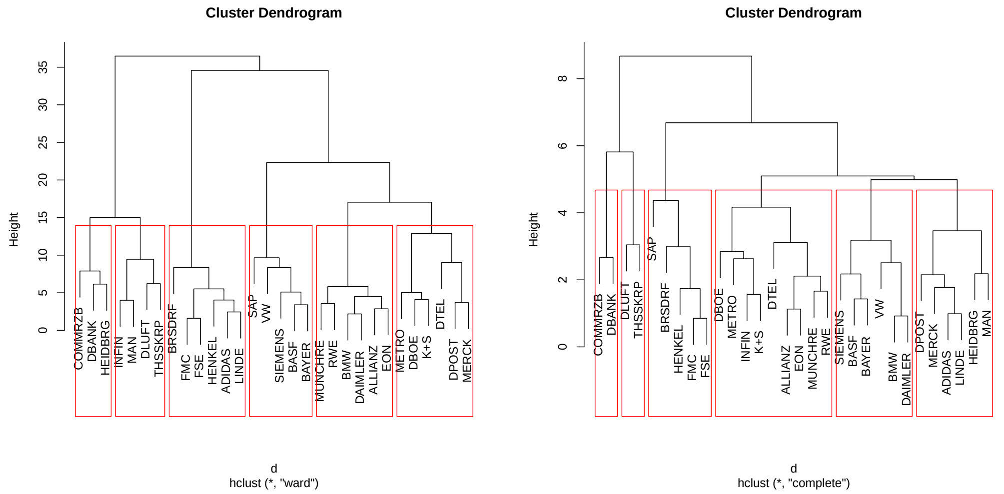

Figure 4.3: Dendrogram (left) for both fundamental and technical parameters with cluster numbering $[4, 6, 1, (\mathbf{3}), (2), (5)]$ and clustering parameters (`manhattan, ward`). Dendrogram (right) for both fundamental and technical parameters with cluster numbering $[5, 6, (4), (2), (\mathbf{3}), 1]$ and clustering parameters (`euclidian, complete`). Clusters in brackets are fundamentally strong and the best one is in bold.

Two main principal components are depicted in Figures C.3 (`tech`, [left]),(`fund`, [right]) and (`fund+tech`, [bottom]). These two components explain 84.46%, 78.64% and 65.13% of the data variability as is shown in the figures, respectively.

4.5 Results and Discussion

After the experiments the achieved results are evaluated and discussed. It is provided some reasoning behind the results. First, there are some facts behind the parameters.

1. The cluster analysis using only technical parameters is not reliable to construct a long-term portfolio. This kind of clustering may suit to day, high-frequency and algorithmic traders.

2. The clustering with fundamental parameters as well as both technical & fundamental parameters is suitable for the long-term portfolio.

3. The set of parameters was not taken very large and the parameters were equally weighted. We assume that more parameters and 'intelligent' weights can provide even better, more intuitive and reliable results.

Let us overview the results. There are clusters evaluated on fundamental metrics (low PE, PB, high DY and ROE). In Table 4.2 [left], the clusters (1), (2) and (5) might be suitable. Assets in the clusters (1), (2) and (**3**) look very good for a robust portfolio in Table 4.2 [right]. The clusters (2), (**3**) and (5) resp. (2), (**3**) and (4) are good candidates for a robust portfolio in Table 4.3 resp. 4.4.

#	MCap	β	Vol	Mom	#	PE	PB	DY	ROE
(1)	17.09	0.63	25.98	1.36	(1)	16.61	2.20	1.69	14.53
2	47.96	1.18	38.29	1.26	(2)	8.39	1.15	5.40	10.33
3	13.54	1.52	55.67	0.78	(**3**)	10.96	2.27	3.72	21.86
(4)	7.13	0.82	37.71	0.91	4	22.28	4.53	1.54	19.56
(5)	57.64	0.79	26.05	1.15	5	11.83	1.17	2.87	2.56
6	11.43	1.20	43.35	1.25	6	25.05	0.83	2.66	−5.08

Table 4.2: Means in each cluster for technical parameters (`manhattan, ward`) [left]. Means in each cluster for fundamental parameters (`manhattan, ward`) [right]. Clusters in brackets are (may be) fundamentally strong and the best one is in bold.

#	PE	PB	DY	ROE	MCap	β	Vol	Mom
1	18.52	2.60	1.58	13.16	17.88	0.57	23.99	1.31
(2)	8.27	1.11	5.39	10.66	34.01	1.14	38.55	1.23
(**3**)	11.92	2.66	3.05	22.89	61.12	1.00	31.66	1.30
4	8.54	0.43	1.38	3.10	13.28	1.55	56.15	1.01
(5)	12.04	1.90	4.66	11.21	14.99	0.82	33.24	1.12
6	20.88	1.38	2.43	0.89	07.68	1.05	42.48	0.98

Table 4.3: Means in each cluster for fundamental and technical parameters (`manhattan, ward`). Clusters in brackets are fundamentally strong and the best one is in bold.

#	PE	PB	DY	ROE	MCap	β	Vol	Mom
1	14.72	1.67	2.34	5.95	13.35	0.96	33.96	1.37
(2)	10.76	1.52	5.18	11.75	21.07	0.97	37.36	1.09
(**3**)	9.78	1.83	3.84	19.25	54.59	1.12	35.85	1.23
(4)	19.14	3.35	1.42	16.83	26.99	0.53	22.26	1.35
5	6.05	0.36	1.38	3.27	16.10	1.59	60.46	0.80
6	25.05	0.83	2.66	−5.08	6.50	0.99	42.05	0.83

Table 4.4: Means in each cluster for fundamental and technical parameters (`euclidian, complete`). Clusters in brackets are fundamentally strong and the best one is in bold.

Based on the results above, the clusters optimized and grouped fundamentally provide coherent groups of stocks to select from. The promising groups of stocks are: i) BASF, BAYER ii) BMW, DAIMLER, SIEMENS, VW, iii) ALLIANZ, MUNCHRE and iv) DTEL, EON, RWE. These stocks delivered acceptable operating results and robust performance over the period from September 2012 to July 2014.

Our original idea was to use one of the data explorative methods (such as cluster analysis) in portfolio management. At that time, while compiling experimental data sets, programming, testing and analyzing the results, we assumed that the idea had been original, innovative and evolutionary. However, during writing this chapter and reading related research publications, we realized that the idea was not a new one as we had assumed and that there were several published references about cluster analysis in the process of portfolio management [Har75]. There are no available references related to what we did exactly, but there are some published references[5] in the domain of portfolio analysis, construction and optimization.

4.6 Summary

Cluster analysis or clustering is the task of assigning a set of objects into groups (called clusters) so that the objects in the same cluster are more similar (in some sense or another) to each other than to those in other clusters [Har75].

Portfolio managers have employed many quantitative, statistical and data exploratory techniques to build better (robust) portfolios. The possible set of methods and techniques is wide, therefore we selected the cluster analysis as an illustrative example.

The empirical results showed that this type of clustering is very explorative and the achieved results were satisfactory. On the other hand more descriptive parameters of each classified company would have delivered results of the higher quality.

The practical use of the clustering method could be exploited in portfolio analysis, construction and optimization. One should select companies into a portfolio from different clusters to avoid over-concentration of the same or similar industry risk. One can recognize if two companies are correlated as well as share similar risk profiles.

Clustering might be one of the possible explorative techniques in a quest for robust portfolios. The cluster analysis provides sufficient insight into potential building blocks (equities) of the robust portfolios. Fundamental, technical, combination of the both and even behavioral parameters can be included in the clustering process.

[5] e.g. `http://post.nyssa.org/.../cluster-analysis-as-a-funds-of-hedge-funds`

Chapter 5

On Portfolio Robustness

The chapter is about robustness, specifically, about portfolio robustness, drawdowns and risk-return measures of a portfolio. *Robustness* [Wil07] is defined as "the ability [of a system] to resist change without adapting its initial stable configuration".

Robustness is a characteristic that is desirable in many scientific fields such as robust statistics[1] [Hub04, HRRS05], optimization, decision making and control [MMY06]. For example, a control system has to provide or ensure the stability and robustness of outputs. Although input data to the system are delivered with outliers, inconsistencies, disturbances and even errors. From that perspective, we can view an investment portfolio as the control system that reacts on positive and negative incoming market signals from the economy and capital markets. The signals could be changes in interest rates, exchange rates, commodity prices, bond prices, retail sales, data regarding slowdowns or recessions, macro data and so on. The robust portfolio has to be intelligently constructed. The idea is to control the portfolio as a control system where control actions are portfolio trades. Clearly, it is desirable to make minimum portfolio changes (trades) to keep overhead costs low.

Robustness of a portfolio [Wil07] can be measured with several metrics such as the largest drawdown, the frequency of drawdowns per a specified period, portfolio sensitivity and volatility, period of the full recovery from the drawdown and several risk-return ratios. The drawdowns, time to recovery and risk-return ratios provide feedback on the portfolio robustness in historical financial data.

The remainder of the chapter is structured as follows: **Section 5.1** gives details on robustness, robust portfolio construction, drawdowns and risk-return ratios. **Section 5.2** is about experimental verification. **Section 5.3** adds some thoughts on robustness in capital markets. **Section 5.4** summarizes the results and concludes the chapter.

[1]Robust statistics are statistics with good performance for data drawn from a wide range of probability distributions, especially for distributions that are not normally distributed. Robust statistical methods have been developed for many common problems, such as estimating location, scale and regression parameters.

5.1 Background

This section explains more on robustness of a portfolio, mainly on drawdowns, reward-to-variability ratios and robust portfolio construction. As already mentioned, some measures such as drawdowns, time to a full recovery and risk-return ratios provide feedback on the portfolio robustness in the context of historical as well as current financial events.

We argue that portfolio robustness is essential and crucial from the several points of view. Robust portfolios are frequented by investors during market crashes and recessions, when capital markets are volatile, panicky and downtrending. Robust portfolios provide not only hedging capabilities but also low operational costs due to infrequent adjustment trades. Robust portfolios combine advantages of investable ETFs and value stock-picking. On top of that when constructed correctly, the robust portfolios are able to adapt to changes in the economy and smooth-out business cycles.

The domain of robust portfolios is well published, but one needs to select carefully to reach for an outstanding piece of research or publications. We recommend and appreciate the following references [FFK07, FKPF10, FHZ10, RS09] but there are plenty more elsewhere. The first reference [FFK07] is more general about the robust portfolio theory and management but the second one [FKPF10] is more focused on robust equity portfolios and quantitative equity investing.

In the next three subsections, we share and explain ideas on drawdawns, reward-to-variability ratios and robust portfolio construction.

5.1.1 Portfolio Drawdowns

The definitions are derived, merged and adjusted from several sources. The main important references are [FHZ10, FFK07, FKPF10, Wil07].

Generally, a portfolio drawdown (or a drawdown decline) is the measure of the decline from a historical peak in some variable (typically the cumulative profit).

More descriptively, a drawdown is measured from the time a retrenchment begins to a new low is reached. It is important to note that a valley cannot be measured until a previous peak is regained. Once the previous peak is reached, the percentage change from the old peak to the lowest trough is recorded. See in Figure 5.1.

The peak-to-trough is a decline during specifically recorded period of an investment, fund or commodity. A *drawdown* is usually quoted as the percentage between the peak and the trough.

Drawdowns help to determine an investment's financial risk. Both the Calmar and Sterling ratios[2] [Wil07] use this metric to compare a security's possible reward

[2] The Calmar ratio is the annualized return for the last t years divided by the maximum drawdown during these years. The Sterling ratio is the annualized return for the last t years divided by the

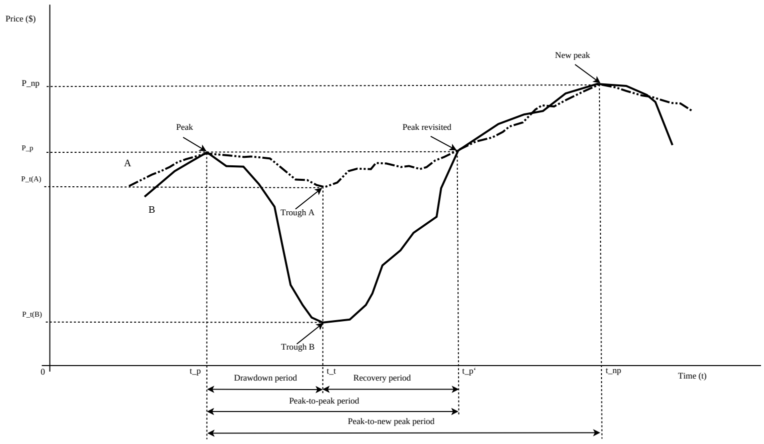

Figure 5.1: A drawdown explained. It shows a description of the drawdown of A and B portfolios. There are depicted the previous peak, trough, revisited peak and the new peak (new high).

to its risk. Intuitively, a portfolio A with smaller or less frequent drawdowns is better than a portfolio B with larger or more frequent ones. Therefore investors prefer the portfolio A to B (CR: $A \succeq B$, $t \in [0, T]$) in Figure 5.1.

5.1.2 Reward-To-Variability Domain

The robust portfolios repeatedly deliver *high reward-to-variability* ratios (such as Sharpe, Sortino or Treynor ratios [Wil07]). The most popular ratio is the Sharpe ratio (SR^3) which is defined as,

$$SR(\mathbf{x}) = \frac{\mu^T \mathbf{x} - r_f}{\sqrt{\mathbf{x}^T \mathbf{\Sigma} \mathbf{x}}}, \tag{5.1}$$

where μ is the return vector on the strategy over a specified period, r_f is the risk-free rate over the period and $\sqrt{\mathbf{x}^T \mathbf{\Sigma} \mathbf{x}}$ is the standard deviation of returns. The Sharpe ratio is quoted in annualized terms. The high Sharpe ratio highlights a good risk-return investment strategy. As a rule of thumb, the Sharpe ratio between $0.30 - 0.48$ is a long-term average of the S&P 500, some hedge funds have experienced the Sharpe ratios over 1.00.

The reward-to-variability ratio[4] is a measure of the excess return (or risk premium) per unit of deviation in a portfolio. Such characteristics of general asset

average of the maximum drawdown (in absolute terms) in each of the preceding 3 years, less an arbitrary 10%. An extra 10% is subtracted from the drawdown as one assumes that all maximum drawdowns will be exceeded. For more see [FKPF10].

[3]We note that it is better to see Sharpe ratio as time dependent $SR(\mathbf{x}, t) = \frac{\mu(t)^T \mathbf{x}(t) - r_f(t)}{\sqrt{\mathbf{x}(t)^T \mathbf{\Sigma}(t) \mathbf{x}(t)}}$, because the input variables are time dependent and fluctuate. This idea was not implemented.

[4]Other reward-to-variability ratios are enclosed for reference purposes in Appendix D.

portfolios are often appreciated in active management of mutual bond and equity funds, private equity and hedge funds.

5.1.3 Robust Portfolio Construction

We propose a portfolio construction with robust characteristics. First, we provide **key factors** that we assume are quite influential in the process of robust portfolio construction. Second, we describe an *allocation procedure*. The description of the *key factors* follows.[5]

1. *Long value stocks, short weak stocks, the relevant index or hold some cash* – The long-short strategy is one of many hedge fund strategies. The long-short hedge funds are stable (robust) during market weaknesses. A short side of a portfolio can be created with a dedicated short bias fund. Shorting is an aggressive and risky approach, so one can held some uninvested cash instead (e.g. 10-30% of the portfolio).
2. *Overweight less cyclical stocks, low beta stocks* – The less pro-cyclic stocks are often low beta stocks. These stocks tend to generate profits even during troubled times. The companies are from diverse industries such as health care, pharmaceutical, retail, telecommunication, utilities, tobacco, food and beverage sectors.
3. *The mix of growth stocks and defensive dividend stocks* – The intelligent mix of two types of stocks provides less volatility, defensiveness and growth. The mix of stocks can be adjusted based on the economic cycle.
4. *Blue chips* – Blue chips are high-quality stocks. They tend to be mature, multinational, large capitalization companies, dominant players in their industries and regular dividend payers.
5. *Stocks from developed markets (less or none from emerging or frontier markets)* – Generally, companies in developed markets are less volatile, stable and better managed than emerging and frontier market companies.
6. *Preference to 'Large Cap'* – Large capitalization companies tend to be more economically stable, established, profitable and less volatile than smaller capitalization companies.
7. *Managers* – Excellent managers of a company deliver stable, consistent and outstanding financial results. The manager ought to be a minority owner of the company and dependable on the delivered performance results.

[5] We see stocks as a proxy of a company, but we may use stocks and company interchangeably.

8. *Diversification (over capitalizations, industries, industry sectors, currencies, countries and continents)* – In academia, it seems that the term diversification is over-stated. Nevertheless, diversified portfolios over many diversification factors are more stable with some capacity for error.

9. *Regular rebalancing, position sizing and optimization* – The additional stability of portfolios could be improved with position trimming, rebalancing, sizing and allocation to riskless asset (cash). There are many portfolio management and robust optimization technigues to achieve this.

10. *Risk management and robust hedging* – The investment world is fairly coincidental and complex. There have been many periods (bubbles, recessions) when the financial markets did not behave efficiently. Correct risk management and robust hedging should provide an extensive support and protection during these times.

We describe an **RMV allocation procedure** how to construct the robust portfolio, practically. We can run an equity selection screener[6] or explorative analysis, so these help to find desired stocks for long and short sides. After the screener provides a list of prospective stocks, one has to handpick the best/worst stocks. It is expected that long positions of the portfolio are larger and less fragmented than short positions (long bias) if we do short-selling. The selected 'long' stocks are diverse and uncorrelated. Then we implement the *two-step robust MV allocation* procedure.

1. We estimate $(\hat{\mu}, \hat{\mathbf{\Sigma}})$ of the selected stocks and compute their weights $(\mathbf{x})$.

2. We reiterate the previous step and include investor's view $(\hat{\mu}_{BL}, \hat{\mathbf{\Sigma}}_{BL})$ on the selected stocks. Finally, we allocate the seed capital into the portfolio of investable assets.

Six times a year one checks whether any rebalancing is needed or any changes in positioning, positions, risk management and hedging are required. During the market stress, this might be done far more frequently (i.e. daily).

5.2 Experimental Verifications

Several experimental cases regarding portfolio parametrization, drawdowns and reward-variability ratios were set-up. Financial data were retrieved from Yahoo [YA] and Bloomberg [BL]. A new computation script was implemented in Financial toolbox of

[6] These investment screeners are freely available on the web (e.g. `http://www.finviz.com`). One selects descriptive, fundamental and technical parameters to get a group of adequate stocks with desired characteristics.

Matlab R2011a. The Financial toolbox offered a lot of the functionality for our numerical and financial analysis. Three investable securities created an equally weighted portfolio and were compared against the benchmark index (the S&P 500). The size of drawdowns and reward-variability ratios were calculated with developed computation scripts.

5.2.1 Portfolio Parametrization

The basic setup of an experimental equity portfolio gave an equal weighting to three securities. These securities, namely Coca-Cola (KO), Procter&Gamble (PG) and IBM (IBM), are U.S. stalwarts dominating the international markets. Their basic performance characteristics are in Table 5.1. Based on the previous hints, these stocks are the building blocks for our built robust portfolio.

Basic Metrics

Stock	Return (μ)	Std (σ)
SP500	0.0599	0.1944
KO	0.0636	0.1786
PG	0.1006	0.1678
IBM	0.1259	0.2669

Table 5.1: Basic performance metrics. The statistics of selected securities were calculated for the 1.1.1995-2.1.2014 period.

5.2.2 Drawdowns

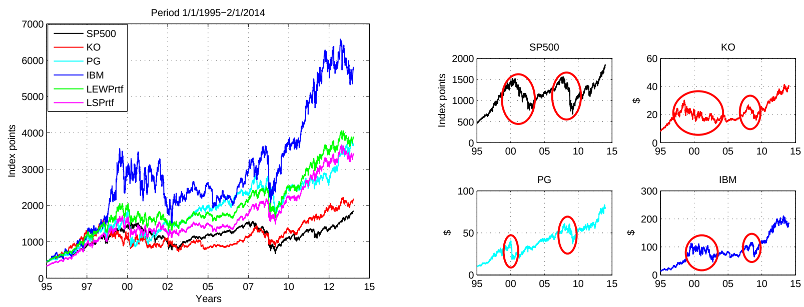

Figure 5.2: Drawdowns. There are depicted the S&P 500, Coca-Cola (KO), Procter&Gamble (PG), IBM (IBM), Long-Equally-Weighted (LEW) and Long-Short (LS) portfolios for the 1.1.1995-2.1.2014 period (left). There are significant price drawdowns of the selected securities over the period (right).

We analyzed and computed drawdowns. In Figure 5.2, there are time series of the selected equities, price-adjusted towards the index, to see their drawdowns and maximum drawdown period.[7] The maximum drawdown periods are in Table 5.2. As one can see, the maximum drawdown (MaxDD) experienced the IBM, the longest drawdown period (DDp) is the result of the Coca-Cola Company and the robust portfolio (LSPrtf) experienced the lowest drawdown. There are estimated forecasts regarding drawdowns – geometric MaxDD (MaxDDG) and the estimation of geometric MaxDD (EMaxDDG). The EMaxDD for IBM seems questionable (107.37%).

Drawdowns

Stock	MaxDD[%]	DDp[days]	MaxDDG[%]	EMaxDDG[%]
SP500	56.78	355	83.88	74.32
KO	55.26	**1169**	80.44	95.95
PG	54.23	41	78.15	86.98
IBM	**59.36**	815	90.04	107.37*
LEWPrtf	41.30	773	77.87	93.76
LSPrtf	34.55	773	**76.66**	**89.22**

Table 5.2: Drawdowns and maximum drawdown periods. The statistics were calculated for the 1.1.1995-2.1.2014 period.

5.2.3 Reward-To-Variability Ratios

In this subsection, apart from the Sharpe ratio, we also included Information ratio and Tracking error. The last two are partly described here and fully explained in Appendix D. In Table 5.3, there were calculated the Sharpe ratios for all our securities, the index as well as the robust portfolios. We see that the robust portfolios experienced the highest Sharpe ratio over periods. The lowest Sharpe ratio is for the index but the shortest period contradicts. Our reasoning for low Sharpe ratio is that it is due to the over-diversification effect of the index.

Information ratios are calculated in Table 5.4. Information ratio is an active return divided by tracking error. It has been used to gauge the skill of active portfolio managers. The higher the Information ratio, the higher the active return of the portfolio, and the better is the manager. One can see that the risky assets and both portfolios underperformed the tracking index in the last 3 and 5 years.

Tracking errors are included in Table 5.5. Tracking error is a measure of how closely a portfolio follows a benchmark index. Passively managed portfolios minimize tracking error. On the other hand actively managed portfolio would normally have a higher tracking error. In Table 5.5 Tracking error decreases as the time period shortens. In our case that is not favorable because we are active portfolio managers.

[7] Maximum drawdown period (DDp) is a period of the largest loss from a previous high in the given time period.

Sharpe ratios

Stock	3y	5y	10y	15y
SP500	2.59	1.22	0.15	0.10
KO	1.98	**2.98**	0.52	0.20
PG	1.37	1.40	**0.59**	**0.30**
IBM	0.14	0.92	0.33	0.14
LEWPrtf	2.64	2.22	0.48	0.23
LSPrtf	**2.81**	1.65	0.53	0.26

Table 5.3: Sharpe ratios. The Sharpe ratio was calculated on different time intervals for the securities, LEW and LS portfolios. The data were calculated for the date 2.1.2014.

Information ratios

Stock	3y	5y	10y	15y
SP500	N/A	N/A	N/A	N/A
KO	-3.57	**-0.11**	**0.51**	0.23
PG	-2.85	-0.43	0.18	0.22
IBM	**-1.80**	-0.12	0.21	0.15
LEWPrtf	-2.51	-0.16	0.28	**0.24**
LSPrtf	-2.47	-0.16	0.28	**0.24**

Table 5.4: Information ratios. There are the Information ratios on the different time intervals for the picked securities, LEW and LS portfolios. The data were calculated for the date 2.1.2014.

Tracking errors

Stock	3y	5y	10y	15y
SP500	**0.00**	**0.00**	**0.00**	**0.00**
KO	0.02	0.08	0.11	0.14
PG	0.02	0.05	0.14	0.17
IBM	0.09	0.18	0.19	0.21
LEWPrtf	0.05	0.12	0.13	0.14
LSPrtf	0.06	0.14	0.15	0.17

Table 5.5: Tracking errors. There are depicted the Tracking errors against the S&P 500 on different time intervals for the picked securities, LEW and LS portfolios. The data were calculated for the date 2.1.2014.

5.3 Experience With Robust Portfolios

5.3.1 Value-Oriented Portfolios

Not surprisingly, core portfolios of many value-oriented investors[8] are coherent with the robust investment thesis. These investors are managers of retail funds, hedge funds, insurance companies and special purpose vehicles. During past capital market drawdowns, their portfolios performed from fairly to extremely well.

5.3.2 Warren Buffet's Portfolios

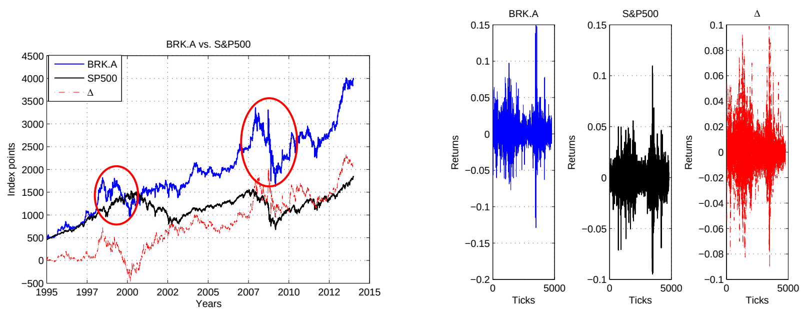

Figure 5.3: Drawdowns and returns – BRK.A vs. the S&P 500 for the 1.1.1995-2.1.2014 period (left) and daily returns (right).

Warren Edward Buffet[9] has constructed robust portfolios for many decades. He has been the Chairman and CEO of Berkshire Hathaway since 1965. Berkshire Hathaway, an investment and insurance holding, comprises of many diversified private and public U.S. businesses with North American as well as global exposure. After reviewing these results, we can state that Berkshire Hathaway (tickers BRK.A, BRK.B at NYSE) has been an extraordinarily successful compounder on a risk adjusted basis. The comparison results are shown in Figure 5.3 and in Table 5.6. Therefore, we can claim that Warren Buffet's investment strategy provides a guide to the robust portfolio construction.

5.3.3 Personal Experience

Since January 1999 I have been active as a private investor in the capital markets. Since then I moved from retail mutual funds to direct security investments, which seemed like a step getting me into a more risky game. The change was from investing with a trained and dedicated professional portfolio manager to investing on my own.

[8] Value-oriented investors looks for undervalued assets to buy and overvalued assets to sell or sell-short. This investment strategy has been popularized at Columbia Business School in the US since 1930s but needed to evolve due to technological and business advancements since 1970s.

[9] Warren Edward Buffet (*August 30, 1930 in Omaha, Nebraska) has been a U.S. business magnate, philanthropist and one of the greatest investors ever.

My investing skill set has improved and direct security investments have been less scary for me since then. During the bursting of the dot-com bubble and the Great recession, both quite disturbing capital market events, I was impacted but fared not as badly as the general investment public nor the global capital market indices. The core investment strategy has been a low turnover value-oriented strategy. The strategy used a low leveraged long-bias approach with securities from emerging and developed markets. Risk-adjusted returns of the general investment strategy has improved gradually. As my private experience suggests this investment approach looks consistent and robust with some minor setbacks over the economic cycle.

5.4 Summary

The chapter was about portfolio robustness, drawdowns of a portfolio and risk-return measures. The robustness of a portfolio can be measured with the largest drawdown, the steepness & frequency of drawdowns, time to a full recovery, risk-returns ratios and so on.

We ran several experiments on the financial time series of the three U.S. large cap stocks, the S&P 500 index and two constructed portfolios. The time period was from 1st January 1995 to 2nd January 2014. One could see clearly that the selected stocks experienced drawdowns as high as 59.36%. The long only portfolio provides drawdown of 41.30% magnitude. As well as expected, the long-short portfolio ensures lower drawdowns (34.55%), offers the lower upside in bull markets but provides the higher Sharpe ratio. The diversification of long investable assets and partially short-selling, limits the size of drawdowns and improves the portfolio performance.

The selected securities and experimental portfolios were tested against high risk-return ratios (Sharpe and Information). The long-short portfolio was the best among the portfolios. Low diversification portfolios provided superior ratios. The index suffers on over-diversification of the miscellaneous quality of enlisted companies.

We shared some thoughts and references on the robust portfolios in capital markets. There have been many investors who successfully built robust portfolios in the past.

Performance statistics

Stock	Return (μ)	Std (σ)	SR	IR	TE	MaxDD[%]
BRK.A	**0.0923**	**0.1582**	**0.5834**	0.2233	0.1447	**51.47**
SP500	0.0599	0.2000	0.3087	N/A	0.0000	56.78

Table 5.6: BRK.A vs. the S&P 500. There are performance ratios and statistics of BRK.A vs. the S&P 500. The BRK.A ticker is better. Calculated for the 1.1.1995-2.1.2014 period.

Chapter 6

Summary

Our work is summarized into the following seven paragraphs. Each of the following paragraph highlights the key investigation, result or contribution.

1. We familiarized ourselves with general and elementary ideas of the financial markets, assets and portfolios. We described basic features of the Mean-Variance Model (MVM), its weaknesses, the portfolio robustness and its basic features.

2. The MVM provides a simple, elegant and intuitive framework for portfolio construction and optimization. It offers to work with portfolio leverage, short-selling, correlated and uncorrelated assets, with various asset classes and so on. The MVM was tested with 3-stock portfolios against the capital market data for the 1995-2014 period. The asset estimation of expected returns and related covariance matrix is vital to the MVM. Small inaccuracies in the estimation of asset parameters lead to incorrect estimation results.

3. There were researched four elemental approaches such as Bayesian, Black-Litterman, Robust and Robust Bayesian approaches to deal with the MVM issue. There were described several experiments to demonstrate the ability of the approaches.

   This empirical results support the empirical and theoretical evidence that the robust portfolio limits drawdowns, improves worst-case performance and provides stable returns in the long horizons.

4. The equity classification as a tool of equity selection was proposed, analyzed and implemented. All included equities were comprised of the German DAX index. The empirical results showed that the clustering analysis is quite explorative and provides a significant fundamental insight for investment managers. On the other hand, results only based on technical parameters were vague,

more fundamental parameters of each classified company with correct setups are needed to achieve high quality results.

5. Robustness, portfolio robustness, drawdowns of a portfolio and risk-return measures were clarified. We confirm that the features such as drawdowns, their frequency and risk-return ratios provide indicative feedbacks on the portfolio robustness.

   From the experimental verification we can state that the long-short portfolios were the best among the portfolios during meltdowns and obviously, the long portfolio during bull markets. The diversification of investable assets, hedging and shorting, limits the size of drawdowns and improves portfolio performance.

   There are many investors who successfully have built robust portfolios for decades. Their approaches, methods and strategies do vary but their long-term performance results have been astonishingly consistent, up-trended and highly robust (W. Buffet, J. Hussman).

6. However, all our experiments finished as anticipated, we supply several ideas where we see inconsistencies and inaccuracies as well as the ideas for further improvements.

   The case studies and experiments were tested on the *ex-post* data. That means we worked with successful companies that survived temporary financial setbacks, but healed and improved themselves. The experiments also omitted transaction costs, fees, taxes, bid-ask spreads and trade-based factors such as a trade size, order type, trade timing, market liquidity and so on.

   While forming the long-term investment portfolio [Cip00, Hul08, Wil07] there are many other factors impacting overall portfolio returns and the covariance matrix such as a macroeconomic situation, exchange rates, capital market discrepancies, market volatility, bond vs. stock ratios, the situation in the commodity sector and so on.

7. One has to be able to identify promising companies (common stock, corporate bonds or warrants) to construct a robust portfolio (in the *ex-ante* manner). A possible methodology was proposed, where the key factors and the RMV allocation procedure were described. Our primary regional focus concentrated on developed markets (Germany and the USA), but these ideas are also applicable in emerging and frontier markets.

Generally, the robust optimization reduces a portfolio drawdowns, volatility, turnover and transaction costs, improves the worse-case performance and delivers more stable returns.

Chapter 7

Conclusion

The objective of the thesis was to investigate *the properties of robust portfolios.* The thesis was primarily focused on equities, but most of the results are applicable to other investment instruments with minor changes. The following paragraphs highlights the key contributions.

1. The thesis explains the MVM (Mean-Variance Model) and its sensitivity. The sensitivity is a product of estimation errors of $(\mu, \mathbf{\Sigma})$. This has been shown in the experiments.

2. The thesis shows mitigating and robust approaches to deal with the MVM issues. We demonstrated that the approaches limit drawdowns, improve worst-case performance and provide stable returns in the long-term horizons.

3. The thesis contributes to an asset selection for robust portfolios via cluster analysis. The classification was tested on elements of the DAX equity index. The clustering delivered a subset of the DAX equities suitable for a robust investment portfolio.

4. Drawdowns, drawdown periods and reward-to-variability ratios are important measures of the portfolio robustness. The thesis proposed the process of robust portfolio construction and its verification. The thesis also identified the robustness in portfolios of several well-known managers.

5. Nevertheless, we also argue that the robustness of portfolios is primarily dependent on manager's investment skills and his market insight. The robust approach is not simply applicable.

Even though portfolio theory and management have developed significantly over the years, we can still find many successful but insufficiently diversified and over-concentrated portfolios in the investment industry. This might be the research area for future work.

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Appendix A

General Facts

This part of Appendix is connected to Chapter 1 and chapters working with the S&P 500. We present details on Hussman[1] Strategic Growth Fund, the S&P 500 and the Dow 30.

A.1 Strategic Growth Fund

Hussman Strategic Growth Fund The Fund [HSF] seeks to achieve long-term capital appreciation, with added emphasis on the protection of capital during unfavorable market conditions. It pursues this objective by investing primarily in common stock, and uses hedging strategies to vary the exposure of the Fund to the general market fluctuations. The performance chart of the fund is in Figure A.1.

A.2 Standard & Poor's 500

The **S&P 500**, or the Standard & Poor's 500 [YA], is a stock market index based on the common stock prices of 500 top publicly traded American companies, as determined by S&P. It differs from other stock market indices like the Dow Jones Industrial Average and the Nasdaq Composite because it tracks a different number of stocks and weights the stocks differently. It is one of the most commonly followed indices and many consider it the best representation of the market and a bellwether for the U.S. economy. The National Bureau of Economic Research has classified common stocks as a leading indicator of business cycles. It is a free-float capitalization-weighted index. The index is maintained by Standard & Poor's, a division of McGraw-Hill that publishes a variety of other stock market indices such as the S&P 1500 and S&P Global 1200. The S&P 500 index has several ticker symbols: GSPC, INX, and SPX.

[1] John Hussman (*October 15, 1962) has been a stock analyst and mutual fund owner. He is a former professor of economics and international finance at the University of Michigan.

In Figures A.2, A.3 and A.4, we calculated some embedded index characteristics such as daily returns, volume, Autocorrelation function (ACF), Parcial Autocorrelation function (PACF) and forecasted returns and conditional variances. These are used when you investigate capital market time series. One can see the increase of volatility in the 2008-2009 period.

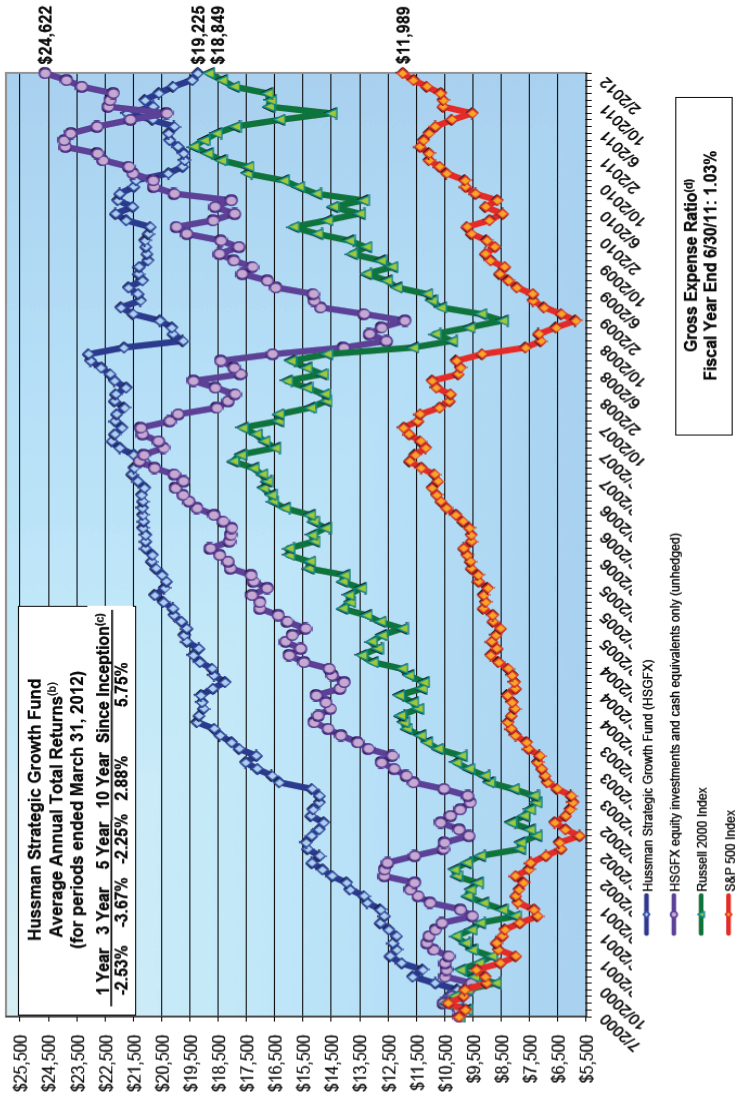

Figure A.1: This is a chart of achieved performance of the Hussman Strategic Growth Fund against the U.S. index equity benchmarks (the S&P 500, Russell 2000). The chart demonstrates the expected quality of the robust equity portfolio. Please, follow the HSGFX time series in the chart. The chart was extracted from the company web side (Hussman Funds, [HSF]).

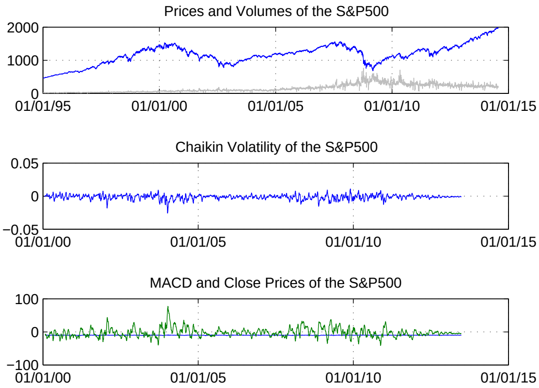

Figure A.2: Prices and Volumes (top), Volatility (middle) and MACD (bottom) of the S&P 500 during the 1.1.1995-25.7.2014 period.

A.3 Dow Jones Industrial Average

The Dow Jones Industrial Average [YA], also called the Industrial Average, the Dow Jones, the Dow 30, or simply the Dow, is a stock market index, and one of several indices created by Charles Dow[2]. It was founded on May 26, 1896, and is now owned by Dow Jones Indexes, which has its majority owned by the CME Group. The average is named after Dow and one of his business associates, statistician Edward Jones. It is an index that shows how 30 large publicly owned companies based in the

[2]Charles Henry Dow (*November 6, 1851 – †December 4, 1902) was an American journalist, Wall Street Journal editor and Dow Jones & Company co-founder.

Top 10 of the S&P 500

#	Ticker	Name	Sector	Weight [%]
1	AAPL	Apple Inc	IT	3.2
2	XOM	Exxon Mobil Corp.	Energy	2.47
3	MSFT	Microsoft Corp.	IT	1.79
4	JNJ	Johnson & Johnson	Health Care	1.69
5	GE	General Electric Co	Industrials	1.50
6	WFC	Wells Fargo & Co	Financials	1.44
7	CVX	Chevron Corp	Energy	1.42
8	BRK.B	Berkshire Hathaway Inc. Class B	Insurance	1.30
9	JPM	JP Morgan Chase	Financials	1.25
10	PG	Procter & Gamble Co	Cons. Staples	1.21

Table A.1: Top 10 constituents of the S&P 500 by index weight. Ticker, Name, Sector and Weight are included. Source: 25th July 2014 [YA].

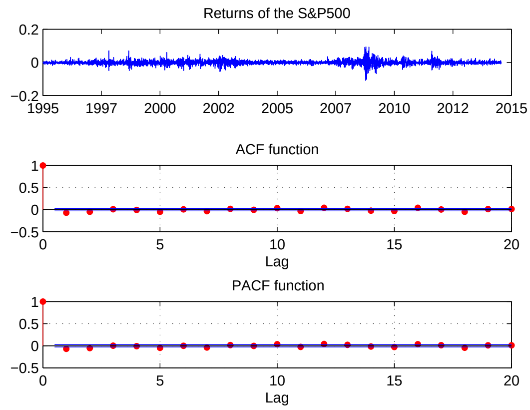

Figure A.3: the S&P 500 – Daily returns (top), Autocorrelation function (ACF, middle), Parcial Autocorrelation function (PACF, bottom).

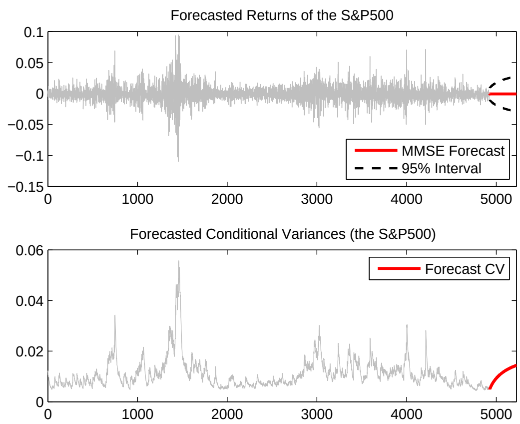

Figure A.4: the S&P 500 – Forecasted Returns (top) and Conditional Variances (bottom).

United States have traded during a standard trading session in the stock market. It is the second oldest U.S. market index after the Dow Jones Transportation Average, which was also created by Charles Dow.

The Industrial portion of the name is largely historical, as many of the modern 30 components have little or nothing to do with traditional heavy industry. The average is price-weighted, and to compensate for the effects of stock splits and other adjustments, it is currently a scaled average. The value of the Dow is not the actual average of the prices of its component stocks, but rather the sum of the component prices divided by a divisor, which changes whenever one of the component stocks has a stock split or stock dividend, so as to generate a consistent value for the index.

Appendix B

VaR and MVM

This part of Appendix is connected to Chapters 1 and 2 in relation to the VaR (Value-at-Risk) and the MVM (Mean-Variance Model).

B.1 VaR – Evaluate Market Risk

The market risk [MWS] of a hypothetical global equity index portfolio is modeled with a filtered historical simulation (FHS) technique. FHS, which has recently received much attention in the risk management literature, is an alternative to traditional historical simulation and Monte Carlo simulation approaches. FHS combines a relatively sophisticated model-based treatment of volatility (GARCH) with a nonparametric specification of the probability distribution of assets returns.

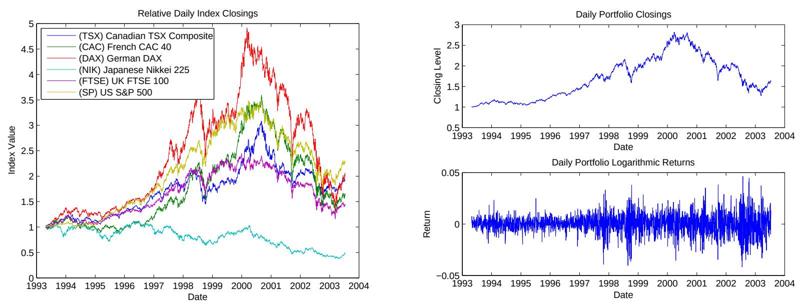

Figure B.1: VaR test. International Indexes – stock market indexies (left) vs. Log returns (right).

This demonstration first extracts the filtered model residuals and conditional volatilities from the portfolio return series with an asymmetric GARCH model from which the series of independent and identically distributed (i.i.d.) standardized residuals is formed. FHS retains the nonparametric nature of historical simulation by bootstrapping (sampling with replacement) from the standardized residuals. These bootstrapped standardized residuals are then used to generate time paths of future

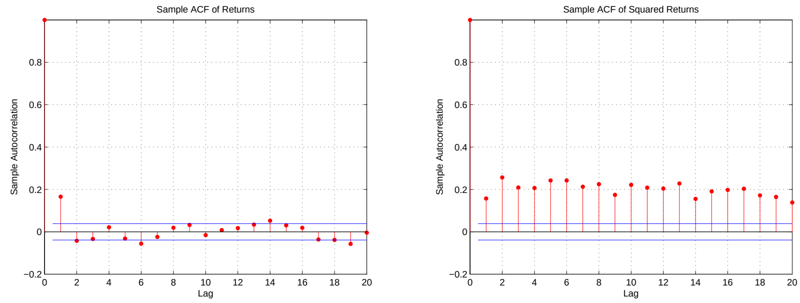

Figure B.2: VaR test. ACF (left) vs. ACFSR (right).

asset returns. Finally, the simulation assesses the **Value-at-Risk (VaR)** of the hypothetical global equity portfolio over a one month horizon.

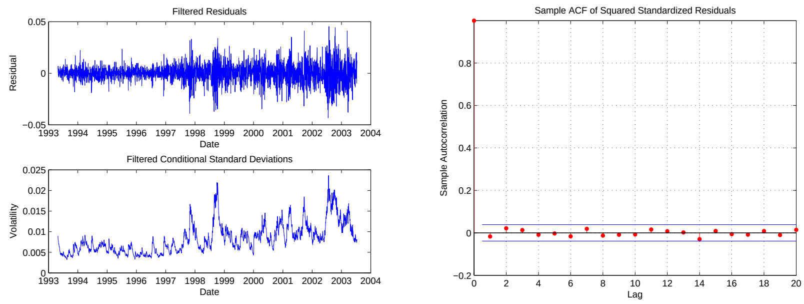

Figure B.3: VaR test. Returns, Volatility (left) vs. SACFSSR (right).

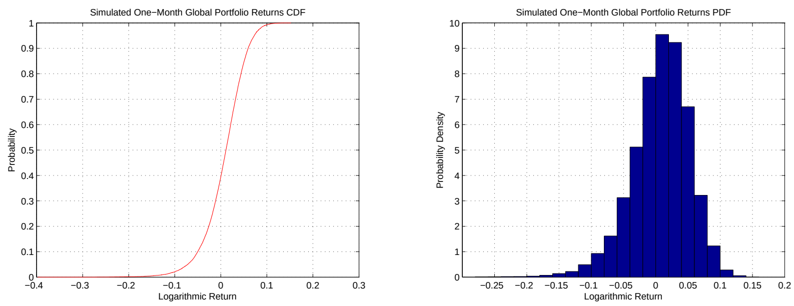

Figure B.4: VaR test. CDF (left) vs. PDF (right).

One of the appealing features of FHS is its ability to generate relatively large deviations (losses and gains) not found in the original portfolio return series. Now we only deal with a set of Figures depicting the procedure (Figures B.1, B.2, B.3 and B.4).

B.2 Experimental Results

There are additional results as figures from the MV model. Figures B.5 and B.6 present optimal capital allocation. The differences of risk aversion (RA) from 1 to 3.5 are presented. The lower number for RA means lower risks and lower expected return.

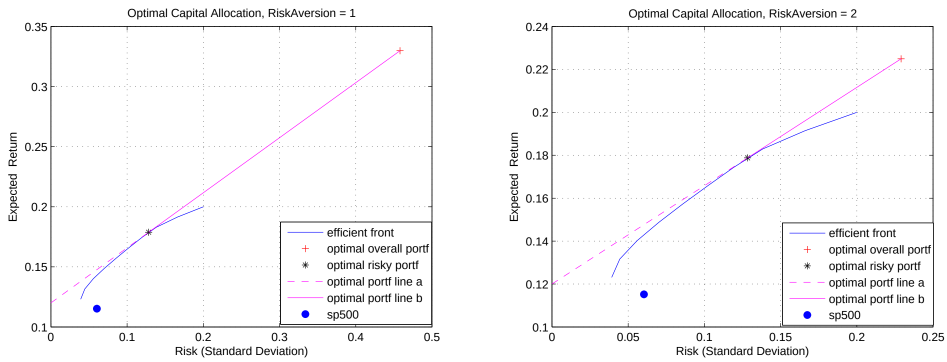

Figure B.5: Optimal Capital Allocation I. The differences of risk aversion (RA) in the presented setups such as RA = 1 (left) and RA = 2 (right).

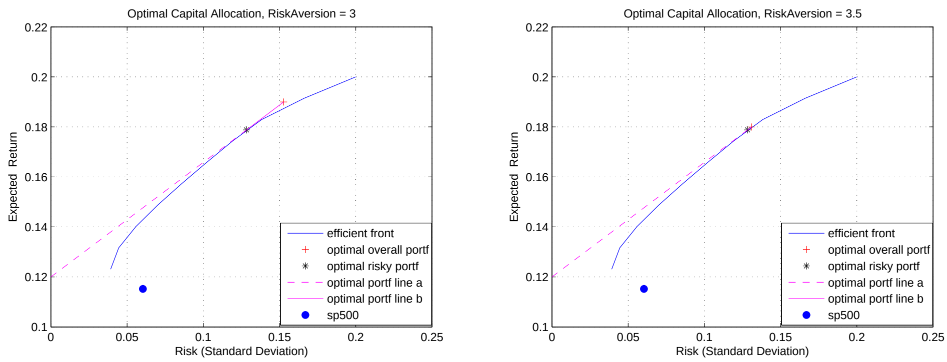

Figure B.6: Optimal Capital Allocation II. The differences of risk aversion (RA) in the presented setups such as RA = 3 (left) and RA = 3.5 (right).

Appendix C

Equity Classification

This part of Appendix supports Chapter 4 in relation to the equity classification. Two clustering methods, principal component analysis (PCA) and the experimental index are described. Then there are visual outputs of the clustering analysis and PCA from the experiments.

C.1 Clustering and PCA Algorithms

Hierarchical clustering [Har75] does not find a single partitioning of the data, but a hierarchy (represented by a tree) of partitionings which may reveal interesting structure in the data at multiple levels of granularity. Hierarchical clustering algorithms may be of two types, *agglomerative algorithms* look at ways of merging data points together to form a hierarchy, while *divisive methods* separate the data repeatedly into finer groups.
K-means [Har75] is a method for clustering a data set, $\mathcal{D} = \{x_1, x_2, \ldots, x_N\}$, of N unlabelled data points into K clusters, where K is specified by the user. The objective of the K-means algorithm is to minimize the following cost function V,

$$V = \sum_{i=1}^{K} \sum_{x_j \in C_i} (x_j - \mu_i)^2$$

where the C_i are each of the K clusters and μ_i are their respective cluster centers (means). The algorithm starts by randomly placing each of the K centers and assigning each data point to the cluster with the closest center. It then iteratively recalculates the cluster center based on the new assignments of data points to clusters, and then reassigns each data point, until convergence. This cost function has many local minima and several runs of the algorithm may be needed.
Principal Component Analysis [Jol02] is a statistical procedure that uses orthogonal transformation to convert a set of observations of possibly correlated variables into a set of values of linearly uncorrelated variables called *principal components*.

This transformation is defined in such a way that the first principal component has the largest possible variance (that is, accounts for as much of the variability in the data as possible), and each succeeding component in turn has the highest variance possible under the constraint that it is orthogonal to (i.e., uncorrelated with) the preceding components.

C.2 DAX Index

#	Company	Industry	Price [€]
1	ADIDAS AG	Apparel	63.43
2	ALLIANZ SE	Insurance	88.13
3	BASF SE	Chemicals	62.32
4	BAYER AG	Spec. Chemicals	62.66
5	BMW AG	Automotive	61.32
6	BEIERSDORF AG	Cosmetics	57.09
7	COMMERZBANK AG	Banks	1.24
8	DAIMLER AG	Automotive	41.68
9	D. BANK AG	Banks	26.17
10	D. BÖRSE AG	Finan. Services	42.15
11	D. LUFTHANSA AG	Transport	9.85
12	D. POST AG	Transport	15.75
13	D. TELEKOM AG	Telecom	9.47
14	E.ON AG O.N.	Utilities	18.10
15	FRESENIUS MED.	Medical Equip.	57.79
16	FRESENIUS SE & CO.	Medical Equip.	87.11
17	HEIDLBRGCMNT AG	Materials	40.51
18	HENKEL AG & CO.	Cons. Staples	60.50
19	INFINEON TECH. AG	Semiconductors	5.90
20	K+S AG	Chemicals	40.57
21	LINDE AG	Energy	123.10
*22	MAN SE	Automotive	76.04
23	MERCK KGAA	Pharma	90.38
*24	METRO AG	Stores/Retail	24.62
25	MUENCHENER RE AG	Insurance	119.05
26	RWE AG	Utilities	33.44
27	SAP AG	Software	52.66
28	SIEMENS AG	Machinery	75.44
29	THYSSENKRUPP AG	Iron/Steel	16.17
30	VOLKSWAGEN AG	Automotive	146.05

Table C.1: The DAX 30. There are the order number, company name, industry and closing price at Xetra börse in Frankfurt am Main (Germany) on 20th August 2012. Companies with '*' (star) were replaced by CONTINENTAL AG (Automotive) and LANXESS AG (Chemicals) on 6th September 2012. The experiments were run on the old DAX.

C.3 Experimental Results

Description of the results contains a tuple like (`manhattan, ward`). This tuple means the type of a distance and the type of a clustering algorithm (`distance, algorithm`).

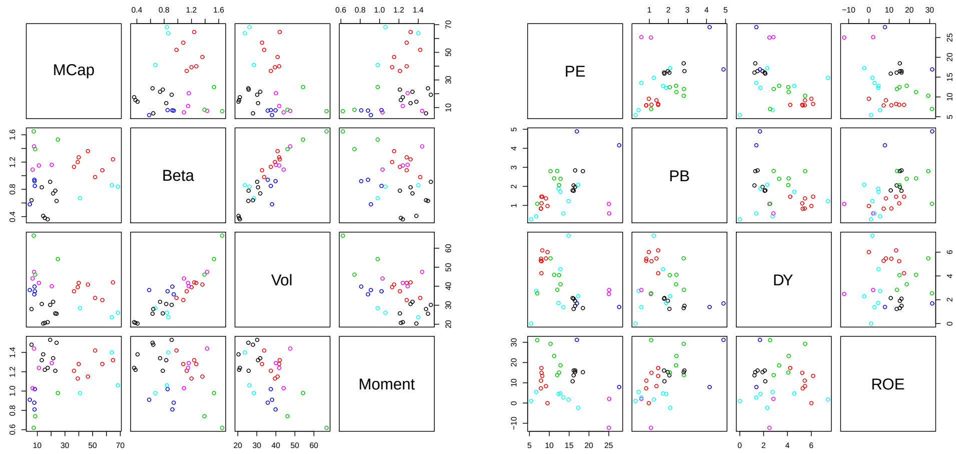

Figure C.1: Scatterplot matrix. Relationship (left) between technical parameters with clustering parameters (`manhattan, ward`). Relationship (right) between fundamental parameters with clustering parameters (`manhattan, ward`).

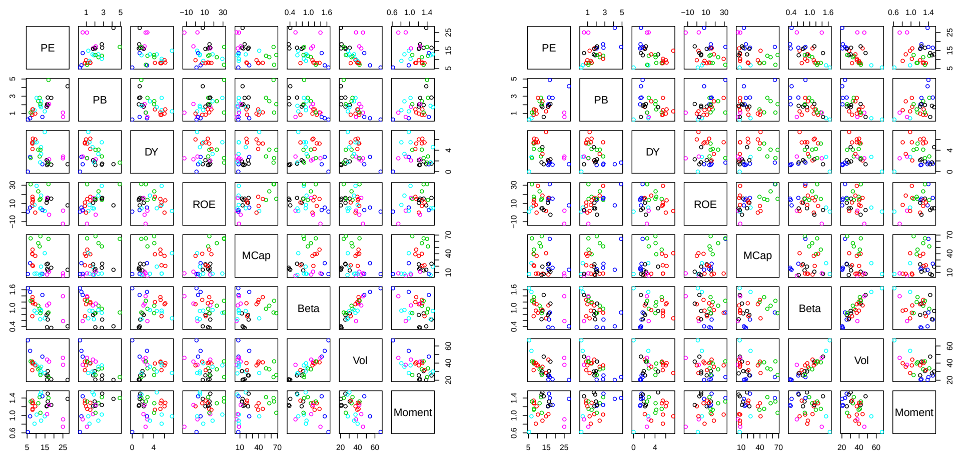

Figure C.2: Scatterplot matrix. Relationship (left) between fundamental and technical parameters with clustering parameters (`manhattan, ward`). Relationship (right) between fundamental and technical parameters with clustering parameters (`euclidian, complete`).

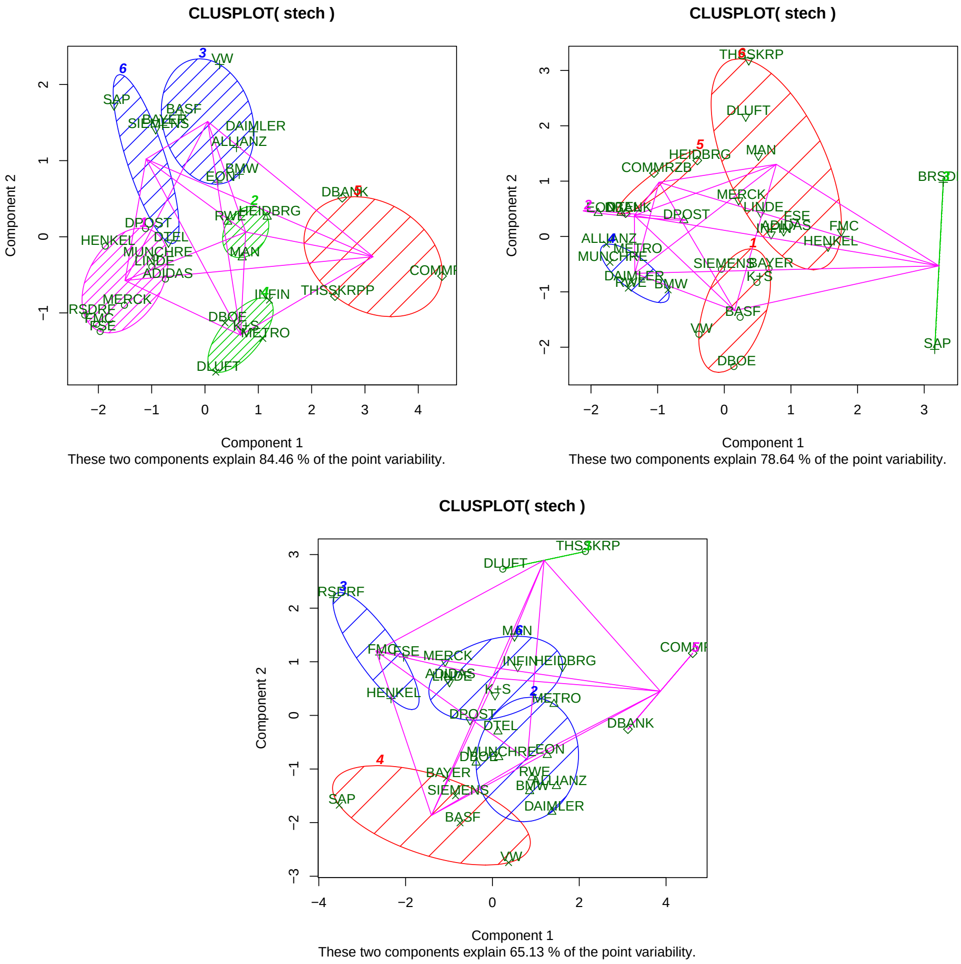

Figure C.3: K-Means clustering with 6 clusters. There are two principal components of technical data with clustering parameters (`manhattan, ward`) described. These two components explain 84.46% of the point variability (left). There are two principal components of fundamental data with clustering parameters (`manhattan, ward`) described. These two components explain 78.64% of the point variability (right). There are two principal components of fundamental and technical data with clustering parameters (`manhattan, ward`) described. These two components explain 65.13% of the point variability (bottom).

Appendix D

On Robustness

This part of Appendix is connected to Chapter 5 in relation to the robustness.

D.1 Definitions of Reward-To-Variability Ratios

Apart from the Sharpe ratio D.1, other relevant *reward-to-variability* ratios are Modigliani-Modigliani, Sortino, Treynor and Information ratios [Wil07]. The previously mentioned Sharpe ratio has been criticized for attaching equal weight to upside 'risk' as downside risk, this is not good for a skewed returns.

$$SR = \frac{\mu - r}{\sigma}, \tag{D.1}$$

where μ is return, r is a risk free rate and σ is variance. The **Modigliani-Modigliani ratio** is defined as,

$$M2 = r + \nu \times SR, \tag{D.2}$$

where ν is the standard deviation of returns of the relevant benchmark and SR is the Sharpe ratio. The **Sortino ratio** (SoR) is calculated the same was as the Sharpe ratio, except that it uses the square root of the semi-variance as the denominator measuring risk. The semi-variance is measured as the variance except that all data points with positive return are replaced with zero, or with some target value.

$$SoR = \frac{\mu - r}{dr}, \tag{D.3}$$

$$dr = (\frac{1}{N}\sum_{k=1}^{N}(r_k - T)^2 f(r_k))^{\frac{1}{2}},$$

where T is a target rate of return, r_k is the k^{th} return, $f(r_k) = 1$ if $r_k < T$ and $f(r_k) = 0$ if $r_k > T$. The Sortino measure ignores upside 'risk' completely. Another

Sharpe-like measure is the **Treynor ratio**, but in denominator there is the portfolio's beta (β, systemic risk)

$$TR = \frac{\mu - r}{\beta}. \tag{D.4}$$

The Treynor (to Sharpe) ratio is more relevant for less diversified portfolios or individual stocks. The **Information ratio** is a different type of performance measure,

$$IR = \frac{\mu - r}{TE}. \tag{D.5}$$

It uses the idea of a tracking error. The ratio gives a measure of the value added by a manager relative to her benchmark. The numerator is the return in excess of a benchmark. but denominator is the standard deviation of differences between the portfolio returns and benchmark returns, the **Tracking error** (TE).

$$TE = \sqrt{E[(R_p - R_b)^2]}, \tag{D.6}$$

where $R_p - R_b$ is the active return. It is the difference between the portfolio return and the benchmark return.

D.2 Experimental Results

There are experimental results of the MVM, statistics of stocks and correlation tables between a group of common stocks. The stocks are represented by tickers. Tickers of indexes and common stocks are 'SPX', 'SPY', **'KO'**, **'IBM'**, **'PG'**, 'WMT', 'WFC', 'AXP' and 'XOM'. The companies and their statistics are described in Tables D.1, D.2, D.3, D.4 and D.5.

#	Company	Industry	Ticker
1	S&P500	US Large Cap Index	SPX
2	S&P500*	US Large Cap ETF	SPY
3	Coca-Cola	Cons. Staples	**KO**
4	IBM	IT	**IBM**
5	Procter&Gamble	Cons. Staples	**PG**
6	Walmart	Stores/Retail	WMT
7	Wells Fargo & Co	Banks	WFC
8	American Express	Cards	AXP
9	Exxon Mobile	Energy	XOM

Table D.1: A complete list of the selected US index and companies. The order number, company name, industry and ticker are provided. ('*'): SPY is a SPX tracking ETF.

#	Ticker	Mean	Std	SR	Cor	MaxDD	MaxDDG	EMaxDD
1	SPX	0.059999	0.200001	0.308693	1.000000	0.567754	0.838760	0.744417
2	SPY	0.077059	0.197811	0.400854	0.959397	0.551885	0.802706	0.703311
3	KO	0.064029	0.183805	0.358450	0.641430	0.552623	0.804354	0.959961*
4	IBM	0.125850	0.274672	0.471468	0.729438	0.593610	0.900441	1.073931*
5	PG	0.100135	0.172367	0.597783	0.707754	0.542183	0.781286	0.876472
6	WMT	0.122277	0.237584	0.529589	0.757205	0.374933	0.469897	1.000701*
7	WFC	0.118300	0.179150	0.679488	0.767050	0.790037	1.560822	1.767826*
8	AXP	0.122347	0.368376	0.341755	0.898199	0.839063	1.826741	1.637375*
9	XOM	0.108642	0.125360	0.891765	0.685802	0.373075	0.466928	0.868777

Table D.2: US. Stock Ratios I. There are ticker, Mean (Mean), Standard deviation (Std), Sharpe ratio (SR), Correlation with the S&P 500 (Cor), Maximum drawdown (MaxDD), Maximum drawdown geometric (MaxDDG) and Expected Maximum Drawdown (EMaxDD) for the period from Jan 1 1995 till Jan 2 2014. Values with '*' (star) are suspicious.

#	Ticker	Mean	Std	SR	Cor	MaxDD	MaxDDG	EMaxDD
1	SPX	0.188421	0.094639	2.815621	1.000000	0.193882	0.215526	0.253870
2	SPY	0.209331	0.093220	3.175716	0.998441	0.186092	0.205908	0.239449
3	KO	0.103604	0.061082	2.398740	0.833213	0.128030	0.137000	0.249818
4	IBM	0.018580	0.037536	0.700044	0.480356	0.191175	0.212173	0.338148
5	PG	0.129259	0.116483	1.569329	0.951599	0.125578	0.134192	0.229833
6	WMT	0.159624	0.009298	24.279569	0.863884	0.141439	0.152497	0.209417
7	WFC	0.271228	0.051946	7.384082	0.970647	0.321663	0.388111	0.480414
8	AXP	0.330375	0.178243	2.621256	0.963544	0.201462	0.224973	0.317344
9	XOM	0.103831	0.090372	1.624829	0.895186	0.217244	0.244934	0.300453

Table D.3: US. Stock Ratios II. There are ticker, Mean (Mean), Standard deviation (Std), Sharpe ratio (SR), Correlation with the S&P 500 (Cor), Maximum drawdown (MaxDD), Maximum drawdown geometric (MaxDDG) and Expected Maximum Drawdown (EMaxDD) for the period from Jan 1 2013 till Jan 2 2014.

#	Ticker	SPX	SPY	KO	IBM	PG	WMT	WFC	AXP	XOM
1	SPX	1.000000	0.959397	0.641430	0.729604	0.708014	0.757205	0.767050	0.898199	0.686005
2	SPY		1.000000	0.763242	0.859072	0.860718	0.848271	0.887371	0.958031	0.844829
3	KO			1.000000	0.850411	0.770489	0.667230	0.667603	0.698355	0.742308
4	IBM				1.000000	0.850180	0.885458	0.800175	0.808511	0.851301
5	PG					1.000000	0.839447	0.944930	0.855444	0.953992
6	WMT						1.000000	0.856632	0.828938	0.790513
7	WFC							1.000000	0.915846	0.908759
8	BAC								1.000000	0.810727
9	XOM									1.000000

Table D.4: US Stock Correlations I. The table shows mutual price dependency of selected stocks for the period from Jan 1 1995 till Jan 2 2014.

#	Ticker	SPX	SPY	KO	IBM	PG	WMT	WFC	AXP	XOM
1	SPX	1.000000	0.999666	0.477663	-0.638266	0.878254	0.812196	0.941114	0.973715	0.690344
2	SPY		1.000000	0.463542	-0.650794	0.874017	0.803826	0.942842	0.974347	0.683384
3	KO			1.000000	0.102228	0.619353	0.845941	0.429097	0.506424	0.468287
4	IBM				1.000000	-0.439498	-0.289291	-0.626308	-0.626306	-0.300237
5	PG					1.000000	0.875565	0.817347	0.838589	0.620525
6	WMT						1.000000	0.739515	0.791906	0.604659
7	WFC							1.000000	0.947417	0.634205
8	AXP								1.000000	0.700083
9	XOM									1.000000

Table D.5: US Stock Correlations II. The table shows mutual price dependency of selected stocks for the period from Jan 1 2013 till Jan 2 2014.

Appendix E

Data Sources

Data Sources The financial data were used in the thesis. The data were retrieved from financial portals and financial data providers. We hinted the resources where it was necessary. We also provide a complete list here.

1. `http://www.bloomberg.com` [BL],
2. `http://www.dax-indices.com` [DI],
3. `http://deutsche-boerse.com` [DB],
4. `http://finance.yahoo.com` [YA],
5. `http://www.ft.com` [FT],
6. `http://hussmanfunds.com` [HSF] and
7. `http://mathworks.com` [MWS].

DTP The master's thesis utilized the IES FSV UK thesis template (v1.8.09.02.16-beta) for LaTeX. The thesis was compiled with MiKTex (v2.9.4533). Calc (v4.0.2.2), GNU R (v2.15.1) and Matlab 2011a (v7.12.0.635) were modeling software packages.