

**Charles University in Prague**

Faculty of Social Sciences  
Institute of Economic Studies



MASTER'S THESIS

**Time-frequency analysis of technology  
IPOs**

Author: **Bc. Martin Kuš**

Supervisor: **Mgr. Lukáš Vácha, Ph.D.**

Academic Year: **2014/2015**

## **Declaration of Authorship**

The author hereby declares that he compiled this thesis independently, using only the listed resources and literature, and the thesis has not been used to obtain a different or the same degree.

The author grants to Charles University permission to reproduce and to distribute copies of this thesis document in whole or in part.

Prague, May 15, 2015

---

Signature

## **Acknowledgments**

I would like to thank my supervisor, Mgr. Lukáš Vácha Ph.D., for his time, patience and guidance throughout the process of writing my thesis. Moreover, I would like to express my never-ending gratitude to my girlfriend, family and friends for their patience and support in life.

## Abstract

In our work, we focus on the dynamics of the volatility and co-movement during the first year of public trading. We use the wavelet analysis to investigate the return volatility of the technology stocks and their co-movement with the market in the time-frequency space. We employ the data sampled on multiple frequencies, ranging from 1 second high-frequency to daily data. We present three main findings. First, we identify gradual decline of the return volatility on all but the shortest investment horizons. Second, we do not find a convincing evidence that the technology stocks synchronize with the rest of the market as they get mature. Third, the different nature of the synchronization with the NASDAQ and S&P 500 indices is also not confirmed.

**JEL Classification** C22, C32, C58, G19  
**Keywords** IPO, technology stocks, wavelet analysis

**Author's e-mail** martin.kus@outlook.com  
**Supervisor's e-mail** vachal@utia.cas.cz

## Abstrakt

V naší práci se soustředíme na dynamiku volatility a společného pohybu akcií v průběhu prvního roku veřejného obchodování na burze. Používáme vlnkovou analýzu ke zkoumání volatility výnosů technologických akcií a jejich pohybu s trhem v čase a na jednotlivých frekvencích. Pro naši analýzu používáme data na různých úrovních granularity, od sekundových vysokofrekvenčních dat po denní data. Z naší práce plynou tři hlavní závěry. Zaprvé, identifikovali jsme postupný pokles volatility výnosů akcií na všech investičních horizontech kromě nejkratšího. Zadruhé, nenašli jsme přesvědčivý důkaz toho, že výnosy technologických akcií se s postupem času synchronizují s trhem. Zatřetí, nepodařilo se nám potvrdit odlišnou povahu synchronizace technologických akcií s NASDAQ a S&P 500 akciovými indexy.

**Klasifikace JEL** C22, C32, C58, G19  
**Klíčová slova** IPO, technologické akcie, vlnková analýza

**E-mail autora** martin.kus@outlook.com  
**E-mail vedoucího práce** vachal@utia.cas.cz

# Contents

List of Tables	vii
List of Figures	viii
Acronyms	ix
Thesis Proposal	x
<b>1 Introduction</b>	<b>1</b>
<b>2 Technology IPOs: Past and Present</b>	<b>3</b>
2.1 The dot-com bubble . . . . .	3
2.2 Recent technology IPOs . . . . .	5
2.3 Conjectures . . . . .	7
<b>3 Literature Review</b>	<b>11</b>
3.1 Introduction to wavelets . . . . .	11
3.2 Discrete wavelet transform . . . . .	12
3.3 Continuous wavelet transform . . . . .	15
<b>4 Methodology</b>	<b>19</b>
4.1 Wavelet function . . . . .	20
4.2 Continuous Wavelet Transform . . . . .	21
4.2.1 Wavelet power spectrum . . . . .	23
4.2.2 Cross-wavelet power . . . . .	23
4.2.3 Wavelet coherence . . . . .	24
4.2.4 Wavelet phase analysis . . . . .	24
4.3 Discrete Wavelet Transform . . . . .	25
4.3.1 Wavelet correlation . . . . .	26

---

<b>5</b>	<b>Data</b>	<b>27</b>
5.1	Stocks and indices . . . . .	27
5.1.1	Facebook . . . . .	27
5.1.2	LinkedIn . . . . .	28
5.1.3	Pandora . . . . .	29
5.1.4	Twitter . . . . .	29
5.1.5	Yelp . . . . .	30
5.1.6	Zynga . . . . .	30
5.1.7	S&P 500 . . . . .	31
5.1.8	NASDAQ Composite . . . . .	31
5.2	Data description . . . . .	32
<b>6</b>	<b>Results</b>	<b>35</b>
6.1	1s sampling frequency . . . . .	37
6.2	30s sampling frequency . . . . .	40
6.3	5min sampling frequency . . . . .	43
6.4	Daily sampling frequency . . . . .	49
6.5	Summary of results . . . . .	52
<b>7</b>	<b>Conclusion</b>	<b>55</b>
	<b>Bibliography</b>	<b>60</b>
<b>A</b>	<b>Complementary tables and figures</b>	<b>I</b>

# List of Tables

5.1	IPO characteristics . . . . .	31
5.2	Descriptive statistics – 30s data . . . . .	33
5.3	Descriptive statistics – 5min data . . . . .	34
6.1	Realized volatility at 5min . . . . .	45
A.1	Descriptive statistics – daily data . . . . .	I
A.2	Descriptive statistics – 1s data, first week of trading . . . . .	I
A.3	Descriptive statistics – 1s data, last week of trading . . . . .	VII

# List of Figures

6.1	Wavelet power – 1s sampling frequency, first versus last week of trading (FB, LNKD, P) . . . . .	38
6.2	Wavelet power – 1s sampling frequency, first versus last week of trading (TWTR, YELP, ZNGA) . . . . .	39
6.3	Wavelet power – 30s sampling frequency . . . . .	41
6.4	Wavelet coherence – 30s sampling frequency . . . . .	42
6.5	Wavelet coefficient correlation – 30s sampling frequency . . . . .	44
6.6	Wavelet power – 5min sampling frequency . . . . .	46
6.7	Wavelet coherence – 5min sampling frequency . . . . .	47
6.8	Wavelet coefficient correlation – 5min sampling frequency . . . . .	48
6.9	Wavelet power – daily sampling frequency . . . . .	50
6.10	Wavelet coherence – daily sampling frequency . . . . .	51
6.11	Wavelet coefficient correlation – daily sampling frequency . . . . .	52
A.1	Plots of logarithmic returns – 30s sampling frequency . . . . .	II
A.2	Plots of logarithmic returns – 5min sampling frequency . . . . .	III
A.3	Plots of logarithmic returns – daily sampling frequency . . . . .	IV
A.4	Plots of logarithmic returns – 1s sampling frequency, first week of trading . . . . .	V
A.5	Plots of logarithmic returns – 1s sampling frequency, last week of trading . . . . .	VI

# Acronyms

<b>BUX</b>	Budapest Stock Exchange Index
<b>CAPM</b>	capital asset pricing model
<b>CWT</b>	continuous wavelet transform
<b>DAX</b>	German Stock Index
<b>DWT</b>	discrete wavelet transform
<b>GARCH</b>	generalized autoregressive conditional heteroskedasticity
<b>IPO</b>	initial public offering
<b>LTM</b>	last twelve months
<b>MODWT</b>	maximum overlap discrete wavelet transform
<b>MSCI</b>	Morgan Stanley Capital International
<b>NYSE</b>	New York Stock Exchange
<b>PX</b>	Prague Stock Exchange Index
<b>WIG</b>	Warsaw Stock Exchange Index
<b>STFT</b>	short-time Fourier transform
<b>COI</b>	cone of influence

# Master's Thesis Proposal

---

<b>Author</b>	Bc. Martin Kuš
<b>Supervisor</b>	Mgr. Lukáš Vácha, Ph.D.
<b>Proposed topic</b>	Spectral Analysis of Central European Financial Markets

---

**Topic characteristics** The aim of this work is to use continuous wavelet transform (CWT) tools to estimate the effect of the recent financial crisis on the volatility of financial markets with the focus on central Europe. Our approach is based on the most recent trends in the area of time series analysis and has several advantages over traditional methods of spectral decomposition like the Fourier transform. We have set two main goals. The first is to describe the effect of the financial crisis in 2008 on the volatility of financial markets in the time-frequency domain. The second is to compare these effects across the examined countries and look for eventual interconnections.

For our analysis, we are going to use high-frequency data that describe common stock market indices (DAX, SP500, PX and WIG) and prices of various commodities (namely gold, crude oil, heating oil and natural gas) until the end of the year 2011.

## Hypotheses

1. The influence of the financial crisis on the volatility of financial markets changes with scale, i.e. the frequency component of the volatility.
2. The volatility on comparable markets across the central European countries is codependent, the degree of dependence changes with scale.
3. Employing the continuous wavelet transform methodology can improve our understanding of the influence of the financial crisis on the financial markets volatility and generally improve the interpretability of results (as compared with traditional methods).

**Methodology** Throughout our work, we are going to use analytical tools associated with the continuous wavelet transform. Generally, wavelet analysis enables us to describe how the periodic components of the particular time series evolve over time. The advantage of wavelet analysis over other methods of spectral decomposition (like the Fourier transform) is that it preserves the time dimension of the data. This property makes it suitable for working with non-stationary time series or time series that likely contain structural breaks. Since both of these properties are typical for the data gathered during the period of crisis or other economic disturbances, the wavelet analysis appears to be most suitable for our purposes. To our knowledge, the focus on the continuous wavelet transform differentiates our work from the majority of existing articles dealing with similar topic and allows for innovative interpretation of results.

## Outline

1. Introduction
2. Literature review
3. Data description
4. Wavelet methodology overview
5. Estimation
6. Empirical results
7. Conclusion

## Core bibliography

1. AGUIRAR-CONRARIA, L., SOARES J.M. (2011): "Business cycle synchronization and the Euro: A wavelet analysis." *Journal of Macroeconomics* (2011)
2. CAPOBIANCO, E. (1999): "Analysis of Financial Volatility by Wavelet Shrinkage." *Methodology and Computing in Applied Probability*, **Vol.1(4)**: pp. 423–443.
3. CROWLEY, P. (2005): "An intuitive guide to wavelets for economists." *Discussion paper 1-2005*, Bank of Finland, Helsinki, Finland.
4. DAUBECHIES, I. (1992): "Ten Lectures on wavelets." *CBMS-NSF Regional Conference Series in Applied Mathematics*, **Vol.61**. SIAM, Philadelphia.
5. RAMSEY, J.B. (2002): "Wavelets in economics and finance: Past and future." *Studies in Nonlinear Dynamics & Econometrics*, **6(3)**: Article 1.
6. RUA, A., NUNES L.C. (2012): "A wavelet-based assessment of market risk: The emerging markets case." *The Quarterly Review of Economics and Finance*. **Vol. 52, Issue 1**: 84–92.

7. VACHA, L., BARUNIK J. (2012): "Co-movement of energy commodities revisited: Evidence from wavelet coherence analysis." *Energy Economics*. **34(1)**: 241-247.
8. VUORENMAA, T. (2004): "A Multiresolution Analysis of Stock Market Volatility Using Wavelet Methodology." *Draft Bank of Finland discussion paper*.

---

Author

---

Supervisor

# Chapter 1

## Introduction

Recent astronomical valuations of the technology start-ups are raising eyebrows of financial analysts around the world. Some fear the return of the 1990s dot-com crisis, some claim that the technology companies today are far more mature than in the past and there is nothing to be worried about. Either way, the current activity on the technology market more than justifies labelling the technology market as “hot”. The valuations of privately held technology companies with close to no profit are going through the roof and the technology initial public offering (IPO) market is the strongest in a decade.<sup>1</sup> We argue that information asymmetry, typical for the immature technology companies, and the current mood on the technology markets influence the return volatility dynamics after the company goes public, particularly on the trading frequencies that represent investors with shorter investment horizons. Inspired by the stream of literature that deals with the IPO volatility, we employ the methods from the fields of spectral analysis and market co-movement to analyse the technology IPO stocks.

The aim of this work is to employ the state-of-art method of spectral analysis, the wavelet transform, to investigate the volatility dynamics of a financial time series in a time-frequency space. We focus on six technology stocks and examine their returns from the moment they went public to the end of the first year of public trading. Using the high-frequency data, we try to describe how the volatility evolved in time and on different frequencies. In the spectral analysis framework, the frequencies can be interpreted as the investment horizons of investors with heterogeneous beliefs. Understanding the frequency dimension of the financial time series is crucial for better understanding of fi-

---

<sup>1</sup>For details and sources, see Chapter 2.

nancial markets and possibly for predicting future crises like the dot-com crisis of 1990s. Moreover, we try to estimate how the returns of the new technology stocks synchronize with the rest of the market, represented by the NASDAQ and S&P 500 indices.<sup>2</sup> The analysis of co-movement has implications for investors' diversification opportunities. Our main contribution is that to our knowledge, we pioneer the use of time-frequency analysis for the investigation of volatility and co-movement of the IPO companies in the specific period after they go public.

The rest of our work is structured as follows. In Chapter 2, we introduce reader to the technology IPOs. We describe the dot-com bubble of 1990s, elaborate on the current state of the technology IPO market and propose our hypotheses. Chapter 3 provides a brief review of the related literature, particularly from the field of wavelet analysis. In Chapter 4, we introduce the wavelet analysis framework and define theoretical concepts necessary for the empirical part of our thesis. Chapter 5 contains description of our data, including brief description of each stock. Chapter 6 provides a discussion of results and Chapter 7 summarizes our findings and hints at the possible extensions of our research.

---

<sup>2</sup>We use NASDAQ Composite, for details see Chapter 5

# Chapter 2

## Technology IPOs: Past and Present

The primary aim of this chapter is to introduce the reader to the technology IPOs, both from the historical and the current perspective. We focus on two specific periods. First, the dot-com bubble that culminated in the year 2000 and represents an important milestone in the history of the technology stocks market. Second, we look at the last few years, which share some characteristics with the bubble period. We use these two perspectives to illustrate the relevance of our topic and to arrive at our research questions.

The structure of this chapter is as follows. First, we provide a brief overview of the historical activity related to the technology IPOs, with the focus on the dot-com bubble of late 1990s. Second, we look at the recent technology IPOs and discuss the connection between both periods. Third, we propose our research questions and formulate them as testable hypotheses.

### 2.1 The dot-com bubble

At the beginning of the 1990s, the new technologies formerly exclusive to the academic and military sphere like personal computers and the internet began to find their way into ordinary households and offices. Suddenly, new horizons in productivity, education and entertainment opened up and financial markets became flooded with excitement and optimism, expecting virtually unlimited growth of the newly discovered industry.

This prosperous market environment gave birth to thousands of new internet-related companies, so called “dot-coms”. These companies were often set up

by young, recently graduated entrepreneurs eager to build the next Microsoft. The business plan of a typical dot-com company was simple: to grow fast and get as many customers to use its product or service as possible.<sup>1</sup> The technology craze on the financial markets allowed many dot-coms to go public and multiply the price of their stocks on the first day of trading before they actually started to earn any money, often even without any tangible plan of how to monetize their service, product or user base.

Mesmerised by the new possibilities, even respected financial media chose to contribute to the public excitement. For example, in the year 1999 *The Wall Street Journal* publicly defended internet companies' lack of profit, stating that their ability to finance themselves with investors' money is the sign of a financial health (Lowenstein 2004, pg. 10). Indeed, the big ideas and promises of the future earnings were enough for many investors to invest their money in the new industry. Stocks of technology companies grew steeply and many dot-coms managed to collect a tremendous amount of money from going public, only to spend it purposelessly and file for bankruptcy after the bubble burst.

This combination of market optimism and rapidly growing prices of technology shares created the so-called dot-com bubble that existed approximately from the year 1997 to the year 2000.<sup>2</sup> According to Kindleberger & Aliber (2011), the bubble can be defined as an upward price movement over some extended period that then collapses. Indeed, during the dot-com bubble the NASDAQ technology index grew by hundreds of percent, reaching its peak on the March 10, 2000 when the price reached the high at \$5132.52.<sup>3</sup> The bubble burst shortly afterwards and millions of investors' money were lost almost overnight. In the aftermath, many companies disappeared completely; frequently mentioned example of the dot-com company that did not make it through the crisis that followed is the online pet supplies retailer Pets.com. On the other hand, many companies survived. For example, Google and Amazon recovered fully and later even surpassed their stock prices from late 1990s.

The opinions on what inflated the dot-com bubble and why did it burst differ across the literature. Popular explanations simply point to the general market euphoria and an abundant venture capital of the dot-com period that in conjunction with inadequate business plans and excessive spending habits

---

<sup>1</sup>For a comprehensive analysis of the Get Big Fast business strategy during the dot-com bubble, see for example Goldfarb *et al.* (2007).

<sup>2</sup>According to some sources, the dot-com period actually started as early as in the year 1995, when the pioneering internet company Netscape had its IPO (Goldfarb *et al.* 2007).

<sup>3</sup>Source: finance.yahoo.com, accessed 11.1.2015.

of most dot-com companies caused the bubble to inflate and burst, after they run out of the investors' money. The academic sphere offers more sophisticated explanations of the events of the dot-com crisis, notably short-sales restriction (Ofek & Richardson 2003), irrational valuation of dot-com companies (Ofek & Richardson 2002) or Taxpayer Relief Act of 1997 (Dai *et al.* 2013). According to the renown economist and a Nobel prize holder Joseph Stiglitz, during the two years after the burst of the dot-com bubble, stockholders lost approximately 8.5 trillion dollars, making it one of the biggest financial crises of the modern history (Stiglitz 2003, pg. 6).

## 2.2 Recent technology IPOs

Nowadays, in 2015, some aspects of the technology market bear an uncomfortable resemblance to the dot-com era. After the dot-com bubble burst, the IPO activity in the technology sector cooled down. In a stark contrast with 371 technology IPOs in 1999 and 261 technology IPOs in 2000, only 23 technology companies (approximately 6% of the 1999 number) went public in 2001 (Ritter 2014). As of today, the unbelievable numbers of technology IPOs from the dot-com era are yet to be matched, but other indicators signal an ongoing renaissance in the technology sector. One of the most prominent signs is the steady growth of the NASDAQ index, recently breaching the \$5000 closing price mark that was reached at the peak of the dot-com period for the last time.

The IPOs of LinkedIn and Facebook from 2011 and 2012 respectively can be labelled as the harbingers of the technology IPO revival. Both IPOs were heavily covered by media and paved the way for the next wave of the technology IPOs, for example Twitter's in 2013. The trend continues, as during the last year many technology companies decided to go public. Among the most anticipated IPOs were the Chinese online retailer and e-commerce giant Alibaba, the outdoor and sport gadgets maker GoPro and mobile game producers King and iDreamSky. The year 2015 looks promising as well. So far, the cloud storage and collaboration software company Box and established online marketplace Etsy had their IPOs and the prices of both shares have grown significantly during the first day of trading. Also, many high-profile technology start-ups that are currently funded from the venture capital are expected to go public sometime this year. Among them for example online payment service provider PayPal, controversial ride sharing service Uber, popular flat sharing

intermediary Airbnb, established cloud storage service Dropbox or trendy on-line radio Spotify. The valuations of some of these start-ups raise eyebrows, since companies with little to no profit are valued astronomically high.<sup>4</sup> An interesting thing to notice is that the median age of the companies that go public is currently more than double than in the dot-com period. In the 1999 the median age of the IPO company was 4 years but in the 2014 it was 11 years (Ritter 2014).

A quick glimpse at the technology IPO report by PricewaterhouseCoopers LLP from January 2015 reveals that the year 2014 turned out to be the best year for the technology IPOs in the last decade. In 2014, the total of 118 companies went public, which accounts for the 84% growth compared to the year 2013 and is the highest number in the last ten years. Moreover, the record breaking total proceeds of \$51.2 billion are by 347% higher than in the year 2013. Of this number, \$21.8 billion was raised by IPO of Chinese online retailer Alibaba, making it the largest IPO of all times. According to the report, these record proceedings reflect the optimistic mood on the stock markets and a strong investor faith in the future of the technology sector. Particularly strong are the Internet Software & Services and Software subsectors that increased their share of the total number of technology IPOs from 44% in 2011 to 69% in 2014. These subsectors have held their position on top of the technology IPOs for the last five years, reflecting the growing importance of the social media, mobile platforms and cloud computing in the everyday life and business. One more highlight from the report is that this time the technology mania is global, since unlike the years before, companies from 19 countries went public in 2014.

Lately, analysts writing for financial media outlets like Business Insider, Fortune or Forbes started addressing the possibility of second dot-com bubble.<sup>567</sup> Among often mentioned indicators of a possible bubble are current valuations of technology start-ups like Snapchat, which, while not generating any revenue, in 2013 reportedly refused the Facebook's acquisition offer of a 3 billion USD.<sup>8</sup> Another example could be text messaging application WhatsApp, purchased

---

<sup>4</sup>The slang term for the technology start-up that is valued at \$1 billion or higher is "Unicorn", see for example <http://fortune.com/2015/01/22/the-age-of-unicorns/>.

<sup>5</sup>Source: <http://www.businessinsider.com/evidence-that-tech-sector-is-in-a-bubble-2013-11>, accessed 4.1.2015

<sup>6</sup>Source: <http://fortune.com/2014/05/08/yes-were-in-a-tech-bubble-heres-how-i-know-it/>, accessed 4.1.2015

<sup>7</sup>Source: <http://www.forbes.com/sites/hbssworkingknowledge/2014/03/05/when-will-the-next-dot-com-bubble-burst/>, accessed 4.1.2015

<sup>8</sup>Source: <http://blogs.wsj.com/digits/2013/11/13/snapchat-spurned-3-billion-acquisition-offer-from-facebook/>, accessed 4.1.2015

by Facebook for \$19 billion in October 2014 or online photo-sharing service Instagram, purchased by Facebook for \$1 billion in April 2012.<sup>910</sup> The wild growth of the NASDAQ index mentioned above also supports the hypothesis of a new bubble, as does the low profitability of the technology companies that currently go public. In 2014, only 20% of the IPOs were profitable according to the LTM earnings metric. This number is the lowest since the 2000's 14% and represents a significant drop from the 2012's 44% (Ritter 2014). On the other hand, some analysts discourage from comparing current technology mania with the dot-com bubble.<sup>1112</sup> Also the current statistics on median age of technology companies going public and annual total number of IPOs as mentioned above suggest that the IPO market is much more mature than in the bubble period and the markets should not fear the technology stocks crisis like the one more than decade ago, at least for now.

To conclude this section, there has not been much of an academic evidence so far on whether we are in a middle of a new technology bubble or not. There is a whirlwind of opinions from notable financial media and popular high profile investors, but none of them brings any hard proof of either theory. All we can say is that massive media hype and thus market momentum generated by potentially game-changing products and services like Apple Watch or Apple Pay suggests that technology mania more and more resembling the one almost two decades ago is not going anywhere.

## 2.3 Conjectures

In the previous two sections, we have provided the background for the current state of the technology IPO market. In this section, we illustrate one of the many reasons why the current technology IPO mania might be of academic interest, provide the motivation for our research approach and formulate our conjectures.

Among the many IPO issues that are subject of academic interest (for a brief review of the IPO literature, see Ritter & Welch (2002)), one is particu-

---

<sup>9</sup>Source: <http://www.wsj.com/articles/facebook-completes-acquisition-of-whatsapp-1412603898?KEYWORDS=whatsapp>, accessed 4.1.2015

<sup>10</sup>Source: <http://www.wsj.com/articles/SB10001424052702303815404577333840377381670>, accessed 4.1.2015.

<sup>11</sup>Source: <http://money.cnn.com/2015/02/20/investing/tech-stocks-bubble-nasdaq/?iid=EL>, accessed 3.5.2015

<sup>12</sup>Source: <http://www.bloombergvew.com/articles/2015-03-11/a-tech-bubble-or-just-a-mistake-/>, accessed 3.5.2015

larly relevant to our topic – the volatility of IPO returns in the “hot” markets. Lowry *et al.* (2010) build on the stylized fact that the difficulty of pricing the IPO company is correlated with the IPO initial return volatility and shows that this volatility is higher when the share of difficult-to-value firms (particularly immature technology firms with high degree of information asymmetry between the firm and the investors) that go public is high. This volatility also seems higher on “hot” markets, specifically during the dot-com bubble we have introduced earlier.

We use this result as a starting point and look at the IPO volatility problem through the lens of the time-frequency analysis. Motivated by the unique dataset we have at our disposal, we take a detour from the usual approach and instead of analysing a dataset constructed as a time series of individual IPO events, we take a thorough look at the 6 high-frequency time series, each telling a story of one particular technology stock from the moment it went public to the end of the first year of trading. We use the state-of-art method of time series analysis that is suitable for this format of data, a wavelet transform, that allows us to observe the dynamics of volatility in time and on different frequencies. When working with financial time series, it is customary to interpret the frequency domain as investment horizons of investors with heterogeneous beliefs. The length of the investment horizon ranges from minutes for noise traders through days for technicians, weeks or months for fundamentalists and years for investment funds (Barunik & Vacha 2013).

The heterogeneity of investors illustrates the importance of understanding the time series dynamics in both time and frequency dimensions. It is also a theme that links our research with the literature stream that analyses the investors’ behaviour during the dot-com crisis. For example Griffin *et al.* (2011), who present the evidence that it were sophisticated institutional investors, not unsophisticated individuals, who drove the collapse of the dot-com bubble. Another related study by Ofek & Richardson (2003) find that the inflation of the bubble was caused by the optimistic investors that had overrun pessimistic ones, who, due to the short trade restrictions, were not able to sell stocks and therefore bring the market prices back to reasonable levels. We believe that the time-frequency analysis of the market activity on different investment horizons could be important for better understanding of market crises. To determine the link between the type of investors and their investment horizon is out of scope of this thesis. However, we believe that there could be a connection and that it changes in time. For example high frequency trading as a technically

demanding trading tool is only recently becoming available both for institutions and individuals.

Inspired by Hasbrouck (2012), who successfully used wavelet tools to analyse the variance of the high-frequency financial data (to our knowledge the first author to do so on a tick sampling frequency of data) and given the chance to work with the high-frequency dataset obtained from the financial data vendor Tickdata, we employ high-frequency financial data up to 1s sampling frequency to inspect the detailed patterns in the IPO return volatility.

To construct our hypotheses, we start with the simple premise. We expect the information asymmetry between investors and the company decrease after IPO, because from the moment the company is publicly traded, the investors' demand for shares becomes known and also the information about the financial health of the company and its performance becomes available. This, in turn, should reduce the speculative traders' incentive to participate, as the opportunities to earn profit decrease. We argue that the speculative trades are typical for investors with shorter investment horizon – higher frequencies in the time-frequency analysis framework. Moreover, we argue that on the “hot” markets (like the technology one), there is an excitement of investors about the new stock, that manifest itself by intensive trading in the beginning of the share's life, therefore higher share of volatility on the higher frequencies in the beginning of the sample. As the stock becomes established on the stock exchange, this excitement and thus part of the trades on the higher frequencies eventually die out. We intend to test this hypothesis by the wavelet power plots.

The next thing of our interest is if the new technology stock gradually becomes synchronized with the rest of the market represented by the NASDAQ and S&P 500 stock indices. Also, we would expect the technology stocks to synchronize differently with each of the two indices, since one represents general market and the other technology one. This hypothesis will be tested by the wavelet coherence and wavelet coefficient correlation.

Finally, we are curious about the return volatility dynamics of the new technology stocks in time and on different frequencies. We are interested in finding common patterns in the time series we use for the analysis. To sum up our conjectures, we propose the following:

**Conjecture 1** *At the beginning of their life, the new technology stocks are plagued by the significant high-frequency volatility component that gradually dies out. The volatility on other frequencies also evolves in time.*

**Conjecture 2** *The volatility of the new technology stocks gradually synchronize with the rest of the market.*

**Conjecture 3** *This synchronization effect is different for the NASDAQ and the S&P 500 indices.*

We are aware that our analysis is not exhaustive, since we draw inspiration from multiple areas of academic research (namely market synchronization, IPO theory, time-frequency analysis) and to incorporate every aspect from every related area to our research would be beyond a scope of a diploma thesis. Our main contribution to the literature is that we try to pioneer a use of wavelet transform from the time-frequency analysis toolbox on the IPOs, particularly in the “hot” technology markets. This approach is, to our knowledge, original and relevant, given the current market conditions.

# Chapter 3

## Literature Review

In this chapter, we are going to provide a brief review of the existing academic literature related to our topic. Since the stream of literature that would deal with the time-frequency analysis of the initial public offerings to our knowledge does not exist, instead of focusing on one particular stream of wavelet literature we provide broader view on the various applications of wavelet methodology in economics, with the emphasis on the co-movement of financial time series. Since we do not build directly on the IPO literature and the main works that provided motivation for our topic are already mentioned in Chapter 2, instead of providing a full review of IPO literature, we refer our reader to the comprehensive review of IPO activity by Ritter & Welch (2002).

This chapter is structured as follows. First, we provide very brief introduction to wavelets, necessary for easier comprehension of the chapter. We follow with review of the wavelet literature splitting it into discrete and continuous wavelet transform sections.

### 3.1 Introduction to wavelets

The wavelet analysis, despite the fact that it is relatively young, has been very popular lately, not only in the economics and finance, but also in the fields like biology, geography or among physicists. Unlike Fourier transform, wavelet analysis allows us to decompose time series into distinct time and frequency components, preserving the time dimension of the data. This is crucial for the analysis of economic and financial time series, which are typically non-stationary and therefore the frequency component is not stable in time.

There are two principal branches of wavelet analysis, discrete and continu-

ous. Discrete wavelet analysis has been widely adopted among economists in the early 2000s. In the case of the discrete wavelet transform (DWT), the time series is decomposed into some discrete number of frequencies. This allows for an intuitive interpretation of results, since the discrete wavelet scales can be interpreted as the proxy for the different investment horizons of investors. However, this advantage can be restrictive in some situations, because only predefined frequency bands are analysed. That is why the continuous wavelet transform (CWT) is becoming popular recently. When using this tool, instead of discrete scales, the entire frequency spectrum is utilized. Therefore this technique can uncover patterns that would not be possible to find with other methods.

For the easier comprehension of the following text, the relation between the frequency, scale and investment horizon should be defined ahead of the Chapter 4 that covers the wavelet methodology in detail. In the context of wavelet analysis, it is customary to use the term scale instead of frequency. The scale is inverse to frequency and low scale implies short investment horizon. Therefore the high frequencies correspond to low wavelet scales and to short investment horizons. Accordingly, the low frequencies correspond to high wavelet scales and to long investment horizons.

## 3.2 Discrete wavelet transform

As mentioned above, the discrete wavelet analysis was the first time-sensitive method of spectral analysis widely adopted by economists.<sup>1</sup> One of the pioneering works is Capobianco (1999). The author estimates the market volatility on the daily observations of the Nikkei stock index. The volatility coefficients are estimated by the generalized autoregressive conditional heteroskedasticity (GARCH) model, both on the original time series and on the time series reconstructed from the periodic components estimated by the DWT on the original time series and smoothed by the wavelet de-noising method. The conclusion of the paper is that the GARCH model estimated on the wavelet de-noised time series has bigger volatility prediction power than the model estimated on regular data.

---

<sup>1</sup>More precisely, the discrete wavelet analysis was the first widely adopted method, where the frequency resolution accommodates to the examined frequency. The short-time Fourier transform that was used before is also time-sensitive, but the estimates are inefficient because the frequency resolution is the same across all frequencies.

One of the popular applications of discrete wavelet methods is the estimation of the relationship between stock markets and macroeconomic indicators. For example Kim & In (2005) use discrete wavelet transform methods to test the Fisher hypothesis.<sup>2</sup> Empirical testing of this hypothesis uncovered that it holds only for the shortest and the longest investment horizons, where the shortest investment horizon corresponds to one month period and the longest investment horizon corresponds to 128-month period. The effect on the intermediate scales was found to be negative. This is considered to be a proof of the hypothesis that during the observed period, stock returns were not viable inflation hedge, except for the scales mentioned above. Another example could be Gallegati (2008), who examines the relationship between the stock market returns and aggregate output. The author employs the DWT tools, mainly cross-correlation analysis and wavelet variance, to determine the scaling properties of both variables and to identify which of the time series leads the other. He finds out that the relationship between the stock returns and output varies across scales and that the returns are leading the aggregate output at the scales corresponding to the period of 16 months or longer. This finding supports the stylized fact that particularly the activity of the investors with longer term investment horizons is linked to the macroeconomic performance of the country.

Next important stream of discrete wavelet literature examines the co-movement of economic indicators and traded commodities. For example Naccache (2011) uses DWT to estimate the relationship between oil price and global economy, which is represented by the Morgan Stanley Capital International (MSCI) Index. The author uses monthly dataset that covers the period from January 1946 to May 2008. The main finding of this paper is that the dependence between global economy and oil prices changes both in time and in frequency. The biggest contribution to the variance in the oil prices in the last fifty years is found in the 20 to 40 year cycles. Another study that covers the oil prices is Barunik *et al.* (2013). The authors perform an analysis of the correlations between the gold, oil and stocks (represented by S&P 500 index) pairs. The analysis performed on daily and intra-day data covering the period from 1987 to 2012 shows that during the times of economic crises, there is prevalent heterogeneity in the correlation between the asset pairs. However, after the 2008 financial crisis, the correlation among the assets becomes stronger and homogeneous, which means that the assets become similarly synchronised on all

---

<sup>2</sup>In a nutshell, the Fisher hypothesis states that the inflation and nominal stock returns are positively correlated.

examined investment horizons.

An alternative perspective to stock market behaviour during the time of crisis is provided by Barunik & Vacha (2009). Authors use wavelet analysis to compare the reaction of Czech, Polish, Hungarian, German and USA stock markets to the financial crisis of 2008. The main finding of their study is that during the most critical two week period in October 2008, the volatility structure at different frequency levels, estimated by the wavelet variance, changed significantly as compared to the whole period. Moreover, the reaction of the Central European markets was found to be noticeably different from the USA. Another view on the stock markets is provided by Mulligan (2004), who uses five different fractal analysis methods, including wavelets, to examine the long memory properties in a sample of 54 technology companies. Based on the estimated long memory parameters, the author concludes that markets are not able to efficiently value at least some of the technology stocks. This represents an evidence against the weak form of efficient market hypothesis that states that in the long run, the future prices of stocks cannot be predicted from their past prices. Recently, Kravets & Sytlenko (2013) performed the comparative analysis of crisis effects on selected national stock indices in time-frequency space. Authors used discrete wavelet transform to estimate how the crisis affected each country on different scales and then used singular spectrum analysis and neural networks methods to forecast the returns of the stock indices. They found that forecasting with neural networks, with wavelet coefficients of the decomposed series as the starting data, generally provides the best results.

The use of DWT in economics is not limited to the analysis of macroeconomics and stock markets. One of the works that illustrates an alternative usage of the discrete wavelet transform in economics is Gençay *et al.* (2005). The authors use the wavelets to estimate the beta of an asset in the capital asset pricing model (CAPM) framework, where it represents the measure of risk that results from the asset's exposure to market (so called systematic or market risk). It is found that the relationship between the beta of the stock and its return is stronger with increasing wavelet scale. This result suggests that the CAPM is more reliable for the longer investment horizons. Another example is Fan & Gençay (2009), who propose a unit root test that utilizes wavelet transform. To test the unit root hypothesis against the near unit root alternative, the variance of the time series is decomposed via discrete wavelet transform into its low and high frequency components. The robustness of the test is assessed with the Monte Carlo simulations.

### 3.3 Continuous wavelet transform

The use of the continuous wavelet transform in economics is pioneered by Aguiar-Conraria *et al.* (2008). The authors use the cross-wavelet transform, the cross-wavelet coherency and the phase difference to show that between years 1947 and 2007, the relation between monetary policy variables (money aggregates and interest rates) and macroeconomic variables (industrial production index and inflation) in the US has evolved in time and the changes are not equivalent across different frequencies.<sup>3</sup> This result points out the possibility that the monetary policy has different impact on the economy in the long and in the short term and this inconsistent effect should be taken into consideration while designing such policy. Another contribution to the branch of wavelet literature that examines macroeconomic patterns is Aguiar-Conraria & Soares (2011a). The authors use CWT to study the degree of business cycle synchronization across European countries that, at the time of the analysis, already had adopted or were planning to adopt Euro. As a proxy for the business cycle, the monthly data that represent the industrial production index of the respective countries are used. The main finding of this paper is that in terms of business cycle synchronization, France and Germany are the core members of the Eurozone, with France leading the European business cycle. Moreover, the strong correlation between geographical proximity and the degree of the business cycle synchronization was found, showing that the business cycles of the adjacent countries are synchronized more than those of a more distant ones. The results demonstrate the importance of the spectral analysis for the policy making related to Euro adoption.

Important insights into the macroeconomic dynamics can be also obtained from the analysis of co-movement of macroeconomics and commodity prices. Such point of view is adopted for example in Aguiar-Conraria & Soares (2011b). Authors use CWT to estimate the relationship between the oil price growth rate, the industrial production growth rate and the inflation in the time-frequency space. On the US monthly data sample from 1946 to 2007, authors show that the causal relation between the oil prices and industrial production is unclear, as the causality changes in time. On the other hand, the oil price growth rate seems to lead the inflation during the whole period across all frequencies. Another finding is that after the mid 1990s, the oil price changes were driven by demand, instead of supply.

---

<sup>3</sup>For more details on CWT methodology, see Chapter 4.

Tiwari (2013) performs similar analysis on the German dataset. For the estimation, author uses oil price returns, industrial production index and German inflation in the period from 1958 to 2009 on the monthly sampling frequency. Similarly to the previous paper, the relation between oil price returns and industrial production is found to be inconclusive. As for the relation between the oil price returns and inflation, the inflation seems to be the leader for most of the time. Moreover, the conclusion of Aguiar-Conraria & Soares (2011b) about the oil price changes being driven by demand is confirmed.

At the intersection of the macroeconomics and stock market co-movement analyses lies the stream of literature that examines the co-movement of the energy commodities. As mentioned by (Vacha & Barunik 2012, pg. 241), the prices of energy commodities are of interest for financial as well as industrial market participants. Moreover, the importance of commodities as a risk diversification tool has grown recently, since they tend to have different statistical properties as other traded assets. Vacha & Barunik (2012) examine the relation between crude oil, heating oil, natural oil and gasoline in the time-frequency space and compare the results to those from the multivariate GARCH model. They use daily data from the 1993 to 2010 period and arrive to some interesting results. First, there seems to be a strong relation between all pairs of commodities except natural gas; this relation evolves in time and across frequencies. Second, the dependence seems to be the strongest in times of economic downturn. Another example of energy commodity study is Sousa *et al.* (2014). The main contribution of this work is that the authors pioneer the use of the multivariate wavelet analysis in economics. Unlike the commonly used bivariate method, multivariate wavelet analysis allows to estimate the co-movement between two time series while isolating the influence of other time series. Authors examine the dependence between carbon and energy prices and economic activity index on a daily sample that covers European markets in the period from 2008 to 2013. A strong relation between carbon and electricity prices is uncovered, with carbon being the leading variable. Also the strong relation between carbon and coal prices is uncovered, but this time the coal is the leading variable.

Another stream of literature deals with the co-movement of the stock market returns. This topic represents an important issue in several areas of finance, most notably in risk management and asset allocation. The reason behind this is that the co-movement of stocks effectively reduces benefits from diversification. Moreover, in the wavelet framework, this effect is different for investors

with heterogeneous investment horizons (Rua & Nunes 2009, pg. 632). A representative study is Rua & Nunes (2009). The authors examine the co-movement of German, Japanese, the UK and the US stock markets on aggregate and sectoral levels. They use monthly data spanning from 1973 to 2007 that cover the stock market indices for the respective countries and ten industrial components of these indices (for example healthcare, utilities or financials). The main finding of this paper is that the degree of the co-movement of international stock returns varies and is dependent on the examined scale. The same holds for the industrial sector components. Moreover, it was found that this relation is generally stronger on higher scales (lower frequencies) so the diversification may be less effective for the long-term investors. A similar approach is adopted by Barunik *et al.* (2011). Authors examine co-movement of the Central European and Western European stock markets, particularly Czech (PX), Hungarian (BUX), Polish (WIG) and German (DAX) stock market indices. In contrast with Rua & Nunes (2009) and similarly to our own study, authors use 5 minute high frequency data, which allows them to examine the activity on the ultra-short investor horizons. The analysis of the sample that spans the period from 2008 to 2009 uncovered the significant change of the co-movement dynamics in time and across frequencies. The strongest relation was found between the Czech and Polish indices, where the significant correlations for the range of wavelet scales from intra-day to three months were identified. The related study is Barunik & Vacha (2013). Authors use similar sample (5 minute high frequency dataset that covers four Central European stock market indices) but focus on the contagion aspect of the stock market crash in September 2008. The contagion is defined as a change in correlation structure of the two time series between and after the crash. To quantify it, authors estimate the wavelet correlation from the DWT family on the two time periods (before and after the stock market crash). The significant contagion was identified between DAX and PX indices. The correlation between higher frequencies unexpectedly decreased after the crash, which implies that the interconnections on the short investment horizons decreased during the period of market stress. This finding is complemented by the expected outcome from the CWT co-movement analysis, which shows that the interconnections between the examined stock market indices changed substantially during the examined period and are different across all wavelet scales.

Similarly to DWT, the use of the continuous wavelet transform in economics is not limited to the analysis of macroeconomics and stock markets

co-movement. For example Rua & Nunes (2012) follow the stream of literature that uses wavelets to assess market risk in time-frequency space. Authors propose continuous wavelet measure of total market risk, represented by the variance of returns, and of the systematic risk, represented by the beta coefficient. To put these measures to test, authors examine the monthly Morgan Stanley Capital International (MSCI) world and emerging markets indices in the period from 1988 to 2008. The main finding is that the systematic risk plays a big role in the risk profile of the emerging countries during the whole period, particularly at the lower frequencies. On the shorter frequencies, the contribution of the systematic risk becomes notable in the 1990s and is gradually growing towards the end of the sample. Second finding is that the variance of monthly returns is driven by the activity at the higher frequencies, that means shorter investment horizons. Moreover, more volatile periods coincide with the periods of economic crises on the emerging markets.

# Chapter 4

## Methodology

In this chapter, we introduce the theoretical concepts essential for the time-frequency analysis, particularly the continuous and the discrete wavelet transform. The foundations of the spectral analysis were laid in the late 19<sup>th</sup> century when the Fourier transform was first introduced for the purposes of geophysical research. The Fourier transform decomposes given signal into the linear combination of sine and cosine functions but discards the time dimension of the data. This makes its use in economics and finance problematic, since the time series from these areas are typically non-stationary and plagued by structural break and sudden jumps. To remedy this issue, Gabor (1946) proposed so called short-time Fourier transform (STFT). In a nutshell, this method breaks a signal into sub-samples (windows) of fixed length and then applies the Fourier transform to each of them. The problem with this approach is that the frequency resolution of the STFT does not adapt to the examined frequency, which makes it inefficient. The wavelet transform solves this problem, as the width of the window accommodates to the measured frequency. This is possible, because the base function, the so called “mother” wavelet, can be stretched and shifted in time. To measure the low-frequency signal components, the wavelet function is stretched. To measure the high-frequency signal components, the wavelet function is compressed. We can say that the wavelet transform locally decomposes frequency of the signal and returns the information about the particular frequency components. (Vacha & Barunik 2012)

This chapter is organized as follows. First, we define the wavelet function and list all of its essential properties. Second, we introduce the continuous wavelet transform (CWT) and we look at the possibilities that it brings both to the univariate and bivariate time series analysis. Finally, we briefly define

the discrete wavelet transform (DWT) and maximum overlap discrete wavelet transform (MODWT) and derive wavelet correlation necessary for our analysis. Throughout this chapter, we follow multiple sources. To describe CWT theory, we follow mainly Aguiar-Conraria *et al.* (2008) and Vacha & Barunik (2012). In case of DWT, we follow Gallegati (2008) and Gençay *et al.* (2002). Readers interested in deeper analysis should see for example Daubechies (1992) or Percival & Walden (2000).

## 4.1 Wavelet function

It feels appropriate to start this section with an intuitive definition of a wavelet function. As hinted by its name, we can imagine a wavelet as a small wave that grows and shrinks in a limited time interval. To define the wavelet function properly, we need to start with the definition of the square integrable function from the set  $L^2(\mathbb{R})$  for which holds that

$$\int_{-\infty}^{\infty} |x(t)|^2 dt < \infty. \quad (4.1)$$

This expression represents the energy of the function  $x$  and thus this space defines the space of functions with finite energy. In the next step, we can define the inner product in  $L^2(\mathbb{R})$  as

$$\langle x, y \rangle := \int_{-\infty}^{\infty} x(t)y^*(t)dt \quad (4.2)$$

and its associated norm  $\|x\| := \langle x, x \rangle^{\frac{1}{2}}$ .<sup>1</sup>

Having a function  $x(t) \in L^2(\mathbb{R})$ , we can define the Fourier transform of function  $x(t)$  as

$$X(f) := \int_{-\infty}^{\infty} x(t)e^{-i2\pi ft} dt. \quad (4.3)$$

Next, we employ the standard definition of Parseval relation that holds for all  $x(t), y(t) \in L^2(\mathbb{R})$ :

$$\langle x(t), y(t) \rangle = \langle X(f), Y(f) \rangle. \quad (4.4)$$

From the equation above, we can derive the Plancherel identity

---

<sup>1</sup>It should be noted that in this chapter, the complex conjugate is represented by the asterisk superscript (\*) and the symbol := stands for “by definition”.

$$\|x(t)\|^2 = \|X(f)\|^2 \quad (4.5)$$

that states that the Fourier transform preserves the function's energy.

For a function  $\psi(t)$  to pass as a mother wavelet (sometimes also called analyzing or admissible wavelet), it must hold that  $\psi \in L^2(\mathbb{R})$  and

$$0 < C_\psi := \int_{-\infty}^{\infty} \frac{|\Psi(f)|}{|f|} df < \infty, \quad (4.6)$$

where the equation above represents the so called admissibility condition and  $\Psi(f)$  is the Fourier transform of  $\psi(t)$ .

It is customary to normalize the wavelet  $\psi$  so it has unit energy:

$$\|\psi\|^2 = \int_{-\infty}^{\infty} |\psi(t)|^2 dt = 1. \quad (4.7)$$

The fact that the wavelet  $\psi$  is square integrable represents an example of a mild decay condition. For functions that decay at sufficient rate, the following expression is equivalent to the admissibility condition (Equation 4.6):

$$\Psi(0) = \int_{-\infty}^{\infty} \psi(t) dt = 0 \quad (4.8)$$

The equations above imply that the wavelet function goes up and down around the time axis and decays in limited time. Therefore we have arrived to our intuitive definition from the beginning of this section.

## 4.2 Continuous Wavelet Transform

To obtain the family of “daughter” wavelets  $\psi_{s,\tau}$ , we take the mother wavelet  $\psi$  and transform it by the scale and translation parameters  $s$  and  $\tau$ :

$$\psi_{s,\tau}(t) := \frac{1}{\sqrt{|s|}} \psi\left(\frac{t-\tau}{s}\right), \quad s, \tau \in \mathbb{R}, s \neq 0. \quad (4.9)$$

The scale parameter  $s$  is associated with frequency and denotes how the mother wavelet is stretched ( $|s| > 1$ ) or compressed ( $|s| < 1$ ). It can be stated that scale is inverse to frequency. Higher (lower) scale implies less (more) compressed wavelet suitable for detection of lower (higher) frequencies of a signal. The translation parameter  $\tau$  defines the position of wavelet in time.

Note that the normalization factor  $\frac{1}{\sqrt{|s|}}$  has to be applied to ensure that the daughter wavelet preserves unit energy.

The continuous wavelet transform (CWT) of a time series  $x(t) \in L^2(\mathbb{R})$  is defined as a projection of  $x(t)$  onto the family of daughter wavelets  $\psi_{s,\tau}$ :<sup>2</sup>

$$W_x(s, \tau) = \langle x, \psi_{s,\tau} \rangle = \int_{-\infty}^{\infty} x(t) \frac{1}{\sqrt{|s|}} \psi^* \left( \frac{t - \tau}{s} \right) dt \quad (4.10)$$

One of the key features of CWT is that the admissibility condition defined in the previous section (Equation 4.6) guarantees that the reconstruction of the original time series  $x(t)$  from its wavelet transform is possible:

$$x(t) = \frac{1}{C_\psi} \int_{-\infty}^{\infty} \left[ \int_{-\infty}^{\infty} W_x(s, \tau) \psi_{s,\tau}(t) d\tau \right] \frac{ds}{s^2}. \quad (4.11)$$

Moreover, Equation 4.12 and Equation 4.13 show that the wavelet transform preserves the energy of the time series and that the Parseval type identity holds:

$$\|x\|^2 = \frac{1}{C_\psi} \int_{-\infty}^{\infty} \left[ \int_{-\infty}^{\infty} |W_x(s, \tau)|^2 d\tau \right] \frac{ds}{s^2}, \quad (4.12)$$

$$\langle x, y \rangle = \frac{1}{C_\psi} \int_{-\infty}^{\infty} [W_x(s, \tau) W_y^*(s, \tau) d\tau] \frac{ds}{s^2}. \quad (4.13)$$

In economics, one is often interested in a phase of the time series, in a sense of position in its pseudo-cycle. To be able to determine the phase, it is important to differentiate between complex and real-valued wavelet functions and wavelet transforms. The complex wavelet transform consists of real and imaginary part,  $\Re\{W_x\}$  and  $\Im\{W_x\}$  respectively. Alternatively, the complex wavelet transform can be defined in terms of its amplitude  $|W_x|$  and phase  $\phi_x(s, \tau) = \tan^{-1} \left( \frac{\Im\{W_x\}}{\Re\{W_x\}} \right)$ . Since for real-valued wavelet functions, the imaginary part is equal to zero and therefore the phase of the wavelet transform is not defined, we need to employ complex wavelet function (this results in a complex wavelet transform) to be able to differentiate between the phase and the amplitude component of the time series. Moreover, Aguiar-Conraria *et al.* (2008) recommend to use analytic wavelet function for which holds that  $\Psi(f) = 0$  for  $f < 0$ .

It is clear that the wavelet function should be chosen carefully, with the particular application in mind. There are many types of wavelets with different

---

<sup>2</sup>Alternatively, we can say that CWT is a convolution of the signal and the family of daughter wavelets.

properties to choose from, for example Morlet, Mexican hat or Daubechies, each type being used for different purposes. For a comprehensive review, see for example Addison (2002).

In our thesis, we use the Morlet wavelet in its simplified form, defined as

$$\psi_{\eta}(t) = \pi^{-1/4} e^{i\eta t} e^{-t^2/2}. \quad (4.14)$$

Parameter  $\eta$  represents the central frequency of the wavelet. (Vacha & Barunik 2012) Following many economic applications (see for example Rua & Nunes (2009)), we use  $\eta = 6$ . For this setting, the inverse relation between scale and frequency ( $f \approx 1/s$ ) holds and makes the interpretation of results considerably more intuitive. (Aguiar-Conraria & Soares 2011a) Moreover, Morlet wavelet provides a good trade-off between scale and time localization (Grinsted *et al.* 2004). For a discussion about the localization of the wavelet function, see Aguiar-Conraria *et al.* (2008).

### 4.2.1 Wavelet power spectrum

We can define the wavelet power spectrum as follows:

$$WPS_n = |W_n^x|^2. \quad (4.15)$$

The wavelet power spectrum measures how is the local volatility of signal distributed across its frequencies. In our work, we use this measure to estimate the dynamics of return volatility. Torrence & Compo (1998) propose testing the statistical significance of the wavelet power against the null hypothesis that the data generating process is actually stationary and is characterized by specific background spectrum ( $P_f$ ). (Aguiar-Conraria *et al.* 2008)

### 4.2.2 Cross-wavelet power

Given two time series,  $x = \{x_n\}$  and  $y = \{y_n\}$ , we can define their cross-wavelet transform as

$$W_n^{xy} = W_n^x W_n^{y*}. \quad (4.16)$$

From the equation above, we can define the cross-wavelet power as  $|W_n^{xy}|$ . The cross-wavelet power represents the frequency-based local covariance of two time series. More intuitively, it uncovers the areas in the time-frequency space,

where the two series share high common power. (Vacha & Barunik 2012) For the theoretical distribution of the cross-wavelet power, see (Torrence & Compo 1998).

### 4.2.3 Wavelet coherence

The wavelet coherence uncovers areas in the time-frequency space, where the two time series co-move but do not necessarily share a high common power. (Vacha & Barunik 2012) It represents the local correlation and thus its values has to be between zero and one. To provide a formal definition of the wavelet coherence of  $x = \{x_n\}$  and  $y = \{y_n\}$ , we follow Torrence & Webster (1999):

$$R_n(s) = \frac{|S(s^{-1}W_n^{xy}(s))|}{S(s^{-1}|W_n^x|)^{\frac{1}{2}}S(s^{-1}|W_n^y|)^{\frac{1}{2}}}. \quad (4.17)$$

$S$  stands for the smoothing operator that ensures that the coherence is not constant and equal to one in the whole time-frequency space. The smoothing is done via convolution, for details see Grinsted *et al.* (2004). In our work, we use the wavelet coherence to estimate the dynamics of the synchronization of the stock with the market.

It should be mentioned that the wavelet analysis suffers from the problem common for all filter-based transformations. If we use the wavelet analysis on a time series with finite length, there is an issue with boundary conditions. Following Grinsted *et al.* (2004), we overcome this problem by padding the time series with zeros. However, this creates regions in the time-frequency space, where the errors caused by the discontinuities in the wavelet transform influence the reliability of estimates. This area is larger on lower frequencies, because the wavelet positioned on the edge of the dataset is stretched more and thus includes more zeros from the padding. We call this area where the edge effects become problematic the cone of influence (COI). (Grinsted *et al.* 2004)

### 4.2.4 Wavelet phase analysis

To finish the section dedicated to CWT, we define the phase difference between the two time series as

$$\phi_{x,y} = \tan^{-1} \left( \frac{\Im \{W_n^{xy}\}}{\Re \{W_n^{xy}\}} \right), \quad (4.18)$$

where  $\phi_{x,y} \in [-\pi, \pi]$ . This measure represents delays of the mutual oscilla-

tions of the two time series, estimated in the time-frequency space. It can be used to uncover which time series leads and which follows the other. Therefore, it can help us define the causality between the two time series. Since in our analysis we are not interested in causality, for details and interpretation of the phase difference results, we refer our reader to Grinsted *et al.* (2004).

### 4.3 Discrete Wavelet Transform

In this section, we provide very basic definitions of DWT and maximum overlap discrete wavelet transform (MODWT), leading to intuitive definition of wavelet correlation. Formally, DWT was defined by Mallat (1999) and Daubechies (1992). In our work, we use maximal overlap DWT (MODWT) variation of classical DWT. The advantage is that unlike DWT, the signal we feed to the transform does not have to be of dyadic length.<sup>3</sup> Moreover, the scaling and wavelet MODWT coefficients are time-invariant. As in the case of CWT, there are many types of discrete wavelets. In our work we use Daubechies' Least Asymmetric wavelet – LA(8).

To define DWT, let us begin with two basic functions – father and mother wavelet. Following Daubechies (1992), we can define all filter coefficients where  $\{h_l\}_{l=0}^{L-1}$  is the wavelet (mother) filter corresponding to high-pass filters,  $\{g_l\}_{l=0}^{L-1}$  is the scaling (father) filter corresponding to low-pass filters and  $L$  is the width of the filters (it is necessary for  $L$  to be even).

It must hold that

$$h_l = (-1)^l g_{L-1-l}, \quad l = 0, \dots, L-1. \quad (4.19)$$

The equation above states that the two filters must represent a quadrature mirror. Then there are three properties that must hold for the wavelet filter coefficients – zero mean, unitary energy and orthogonality to the even shifts:

$$\sum_{l=0}^{L-1} h_{1,l} = 0, \quad (4.20)$$

$$\sum_{l=0}^{L-1} h_{1,l}^2 = 1, \quad (4.21)$$

---

<sup>3</sup>In this context, dyadic refers to the sample of such size, that it is divisible by  $2^j$ .

$$\sum_{l=0}^{L-1} h_{1,l} h_{1,l+2n} = 0, \quad \forall n \in Z, n \neq 0. \quad (4.22)$$

As proposed by Mallat (1999), the pyramid algorithm is used to obtain the wavelet  $W_{j,t}$  and scaling  $V_{j,t}$  coefficients. At each of  $j$  levels of the algorithm  $j = 1, \dots, J$ , the input is decomposed and the signal is obtained. For detailed procedure, see Mallat (1999).

So far, we have described DWT. For MODWT, the coefficients need to be rescaled, because the length of the signal does not have to be dyadic:

$$\tilde{W}_{j,t} = \sum_{l=0}^{L-1} \tilde{h}_{j,l} x_{t-l}, \quad (4.23)$$

$$\tilde{V}_{j,t} = \sum_{l=0}^{L-1} \tilde{g}_{j,l} x_{t-l}. \quad (4.24)$$

Also the mother and father filters need to be transformed as  $\tilde{h}_{j,l} = \frac{h_{j,l}}{2^{j/2}}$  and  $\tilde{g}_{j,l} = \frac{g_{j,l}}{2^{j/2}}$ . The ability of MODWT to use the whole length of the signal, as well as more efficient wavelet variance estimates are great advantages over classical DWT. (Gallegati 2008)

As in the case of CWT, the discrete wavelet transform has to preserve the energy of the signal. For the precise definition and discussion, see for example Gençay *et al.* (2002) or Percival & Walden (2000). For the definitions of discrete wavelet variance and sample variance, consult (Gallegati 2008) and Gençay *et al.* (2002).

### 4.3.1 Wavelet correlation

We conclude this chapter by the definition of wavelet correlation. Following Barunik & Vacha (2013), we can define the wavelet correlation of  $x(t)$  and  $y(t)$  at wavelet scale  $j$  as

$$\rho_j = \frac{Cov[w_x(u, j), w_y(u, j)]}{\sqrt{Var[w_x(u, j)] Var[w_y(u, j)]}}, \quad (4.25)$$

where  $w_x(u, j)$  and  $w_y(u, j)$  are the vectors of discrete wavelet coefficients. The wavelet correlation represents a measure of the relationship between the two time series on scale-by-scale basis. For details and treatment of confidence intervals, see Gençay *et al.* (2002).

# Chapter 5

## Data

In our work, we use the data that represent closing prices (subsequently transformed to logarithmic returns) of six technology stocks that recently went public – Facebook, LinkedIn, Pandora, Twitter, Yelp and Zynga. For each stock, we work with four time series that were sampled at different frequencies. The time series at 1s, 30s and 5min sampling frequency were obtained from financial data outlet Tick Data.<sup>1</sup> The daily data were downloaded from Yahoo Finance.<sup>2</sup> The data for each stock cover the period from the particular IPO date to the 250th day of trading (approximately one trading year). We also use NASDAQ Composite and S&P 500 indices to estimate stock's co-movement with the market.

In the first section of this chapter, we provide a brief introduction of each company and describe background of each IPO. In the second section, we describe the data from the statistical point of view and describe how the data were transformed.

### 5.1 Stocks and indices

#### 5.1.1 Facebook

Facebook, Inc. is a global online social networking company. It was launched on February 4, 2004 and has seen rapid growth since then - as of September 30, 2014, it had 1.35 billion monthly active users and employed 8348 employees, which places it among the biggest global social networks.<sup>3</sup> The company's main products are its Facebook website and mobile app, which enable users

---

<sup>1</sup>Website: <https://www.tickdata.com/>, last version of data retrieved 13.11.2014.

<sup>2</sup>Website: <http://finance.yahoo.com/>, last version of data retrieved on 7.12.2014.

<sup>3</sup>Source: <http://newsroom.fb.com/company-info/>, accessed 23.12.2014.

to connect and share content, Messenger, a messaging app available across all three leading mobile platforms and recently purchased Instagram, a service that enables users to share photos and videos and modify them by the distinct set of image filters.

Facebook went public on May 18, 2012. Traded on the NASDAQ stock exchange under the symbol FB, the closing price on the first trading day was \$38.23, only \$0.23 above the IPO price of \$38. Facebook's debut was afflicted by the delayed start of the trading session and initial trading glitches, yet it raised \$16 billion, making Facebook valued at \$104.2 billion and becoming second biggest technology IPO in the 2010–2014 period.<sup>4</sup> The IPO was led by Goldman Sachs, Morgan Stanley and JPMorgan Chase.<sup>56</sup>

### 5.1.2 LinkedIn

LinkedIn Corporation runs a global online professional networking service. Its users are able to create and manage profiles, showcase their professional skills and connect with other users and professional networks. LinkedIn offers both website and mobile apps and also provides services for corporations. Corporate human resources departments can use LinkedIn to market new job offers, connect with candidates and eventually hire them. Its site was officially launched on May 5, 2003. As of third quarter of 2014, LinkedIn employs more than 6000 people and has over 332000 million members which makes it the world's largest online professional network.<sup>7</sup>

On May 19, 2011, LinkedIn went public on the New York Stock Exchange (NYSE) under the symbol LNKD. During the first day of trading, the price of shares more than doubled, rising from the \$45 IPO price through the \$83 opening price to the final \$94.25 closing price. This made LinkedIn worth \$8.9 billion at the end of the first trading day, while raising \$352.8 million from its IPO. Investors generally perceived the first day returns as a huge success. However, massive first day increase in price had drawn commentaries that compared LinkedIn's IPO to IPOs of dot-com era. The IPO was orchestrated by

---

<sup>4</sup>Source:<http://www.pwc.com/gx/en/technology/publications/global-technology-ipo-review.jhtml?overlay=hero3a>, accessed 29.12.2014.

<sup>5</sup>Source: <http://www.wsj.com/articles/SB10001424052702303448404577411903118364314>, accessed 29.12.2014.

<sup>6</sup><http://www.bloomberg.com/news/2012-05-17/facebook-raises-16-billion-in-biggest-technology-ipo-on-record.html>, accessed 29.12.2014.

<sup>7</sup>Source: <https://press.linkedin.com/about-linkedin>, accessed 23.12.2014.

three big investment banks - Morgan Stanley, JPMorgan and Bank of America Merrill Lynch.<sup>89</sup>

### 5.1.3 Pandora

Pandora Media, Inc. runs an internet radio service that enables users to create custom radio stations and find new songs via unique music recommendation system. Pandora was founded in 2000 and is only accessible in United States, Australia and New Zealand, via mobile app or a website. Basic service is free, though premium paid content is available. As of May 2014, it had 77 million active listeners and 9.13% share of total radio listening in the United States.<sup>10</sup>

Pandora went public on June 15, 2011 on the New York Stock Exchange. Traded under the symbol P, the price of its stocks closed at \$17.42, \$1.42 above the IPO price of \$16, making Pandora worth approximately \$2.8 billion. During the IPO, \$234.9 million was raised. The offering was managed primarily by Morgan Stanley, Citigroup and JP Morgan Chase.<sup>11</sup>

### 5.1.4 Twitter

The main product of Twitter, Inc. is Twitter, a popular online platform for real-time sharing of content and information. Users are able to curate their news feeds and receive updates from the accounts they follow. Unlike Facebook, Twitter emphasizes simple interface created for sharing of information rather than connecting people. Twitter, inc. also offers other online services – a short video loops application Vine and #Music, a music discovery platform. The company was incorporated in April 2007 and its main service, Twitter, has currently 284 million active users per month.<sup>12</sup>

Twitter held its IPO on November 7, 2013 on the New York Stock Exchange. Traded under the symbol TWTR, the IPO raised \$1.82 billion and made Twitter valued at \$14.2 billion. The IPO share price was \$26 and nearly doubled during the first day of trading, reaching the closing price of \$44.90.

---

<sup>8</sup>Source: <http://www.reuters.com/article/2011/05/19/us-linkedin-ipo-risks-idUSTRE74H0TL20110519>, accessed 27.12.2014.

<sup>9</sup>Source: <http://www.wsj.com/articles/SB10001424052748704816604576333132239509622>, accessed 27.12.2014.

<sup>10</sup><http://investor.pandora.com/phoenix.zhtml?c=227956&p=irol-newsArticle&ID=1937243>, accessed 23.12.2014.

<sup>11</sup>Source: <http://www.bloomberg.com/news/2011-06-14/pandora-media-raises-234-9-million-in-ipo-after-pricing-stock-above-range.html>, accessed 27.12.2014.

<sup>12</sup>Source: <https://about.twitter.com/company>, accessed 23.12.2014.

The IPO was managed by Goldman Sachs, Morgan Stanley and JPMorgan Chase.<sup>1314</sup>

### 5.1.5 Yelp

Yelp. Inc runs Yelp, an online community review service that enables users to discover and rate local businesses like restaurants, dentists and repairmen. Yelp exists as a webpage and a mobile application. Special services for businesses are available as well, including business account that offers various business metrics and promotion tools as well as direct interaction with reviewers. Yelp was founded in July 2004 and since then, 67 million reviews were written by users. As of Q3 2014, the service had on average 139 million unique visitors per month.<sup>15</sup>

The initial public offering of Yelp took place on March 2, 2012. Trading on the New York Stock Exchange under the symbol YELP, the stock price reached \$24.58 at the first day of trading, making Yelp valued at \$1.47 billion. Yelp's IPO raised \$107.3 million, the IPO price of the stocks being \$15 a piece. The IPO was managed by Citigroup, Jefferies Group and Goldman Sachs.<sup>16</sup>

### 5.1.6 Zynga

Zynga, Inc. is a online social game developer, known mainly for titles FarmVille, Zynga Casino and Words With Friends. Its games are distributed either as an online services (mostly via Facebook and Zynga's website) or as mobile apps, on iOS and Android platforms. Zynga was founded in 2007 and as of March 2012, it had more than 2900 employees. Its games are played by more than 100 million gamers every month.<sup>17</sup>

Zynga went public on December 16, 2011 on the NASDAQ stock exchange. The shares were offered under the symbol ZNGA for the IPO price \$10, but unexpectedly falling to closing price \$9.5 at the end of the day. The IPO

---

<sup>13</sup>Source: <http://www.forbes.com/sites/hershshEFRIN/2013/11/08/why-twitters-ipo-was-really-a-failure/>, accessed 29.12.2014

<sup>14</sup>Source: <http://www.bloomberg.com/news/2013-11-06/twitter-raises-1-82-billion-pricing-ipo-above-offering-range.html>, accessed 29.12.2014

<sup>15</sup>Source: <http://www.yelp.com/factsheet>, accessed 23.12.2014.

<sup>16</sup>Source: <http://www.bloomberg.com/news/2012-03-02/yelp-surges-in-trade-debut-after-initial-offer-raises-more-than-forecast.html>, accessed 27.12.2014.

<sup>17</sup>Source: <http://investor.zynga.com/faq.cfm>, accessed 26.12.2014.

managed by Goldman Sachs and Morgan Stanley raised \$1 billion, making Zynga valued at \$7 billion.<sup>18</sup>

Table 5.1 provides summary of the information about the IPOs described above.

### 5.1.7 S&P 500

The S&P 500 is a stock market index generally perceived as an indicator of the American economy and one of the best approximations of the American stock market. It consists of 500 top American companies that are publicly traded on NYSE or NASDAQ stock exchange and have market capitalization \$5.3 billion or higher. The index is float-adjusted market capitalization weighted and represents approximately 80% coverage of accessible market capitalization.<sup>19</sup>

### 5.1.8 NASDAQ Composite

The NASDAQ Composite is a market capitalization weighted index that comprises more than 3000 common type stocks listed on the NASDAQ stock exchange, both American and international based. The value of this index is often used as a gauge of the technology and growth companies stocks performance.<sup>20</sup> In our work, we refer to this index simply as “NASDAQ”.

Table 5.1: IPO characteristics

Symbol	Exchange	Offer date	\$M raised	IPO price	FD open	FD close
FB	NASDAQ	18.5.2012	16000	38	42.05	38.23
LNKD	NYSE	19.5.2011	352.8	45	83	94.25
P	NYSE	15.6.2011	234.9	16	20	17.42
TWTR	NYSE	7.11.2013	1820	26	45.1	44.9
YELP	NYSE	2.3.2012	107.3	15	22.01	24.58
ZNGA	NASDAQ	16.12.2011	1000	10	11	9.5

*Source:* Compilation of the sources referenced in this chapter.

<sup>18</sup><http://www.bloomberg.com/news/2011-12-15/zynga-said-to-price-100-million-shares-at-10-each-in-ipo-top-of-range.html>, accessed 28.12.2014

<sup>19</sup>Source: [http://eu.spindices.com/idsenhancedfactsheet/file.pdf?calcFrequency=M&force\\_download=true&hostIdentifier=48190c8c-42c4-46af-8d1a-0cd5db894797&indexId=340](http://eu.spindices.com/idsenhancedfactsheet/file.pdf?calcFrequency=M&force_download=true&hostIdentifier=48190c8c-42c4-46af-8d1a-0cd5db894797&indexId=340), retrieved 30.12.2014.

<sup>20</sup>Source: <http://www.nasdaq.com/markets/composite-eligibility-criteria.aspx>, accessed 30.12.2014.

## 5.2 Data description

In this section, we describe our data as well as the transformations that we performed to make the data suitable for our estimations. We start with the closing prices of six technology stocks. For each stock, the data are sampled at four different sampling frequencies (1s, 30s, 5min and daily sampling frequency). We also employ closing prices of the S&P 500 index (sampled in the same way as our stocks) and the closing prices of the NASDAQ index, sampled daily. All three sampling frequencies were handled equally, with the exception of daily data, where some transformations were not necessary. Hence, to keep this chapter legible, we provide the description of the data transformations and the descriptive statistics only for the 30s datasets. To illustrate the relatively low number of zero returns in the 5min data, we also show the descriptive statistics for 5min datasets. The descriptive statistics and plots for the rest of the datasets can be found in the appendix.

First, it needs to be mentioned that we are working with the data, where every observation is sampled. That means that even if there was no change in price in time  $t+1$ , the observation is logged with the price of the observation in time  $t$  (as opposed to simply leaving it out from the dataset). Naturally, this is not an issue for daily data, but on the 1s, 30s and 5min sampling frequencies, there is significant share of observations that would be discarded if we used different approach. After transforming prices to logarithmic returns, we get the zero return where the price in the interval from  $t$  to  $t+1$  did not change. Table 5.2 shows that on the 30s sampling frequency, there was up to 44% of observations with zero returns (the case of YELP). That means that the sampling resolution of 30s is too high for a symbol with relatively low trading activity and the interpolation as described above was needed to keep the dataset evenly sampled, which is fundamental for all operations with time series. High share of zero observations in the 30s data provides a motivation to run our estimations also on the 5min data, where the share of zero observations is significantly lower (see Table 5.3). In Chapter 6, we compare the results yielded by both approaches and comment on the sensitivity of our analysis to the granularity of the data.

As mentioned above, our base sample is the set of closing prices of six technology stocks. Sampled at the 30s frequency, each time series contains observations from the first moment of trading (the IPO) to the 250th trading day. We have cropped the daily observations to 9:30:30 – 15:00:00 trading hours

to be able to pair the data with the S&P 500 index (for which we have daily observations that end at 15:00:00). Therefore our estimations are based on the trading day 5.5 hours long. According to the Equation 5.1, we have transformed closing prices to log returns, since (unlike prices) returns represent metric that is comparable across stocks. Moreover, we have discarded overnight returns and paired the observations with S&P 500 or NASDAQ indices (both were transformed to logarithmic returns as well) according to the unique combination of time and date.

$$\text{logreturn}_t = \ln(p_t) - \ln(p_{t-1}) \quad (5.1)$$

Table 5.2 displays basic descriptive statistics including number of observations with zero returns. Table 5.3 shows these statistics for the five minute data.

Table 5.2: Descriptive statistics – 30s data

Statistic	FB	LNKD	P	TWTR	YELP	ZNGA
Mean	-2.38E-06	-2.25E-06	-1.19E-05	-7.79E-07	-2.03E-06	-9.61E-06
St. dev	9.94E-04	1.77E-03	2.06E-03	1.19E-03	1.96E-03	2.02E-03
Skewness	-2.11E-01	-1.71E-01	1.48E+00	-7.61E-02	-3.13E-01	4.05E-02
Kurtosis	2.52E+01	5.70E+01	1.62E+02	6.65E+01	5.07E+01	7.39E+01
Min	-2.63E-02	-4.94E-02	-6.72E-02	-5.09E-02	-6.06E-02	-8.28E-02
Max	2.44E-02	5.04E-02	1.29E-01	5.00E-02	6.42E-02	7.98E-02
No. obs.	164510	164689	164743	164592	164719	164560
Null obs.	17885	37623	69692	11448	72228	48972
Null obs. %	11%	23%	42%	7%	44%	30%

*Source:* Author's computations in Matlab.

Table 5.3: Descriptive statistics – 5min data

Statistic	FB	LNKD	P	TWTR	YELP	ZNGA
Mean	-2.46E-05	-1.99E-05	-8.96E-05	-1.33E-05	9.01E-06	-7.00E-05
St. dev	2.89E-03	4.55E-03	5.18E-03	3.32E-03	4.98E-03	4.69E-03
Skewness	-7.04E-02	-5.34E-01	-1.46E+00	6.00E-02	5.15E-01	-1.70E-01
Kurtosis	1.63E+01	5.42E+01	3.96E+01	1.75E+01	1.69E+01	2.56E+01
Min	-5.13E-02	-1.01E-01	-1.33E-01	-5.23E-02	-4.54E-02	-8.28E-02
Max	2.85E-02	9.26E-02	4.16E-02	5.57E-02	7.40E-02	8.42E-02
No. obs.	16226	16244	16250	16235	16247	16231
Null obs.	614	340	1642	395	1192	2409
Null obs. %	4%	2%	10%	2%	7%	15%

*Source:* Author's computations in Matlab.

# Chapter 6

## Results

The aim of this chapter is to examine our data using the wavelet analysis and to interpret the results with respect to the conjectures defined in Chapter 2. For this purpose, we employ three unique tools. We use the wavelet power plots to estimate the volatility dynamics of each time series in time-frequency space. We employ the wavelet coherence tool to estimate the co-movement and possible synchronization of the stocks with the market. Finally, we estimate the wavelet coefficient correlation, which complements the results obtained from the wavelet coherence.

First, some technicalities need to be mentioned. For our estimations and major data transformations, we use Matlab and Wolfram Mathematica. For minor data transformations and manipulations, MS Excel is used. For the estimation of the wavelet power and the wavelet coherence, we use Matlab toolbox “*Crosswavelet and Wavelet Coherence*” by Aslak Grinsted.<sup>1</sup> For the estimation of the wavelet coefficient correlation, we use Matlab toolbox “*WMTSA Wavelet Toolkit for MATLAB*” by Charlie Cornish.<sup>2</sup>

For the wavelet power and wavelet coherence estimations, the Morlet wavelet described in Chapter 4 is employed. Also, let us remind that for the wavelet coefficient correlation, we use Daubechies’ Least Asymmetric wavelet – LA(8). For more details on this kind of wavelet and its uses, see for example Gençay *et al.* (2002).

Before we start with the interpretation of results, it is important to remind the reader of the relationship between scales, frequencies and investment horizons in the wavelet analysis framework. Short investment horizons correspond to high frequencies and low wavelet scales. Similarly, long investment horizons

---

<sup>1</sup>Available here: <http://noc.ac.uk/using-science/crosswavelet-wavelet-coherence>.

<sup>2</sup>Available here: <http://www.atmos.washington.edu/wmtsa/>.

correspond to low frequencies and high wavelet scales. Where possible, we interpret the results in terms of activity on different investment horizons.

Finally, let us comment on the interpretation of the wavelet power and the wavelet coherence plots. In both cases, y-axis represents the investment horizons. Default units for daily data are trading days, for the rest of the datasets the units are labelled. The x-axis represents time in trading days for both types of plots. The areas of the significant wavelet power or the significant co-movement of the two series are marked by the black border. The significance is tested according to Grinsted *et al.* (2004).

For wavelet power plots, the significant areas represent the contribution of the activity at the particular investment horizon in the particular period to the total volatility of the time series. For wavelet coherence plots, the significant areas represent the co-movement of the two series, at the particular investment horizon in the particular period. The red end of the colour spectrum stands for the strongest relationships, the blue end of the spectrum for the weakest.

Since the estimations on the longest investment horizons are unreliable due to the effect of the cone of influence (see Chapter 4), we have intentionally removed them from our plots. Finally, note that we do not comment on the phase difference represented by the small arrows in the wavelet coherence plots. We do so because we are more interested in the strength of the relationship and its localisation in the time-frequency space and less in its causality.

Before we start with the interpretation of results, let us remind tested conjectures:

**Conjecture 1** *At the beginning of their life, the new technology stocks are plagued by the significant high-frequency volatility component that gradually dies out. The volatility on other frequencies also evolves in time.*

**Conjecture 2** *The volatility of the new technology stocks gradually synchronize with the rest of the market.*

**Conjecture 3** *This synchronization effect is different for the NASDAQ and the S&P 500 indices.*

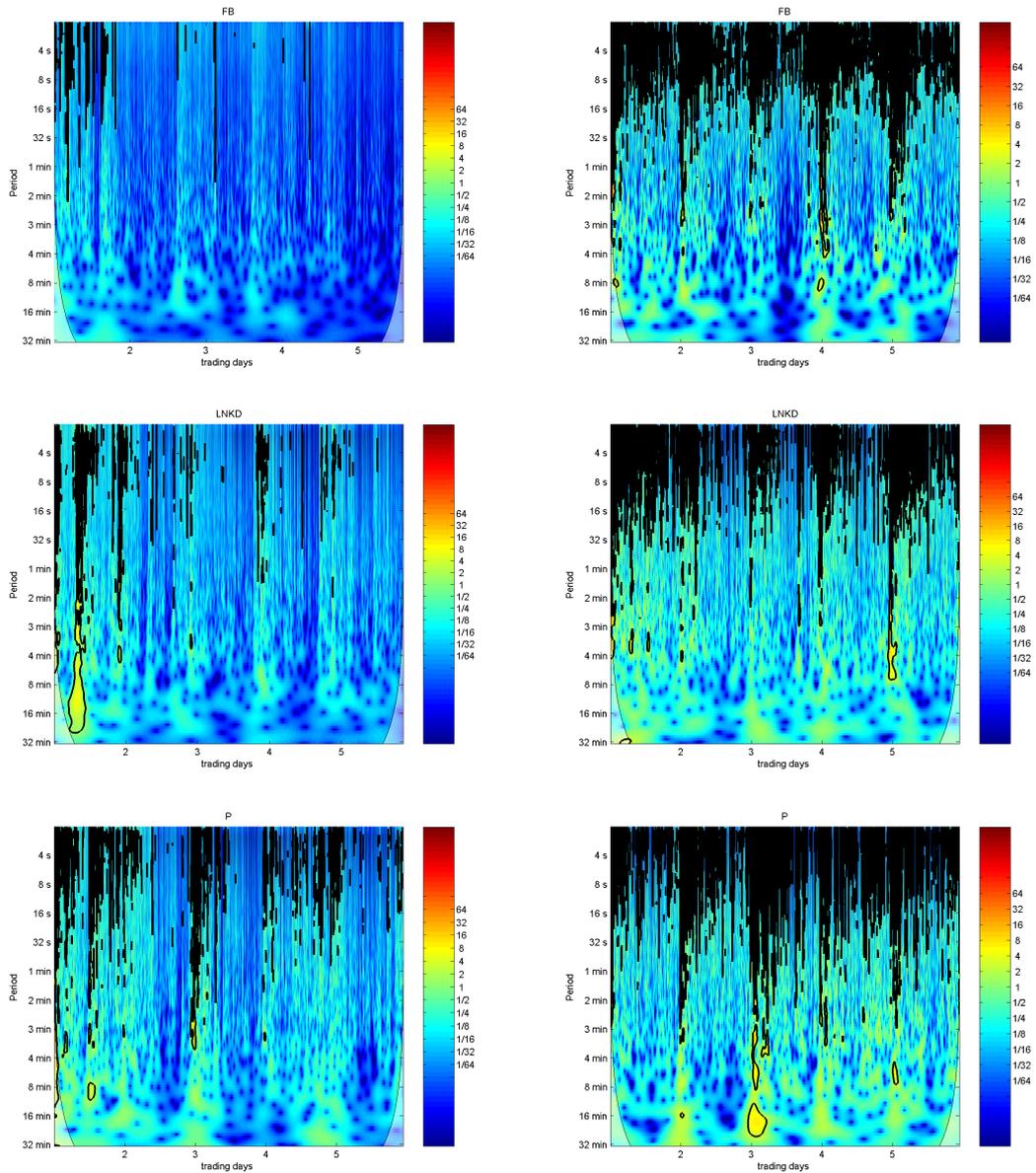
## 6.1 1s sampling frequency

In this section, we investigate the time-frequency volatility dynamics estimated on the data with 1 second sampling frequency. Our first observation is that given the computational power we have at our disposal, the 1 second resolution is too fine to estimate the wavelet power and wavelet coherence on the whole sample (250 trading days) in acceptable computing time. This is caused by high number of wavelet coefficients that needs to be computed. Therefore we estimate the volatility only for the first and last week of trading. Since there is a high share of observations equal to zero (zero returns) in the data (see Table A.2 and Table A.3), these results serve primarily as an introduction for the results estimated on the lower granularity of data in the following sections. The zero returns imply that the price has not changed between the two observations. It is self-evident that the higher the sampling frequency, the higher is the probability that the price between two observations remains constant.

Figure 6.1 and Figure 6.2 shows the volatility during the first and the last week of trading of each stock, in the left and the right column respectively. The dark regions represent small areas of significant wavelet power in the short periods that, due to the limited resolution of our plots, appear as black lines. For the first week, we can observe bursts of volatility at the beginning of each trading day. In the first half of the week, these bursts can be observed across all investment horizons. Towards the end of the first week, the volatility stays significant only at the shorter investment horizons. The exceptions are Facebook and Zynga, where the bursts of volatility are less pronounced after the first day of trading. In the case of Facebook, this could be caused by the glitches on the stock exchange that affected the trading during the IPO (see Section 5.1). We assume that problems during the first day of trading could deter the investors, which resulted in low volatility during the first week.

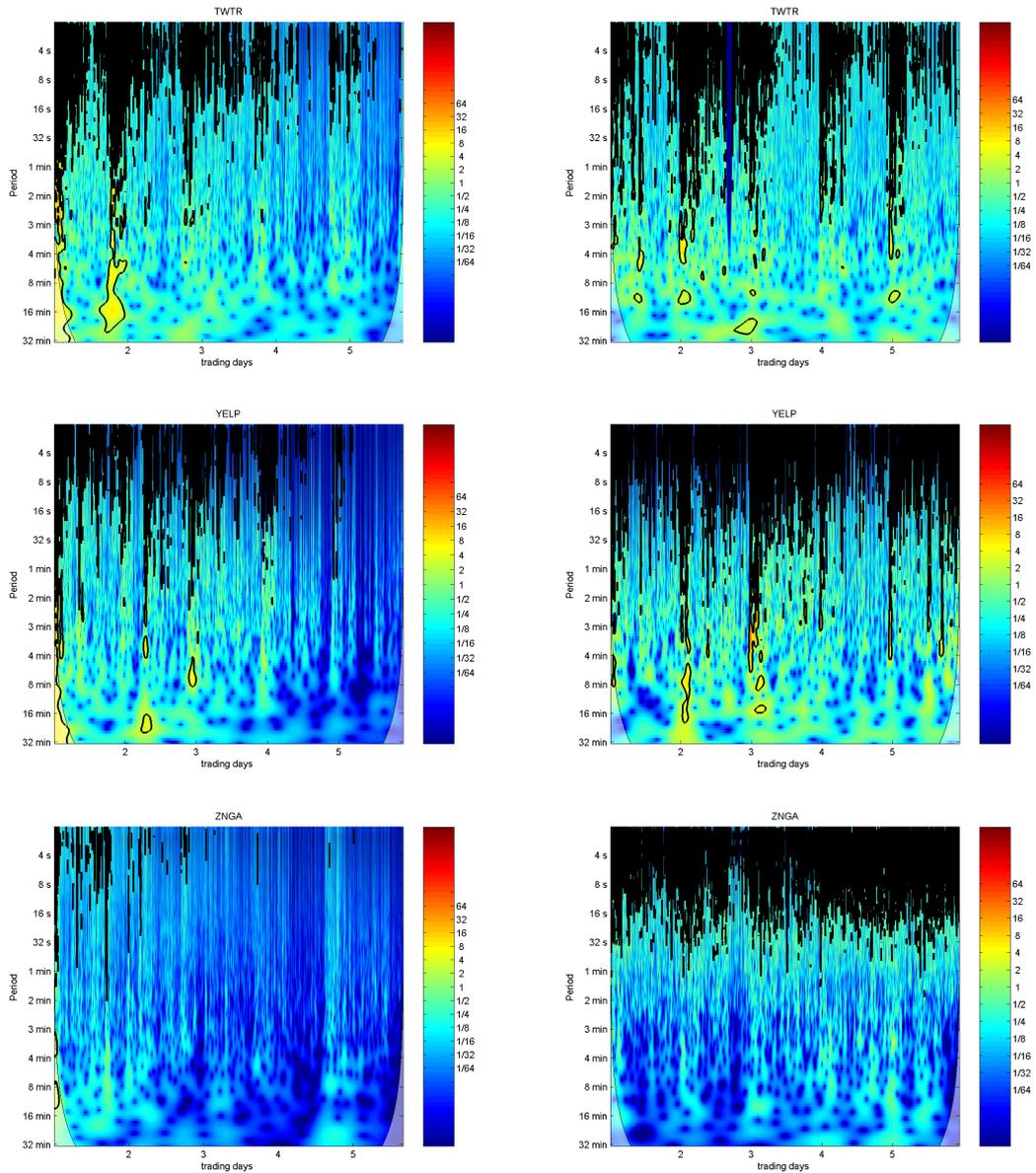
The volatility during the last week of trading exhibits similar structure as the first week. The burst of volatility across all shorter investment horizons (up to approximately 8 minute) can be observed at the beginning of each trading day. The only exception is Zynga, which has stable volatility contribution on the higher frequencies from 4 to 30 seconds.

Figure 6.1: Wavelet power – 1s sampling frequency. First versus last week of trading (left and right column respectively). From top to bottom: Facebook, LinkedIn, Pandora.



*Source:* Author's computations in Matlab with wtc-r16 toolbox

Figure 6.2: Wavelet power – 1s sampling frequency. First versus last week of trading (left and right column respectively). From top to bottom: Twitter, Yelp, Zynga.



*Source:* Author's computations in Matlab with wtc-r16 toolbox

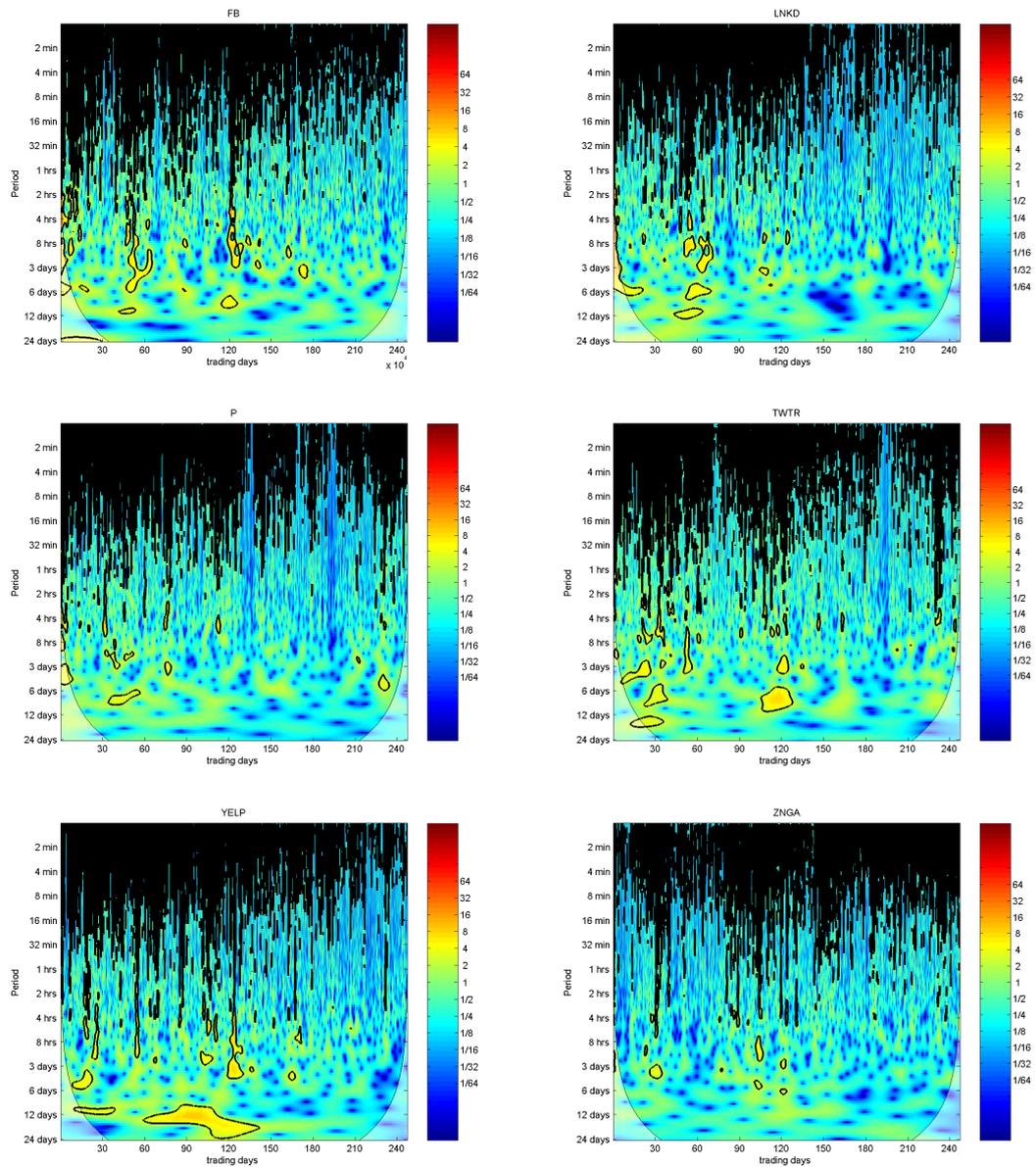
## 6.2 30s sampling frequency

In this section, we investigate the time-frequency volatility dynamics estimated on the data with 30 second sampling frequency. Figure 6.3 shows the wavelet power plots for all our stocks, estimated on the whole sample of 250 trading days. From the plots we can immediately see that the majority of wavelet power (volatility) is located on the higher frequencies that correspond up to the 1 hour investment horizon. In line with *Conjecture 1*, we observe the gradual decay of the volatility component on the scales approximately from 8 minutes to 1 hour. This effect is most pronounced for Facebook and Yelp. The volatility on the shortest investment horizons (scales from 2 to 8 minutes) is present during the whole observed period for all stocks. Moreover, we observe short periods of significant volatility on the lower frequencies (around the 3 day investment horizon) for all stocks. Surprisingly, this volatility is present only in the first half of all samples, then it disappears.

Figure 6.4 shows the plots of wavelet coherence. Each plot represents the co-movement of the particular stock with the S&P 500 index. In line with *Conjecture 2*, we look for the patterns that would tell us something about the synchronization of the stocks with the market. Our first observation is that at the 30 second sampling frequency, the coherence plots do not provide particularly useful information about the dynamics on the higher frequencies. Like in the case of the univariate analysis, there are many short periods of significant co-movement that form the dark area in the upper part of each plot. However, on the scales that represent 4 hour and longer investment horizons, we can observe interesting patterns. First, we can see that Facebook exhibits strong co-movement with S&P 500 at approximately 12 days investment horizon, from 180th day of trading day to the end of the sample. This is in line with *Conjecture 2*. On the other hand, in case of Pandora and Zynga, we observe significant areas of co-movement on the lower frequencies in the first half of the trading year, then it dies out. This could be interpreted as decrease of synchronization with the market and therefore contradicts *Conjecture 2*. Twitter seems to be synchronized with the market in the first half and at the end of the sample. For LinkedIn and Yelp, we observe areas of significant co-movement more or less evenly distributed in the first year of trading, therefore we cannot confirm the increasing synchronization as the stock gets mature.

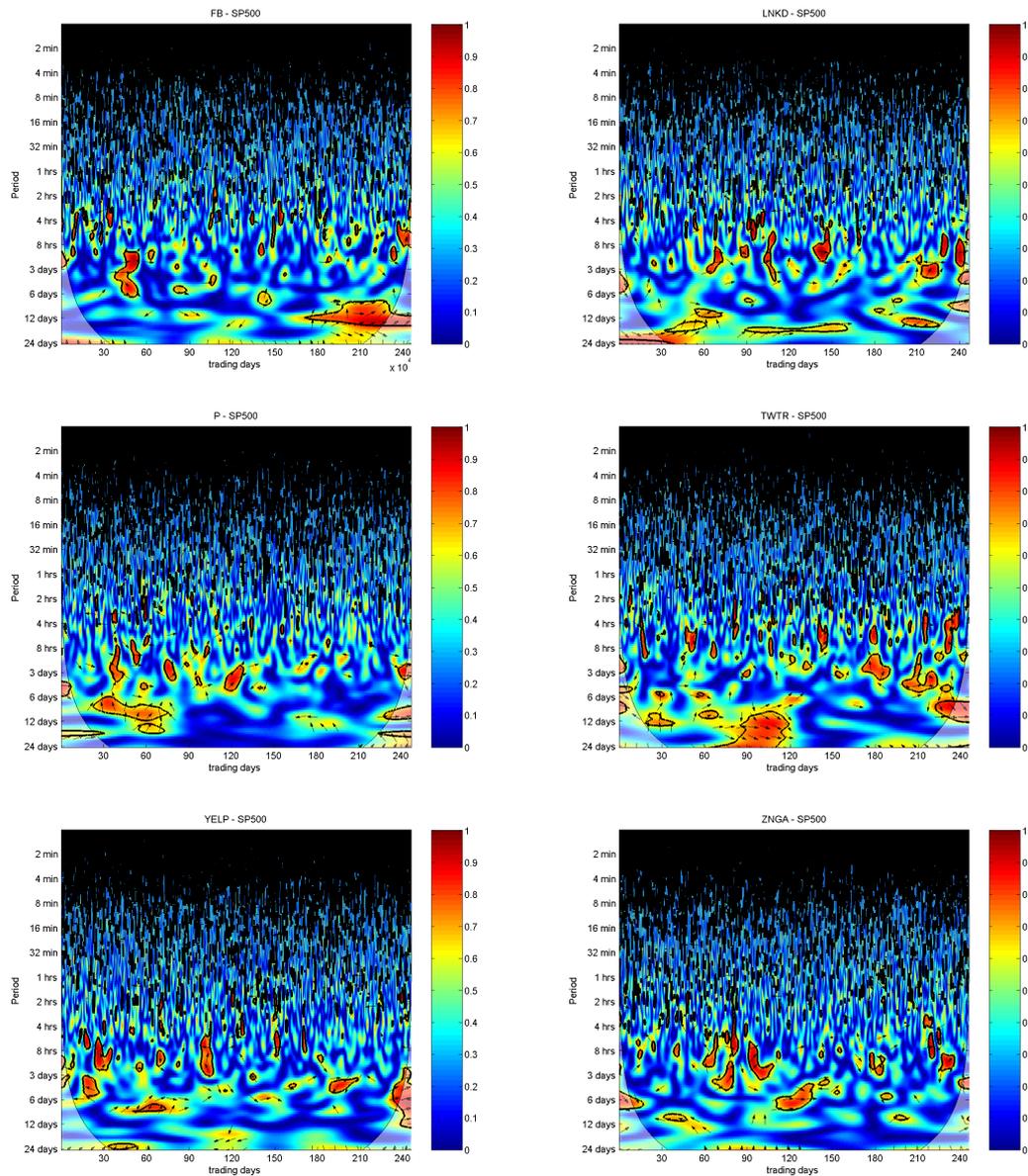
To complement the wavelet coherence analysis we use another tool to measure the synchronization of the stock with the market. Figure 6.5 shows the

Figure 6.3: Wavelet power – 30s sampling frequency. From upper left to bottom right: Facebook, LinkedIn, Pandora, Twitter, Yelp, Zynga.



Source: Author's computations in Matlab with wtc-r16 toolbox

Figure 6.4: Wavelet coherence – 30s sampling frequency. From upper left to bottom right: Facebook–SP500, LinkedIn–SP500, Pandora–SP500, Twitter–SP500, Yelp–SP500, Zynga–SP500.



*Source:* Author's computations in Matlab with wtc-r16 toolbox

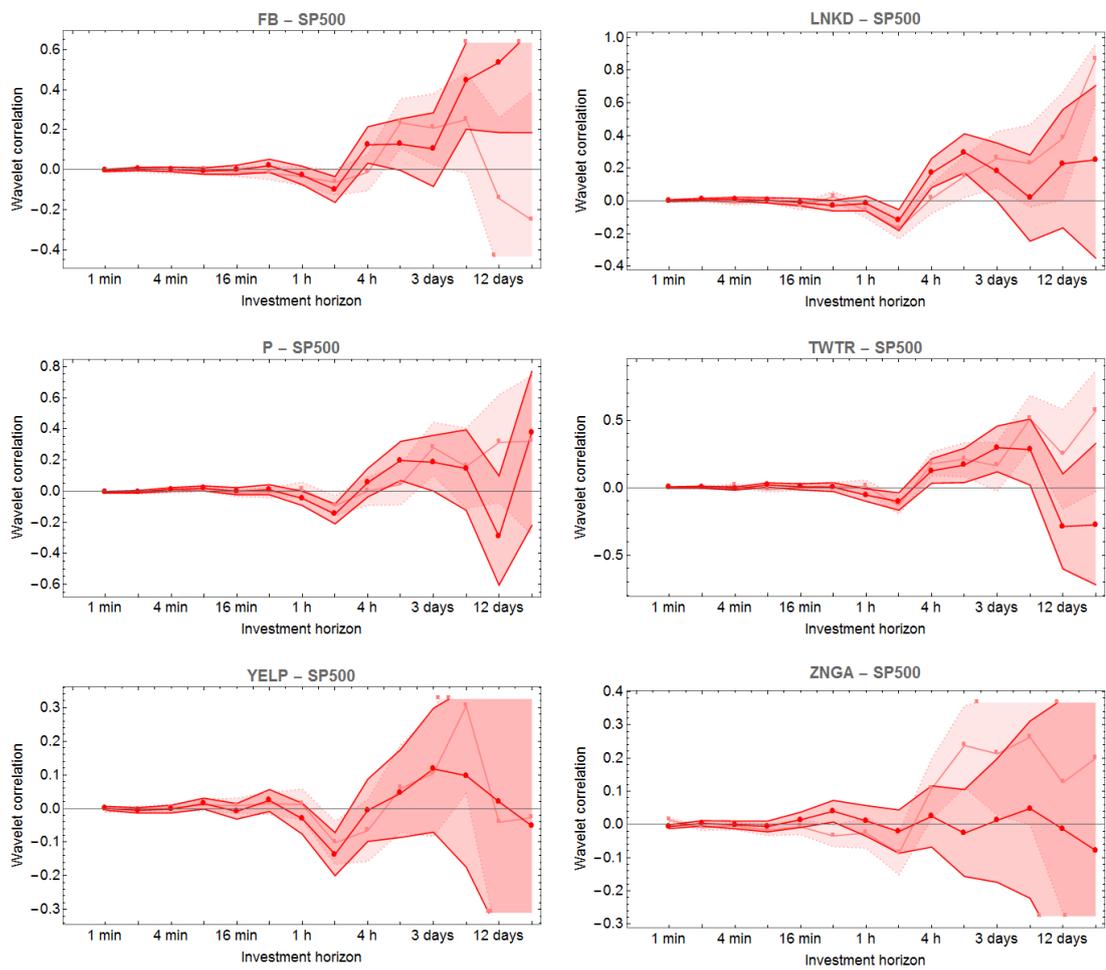
plots of the wavelet coefficient correlation. For each stock, we compare the correlation of wavelet coefficients between the particular stock and S&P 500 index. We measure this correlation for the first and second half of each sample (trading year) and compare them. The plots in pink represent the wavelet coefficients correlation and the respective confidence intervals. The darker red plots represent this correlation coefficients in the second half of the sample. For the 30s data, we compare the correlation of 14 wavelet scales, ranging from the 1 min to 24 days investment horizons. Following Barunik & Vacha (2013), to estimate if the difference between the correlation coefficients in the two time windows is significant, we check if the confidence intervals overlap. The confidence intervals correspond to the 90% confidence level. All other wavelet coefficient correlation estimations in this chapter also employ this confidence level.

As we can see from the plots, the only stock where the significant shift of correlation coefficient can be identified is Zynga, where the correlation at the 32 min investment horizon increased. This is in line with the *Conjecture 2*. For the rest of the stocks, the significant shift of the correlation coefficients cannot be confirmed. It is worth mentioning that for Facebook and Twitter, two biggest stocks with respect to the funds raised during the IPO (see Table 5.1), most of the wavelet coefficients on the scales from the 4 hour investment horizon and longer are positive and significantly different from zero. This implies that there is a positive relationship between these companies and the market, at least in the second half of their first year of public trading. However, we cannot confirm that this relationship changed between first and second half of the year. Similar observation can be made for Pandora and LinkedIn, with the difference that the positive relationship spans across less investment horizons. We failed to identify such relationship for Yelp and Zynga.

### 6.3 5min sampling frequency

In order to determine the sensitivity of our results to the sampling frequency of the data, we employ the approach from Section 6.2 and repeat the estimations on the 5min samples. First, let us briefly mention the additional benefit of this approach. It is known that volatility estimations on very high frequencies are plagued by the microstructure frictions (see for example Andersen *et al.* (2001)). Since we focus more on the general dynamics of the volatility in the time-frequency space than on the statistical properties of its estimate, we

Figure 6.5: Wavelet coefficient correlation – 30s sampling frequency. From upper left to bottom right: Facebook–SP500, LinkedIn–SP500, Pandora–SP500, Twitter–SP500, Yelp–SP500, Zynga–SP500.



Source: Author's computations in Mathematica and Matlab with WMTSA toolbox

consider the microstructure frictions problem to be beyond the scope of our work. However, Andersen *et al.* (2001) mention that the five minute sampling frequency provides a good trade-off between the resolution of the data and the amount of the microstructure noise. This brings additional motivation to employ 5 minute as well as 30 second data.

Figure 6.6 shows the wavelet power plots. For Facebook, LinkedIn and Yelp, the power plots look very similar to their 30s counterparts. In case of Pandora, Twitter and Zynga, the areas of high power on the longer investment horizons (around 4 days) in the first half of the respective trading years are pronounced. Generally, we observe decrease of volatility in the second half of the trading year for all stocks.

To test this observation in a more robust way, we calculate the realized volatility for the first and the last 60 trading days for each stock and compare them. For the sake of simplicity, we use the realised volatility estimator defined as  $RV_T = \sqrt{\sum_{i=1}^T lr_i^2}$ , where  $T$  is the period over which the realized volatility is calculated and  $lr$  are the logarithmic returns. Andersen & Bollerslev (1998) show that it is a consistent estimator of the total quadratic variation under the assumption of zero noise contamination in the price process. As mentioned above, the five minute data provide favourable trade-off between the detail and the noise and therefore for our purposes we consider the noise on the 5 minute data to be negligible. For more details, see Andersen *et al.* (2001). Table 6.1 shows that for all stocks except Zynga, the realized volatility decreased significantly. This is in line with the results from the wavelet power plots.

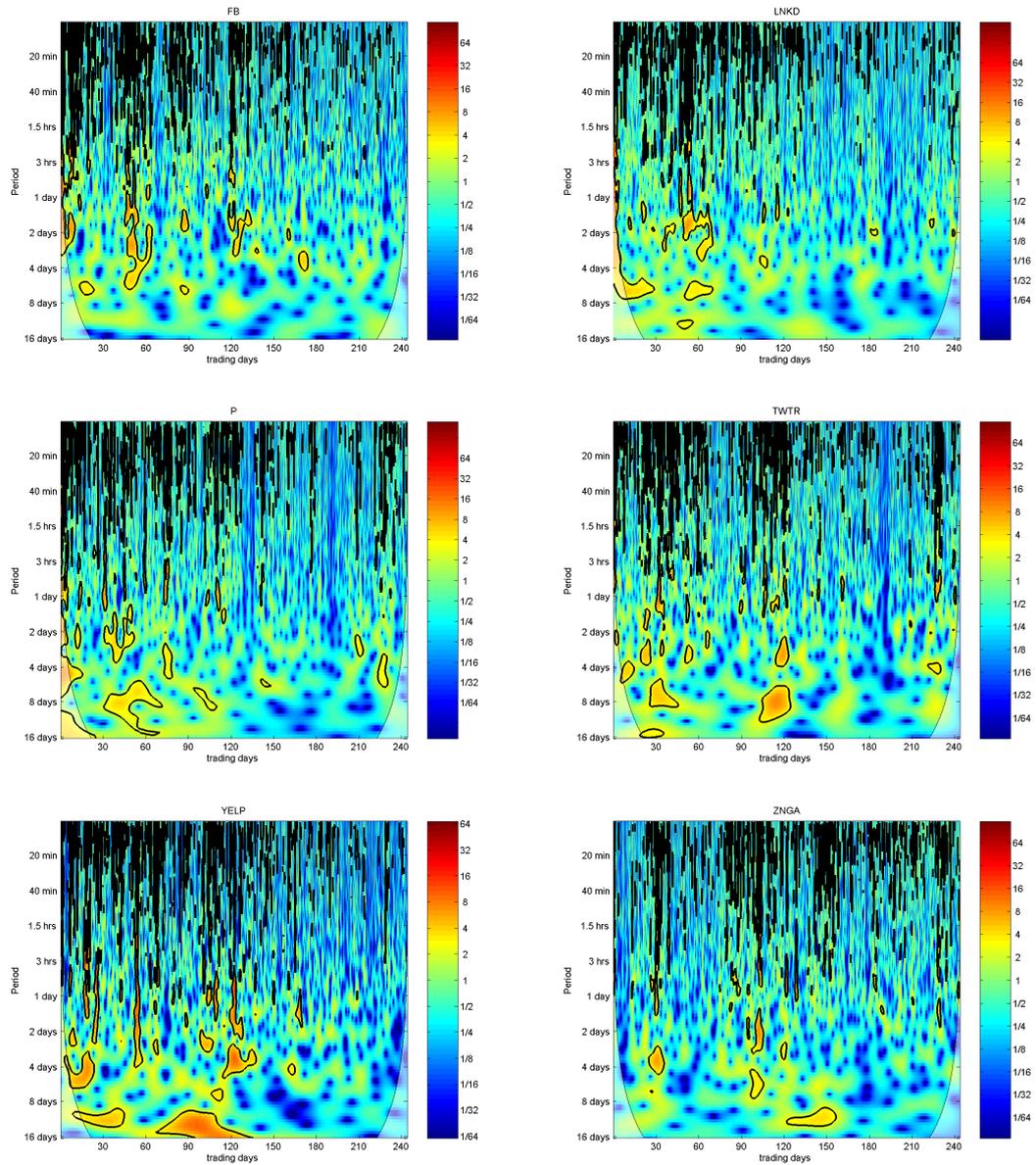
**Table 6.1:** Realized volatility at 5min: first versus last 60 trading days of the first trading year

	FB	LNKD	P	TWTR	YELP	ZNGA
RV first	0.238	0.403	0.404	0.265	0.365	0.278
RV last	0.128	0.198	0.263	0.170	0.219	0.284
Change (%)	-46%	-51%	-35%	-36%	-40%	+2%

*Source:* Author's computations in Matlab.

The wavelet coherence of the 5 minute data is shown in Figure 6.7. For all stocks except Twitter and Zynga, we observe similar time-frequency co-movement dynamics as for the 30s estimates. With respect to 30s estimates, Twitter exhibits weaker co-movement around the 8 day investment horizon in the middle of the trading year. On the contrary, Zynga shows stronger co-

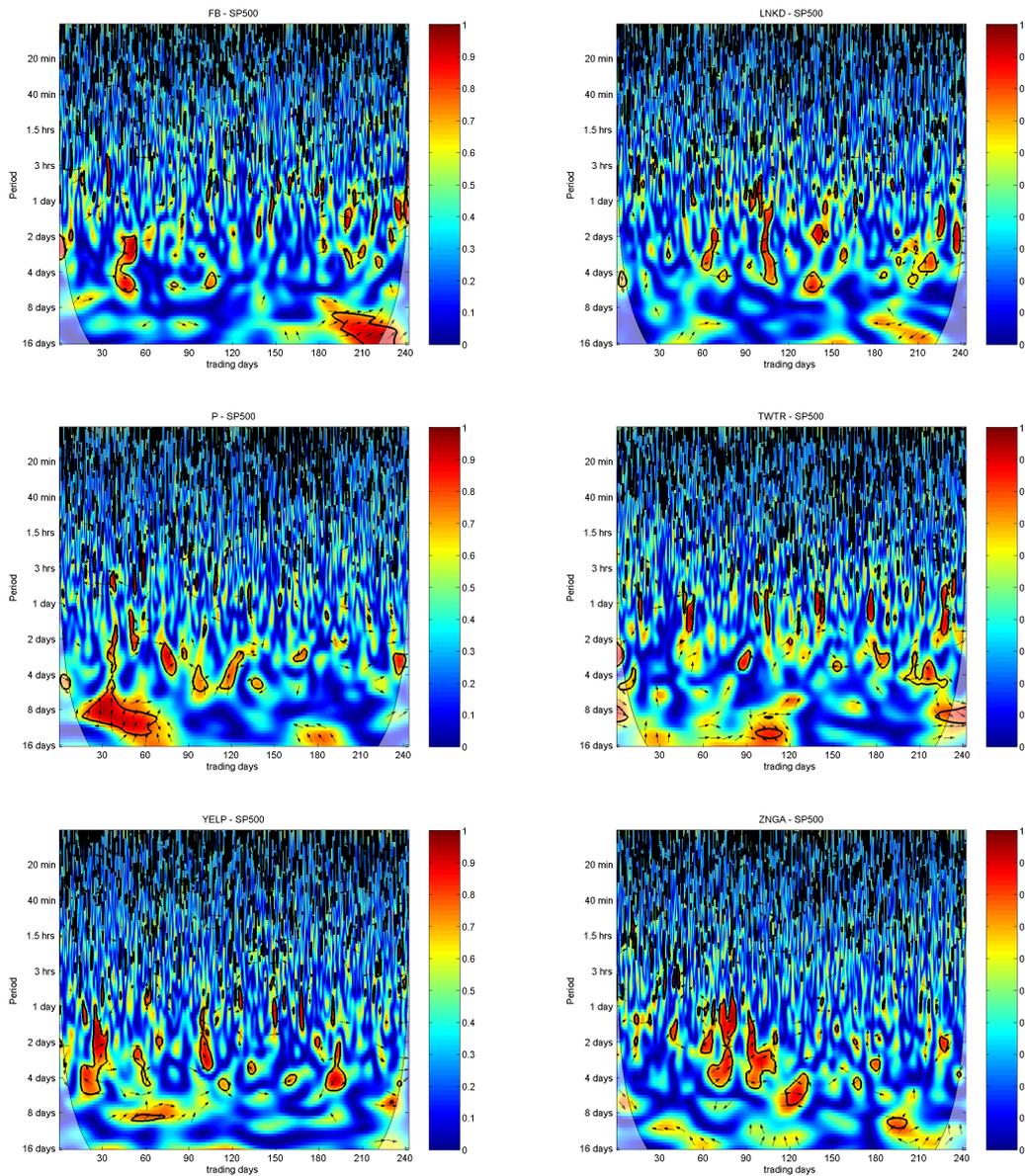
Figure 6.6: Wavelet power – 5min sampling frequency. From upper left to bottom right: Facebook, LinkedIn, Pandora, Twitter, Yelp, Zynga.



Source: Author's computations in Matlab with wtc-r16 toolbox

movement around the 4 day investment horizon in the first half of the sample. In general, we do not observe increase of co-movement between the stocks and the S&P 500 index during the respective trading years.

Figure 6.7: Wavelet coherence – 5min sampling frequency. From upper left to bottom right: Facebook–SP500, LinkedIn–SP500, Pandora–SP500, Twitter–SP500, Yelp–SP500, Zynga–SP500.

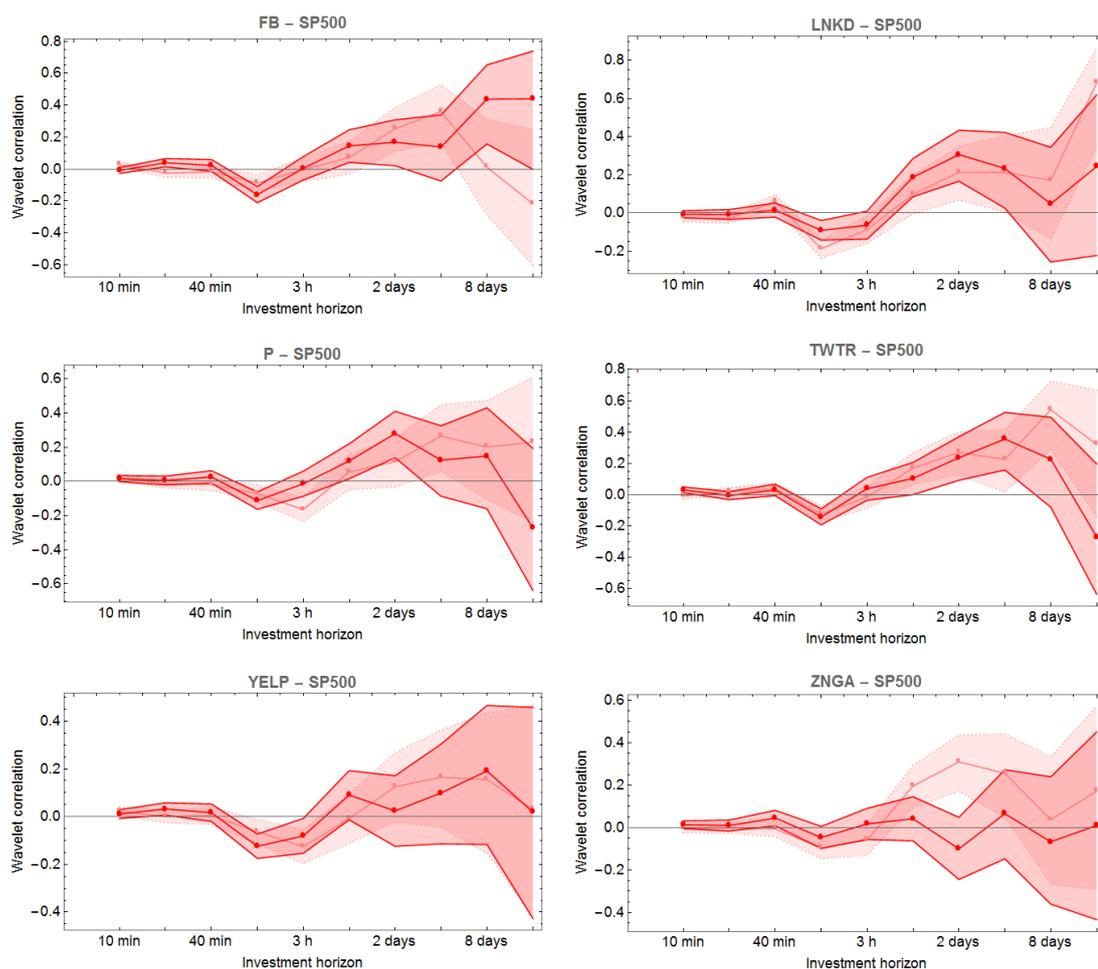


*Source:* Author's computations in Matlab with wtc-r16 toolbox

Figure 6.8 shows the wavelet coefficient correlation. Similarly to the 30s

data, the scales where the correlation coefficient shift is significant are scarce. For Facebook, we can identify significant decrease of the correlation coefficient that represents 10 minutes investment horizon. We can also observe increase of the correlation on the 20 minutes investment horizon for the same stock. For Zynga, we observe significant decrease of correlation on the scale that represents 2 day investment horizon. For the rest of the stocks, we cannot identify significant changes. However, as with the 30s data, for Facebook, LinkedIn, Pandora and Twitter, we observe correlation coefficients on the longer investment horizons being positive and significantly different from zero.

Figure 6.8: Wavelet coefficient correlation – 5min sampling frequency.  
From upper left to bottom right: Facebook–SP500, LinkedIn–SP500, Pandora–SP500, Twitter–SP500, Yelp–SP500, Zynga–SP500.



Source: Author's computations in Mathematica and Matlab with WMTSA toolbox

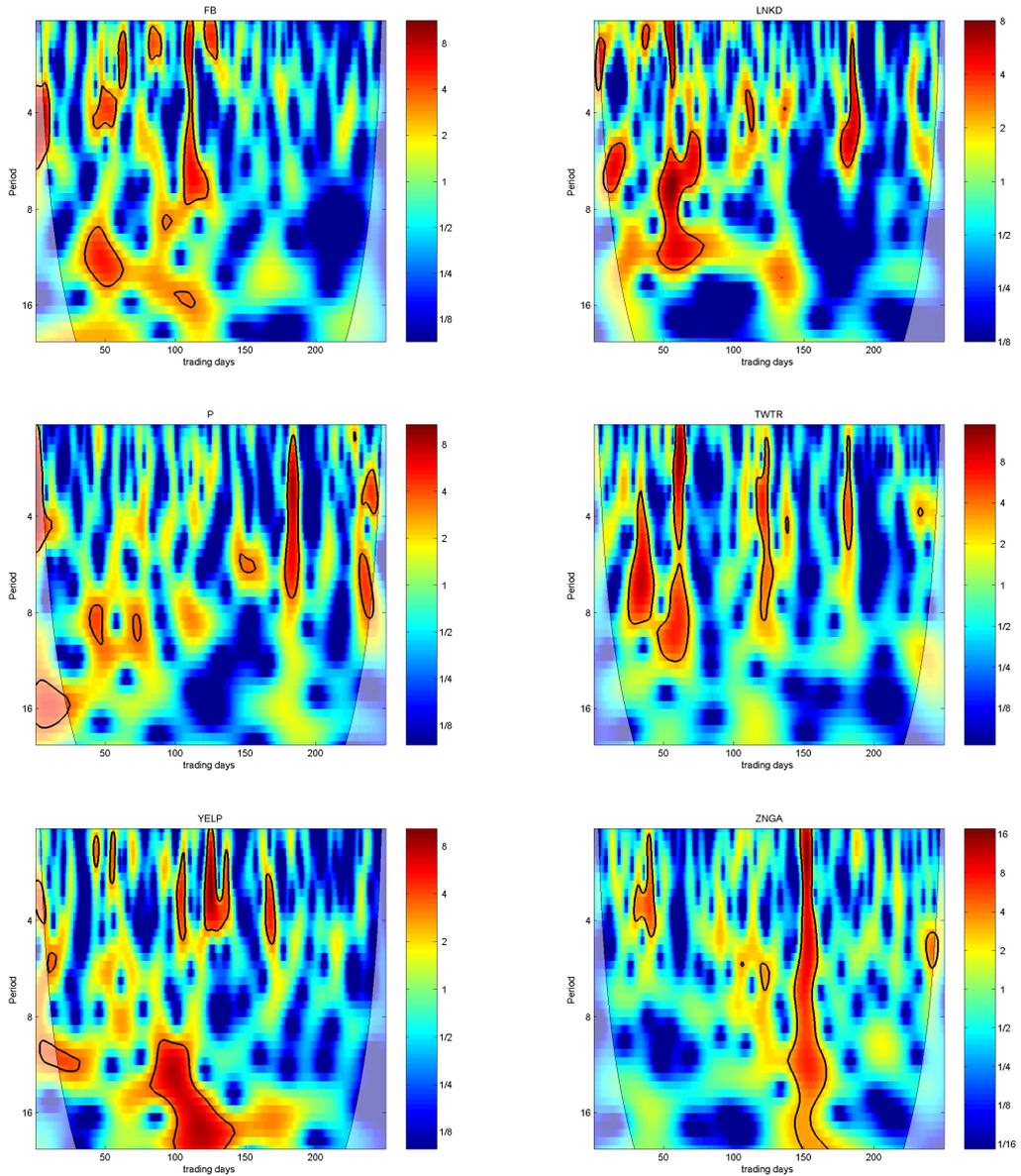
## 6.4 Daily sampling frequency

In this section, we investigate the time-frequency volatility dynamics estimated on the data with daily sampling frequency. For the estimation of the co-movement, we use NASDAQ stock market index as the market proxy, instead of the S&P 500 as in the previous sections. Figure 6.9 shows the wavelet power plots for all stocks. In case of Facebook, we observe sudden drop of volatility across all investment horizons in the second half of the sample. Similar dynamics can be observed for LinkedIn, Twitter and Yelp, with the difference that the volatility dies out later, approximately around the 180th day of trading. These findings are in line with *Conjecture 1*. On the other hand, we observe increase of volatility on higher frequencies towards the end of the sample in case of Pandora. For Zynga, there is clearly visible burst of volatility across all investment horizons in the middle of the sample.

Figure 6.10 shows the plots of the wavelet coherence. Each plot represents the co-movement of the particular stock with the NASDAQ index. As we can see, only Twitter, Zynga and Yelp exhibit increase of co-movement as the stock gets older. Particularly in the case of Zynga, we can observe gradual movement of coherence from shorter to longer investment horizons. That means, that the stock gradually becomes synchronized with the rest of the market on the longer investment horizons. Also for Yelp, we can see a large significant area of co-movement at the longer investment horizons at the end of the sample. For Twitter, we observe increase of common power at shorter investment horizons (around the 4 day frequency) at the end of the sample. These findings are in line with *Conjecture 2*. For other stocks, the results are not conclusive, since we observe areas of significant co-movement on all frequencies during the whole observed period.

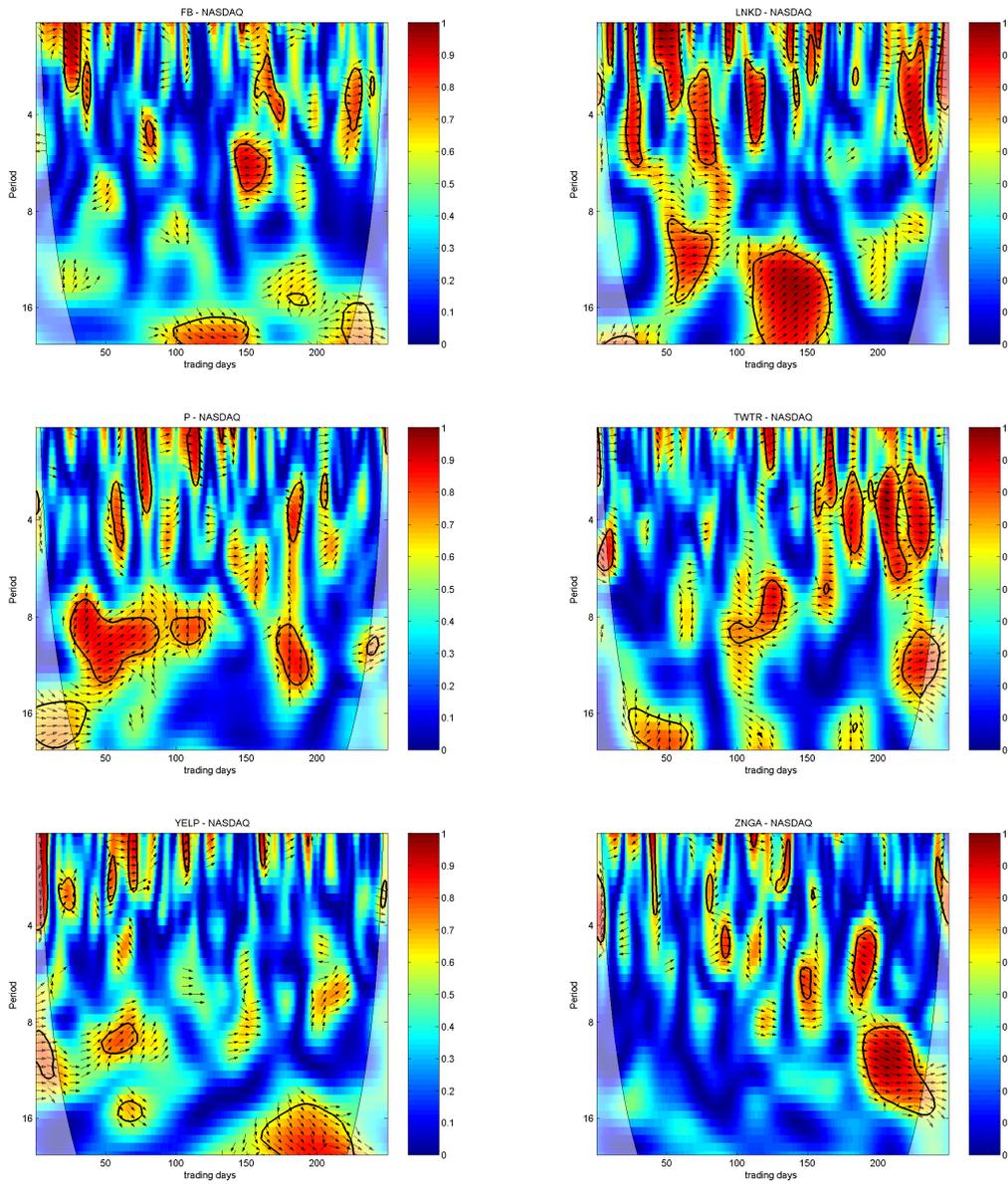
Figure 6.11 shows the plots of the wavelet coefficient correlation. For the daily data, we compare the correlation of 4 wavelet scales, ranging from the 2 to 16 day investment horizons. As we can see from the plots, there are two stocks where we observe significant shift in the correlation structure. In case of Facebook, we observe increase of the correlation at the wavelet scale that represents the 4 day investment horizon. This is in line with the *Conjecture 2*. In case of LinkedIn, we observe decrease of the correlation at the scale that represents 2 day investment horizon. This result is surprising, since we expected the increase of the correlation in the second half of the sample. For the rest of the stocks, the significant shift of the correlation coefficients cannot be con-

Figure 6.9: Wavelet power – daily sampling frequency. From upper left to bottom right: Facebook, LinkedIn, Pandora, Twitter, Yelp, Zynga.



Source: Author's computations in Matlab with wtc-r16 toolbox

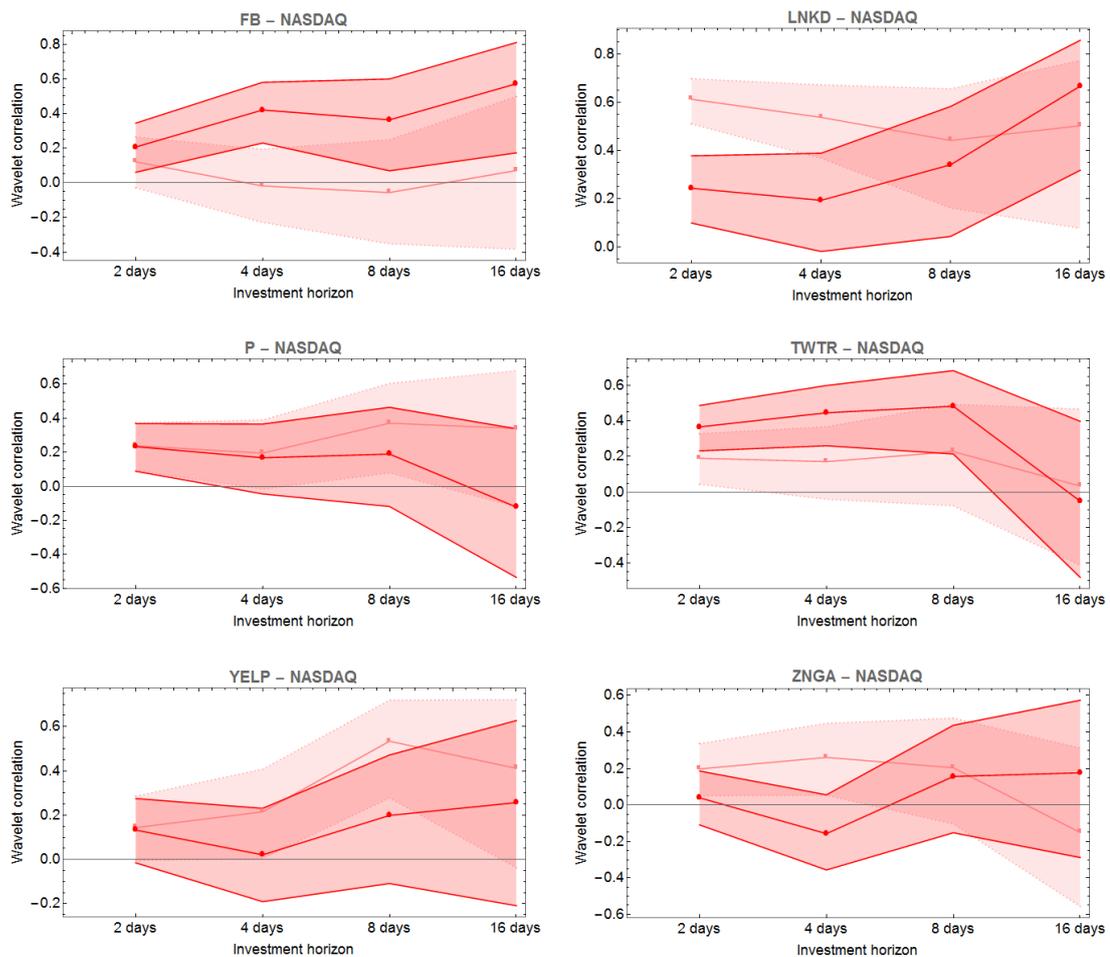
Figure 6.10: Wavelet coherence – daily sampling frequency. From upper left to bottom right: Facebook–NASDAQ, LinkedIn–NASDAQ, Pandora–NASDAQ, Twitter–NASDAQ, Yelp–NASDAQ, Zynga–NASDAQ.



*Source:* Author's computations in Matlab with wtc-r16 toolbox

firmed. Moreover, for Facebook, LinkedIn and Twitter, we observe correlation coefficients in the second half of the trading year being significantly different from zero. This is in line with the 30s and 5min results.

Figure 6.11: Wavelet coefficient correlation – daily sampling frequency. From upper left to bottom right: Facebook–SP500, LinkedIn–SP500, Pandora–SP500, Twitter–SP500, Yelp–SP500, Zynga–SP500.



Source: Author's computations in Mathematica and Matlab with WMTSA toolbox

## 6.5 Summary of results

The main findings can be summed up as follows. To test our conjectures, we have estimated wavelet power, wavelet coherence and correlation at scales. We have done so on the 1 second, 30 second, 5 minute and daily time series that

represent 6 technology stocks during the first year of public trading. First, we have concluded that because of the high share of zero returns, 1 second data are not suitable for our estimations. Therefore we will provide commentary only on the estimations that employ less granular datasets.

Second, we have identified the time-frequency dynamics of volatility in line with *Conjecture 1*. However, this result is not robust across all granularities of data. On the 30 second data, the gradual decrease of volatility is observed for all but the shortest investment horizons, but the visibility of this effect is not equally clear for all stocks. On 5 minute data, this result is confirmed, particularly with the respect to the decrease of volatility on the longer investment horizons. Moreover, we have managed to confirm this result for all but one stock by measuring realized volatility in the first and last 60 trading days. The minor difference between results on 30 second and 5 minute data can be attributed to relatively large number of zero returns in 30 second data. Moreover, we believe that the persistence of the volatility on the shortest investment horizons can be attributed to the microstructure frictions. For the daily data, the results are less conclusive. For most of the stocks, we observe the decrease of volatility at the end of the sample. However, the results are obscured by the relatively low level of detail in the plots, caused by the low sampling frequency of daily data. Nevertheless, we consider *Conjecture 1* to be confirmed.

Third, for some stocks we observe the time-frequency dynamics of co-movement in line with *Conjecture 2*. This result holds across all granularities of data. However, the robustness checks in the form of wavelet coefficient correlations do not confirm our conjecture. This hints at the possibility that the results we obtained from the wavelet coherence are actually spurious and there is no real relation between our stocks and the market. On the other hand, the correlation at scales uncovers additional result outside our conjectures. Although the change of the correlation between the first and the second half of the trading year cannot be confirmed, some of the stocks (particularly the “bigger” ones like Facebook and Twitter) do exhibit significant positive correlation of wavelet coefficients with both market proxies (S&P 500 and NASDAQ) in the second half of the trading year. This is observed for investment horizons of four hours and longer. This result implies that some kind of co-movement between the technology stocks and the markets does indeed exist but we were not able to describe its dynamics (in the sense of gradual synchronization, as the stock matures). With respect to our findings, we cannot confirm *Conjecture 2*.

Fourth, due to the inconclusive nature of previous results, we cannot con-

---

firm *Conjecture 3*. While in particular cases the observed dynamics of the co-movement differs, the findings are not strong enough to confirm our conjecture.

# Chapter 7

## Conclusion

In this thesis, we analyse the return volatility of the technology stocks and their co-movement with the market in the time-frequency space. We focus on the dynamics of the volatility and co-movement during the first year of public trading. In the time-frequency analysis framework, the frequency dimension of the data can be interpreted as investment horizons of investors with heterogeneous beliefs.

To perform the analysis, we use the methods from the wavelet analysis toolbox, particularly wavelet power spectrum, wavelet coherence and wavelet coefficient correlation. We analyse the datasets that represent six technology stocks that recently had their IPOs, particularly Facebook, LinkedIn, Pandora, Twitter, Yelp and Zynga. For our estimations, we employ the data sampled on multiple frequencies, ranging from 1 second high-frequency to daily data. The main contribution of our work is that to our knowledge, we are the first to use the wavelets to analyse the volatility and co-movement dynamics of the technology stocks in the specific period of the first year of public trading. Our results have implications for the understanding of the stock behaviour after the IPO. Moreover, the analysis of stock co-movement has implications for the diversification opportunities of investors.

We present three main findings. First, employing the wavelet power spectra, we identify gradual decline of the return volatility on all but the shortest investment horizons. This finding is in line with our conjecture that the technology stocks are plagued by the high-frequency volatility that dies out as the stock matures and becomes established on the stock exchange. This result holds across most of the tested stocks and all but daily data. Moreover, it is confirmed by the realized volatility analysis on 5 minute data. Second, em-

ploying wavelet coherence, we observe gradual synchronization of some stocks with the market. However, this result is not robustly confirmed by the analysis of correlation at scales. Thus our conjecture that technology stocks gradually synchronize with the rest of the market was not confirmed. However, for some “bigger” stocks, significantly positive correlation with the market on the longer investment horizons was observed. This result implies that even if the dynamics of the relationship between the stocks and the market cannot be estimated, it indeed exists. Third, we cannot find an evidence for our conjecture that the technology stocks exhibit different dynamics of co-movement for NASDAQ and S&P 500 stock market indices.

There are several possible directions in which our work can be extended. The analysis of volatility can be enriched by including the analysis of jumps or deeper incorporation of the realized volatility measures. Also, an interesting addition would be to perform our analysis using the high-frequency dataset of NASDAQ stock index. We believe that in this way more dependencies could be uncovered. Finally, the multivariate approach could be employed, adding more time series to the co-movement analysis and thus controlling for the external influences.

# Bibliography

- ADDISON, P. S. (2002): *The Illustrated Wavelet Transform Handbook: Introductory Theory and Applications in Science, Engineering, Medicine and Finance*. CRC Press.
- AGUIAR-CONRARIA, L., N. AZEVEDO, & M. J. SOARES (2008): “Using wavelets to decompose the time–frequency effects of monetary policy.” *Physica A* **387(12)**: pp. 2863–2878.
- AGUIAR-CONRARIA, L. & M. J. SOARES (2011a): “Business cycle synchronization and the euro: A wavelet analysis.” *Journal of Macroeconomics* **33(3)**: pp. 477–489.
- AGUIAR-CONRARIA, L. & M. J. SOARES (2011b): “Oil and the macroeconomy: using wavelets to analyze old issues.” *Empirical Economics* **40(3)**: pp. 645–655.
- ANDERSEN, T. G. & T. BOLLERSLEV (1998): “Answering the skeptics: Yes, standard volatility models do provide accurate forecasts.” *International economic review* pp. 885–905.
- ANDERSEN, T. G., T. BOLLERSLEV, F. X. DIEBOLD, & H. EBENS (2001): “The distribution of realized stock return volatility.” *Journal of financial economics* **61(1)**: pp. 43–76.
- BARUNIK, J., E. KOCENDA, & L. VACHA (2013): “Gold, oil, and stocks.” *arXiv preprint arXiv:1308.0210* .
- BARUNIK, J. & L. VACHA (2009): “Wavelet analysis of central european stock market behaviour during the crisis.” *IES Working Paper* **23/2009**. IES FSV. Charles University.

- BARUNIK, J. & L. VACHA (2013): “Contagion among Central and Eastern European Stock Markets during the Financial Crisis.” *Czech Journal of Economics and Finance (Finance a uver)* **63(5)**: pp. 443–453.
- BARUNIK, J., L. VACHA, & L. KRISTOUFEK (2011): “Comovement of central european stock markets using wavelet coherence: Evidence from high-frequency data.” *IES Working Paper 22/2011*. IES FSV. Charles University.
- CAPOBIANCO, E. (1999): “Statistical analysis of financial volatility by wavelet shrinkage.” *Methodology and Computing in Applied Probability* **1(4)**: pp. 423–443.
- DAI, Z., D. A. SHACKELFORD, & H. H. ZHANG (2013): “Capital gains taxes and stock return volatility.” *The Journal of the American Taxation Association* **35(2)**: pp. 1–31.
- DAUBECHIES, I. (1992): *Ten Lectures on Wavelets*, volume 61. SIAM.
- FAN, Y. & R. GENÇAY (2009): “Unit root tests with wavelets.” *MPRA Paper No. 15913* .
- GABOR, D. (1946): “Theory of communication. part 1: The analysis of information.” *Journal of the Institution of Electrical Engineers-Part III: Radio and Communication Engineering* **93(26)**: pp. 429–441.
- GALLEGATI, M. (2008): “Wavelet analysis of stock returns and aggregate economic activity.” *Computational Statistics & Data Analysis* **52(6)**: pp. 3061–3074.
- GENÇAY, R., F. SELÇUK, & B. WHITCHER (2002): *An introduction to wavelets and other filtering methods in finance and economics*. Academic Press.
- GENÇAY, R., F. SELÇUK, & B. WHITCHER (2005): “Multiscale systematic risk.” *Journal of International Money and Finance* **24(1)**: pp. 55–70.
- GOLDFARB, B., D. KIRSCH, & D. A. MILLER (2007): “Was there too little entry during the dot com era?” *Journal of Financial Economics* **86(1)**: pp. 100–144.
- GRIFFIN, J. M., J. H. HARRIS, T. SHU, & S. TOPALOGLU (2011): “Who drove and burst the tech bubble?” *The Journal of Finance* **66(4)**: pp. 1251–1290.

- GRINSTED, A., J. C. MOORE, & S. JEVREJEVA (2004): "Application of the cross wavelet transform and wavelet coherence to geophysical time series." *Nonlinear Processes in Geophysics* **11(5/6)**: pp. 561–566.
- HASBROUCK, J. (2012): "High frequency quoting: Short-term volatility in bids and offers." .
- KIM, S. & F. IN (2005): "The relationship between stock returns and inflation: new evidence from wavelet analysis." *Journal of Empirical Finance* **12(3)**: pp. 435–444.
- KINDLEBERGER, C. P. & R. Z. ALIBER (2011): *Manias, panics and crashes: a history of financial crises*. Palgrave Macmillan.
- KRAVETS, T. & A. SYTIENKO (2013): "Wavelet analysis of the crisis effects in stock index returns." *Ekonomika/Economics* **92(1)**.
- LOWENSTEIN, R. (2004): *Origins of the crash: The great bubble and its undoing*. Penguin.
- LOWRY, M., M. S. OFFICER, & G. W. SCHWERT (2010): "The variability of ipo initial returns." *The Journal of Finance* **65(2)**: pp. 425–465.
- MALLAT, S. (1999): *A wavelet tour of signal processing*. Academic press.
- MULLIGAN, R. F. (2004): "Fractal analysis of highly volatile markets: an application to technology equities." *The Quarterly Review of Economics and Finance* **44(1)**: pp. 155–179.
- NACCACHE, T. (2011): "Oil price cycles and wavelets." *Energy Economics* **33(2)**: pp. 338–352.
- OFEK, E. & M. RICHARDSON (2002): "The valuation and market rationality of internet stock prices." *Oxford Review of Economic Policy* **18(3)**: pp. 265–287.
- OFEK, E. & M. RICHARDSON (2003): "Dotcom mania: The rise and fall of internet stock prices." *The Journal of Finance* **58(3)**: pp. 1113–1138.
- PERCIVAL, D. B. & A. T. WALDEN (2000): *Wavelet Methods for Time Series Analysis*. Cambridge Series in Statistical and Probabilistic Mathematics. Cambridge University Press.

- PricewaterhouseCoopers LLP (2015): “Global technology, IPO review full-year and Q4 2014.” [Online] <http://www.pwc.com/gx/en/technology/publications/assets/full-year-and-q4-2014-global-tech-ipo-review.pdf> [Accessed April 27, 2015].
- RITTER, J. R. (2014): “Initial public offerings: Technology stock IPOs.” [Online] <http://bear.warrington.ufl.edu/ritter/IP0s2013Tech.pdf> [Accessed January 12, 2014].
- RITTER, J. R. & I. WELCH (2002): “A review of ipo activity, pricing, and allocations.” *The Journal of Finance* **57(4)**: pp. 1795–1828.
- RUA, A. & L. C. NUNES (2009): “International comovement of stock market returns: A wavelet analysis.” *Journal of Empirical Finance* **16(4)**: pp. 632–639.
- RUA, A. & L. C. NUNES (2012): “A wavelet-based assessment of market risk: The emerging markets case.” *The Quarterly Review of Economics and Finance* **52(1)**: pp. 84–92.
- SOUSA, R., L. AGUIAR-CONRARIA, & M. J. SOARES (2014): “Carbon financial markets: A time-frequency analysis of CO2 prices.” *Physica A* **414**: pp. 118–127.
- STIGLITZ, J. E. (2003): *The Roaring Nineties: A New History of the World’s Most Prosperous Decade*. New York: W. W. Norton & Company.
- TIWARI, A. K. (2013): “Oil prices and the macroeconomy reconsideration for germany: Using continuous wavelet.” *Economic Modelling* **30**: pp. 636–642.
- TORRENCE, C. & G. P. COMPO (1998): “A practical guide to wavelet analysis.” *Bulletin of the American Meteorological Society* **79(1)**: pp. 61–78.
- TORRENCE, C. & P. J. WEBSTER (1999): “Interdecadal changes in the ENSO-monsoon system.” *Journal of Climate* **12(8)**: pp. 2679–2690.
- VACHA, L. & J. BARUNIK (2012): “Co-movement of energy commodities revisited: Evidence from wavelet coherence analysis.” *Energy Economics* **34(1)**: pp. 241–247.

# Appendix A

## Complementary tables and figures

Table A.1: Descriptive statistics – daily data

Statistic	FB	LNKD	P	TWTR	YELP	ZNGA
Mean	-1.51E-03	6.41E-04	-1.79E-03	-3.75E-04	-2.65E-04	-5.30E-03
St. dev	3.47E-02	4.11E-02	5.03E-02	4.14E-02	4.47E-02	5.28E-02
Skewness	1.76E-01	9.63E-02	-1.23E+00	-1.36E+00	2.81E-01	-2.73E+00
Kurtosis	6.57E+00	5.29E+00	8.52E+00	1.37E+01	5.91E+00	2.64E+01
Min	-1.24E-01	-1.91E-01	-2.73E-01	-2.77E-01	-1.58E-01	-4.68E-01
Max	1.75E-01	1.64E-01	1.16E-01	1.82E-01	2.03E-01	1.56E-01
No. obs.	249	249	249	249	249	249
Null obs.	3	0	3	1	1	7
Null obs. %	1%	0%	1%	0%	0%	3%

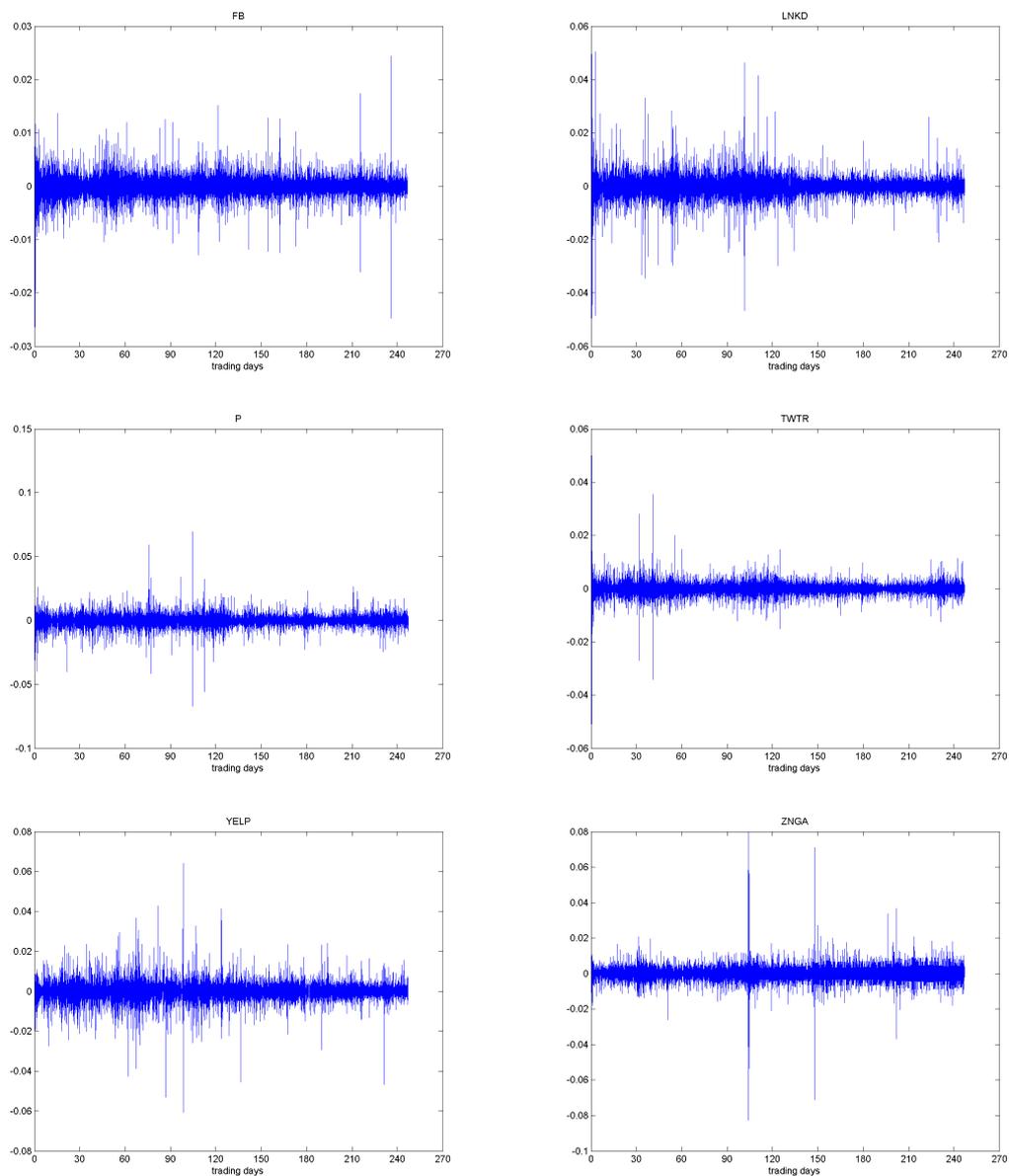
*Source:* Author's computations in Matlab.

Table A.2: Descriptive statistics – 1s data, first week of trading

Statistic	FB	LNKD	P	TWTR	YELP	ZNGA
Mean	-1.61E-06	2.60E-06	-2.68E-06	3.62E-07	-2.97E-07	-1.97E-06
St. dev	2.75E-03	1.97E-03	1.39E-03	6.29E-04	6.59E-04	1.81E-03
Skewness	1.94E-01	-2.70E+00	1.99E+01	-6.24E-01	3.98E+00	4.45E-01
Kurtosis	7.16E+03	8.28E+03	3.44E+03	1.00E+03	7.21E+02	4.62E+03
Min	-3.31E-01	-2.55E-01	-8.29E-02	-5.07E-02	-3.40E-02	-1.48E-01
Max	3.31E-01	2.47E-01	1.82E-01	5.07E-02	4.86E-02	1.51E-01
No. obs.	91786	97153	98763	94230	98053	93290
Null obs.	28943	63900	75341	56734	87480	75410
Null obs. %	32%	66%	76%	60%	89%	81%

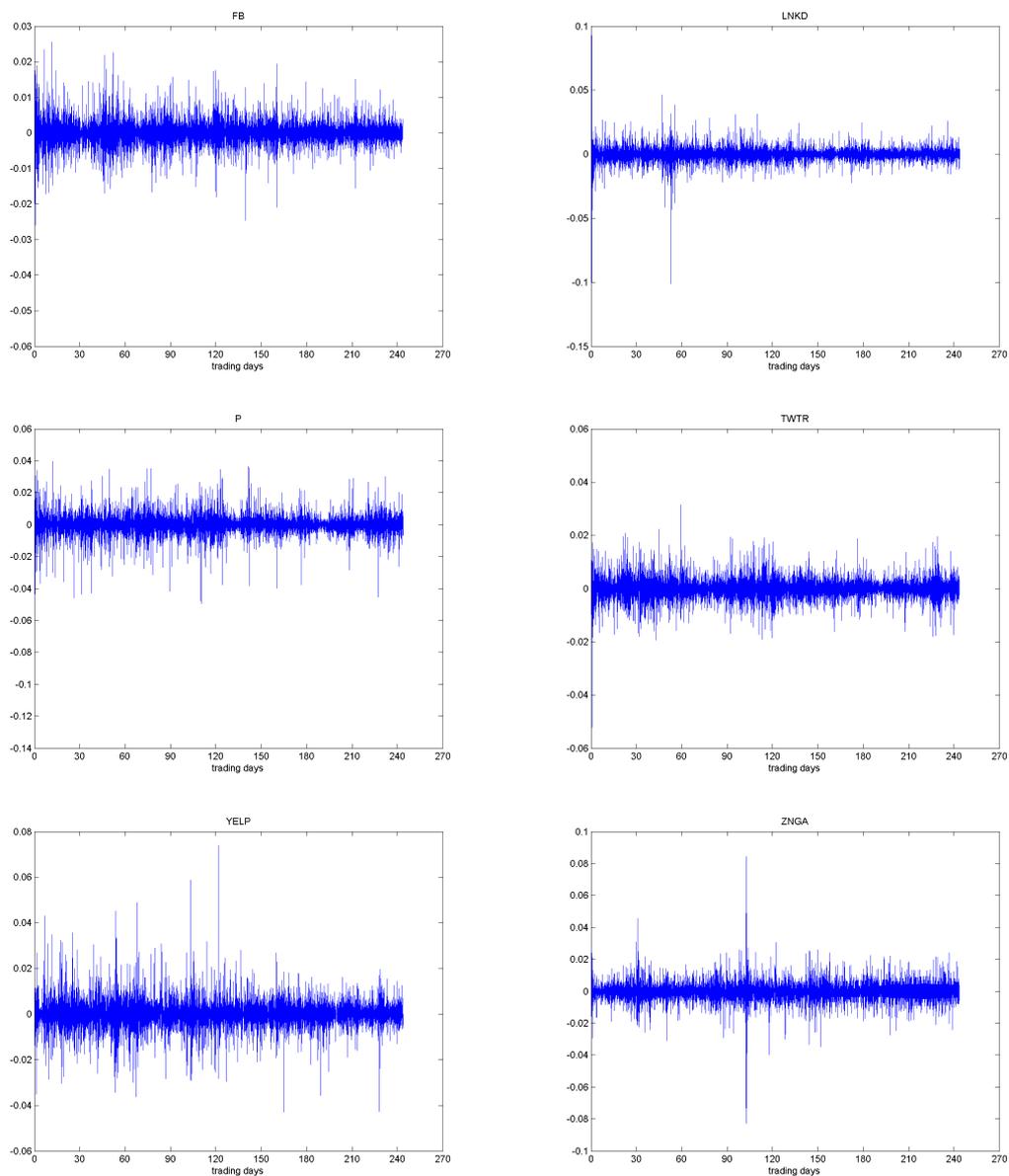
*Source:* Author's computations in Matlab.

Figure A.1: Plots of logarithmic returns – 30s sampling frequency.  
From upper left to bottom right: Facebook, LinkedIn,  
Pandora, Twitter, Yelp, Zynga.



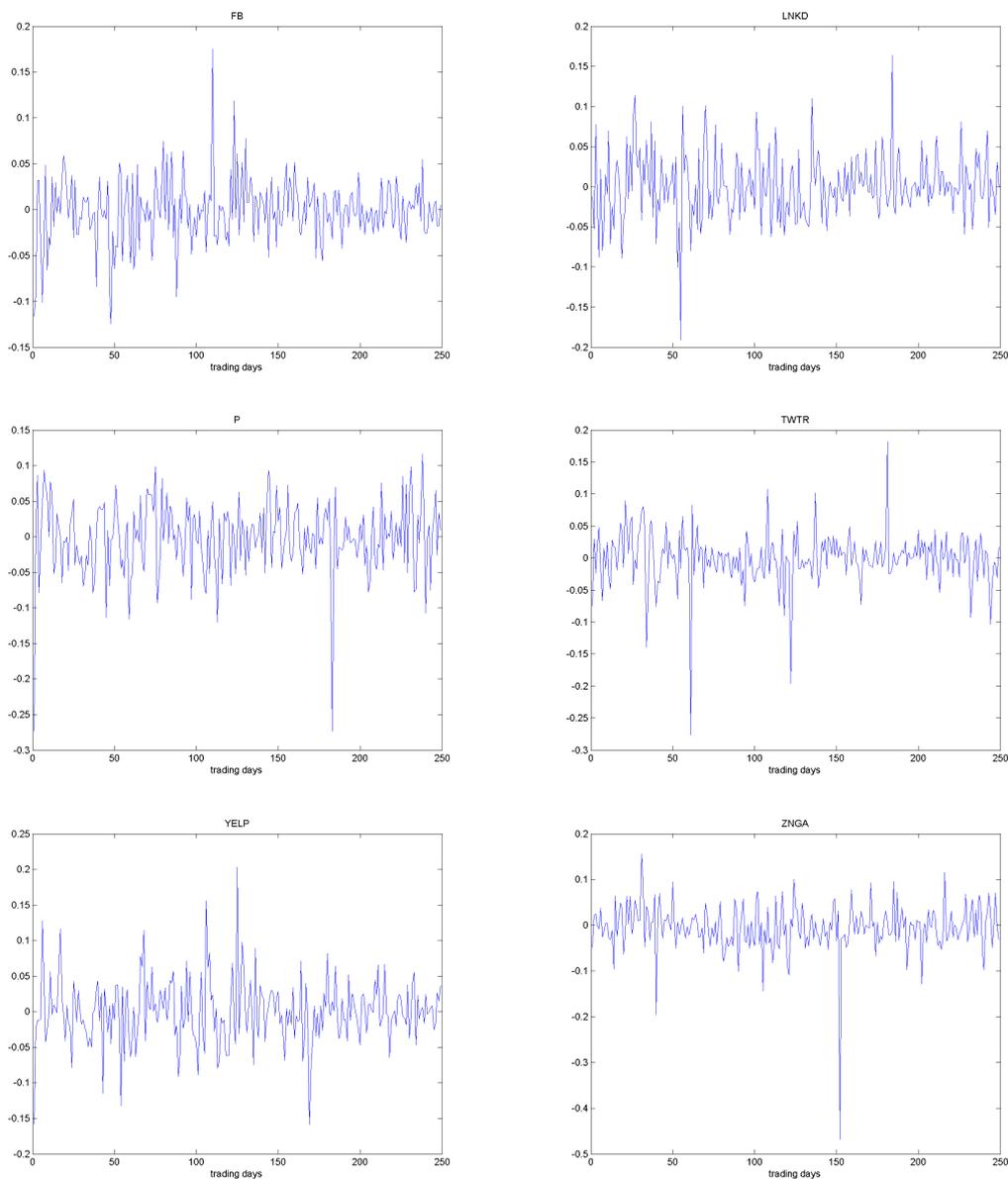
*Source:* Author's computations in Matlab

Figure A.2: Plots of logarithmic returns – 5min sampling frequency.  
From upper left to bottom right: Facebook, LinkedIn,  
Pandora, Twitter, Yelp, Zynga.



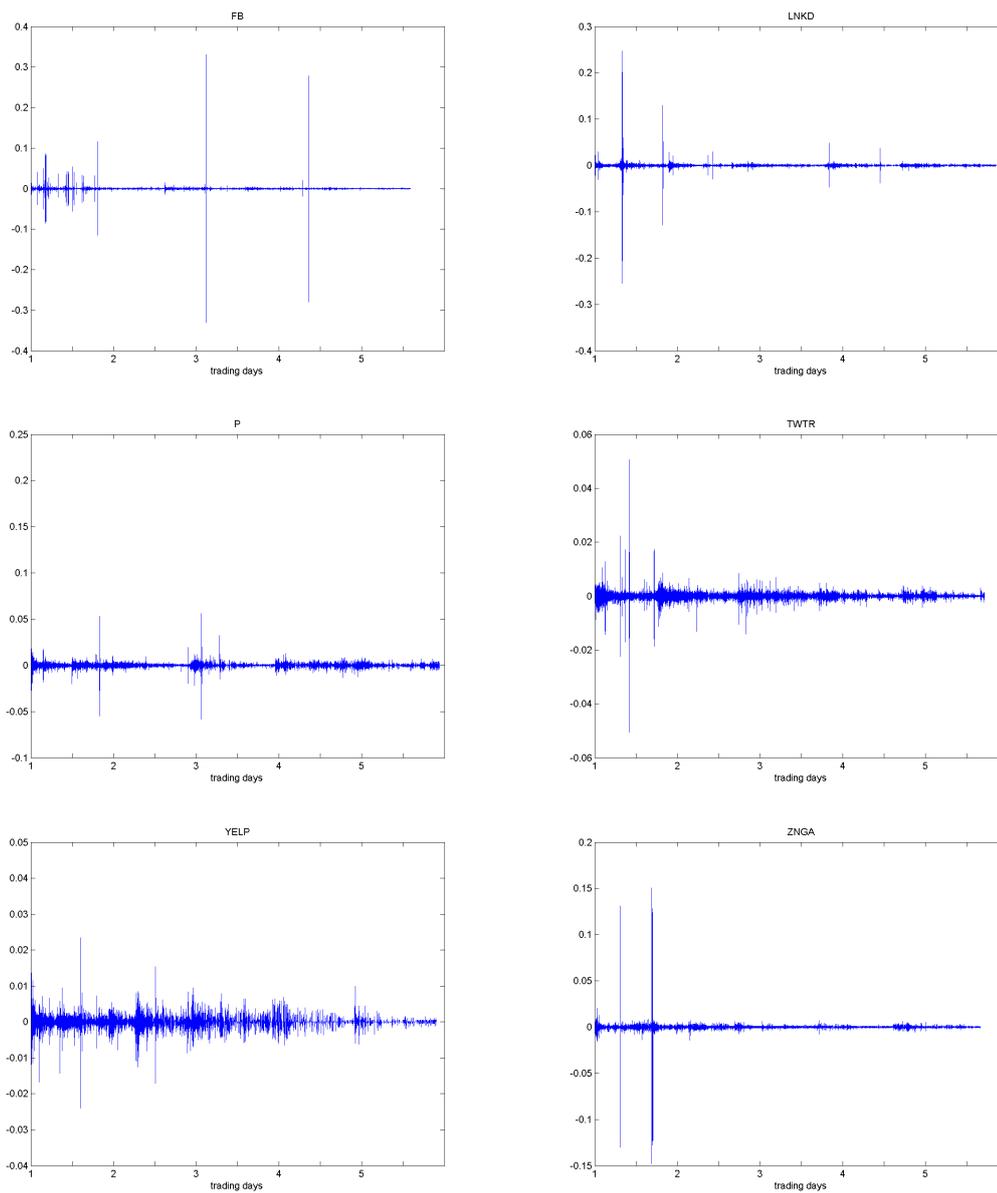
*Source:* Author's computations in Matlab

Figure A.3: Plots of logarithmic returns – daily sampling frequency.  
From upper left to bottom right: Facebook, LinkedIn,  
Pandora, Twitter, Yelp, Zynga.



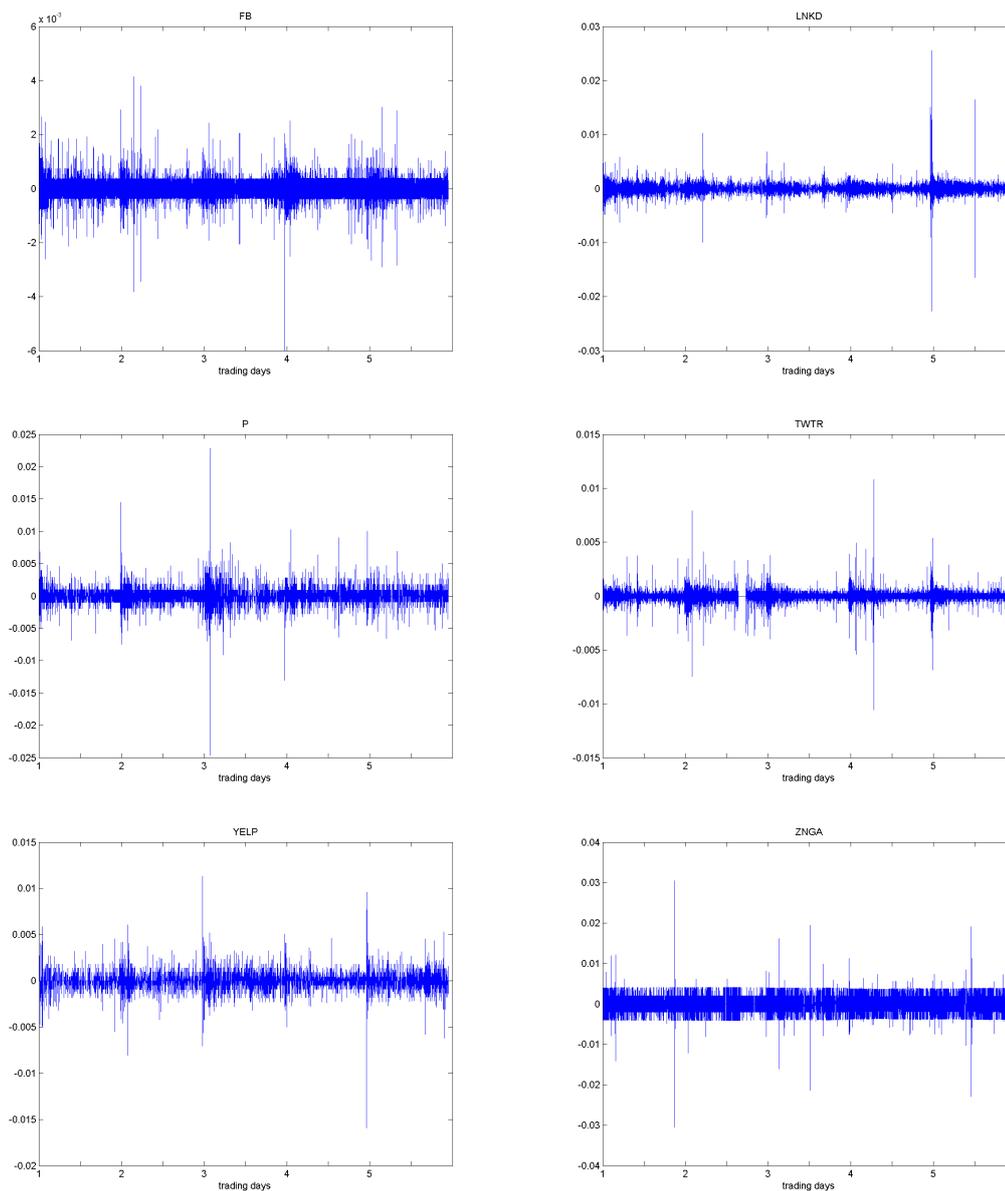
*Source:* Author's computations in Matlab

Figure A.4: Plots of logarithmic returns – 1s sampling frequency, first week of trading. From upper left to bottom right: Facebook, LinkedIn, Pandora, Twitter, Yelp, Zynga.



*Source:* Author's computations in Matlab

Figure A.5: Plots of logarithmic returns – 1s sampling frequency, last week of trading. From upper left to bottom right: Facebook, LinkedIn, Pandora, Twitter, Yelp, Zynga.



*Source:* Author's computations in Matlab

Table A.3: Descriptive statistics – 1s data, last week of trading

Statistic	FB	LNKD	P	TWTR	YELP	ZNGA
Mean	-7.59E-08	3.49E-07	7.90E-07	-5.18E-07	4.52E-07	-5.70E-07
St. dev	1.84E-04	3.89E-04	4.29E-04	2.51E-04	2.73E-04	8.61E-04
Skewness	-2.48E-01	1.58E+00	2.66E-01	6.42E-02	-2.51E-01	-1.76E-01
Kurtosis	4.21E+01	4.56E+02	2.73E+02	1.75E+02	3.02E+02	6.69E+01
Min	-5.99E-03	-2.26E-02	-2.46E-02	-1.05E-02	-1.59E-02	-3.05E-02
Max	4.15E-03	2.56E-02	2.29E-02	1.08E-02	1.13E-02	3.05E-02
No. obs.	98995	98995	98995	98995	98995	98995
Null obs.	65238	78809	90607	51754	93698	87929
Null obs. %	66%	80%	92%	52%	95%	89%

*Source:* Author's computations in Matlab.