Charles University in Prague

Faculty of Social Sciences
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BACHELOR THESIS

Variance structure of the Bitcoin currency

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Declaration of Authorship

The author hereby declares that he compiled this thesis independently, using only the listed resources and literature.

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Abstract

The purpose of this thesis is to explain how Bitcoin works, analyze the Bitcoin total variation and to separate the jump component of realized variance from the continuous part. In order to do so, we use estimates of quadratic variation and integrated variance. We detect jumps using a test which is based on the difference between realized variance and bipower variation. The results for BTC/USD exchange rate are then compared with the results for EUR/USD exchange rate, price of gold and for the S&P 500 index. In case of all datasets, we use data with five-minute frequency. It seems that no other work analyzing the Bitcoin total variation using the same methods to separate the jump component from the continuous part of a price process has been written so far. We found that jumps in the Bitcoin total variation are stronger than for other analyzed instruments. The results also suggest that the duration between jumps for Bitcoin considerably prolonged during the monitored period which may indicate that the behavior of price of bitcoin has stabilized over time. We also found out that the variance of price of bitcoin is higher during the monitored period in comparison with other analyzed instruments.

JEL Classification	C14, C22, C50, C58
Keywords	Bitcoin, Digital Currency, Realized Variance, Realized Volatility, Bipower Variation, High Frequency Data, Jumps

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Abstrakt

Cílem této práce je vysvětlit, jak funguje digitální měna Bitcoin a dále analýza volatility Bitcoinu a oddělení skokové části realizované variace od části spojité. K tomuto využíváme odhadů kvadratické a integrované variace, přičemž cenové skoky detekujeme pomocí testu založeného na rozdílu mezi realizovanou a dvojmocnou variací. Výsledky pro kurz BTC/USD jsou poté srovnány s výsledky pro směnný kurz EUR/USD, pro cenu zlata a také pro hodnotu akciového indexu S&P 500. Pro naši analýzu využíváme data s frekvencí 5 minut. Podle všeho nebyla doposud napsána práce zabývající se analýzou volatility Bitcoinu, jež by využívala výše zmíněné metody k oddělení skokové a spojité části cenového procesu. Zjistili jsme, že skoky jsou v případě Bitcoinu vyšší než v případě ostatních analyzovaných instrumentů. Výsledky dále naznačují, že doba mezi jednotlivými detekovanými skoky se pro Bitcoin ke konci sledovaného období prodloužila. Dále se ukázalo, že variace je v případě Bitcoinu za sledované období vyšší než u jiných instrumentů.

Klasifikace JEL	C14, C22, C50, C58
Klíčová slova	Bitcoin, Digitální měna, Realizovaná variace, Realizovaná volatilita, Dvojmocná variace, Vysokofrekvenční data, Skoky

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Acronyms

ALM Adjusted Lagrange Multiplier

AR Autoregressive

ARCH Autoregressive Conditional Heteroskedasticity

ATM Automated Teller Machine

BPI Bitcoin Price Index

BTC bitcoin

CNY Chinese Yuan Renminbi

ECM Error Correction Model

EUR Euro

GARCH Generalized Autoregressive Conditional Heteroskedasticity

GB Gigabyte

GDP Gross Domestic Product

GMT Greenwich Mean Time

NASDAQ National Association of Securities Dealers Automated Quotations

NFC Near Field Communication

NYSE New York Stock Exchange

OTC Over The Counter

QR Quick Response

US United States

USD United States dollar

VAR Vector Autoregression

VECM Vector Error Correction Model

XAU Gold Ounce

Bachelor Thesis Proposal

Author	Martin Pátek
Supervisor	PhDr. Ladislav Krištoufek Ph.D.
Proposed topic	Variance structure of the Bitcoin currency

Topic characteristics The econometrics of financial volatility has gained a great interest recently. Nevertheless, the number of studies related to the topic of Bitcoin exchange rate volatility remains very limited. The main reason for this is that Bitcoin was introduced in 2009 which implies that it is still a very recent phenomenon. As the number of bitcoins in the circulation grows, the importance of a detailed Bitcoin analysis grows as well. In the theoretical part of the thesis, we would like to examine the benefits and potential risks of Bitcoin and try to explain how this cryptocurrency works. The aim of the empirical part is to analyze Bitcoin exchange rate volatility and the role of jumps in the Bitcoin total variation. The results will then be assessed and compared with volatility of other currencies and financial assets.

Hypotheses

1. Are jumps in the Bitcoin total variation stronger than jumps in the total variation of other currencies and stocks?
2. Is the variance of Bitcoin higher than the variance of other currencies and stocks?
3. Do the characteristics of jumps of Bitcoin get closer to the ones of other currencies and stocks in time?

Methodology To analyze Bitcoin exchange rate volatility, common methods which are based on realized variation measures will be used. We will apply primarily realized variance (sum of squared returns) and bipower variation to separate the Bitcoin total variation into its continuous and jump components.

Outline

1. Introduction
2. Bitcoin
3. Literature review
4. Methodology
5. Results
6. Conclusion

Core bibliography

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Author Supervisor

Chapter 1

Introduction

Bitcoin is a digital currency[1] introduced by Satoshi Nakamoto (possibly a pseudonym) which became fully operational in January 2009 (Nakamoto 2008). The basis for the Nakamoto's paper was a text written by Wei Dai in 1998 called b-money (Dai 1998). In this text, the author describes two protocols and in both of them he assumes "the existence of an untraceable network, where senders and receivers are identified only by digital pseudonyms (i.e. public keys)"[2].

According to Bitcoincharts (2014), on December 28, 2014, Bitcoin[3] market capitalization was 4,288,557,050 USD and total number of bitcoins in circulation was 13,657,825. Bitcoin is currently accepted even by very large companies, such as Dell, Microsoft or Overstock.com.

Bitcoin has many interesting features. As Meiklejohn *et al.* (2013) point out, some of them distinguish Bitcoin from other alternative payments, including PayPal or online payment solution, WebMoney Transfer. These alternative payments are all denominated in current fiat currency (currency which is declared by government to be legal tender, but has no intrinsic value). Unlike Bitcoin, these types of payments also directly link all the transactions with payer's and payee's identity.

One of the advantages of Bitcoin are its low transaction fees which are less than 0.1% (Titus 2013). Low transaction fees are directly connected to

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[1]According to Grinberg (2011), Bitcoin is "a digital, decentralized, partially anonymous currency, not backed by any government or other legal entity... It relies on peer-to-peer networking and cryptography to maintain its integrity." (Grinberg 2011, p. 160)

[2]http://www.weidai.com/bmoney.txt

[3]We will follow the usual convention, as mentioned, for example, in book written by Pagliery (2014), that Bitcoin with a capital "B" labels the protocol and the system in general, while bitcoin with lower "b" labels units of the currency.

irreversible Bitcoin transactions in the way that there is not any financial institution which has to resolve disputes in case of chargeback frauds as can be the case with credit cards (Nakamoto 2008).

Bitcoin has gained a great attention since its creation, but often even in a negative sense. Bitcoin was very closely connected to an online black market Silk Road which used Bitcoin for trading. Silk Road was until its shutdown in October 2013 a marketplace for drug dealers who exploited the anonymity provided by the Bitcoin scheme (Hencic & Gouriéroux 2015). Bitcoin also raised controversy in January 2014 when the most well-known Bitcoin exchange, Mt. Gox, bankrupted and announced that it had lost 850,000 BTC (around 277,185,000 USD as of December 2014) (Raiborn & Sivitanides 2015). In case of the Czech Republic, according to the Financial Analytical Unit of the Ministry of Finance of the Czech Republic, investing of proceeds stemming from illegal activity into Bitcoin increased by approximately 20 to 30 percent in 2014 (Třeček & Česká tisková kancelář 2015).

As Yermack (2013) and Lim *et al.* (2014) warn, one of the disadvantages of Bitcoin is that it is a frequent target of hacking. Moreover, since Bitcoin is not issued and regulated by any trusted third party, users bear the responsibility for lost bitcoins.

Another disadvantage of Bitcoin, which is discussed frequently, is its volatility. One of the reasons for Bitcoin's volatility can be its short existence and that around only 65% of bitcoins have been mined so far. Consequently, the price of bitcoin can be influenced even by comparatively small incidents caused, for example, by speculators or noise traders. Events like bankruptcy of Mt. Gox, shutdown of Silk Road or negative statements about Bitcoin from representatives of the People's Bank of China also play a very important role. As can be seen on bitcoincharts.com, on November 30, 2011, the weighted price of bitcoin (BTC) was 3.19 USD. Exactly one year later, on November 30, 2012, one bitcoin was worth 12.34 USD which rocketed to 1132.29 USD on November 30, 2013 for one bitcoin.

In a broad sense, volatility is described as fluctuations of some phenomenon over a given period of time. Volatility is a crucial factor of risk management, option pricing and portfolio management since volatility serves as a primary indicator of riskiness of an asset. Therefore, volatility is a key factor for investors seeking optimal risk-return trade-off. Analyzing the volatility of BTC/USD exchange rate[4] is interesting, besides other things, due to the fact that Bitcoin

[4]We sometimes refer to BTC/USD exchange rate as a price of bitcoin in this thesis.

exchanges operate 24 hours a day, 7 days a week. That differentiates Bitcoin market from other global financial exchanges such as NASDAQ or NYSE which are open only within stated time frame (Waring 2014). Despite the fact that some papers about volatility of Bitcoin have already been written, it still remains a largely unexplored field. This thesis should generally contribute to this topic and provide answers to hypotheses stated in the proposal.

The objective of this thesis is to analyze the Bitcoin total variation and to separate the jump component from the continuous part of a price process. We use realized variance and bipower variation which utilize high-frequency data (unlike ARCH-type models) to estimate volatility of financial returns. As Andersen *et al.* (2011) note, only a few years ago, authors modeling volatility used primarily GARCH or stochastic volatility models (utilizing data with daily or even lower frequency). However, these parametric models have become excessively restrictive and cumbersome (Zheng *et al.* 2014). Therefore, new nonparametric measures such as realized volatility have become increasingly used as mentioned, for instance, in extensive review paper written by Andersen *et al.* (2010a) and their advantages are pointed out also in works written by Zhang *et al.* (2009) and Bedowska-Sójka & Kliber (2010), among others. Among studies presenting realized volatility belong articles written by Andersen *et al.* (1999; 2001), McAleer & Medeiros (2008) and Barndorff-Nielsen & Shephard (2005). Bipower variation, the key concept used for decomposing variation to jump and continuous component, is introduced in the article written by Barndorff-Nielsen & Shephard (2004) which seems to be the first paper presenting the possibility of this decomposition.

Jumps can be characterized as extreme price movements that happen within short period of time. As stated by Hanousek & Novotný (2014), this price change is significantly higher than the prevailing market volatility. Moreover, jumps are notably less persistent and predictable than continuous component (Andersen *et al.* 2005b). These discontinuities in price process thus considerably influence financial management. There are several explanations for their occurrence such as low market liquidity or release of unexpected news.

The remainder of this thesis is organized as follows. Chapter 2 describes Bitcoin scheme more in detail. We discuss Bitcoin clients with their advantages and drawbacks and also the anonymity which can be provided by Bitcoin scheme. Chapter 3 reviews relevant literature. Chapter 4 introduces the methodology. Chapter 5 describes the datasets used for the analysis and presents the results. Chapter 6 summarizes our findings.

Chapter 2

How Bitcoin works

This chapter focuses on the Bitcoin scheme more in detail, explains how Bitcoin works and presents characteristics of Bitcoin exchanges. This chapter also describes different types of Bitcoin clients along with their advantages and drawbacks. The last section of this chapter deals with the anonymity that Bitcoin scheme can provide.

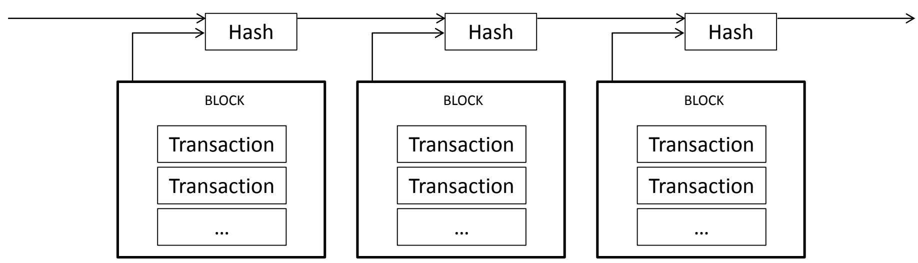
2.1 Bitcoin as a digital currency

Bitcoin is a peer-to-peer based digital currency which implies that there is not any trusted third party mediating and regulating transactions. Based on the market capitalization, Bitcoin is the most popular digital currency (Kristoufek 2013). Other examples of digital currencies are, for example, Ripple, Litecoin or Peercoin. Every Bitcoin transaction is public and recorded in the block chain which serves as a public ledger (Titus 2013). Block chain, including all the past transactions, is downloaded when you start using a desktop Bitcoin client (wallet). Record of all transactions in the block chain prevents double-spending, because only the transaction which was made as first is considered to be valid (Nakamoto 2008) as explained in more detail below. Every transaction is assigned to a block which consists of several transactions.

The process of creating blocks is called "mining". Mining is also a process through which new bitcoins are generated. Successful miners are rewarded by predetermined amount of bitcoins. When the first block was created on January 3, 2009, the reward was 50 BTC per block. After every 210,000 blocks, this amount is decreased to its half. Currently, the reward is 25 BTC. The Bitcoin scheme is set up in a way that new bitcoins are mined at a rate which is predictable and as Raiborn & Sivitanides (2015) mention, no more than 10 BTC should be mined every 10 minutes. As noted by Ron & Shamir (2013), since the reward for mining is halved every 210,000 blocks, the supply of bitcoins follows a geometric series that converges to total number of 21 million supplied bitcoins.

The time of each transaction is uniquely determined since each transaction has its timestamp. Each timestamp has its corresponding hash (which applies to all the transactions in the block) in which the previous timestamp is incorporated. This process enables to connect blocks in a chain. Each hash, which is a fixed-length concise way of representing data of any size (Titus 2013), is unique. Scheme of this process is depicted in Figure 2.1.

Figure 2.1: Block timestamp

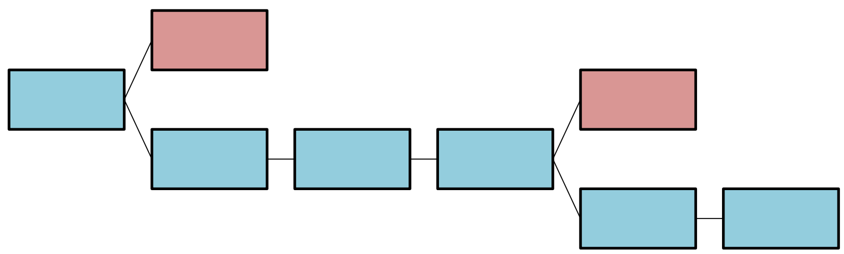


This figure depicts the principle of timestamped hashes. The timestamp generally serves as a proof of existence of a transaction at the stated time. Each block of transactions (including their timestamp) has its corresponding hash which is used to create the hash of the next block. This connects blocks in a chain.

Source: author.

If two blocks are generated almost simultaneously, some clients may receive one version of a block and other clients may receive the other one. Nevertheless, in the block chain, there cannot be two versions of one block. Individual nodes accept particular version of a block by using its hash in process of mining subsequent block in the block chain. In a situation when there are two chains (which is called "fork" in the block chain), the one which becomes shorter over time is considered to be invalid and nodes which accepted this part of chain will switch to the valid one (Nakamoto 2008). In the Figure 2.2, the blue blocks represent the valid chain and the red blocks are the ones that are not used in the block chain.

Figure 2.2: Fork in the chain



This figure presents fork in the chain which can happen when two blocks are generated roughly at the same time. This problem (since there cannot be two paths in the block chain) is resolved in a way that only blocks in the longest chain (i.e. more new blocks were built on top of this chain) are used (blue blocks). All transactions of the blocks of shorter chain (red blocks) are then included in a block in longer chain.

Source: author.

Bitcoin scheme uses public key cryptography. When a wallet is created, users obtain a pair of keys (public and private key) and an address. All transactions carry a digital signature made with a unique private key. This signature serves as a proof that the transaction is valid. Private key allows users to access their wallets and to send money to other users. This number should be kept in secret otherwise users can lose their bitcoins, because when a private key is known, transactions can be digitally signed which implies that the whole network will treat this transaction as valid. Private key is used when generating public key which is then used for creating an address (in a simplified way, address is a hash of public key). This process practically cannot be made in a reverse order. An address has to be known to the user with whom you want to make a transaction. It identifies a payee and a payer and controls validity of the signature. It is usually a matter of just one click in your wallet to generate a new address. That simplifies securing privacy by creating a new address for each transaction.

2.2 Bitcoin exchanges

One of the specifics of Bitcoin exchanges is that unlike other financial markets such as NASDAQ or NYSE, Bitcoin exchanges operate 24 hours a day, 7 days a week. Therefore, users can buy and sell bitcoins whenever they want (which confirms the international character of this digital currency, because it can always be traded, regardless of the time zone). The downside for investors may be that significant price movements of Bitcoin may occur at a time when investors do not track the price (e.g. when they are sleeping). The question is whether this is really a disadvantage when automated trading (algorithmic trading bots) is constantly developing[1]. As Pieters & Vivanco (2015) point out, Bitcoin exchanges have one of the lowest entry costs of any financial market due to uncomplicated access for anyone with a computer.

There exist a large number of Bitcoin exchanges where users can buy and sell bitcoins for major currencies. According to Titus (2013), it is also considered to be the cheapest way of acquiring bitcoins (among other ways belong, for example, person-to-person trading website LocalBitcoins.com, mining and also an online auction site eBay). For example, as of March 6, 2015, fee at Bitstamp starts at 0.25% if you trade less than 20,000 USD in one month and ends at 0.10% if monthly trading volume is greater than 20,000,000 USD. For some, it might also be the easiest way to acquire bitcoins.

Until its bankruptcy in February 2014, Mt. Gox was the most popular Bitcoin exchange (Yermack 2013). Currently, among the largest exchanges belong Bitstamp, BTC-e, BTC China or Bitfinex. As Yermack (2013) notes, Bitcoin exchanges are not very liquid. Moreover, an order may be lost due to a technological error (execution risk). Worth mentioning is also the case from October 2013 when Chinese Bitcoin exchange GBL was shut down. According to BBC (2013), investors lost up to 31.8 million Hong Kong dollars. Yermack (2013) also states that there is a considerable difference between the ask price and the bid price.

In connection with online exchanges, we will also briefly discuss Bitcoin ATM which can be described as an instant exchange. When the first ATM was put into operation in October 2013 in Vancouver, it received a considerable media attention. The greatest disadvantage of Bitcoin ATM is that in comparison with online exchanges, users have to pay much larger fees: between 3% to 5% of the withdrawal amount (Wile 2014). For some, an advantage may be that users of Bitcoin ATM do not have to provide personal information such as bank account, they just have to scan a palm. That would prove useful mainly for those who do not have a bank account. The benefit of an ATM is also the fact that it

[1]More about trading bots can be found, for example, at coindesk.com.

makes Bitcoin more accessible even to those for whom words like cryptography or peer-to-peer network are totally unknown.

2.3 Bitcoin clients

Bitcoin clients enable users to carry out transactions. Users can use either desktop wallet, online wallet or mobile wallet. Desktop clients are very often used for the ease of making a back up of a wallet and also for its security. Needless to say that the security of the desktop wallet is limited by the security of user's computer (Titus 2013). For some, the disadvantage of desktop clients can be that before it is possible to work with the client, users have to download the block chain (or at least its part) the size of which now exceeds 30 GB (Blockchain 2015). Another drawback is that unless users do not store their desktop wallet on a file hosting service, such as Dropbox or Google Drive, they can access it only from one computer. Desktop wallets include, for example, Electrum, Armory, mSIGNA or MultiBit.

Online wallets are by contrast generally very quickly set up and ready for use. Nonetheless, this is counterbalanced by lower security than in case of desktop clients. An example might be the theft from October 23, 2013 when an online wallet Inputs.io was hacked and 1.2 million USD was stolen (Möser *et al.* 2014). Providers of online wallets are, for example, Coinbase, Blockchain, BitGo and Hive.

Mobile wallets leave out some functions which are normally of a minor use (such as sign and verify message function[2]). However, users can take an advantage of new technologies in smartphones, such as using NFC technology to obtain the payee's address or obtaining that by scanning a QR code.

2.4 Anonymity of Bitcoin

As already mentioned, the anonymity that Bitcoin scheme provides can be exploited in many ways. First, Bitcoin can serve to make money laundering easier. There exist mixing services which enable users to pool their transactions with transactions of other people in a shared wallet. Mixing services receive and send back given amount of money from addresses that are not linked and as a result the origin of money is covered. It can be compared to the case of

[2]See, for example, bitcoinmagazine.com.

sending money through offshore accounts for the same purpose. Mixing services are provided by several Bitcoin websites, such as Bitcoin Fog, Bit Laundry or CoinSplitter.org. However, as Meiklejohn *et al.* (2013) point out, the capacity of mixing services was, at least at the time of writing their article, not sufficient to launder large amounts of bitcoins.

Despite the fact that personal information are not stated in the block chain, Ron & Shamir (2013) were able to get detailed overview of the financial activities of Bitcoin users. They also identified large Bitcoin institutions, such as Deepbit, Instawallet and Mt. Gox along with their transactions. Therefore, they were able to characterize typical user of Bitcoin or, for example, average balance of an address (which was found to be 2.4 BTC). Authors revealed several interesting facts about Bitcoin scheme such as:

- balance of roughly 97% of all addresses was less than 10 BTC;
- 93% of all addresses were involved in fewer than 10 transactions;
- there were only 70 addresses with balance larger than 10,000 BTC;
- 40% of all addresses received less than 1 BTC in total; and
- there were only 340 transactions larger than 50,000 BTC until then and the vast majority of these transactions were successors of the transaction of 90,000 BTC made on November 8, 2010.

Another work dealing more closely with analyzing bitcoin transactions is a study written by Meiklejohn *et al.* (2013). Using several heuristics, authors cluster relevant addresses in order to get an image about individual users. They, similarly as Ron & Shamir (2013), identify large institutions and reveal interesting statistics about the activity of Bitcoin users. At the time of the writing their paper, they found that approximately 60% of the overall activity within the Bitcoin scheme was connected to the betting game, Satoshi Dice. Meiklejohn *et al.* (2013) claim that users would have to make a considerable effort to make the heuristics used in this study ineffective.

Ron & Shamir (2014) trace Bitcoin transactions of a founder of the black market Silk Road who is known as Dread Pirate Roberts and is believed to be an American citizen Ross William Ulbricht. The topic of deanonymization of Bitcoin users is also analyzed in a study by Biryukov *et al.* (2014), Koshy *et al.* (2014) and several others. It is also important to say that when one opens an account on some Bitcoin exchange, personal information such as home address, name, bank account numbers, copy of passport or national ID card may be collected[3].

These studies generally show that despite the fact that Bitcoin scheme definitely provides certain level of anonymity, it is not as anonymous as many people think. The main reason is that all transactions are public, recorded in the block chain. It is possible to find out the balance of an address and also all of its transactions. This is not sufficient by itself to uncover one's identity, but once an owner of the address is revealed, it is possible with some degree of certainty to reveal identity of owners of other addresses. That is why Bitcoin scheme is often said to be pseudoanonymous.

[3]See, for instance, Privacy Policy of the Bitcoin exchange Bitstamp at bitstamp.net.

Chapter 3

Literature review

This chapter aims at quantitative research papers about Bitcoin that have been written so far. The vast majority of works about Bitcoin scheme focus on its functioning and legal issues concerning this digital currency such as money laundering and security of trading. However, still very few financial econometrics studies analyzing Bitcoin have been written to date.

Works mentioned in this chapter are primarily ordered according to the approach used to analyze Bitcoin's features. These works are then ordered chronologically. In the first section, works concerning volatility of Bitcoin exchange rate are mentioned. In the second section, works dealing with the relationship between social media and search queries and the price of bitcoin are studied. These works then follow chronological order as well.

3.1 Volatility of Bitcoin exchange rate

Briere *et al.* (2013) study Bitcoin as an investment and show that Bitcoin has high volatility (175% annually) compared with other assets (10% in case of EUR or 17% in case of stocks) and high average return (371% annually). Authors also state that Bitcoin may improve diversification of a portfolio since correlation of Bitcoin with other assets is notably low. Despite this claim, authors do not consider Bitcoin to be a suitable investment for considerably risk-averse investors due to its high volatility. Authors of this study do not describe methods used for investigating the volatility of Bitcoin.

Yermack (2013) compares the volatility of Bitcoin with major currencies, such as Japanese Yen, Euro and Swiss Franc. Yermack (2013) explains the price (exchange rate) trend of Bitcoin in a different way than Sapuric & Kokkinaki (2014) mentioned below. Unlike Sapuric & Kokkinaki (2014), Yermack (2013) disregards the volume of transactions in analyzing the Bitcoin's exchange rate. The author finds that the volatility of Bitcoin is very high in comparison with other currencies making it a currency that cannot be used for regular payments at retailers. The retailers would have to constantly recalculate the price of goods according to current exchange rate of Bitcoin, which would be a very expensive process. The author also states that BTC/USD exchange rate is not correlated with the exchange rates of other analyzed currencies.

Sapuric & Kokkinaki (2014) analyze the volatility of Bitcoin exchange rate against common currencies. What distinguishes this work from other works written on this topic is that it is the first paper which adjusts for volume of transactions when studying the volatility of Bitcoin. Authors assume that the value of BTC/USD exchange rate up to some level coincides with Bitcoin trading volume. When the trade volume of Bitcoin is not taken into account, they consider the volatility of Bitcoin to be significantly higher than volatility of other major currencies, including Euro, Swiss Franc, Russian Ruble or Japanese Yen. Nevertheless, when they take into consideration the Bitcoin trading volume, they find that this normalized volatility greatly decreased. As a result, they concluded that "Bitcoin is not as volatile and risky as widely acclaimed" (Sapuric & Kokkinaki 2014, p. 256). It is worth mentioning that authors of this paper are academics at University of Nicosia, the largest university in Cyprus, which is the first accredited university in the world which accepted Bitcoin for payment of tuition (University of Nicosia 2014). One of the reasons for this decision may be the Cyprus banking crisis of early 2013. At that time, Cypriot government announced a bail-in of its banks which for large depositors meant that they had to participate in the rescue of the banks resulting in huge losses of the depositors. In order to protect their deposits and avoid possible government intervention, many of them exchanged their money into Bitcoin.

The paper written by Hencic & Gouriéroux (2015) represents one of the most extensive studies about the dynamics of Bitcoin exchange rate. Authors state that rapid changes of the exchange rate can be interpreted as bubbles and outline four possible causes of this behavior: speculative trading, asymmetric information and crowd phenomena, lack of regulation and predictable supply of bitcoins. They estimated and predicted the exchange rate using the mixed (causal/noncausal) Autoregressive (AR) process with Cauchy distributed errors.

As mentioned, one of the reasons for bubbles proposed by Hencic & Gouriéroux (2015) is speculative trading. This, up to some level, coincides with an explanation of high volatility of Bitcoin exchange rate given by Kristoufek (2013) whose work is mentioned below.

3.2 Role of social media

The following works analyze the impact of social media and Bitcoin's popularity (measured particularly by search queries on Google and Wikipedia) on different aspects of Bitcoin market. Generally, they find that information obtained from Google Trends (which gather data relating to searches on Google) or from posts on social media can be very useful for nowcasting[1] the Bitcoin market. As noted by Garcia *et al.* (2014), Google searches related to Bitcoin increased in the period from June 2010 to December 2013 by more than 10,000%.

Šurda (2012) analyzes in the empirical part of his diploma thesis price of bitcoin, price volatility, liquidity or velocity. Analysis in this thesis is not as in-depth as, for example, in case of Hencic & Gouriéroux (2015). The reason is that empirical analysis is just one part of the work and also that Šurda (2012) deals with wider scope of Bitcoin's characteristics. Similarly as in the paper written by Kristoufek (2013), author of this thesis studies the relationship between price of bitcoin and search queries on Google Trends (specifically, searches of the term "bitcoin"). The results imply strong positive correlation between bitcoin price in USD and number of Bitcoin searches through Google Search which is consistent with results of Kristoufek (2013) discussed below. Šurda (2012) also examines liquidity of Bitcoin calculated as change in quantity divided by change in price. The author studies correlation between liquidity and price volatility calculated as the relative change of the price during the specific interval. Results indicate medium to strong negative correlation which, as the author states, corresponds with Bitcoin as a medium of exchange with an inelastic supply. Liquidity is also analyzed with relation to the price of bitcoin. Author finds weak negative correlation of liquidity with price. As Šurda (2012) notes, low liquidity associated with high price can indicate bubble behavior and high liquidity related to low price can be explained by resistant nature of Bitcoin scheme preventing its breakdown.

[1] Nowcasting means predicting the present. Nowcasts can be characterized as short-term predictions.

Kristoufek (2013) explains the evolution of price of bitcoin differently than Sapuric & Kokkinaki (2014). Kristoufek (2013) in his work states that the volatility is particularly influenced by speculation of traders and that it cannot be sufficiently explained only by financial and economic theories. This can be compared to the statement given by Yermack (2013) who says that macroeconomic events influencing other currencies essentially do not have an impact on the price of bitcoin.

Kristoufek (2013) analyzes the relationship between exchange rate of Bitcoin and search queries on Google Trends and Wikipedia. For this purpose, the author uses primarily Vector Autoregression (VAR) and Vector Error Correction Model (VECM) which is a generalization of VAR. The results imply substantial positive correlation between search terms (for both Google Trends and Wikipedia) and price level of bitcoin. Moreover, the author finds that not only that searched terms influence the price of Bitcoin, but also that this relationship holds vice versa, i.e. that the price of bitcoin influences searches related to this digital currency.

Kaminski & Gloor (2014) in their work use sentiment analysis based on posts on Twitter to explore if Twitter sentiments are correlated with Bitcoin trading volume and price of bitcoin. They use data from four major Bitcoin exchanges: Bitstamp, Bitfinex, BTC-e and BTC China and find negative correlation between negative Bitcoin tweets or tweets about Bitcoin expressing doubts and the price of bitcoin. On the contrary, correlation with positive tweets does not seem to be significant. On the basis of data from Bitstamp, they find that there is a positive correlation between negative tweets and the trading volume. Finding which may be the most relevant for the purpose of this thesis is that intraday price volatility of bitcoin (represented by the intraday spread) is significantly positively correlated with tweets (about Bitcoin) including signals of uncertainty and negative tweets. Therefore, authors conclude that Twitter sentiments can be helpful in nowcasting Bitcoin market behavior.

Another work dealing with the role of social media in connection with Bitcoin is an article written by Garcia *et al.* (2014). Using VAR, authors analyze the interdependence between price of bitcoin, number of new Bitcoin users and social interactions, including:

- Google searches of the term “bitcoin”;
- daily number of views of the English Wikipedia page about Bitcoin;

- number of tweets about Bitcoin; and
- "popularity" of the Facebook page about Bitcoin.

Authors use data from Mt. Gox, BTC-China and BTC-de. It is the first article about Bitcoin combining technological, social and economic aspects in one analysis. Authors find that search volume (represented by Google searches and views of the Wikipedia page) increases with price of bitcoin, information sharing (represented by tweets and popularity of the Facebook page) increases with search volume and price increases with information sharing. This three-way loop "represents the feedback cycle between social dynamics and price in the Bitcoin economy" (Garcia *et al.* 2014, p. 7). The second, "user adoption", cycle modeling the dependency of price of bitcoin and the number of Bitcoin users is according to authors represented by this loop: search volume increases with price, the number of new users increases with search volume and price increases with number of new users. Authors state that these cycles stand behind the evolution of bubbles on the Bitcoin market.

One of the last works focusing on the bitcoin price is the article written by Kristoufek (2015). This work, unlike the author's already mentioned work in which the author dealt with the price of bitcoin only in relation to searched terms on Google and Wikipedia, now (using wavelet coherence analysis) addresses a broad spectrum of influences on the price of bitcoin (economic, transaction and technical drivers, interest etc.). Based on data of BTC/USD exchange rate from September 2011 to February 2014, the author finds that bitcoin price experienced periods of bubbles after whose bursting the price never returned back to its original level. Author also finds out that increase in use of bitcoins for trade (i.e. for non-exchange transactions) led to price appreciation of bitcoin in the long run and that rising bitcoin price resulted in an increase of bitcoin transactions at exchanges in the short run. Author also reminds already mentioned mining of bitcoins and the fact that in order to keep the bitcoin supply controlled, mining of new bitcoins becomes more difficult with increases in computational power of miners. Consequently, mining becomes more expensive (due investing into more powerful mining hardware and higher demand of electricity as well). Kristoufek (2015) finds that as the price of bitcoin rises, more miners are attracted (and that leads to more difficult and costly mining).

Author also partly follows his article written in 2013 in quantifying the effect of social media on price of bitcoin using data from Google and Wikipedia.

Kristoufek (2015) divides the impact of Bitcoin's popularity into two phases. In the phase when a bubble is growing, the higher the popularity the higher the price of bitcoin. During the phase when a bubble is bursting, the interest in Bitcoin has an opposite effect. The author also concludes that Bitcoin cannot be considered a safe haven and that there is no causal relationship between the Chinese Yuan Renminbi (CNY) market and United States dollar (USD) market.

Similarly as Kristoufek (2013), Li *et al.* (2014) assert that since Bitcoin is not a physical currency, traditional economic theories cannot sufficiently explain behavior of this digital currency. Authors examine the determinants of Bitcoin exchange rate and just as Kristoufek (2015) take into consideration not only economic factors (size of the Bitcoin market and nominal monetary policy factors such as money supply and inflation rate), but also mining technology and technology-related factors such as public interest (based on the number of Google searches and Twitter posts). Using the Error Correction Model (ECM), authors find that change in the US inflation rate negatively affects BTC/USD exchange rate (decreasing the value of Bitcoin) while change in the US money supply has a positive effect. Unlike these two factors which have a significant impact on price of bitcoin, the US GDP and the US inflation rate do not seem to have considerable effect on bitcoin's price. Similarly as the US money supply, also the number of Bitcoin transactions positively affects bitcoin's price. Authors find that interest has a positive impact as well (the higher the interest the higher the Bitcoin exchange rate). The same applies for mining difficulty which also positively correlates with BTC/USD exchange rate.

Chapter 4

Methodology

This section provides an insight into most of the methods that are used in this thesis. We also give reasons for choosing these particular methods. We mainly follow the articles written by Andersen *et al.* (2005a; 2011) and Ruxton (2006) in this chapter.

4.1 Realized variance measures

The availability of high-frequency data induced an important progress in economics of financial volatility. As Zheng *et al.* (2014) propose, volatility calculation techniques can generally be divided into two groups: parametric and nonparametric methods. Parametric procedures (such as ARCH model and stochastic volatility model) rely on explicit functional form assumptions regarding the volatility and can suggest misspecification. Nonparametric approaches (such as absolute return and realized volatility which is used in this thesis) do not depend upon any particular form of actual (but unobservable) volatility (Thomakos & Wang 2003). In case of realized volatility, the focus is on the ex-post measurement of the volatility and can be therefore computed without link to a certain model. Thanks to high-frequency data, we can analyze the variance of price changes on intraday basis. The advantage of nonparametric methods is also that they can be computed relatively easily.

As Barndorff-Nielsen & Shephard (2004) noted, the method proposed by them and used in this thesis for estimating quadratic variation of the jump component seems to be the first method that can be used for separating quadratic variation into continuous and jump components. The idea behind this method is to compare two measures of variance: one includes the contribution of jumps to the total variance (if they are present) and the second measure of variance is robust to the jump contribution. By testing the statistical significance of the difference between these two measures we find out whether jumps are present.

4.2 Theoretical model and realized variance

Trades may occur at any time during the trading hours. This means that volatility measures can be calculated using arbitrarily short time interval. As a result, we model the following price process in continuous time.

Let us consider a univariate logarithmic price process of an asset, $p_t = \ln P_t$ (where P_t is the price process of an asset at day t) defined on a complete probability space $(\Omega,\ \mathcal{F},\ P)$, evolving continuously over time interval $[0, T]$, where T is a finite positive integer. We suppose an information filtration[1], that is an increasing family of σ-algebras, $(\mathcal{F})_{t\in[0,T]} \subseteq \mathcal{F}$[2], satisfying usual conditions of P-completeness[3] and right continuity, i.e. $\mathcal{F}_t = \mathcal{F}_{t+}$, where

$$\mathcal{F}_{t+} = \bigcap_{u>t} \mathcal{F}_u. \tag{4.1}$$

Due to the right continuity, $\mathcal{F}_{t+}$ can be intuitively interpreted as the information set available infinitesimally just after time t. Finally, we assume that the information set at time t, $\mathcal{F}_t$, contains information about all the asset prices (including the relevant state variables) that arose from time 0 to t.

Let $p_{t,i}$ be the ith logarithmic price during day t. Then we define the ith intraperiod return of day t as

$$r_{t,i} = \ln\left(\frac{P_{t,i}}{P_{t,i-1}}\right) = \ln\left(P_{t,i}\right) - \ln\left(P_{t,i-1}\right) = p_{t,i} - p_{t,i-1}. \tag{4.2}$$

The difference in time between observing $p_{t,i}$ and $p_{t,i-1}$ corresponds to the sampling frequency which is in case of this thesis equal to five minutes for all datasets.

Now, we introduce a standard Brownian motion as defined by Fouque *et al.* (2000) and Chang (1999). Standard Brownian motion $\{W_t : t \geq 0\}$ defined

[1]We follow the works written by Coculescu & Nikeghbali (2007) and Tian (2015).

[2]That means that for each t, $\mathcal{F}_t$ is a σ-algebra included in $\mathcal{F}$ and if $s \leq t$, then $\mathcal{F}_s \subset \mathcal{F}_t$.

[3]Completeness of a filtration $\mathcal{F}_t$ means that $\mathcal{F}_0$ contains all null sets (a null set is a set X such that there exists a set Y where $X \subset Y$, $Y \in \mathcal{F}$, and $P(Y) = 0$).

on the probability space $(\Omega, \mathcal{F}, P)$ is a real-valued stochastic[4] process which satisfies the following properties:

1. $W_0 = 0$;

2. the process $\{W_t : t \geq 0\}$ has continuous trajectories: $P(\omega \in \Omega : W(\cdot, \omega)$ is a continuous function$) = 1$;

3. for every choice of nonnegative real numbers $0 \leq s_1 < t_1 \leq s_2 < t_2 \ldots \leq s_n < t_n < \infty$, the increment random variables $(W_{t_1} - W_{s_1}, W_{t_2} - W_{s_2} \ldots, W_{t_n} - W_{s_n})$ are jointly independent;

4. the process $\{W_t : t \geq 0\}$ has stationary increments: for any $0 < s, t < \infty$, the distribution of the increment $W_{t+s} - W_s$ is the same as the distribution of $W_t - W_0 = W_t$; and

5. $W_t \sim N(0, t)$.

We assume that the general representation of an arbitrage-free logarithmic asset price process is a jump-diffusion process

$$\mathrm{d}p_t = \mu_t \mathrm{d}t + \sigma_t \mathrm{d}W_t + \xi_t \mathrm{d}q_t, \tag{4.3}$$

where μ_t is the drift term (predictable, finite-variation component), $\sigma_t > 0$ is the spot (point-in-time) volatility process (measure of the standard deviation of the returns) assumed to be càdlàg[5], W_t is a standard Brownian motion, q_t is a jump indicator ($q_t = 1$ if there is a jump at time t and 0 otherwise) and ξ_t corresponds to the magnitude of the jump (if any occurs at time t).

As stated by Fleming & Paye (2006), studies written by that time found that daily stock returns standardized by realized volatility seemed to have standard normal distribution. That would indicate that assuming presence of jumps may be misleading, because its presence would violate the normality of the standardized returns[6] (see e.g. Andersen *et al.* 2010b). However, it does not correspond with the fact that in the literature about testing for jumps such

[4] "Stochastic" comes from the Greek "stochos": aim, guess, characterized by randomness.

[5] Càdlàg function is a right continuous function which has left limits at each point.

[6] Andersen *et al.* (2010b) analyzed volatility of stocks and found that the first four sample moments of standardized returns "adhere fairly closely to those of the slightly modified Gaussian distribution. Specifically, the implicit null of an underlying continuous-time diffusion is not rejected for nine of the 30 stocks..." However, authors also state that sample kurtosis for standardized returns is "significantly different from the theoretical value of $m_4 = 2.925$ that should obtain for a homogeneous diffusion" (Andersen *et al.* 2010b, p. 253).

as works written by Huang & Tauchen (2005) and Andersen *et al.* (2005b), the presence of jumps is very often detected. Articles written by Fleming & Paye (2006) and Andersen *et al.* (2010b), among others, present a possible resolution of this conflict by stating that it might be caused by bias in realized volatility estimates. It means that even if the standardized returns have distribution which seems to be very close to the normal distribution, the assumption that the price process contains jumps may not be misleading. We discuss this in more detail in the next chapter in the Section 5.5.

According to Andersen *et al.* (2005a), if we assume that price process corresponds to Equation 4.3, then the associated daily return is defined as

$$r_t = p_t - p_{t-1} = \int_{t-1}^{t} \mu_\tau \,\mathrm{d}\tau + \int_{t-1}^{t} \sigma_\tau \,\mathrm{d}W_\tau + \sum_{t-1 \leq \tau < t} J_\tau, \tag{4.4}$$

where the sum cumulates jumps occurring over the day and $J_t = \xi_t \cdot I(q_t = 1)$, $I(\cdot)$ being the indicator function. That means that J_t is not zero only if a jump occurs during day t.

Quadratic variation is a measure of the volatility on day t and is defined as

$$QV_t = \int_{t-1}^{t} \sigma_s^2 \,\mathrm{d}s + \sum_{t-1 < s \leq t} J_s^2, \tag{4.5}$$

which in case of no jumps is reduced to integrated variance which is defined as

$$IV_t = \int_{t-1}^{t} \sigma_s^2 \,\mathrm{d}s. \tag{4.6}$$

Realized variance is defined as sum of all intraday squared returns

$$RV_t = \sum_{i=1}^{n} r_{t,i}^2, \tag{4.7}$$

where n is the total number of return observations per trading day.

Realized variance converges uniformly in probability to quadratic variation as the total number of return observations per trading day increases to infinity as noted by Andersen *et al.* (2010b; 2011) and Huang & Tauchen (2005), among others, i.e.

$$\mathrm{plim}_{n\to\infty} RV_t = QV_t. \tag{4.8}$$

Next, we define realized volatility as the square root of realized variance

$$RVO_t = \sqrt{RV_t}. \tag{4.9}$$

4.3 Bipower variation and estimation of jumps

We want to separate the jump component from the continuous part and as already mentioned above, realized variance is the estimator of quadratic variation. Therefore, we define realized bipower variation

$$RBV_t = \mu_1^{-2} \frac{n}{n-2} \sum_{i=3}^{n} |r_{t,i-2}||r_{t,i}|, \tag{4.10}$$

where $\mu_r = E\left(|Z|^r\right) = 2^{r/2} \frac{\Gamma((r+1)/2)}{\Gamma(1/2)}$ for $r > 0$, $Z \sim N(0,1)$ and $\Gamma(\cdot)$ denotes the Gamma function. In our case, $\mu_1 = \sqrt{2/\pi}$. Bipower variation is crucial for decomposing variation into jump and continuous component since it converges in probability to the integrated variance as noted by Huang & Tauchen (2005) and Andersen *et al.* (2010b), among others, i.e.

$$\operatorname{plim}_{n \to \infty} RBV_t = IV_t. \tag{4.11}$$

Definition of bipower variation in Equation 4.10, which was given by Andersen *et al.* (2011), helps render the estimator robust to certain type of microstructure noise which may be present because of the discreteness of prices, bid-ask bounce[7], and asynchronous (nonsynchronous) trading[8], among other aspects. As a result, using data with the highest frequency does not necessarily have to be the best approach. We discuss the possible noise contamination in the next chapter.

There are several studies written about optimal sampling frequency taking into consideration the accuracy, which is related to the highest possible frequency, and microstructure noise. As McAleer & Medeiros (2008), Chow *et al.* (2009), Maderitsch (2014) and Andersen *et al.* (2011), among others, point out, this optimal sampling frequency is believed to be the five-minute frequency corresponding to 288 observations over trading day (assuming continuous 24-hour trading).

[7]Bid-ask bounce is a situation when trade price oscillates between the bid and ask side of the market.

[8]Asynchronous trading means that the underlying return process is observed at irregular intervals which means that the trading intensity can vary.

Since RV_t is an estimator of total quadratic variation and RBV_t is an estimator of integrated variance, by differencing RV_t and RBV_t, we get a consistent estimator of jumps

$$\text{plim}_{n\to\infty}(RV_t - RBV_t) = \sum_{i=1}^{m_t} J_{t,i}^2, \tag{4.12}$$

where m_t denotes the number of jumps over day t.

We then use a test statistic proposed by Andersen *et al.* (2011) to test for the presence of jumps which is defined as

$$Z_t = \frac{\frac{RV_t - RBV_t}{RV_t}}{\sqrt{\frac{1}{n}\left(\left(\frac{\pi}{2}\right)^2 + \pi - 5\right)\cdot \max\left(1, \frac{RTQ_t}{RBV_t^2}\right)}}, \tag{4.13}$$

where RTQ_t is realized tripower quarticity as defined below. Moreover, under the null hypothesis of no within-day jumps and under sufficient regularity conditions (see e.g. Andersen *et al.* 2011), Z_t has asymptotically standard normal distribution.

The realized tripower quarticity is defined as follows

$$RTQ_t = n\mu_{4/3}^{-3}\left(\frac{n}{n-4}\right)\sum_{i=5}^{n} |r_{t,i-4}|^{4/3}|r_{t,i-2}|^{4/3}|r_{t,i}|^{4/3}, \tag{4.14}$$

where $\mu_{4/3} = 2^{2/3}\frac{\Gamma(7/6)}{\Gamma(1/2)}$.

We now present the realized measure of the jump contribution to the quadratic variation of the logarithmic process as defined by Andersen *et al.* (2011)

$$JV_t = I(Z_t > \Phi_\alpha)(RV_t - RBV_t), \tag{4.15}$$

where Φ_α is a critical value from the $N(0,1)$ distribution and $I(\cdot)$ still denotes the indicator function.

The realized measure of the integrated variance is then defined as

$$C_t = I(Z_t \le \Phi_\alpha)\cdot RV_t + I(Z_t > \Phi_\alpha)\cdot RBV_t. \tag{4.16}$$

As a result, we estimate the continuous component by realized variance on days without detected jumps and by realized bipower variation on days with detected jump contribution to the daily realized variance. That means that the continuous and jump components always add up to the daily realized variance.

In our thesis, we use the 95$^{\text{th}}$ percentile of the standard normal distribution to obtain the results stated in the next chapter. On the basis of definitions in this chapter, we will use the following notation in the remaining part of this work: RV_t will denote realized variance, RBV_t will stand for realized bipower variation, C_t will denote the continuous sample path variation and J_t will denote the sum of the within day squared jumps.

4.4 Unequal variance t-test

We also compare the mean values of continuous and jump components of realized variances using a t-test with independent (unpaired) samples. On the basis of works written by Yermack (2013) and Briere *et al.* (2013), we assume that BTC/USD exchange rate is not correlated with other analyzed instruments.

If μ_{BTC} is the true population mean of continuous/jump components of daily realized variances for BTC/USD exchange rate and μ_i is the true population mean of continuous/jump components for other instruments, then the null hypothesis in our test is

$$H_0 : \mu_{BTC} - \mu_i = 0,$$

where i ={EUR, XAU, S&P500} and the alternative hypothesis is that the mean difference is greater than zero

$$H_A : \mu_{BTC} - \mu_i > 0.$$

As noted by Zimmerman (2004), when the assumption that two distributions have the same variance is violated, Type I error rates are altered, particularly for unequal sample sizes which is our case. Based on the article written by Ruxton (2006), who suggests "avoiding preliminary tests and adopting the unequal variance t-test unless an argument based on logical, physical, or biological grounds can be made as to why the variances are very likely to be identical" (Ruxton 2006, p. 689), we directly use Welch's approximation for the degrees of freedom (referred to as the Welch Approximate Degrees of Freedom). This test is known as unequal variance t-test or the Welch t-test. This approach is also supported by Zimmerman (2004) who states that "when sample sizes are unequal, it appears that the most efficient strategy is to perform the Welch t-test or a related separate-variances test unconditionally, without regard to the variability of the sample values" (Zimmerman 2004, p. 180). Ruxton (2006) also states that even in case when the population variances are equal, the power of the Welch t-test is similar to the power of the Student's t-test which assumes the equality of variances.

Following the definition given by Ruxton (2006), the unequal variance t-test statistic is

$$t = \frac{\bar{X}_{BTC} - \bar{Y}_i}{\sqrt{\frac{s^2_{BTC}}{n_{BTC}} + \frac{s^2_i}{n_i}}}, \tag{4.17}$$

where i ={EUR, XAU, S&P500}, $\bar{X}_{BTC}$ is the sample mean of continuous/jump components for BTC/USD exchange rate, $\bar{Y}_i$ is the sample mean of continuous/jump components for other instruments, s^2_{BTC} and s^2_i are corresponding variances and n_{BTC} and n_i are corresponding sample sizes. We can see that unlike in Student's t-test, the variances of the two samples are not pooled. We perform the unequal variance t-test at the 0.05 significance level.

The number of degrees of freedom is calculated as

$$\nu = \frac{\left(\frac{1}{n_{BTC}} + \frac{u}{n_i}\right)^2}{\sqrt{\frac{1}{n^2_{BTC}\cdot(n_{BTC}-1)} + \frac{u^2}{n^2_i\cdot(n_i-1)}}}, \tag{4.18}$$

where

$$u = \frac{s^2_i}{s^2_{BTC}}. \tag{4.19}$$

Chapter 5

Discussion of the results

This chapter is divided into five sections. The first four sections describe the datasets used for the purpose of this thesis and also present the results for each of the analyzed instruments. We mention the source of the data, the period we analyze and the frequency of sampling. We also present characterization of the distribution of the intraday returns, daily returns and daily returns when standardized by realized volatility (r_t/RVO_t) and when standardized by square root of realized bipower variation ($r_t/\sqrt{RBV_t}$). Next, we report the descriptive statistics of realized variance along with the descriptive statistics of its components for each dataset. The final section is dedicated to the discussion of jumps in the Bitcoin total variation in comparison with jumps in the total variation of EUR/USD exchange rate, in the total variaton of price of gold and of the value of the S&P 500 index. We also discuss the distribution of standardized daily returns in the last section of this chapter. Finally, we focus on the development of jumps of Bitcoin in time and assess if these get closer to the ones of other above-mentioned assets.

As indicated above, we use four datasets of five-minute data. All datasets (apart from the S&P 500 index dataset) capture the period from December 31, 2011 at 0:00 to February 1, 2014 at 23:55. This choice was based on the availability of the data for Bitcoin. The specific reason for choosing this period is given below in Section 5.1. Unlike trading hours for BTC/USD currency pair, trading hours for other instruments cover only a specific part of the day (as mentioned in Chapter 2, Bitcoin exchanges operate 24 hours a day, 7 days a week). Moreover, on national holidays, trading of some instruments may be suspended for several hours. As mentioned by Maderitsch (2014), there is not a widespread agreement on how the overnight (non-trading, that is when the market is closed) data in relation to realized variance estimation should be treated. In this thesis, we follow the works written by Corsi *et al.* (2008), Andersen *et al.* (1999), Hansen & Lunde (2005) and Thomakos & Wang (2003), among others, in which the data that fall outside the period of trading hours are not used for analysis. Andersen *et al.* (1999, p. 8) state: "In order not to confound the distributional characteristics of the various volatility measures by these largely deterministic calendar effects, we explicitly excluded a number of days from the raw 5-minute return series."

Another approach is to find optimal weights in order to use both observations from trading hours and data points from the non-active part of the day as used, for example, in the work by Maderitsch (2014). Advantages of this method are also pointed out in the work written by Ahoniemi & Lanne (2013). The method used in some works is also to discard the overnight observations, but then to increase the resulting value of the calculated realized variance by scaling so that the realized variance captures the whole day.

Following the work written by Hansen & Lunde (2005), we also discarded days with less than five hours of trading (that corresponds to 60 observations per day) from the sample. The reason is that these days could cause inconsistency in our estimations. It can lead to overvalued or oppositely undervalued variation measures if there were a large number of missing observations between two data points.

Andersen *et al.* (2000; 2010b), among others, state that non-standardized high-frequency asset returns are fat-tailed in comparison with the normal distribution. It implies higher probability of extreme deviations than in case when non-standardized returns followed a normal distribution (often referred to as "tail risk"). We present the descriptive statistics of daily returns, we test for normality using Jarque–Bera Adjusted Lagrange Multiplier (ALM) goodness-of-fit test and we present in appendix quantile-quantile plots but leave the in-depth analysis of fat tails for further research.

Just to outline possible forces that generally trigger jumps, we follow the work written by Baker *et al.* (2015) in which authors analyze the causes of jumps in national stock and bond markets and present their summary as written by the authors in Table 5.1. Other explanation of the emergence of jumps is given by Joulin *et al.* (2008) who say that neither released news (both idiosyncratic company news and macroeconomic news affecting the whole market) nor the large transaction volumes are the causes of large price jumps. They state that jumps are caused mainly due to the lack of market liquidity. This explanation does not correspond with the explanation of Andersen *et al.* (2005b) or Lee & Mykland (2008) who state that release of macroeconomic news has an impact on occurrence of jumps and that "except in one or two cases, jumps were always associated with news events" (Lee & Mykland 2008, p. 2553).

Table 5.1: Jumps by reason

Policy categories	**Non-policy categories**
Government spending	Macroeconomic news & outlook
Taxes	Corporate earnings & profits
Monetary policy & central banking	Commodities
Trade & exchange rate policy	Unknown/no explanation
Regulation (other than above)	Foreign Stock Markets
Sovereign military & security actions	Terrorist attack & large-scale violence by non-state actors
Other policy	Other non-policy

This table summarizes the forces that trigger jumps in stock and bond markets as outlined by Baker *et al.* (2015) who also describe some of the categories more in detail. *Government spending* covers news reports and forecasts about stimulus programs, health care, publicly funded pensions and so on. *Taxes* cover news reports about current or planned tax changes. *Monetary policy & central banking* cover actions conducted mainly by the central bank (change of interest rate, inflation control etc.). *Sovereign military & security actions* cover military actions such as war, invasion, or reactions to terrorist actions. *Macroeconomic news & outlook* include news focusing on macroeconomic forecasts (inflation, unemployment etc.).

Source: Baker *et al.* (2015).

The reasons for jumps of Bitcoin may differ from those mentioned in Table 5.1 if only for the reason that Bitcoin is not issued and regulated by any trusted third party. Moreover, there can be several other reasons for jumps of Bitcoin such as Bitcoin exchange bankruptcy, shutdown of some Bitcoin service or large theft of bitcoins. The argument of low market liquidity mentioned above would correspond with the statement given by Yermack (2013) that Bitcoin exchanges are not very liquid.

Using the computed log-returns, we constructed realized variation measures: realized variance (RV_t), realized bipower variation (RBV_t) and tripower quarticity (RTQ_t). Then, we computed jump detection test statistic, Z_t, defined in Equation 4.13 and also components of realized variance, J_t and C_t. As mentioned in Chapter 4, the results, which are presented in the following sections, were obtained using the 95$^{\text{th}}$ percentile of the standard normal distribution.

5.1 Bitcoin to US Dollar exchange rate

One of the Bitcoin's advantages is availability of data about Bitcoin. CoinDesk on its website makes Bitcoin Price Index (BPI) accessible (coindesk.com). It is an index of BTC/USD exchange rate calculated as an average of price of bitcoin on exchanges that meet specific criteria. At this time, five exchanges meet the BPI criteria: Bitfinex, Bitstamp, BTC-e, itBit and OKCoin. For our needs, data freely provided by CoinDesk cannot be used, because we need high-frequency data (e.g. with five-minute frequency) and BPI is available only with daily frequency. The same problem arises with datasets provided by numerical data platform Quandl. There are not only datasets for exchange rates, but also for number of bitcoins in circulation, the market capitalization of Bitcoin and the number of unique Bitcoin addresses, among others (quandl.com). Nevertheless, all datasets include only daily data.

Fortunately, at bitcoincharts.com, there is a possibility to load data even with one-minute frequency. The disadvantage is that the data cannot be easily exported and that in case of five-minute data, it is possible to load only a period of approximately 13 days at a time.

We obtained from bitcoincharts.com the data for BTC/USD exchange rate with five-minute frequency traded, at the time of its existence, at the most widely used Bitcoin currency exchange Mt. Gox (see e.g. Yermack 2013) with the first observation on December 31, 2011 at 0:00 and the last observation on February 1, 2014 at 23:55. The reason for not using the data after February 1, 2014 is that Mt. Gox encountered issues with getting money out of the exchange in February 2014: it halted withdrawals on February 7 and on February 25, Mt. Gox disappeared (Villar *et al.* 2014). Due to the fact that observations on December 25, 2012 (23:55) and on December 26, 2012 (0:00) are missing, we have 220,030 data points from 764 days. Since we use data with five-minute frequency and Mt. Gox operated 24 hours a day, we have 288 observations each day resulting in 287 intraday returns.

Unfortunately, 8890 observations do not contain the information about the closing price. Therefore, the dataset consisted of 211,140 observations with stated closing price representing 95.96% of observations that we could have if all the observations during the monitored period were available and contained information about the closing price. Following the approach used, for example, in the work written by Chow *et al.* (2009), we assigned to the observation without stated closing price the closing price that corresponds to the preceding observed closing price. Table 5.2 summarizes the number of days with related number of observations without stated closing price of bitcoin.

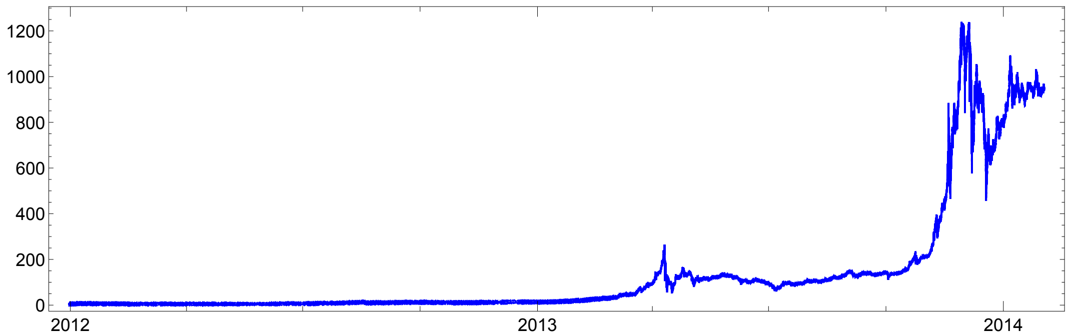
Table 5.2: The number of observations without stated closing price per day and the corresponding number of days

# of "missing" obs. per day	**# of days**
⟨87,124⟩	2
⟨70,86⟩	3
⟨50,69⟩	13
⟨40,49⟩	23
⟨30,39⟩	32
⟨20,29⟩	81
⟨10,19⟩	157
⟨0, 9⟩	317
0	136

Source: author and `bitcoincharts.com`.

The maximum number of observations without stated closing price on one day was equal to 124. There were still another 164 observations on that day with stated closing price so we left even this day in our dataset. We can also see that on the vast majority of days there were no more than 9 observations without quoted closing price. Consequently, we use 220,030 observations of closing price for our analysis.

Figure 5.1: Price of bitcoin (Mt. Gox): 31/12/11 – 1/2/14



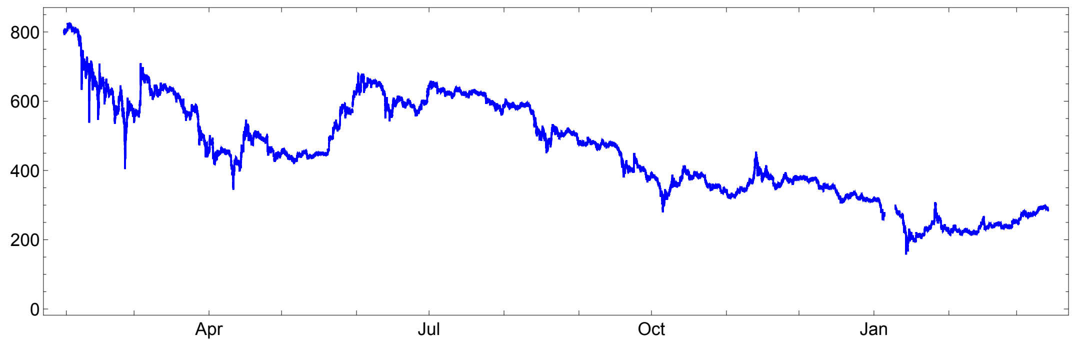
This figure depicts the development of the closing price of bitcoin in USD on the Mt. Gox exchange from December 31, 2011 to February 1, 2014.

Source: author and `bitcoincharts.com`.

Firstly, we present in Figure 5.1 how the closing price of bitcoin in USD was developing during the monitored period. We can see that compared to the price of bitcoin at the end of year 2013, the price remained relatively low until the beginning of 2013 (more specifically, it was on August 2, 2012 at 18:35 for the first time when the price exceeded 10 USD and on April 1, 2013 at 14:20 when the price exceeded 100 USD for the first time). Figure 5.1 shows that the price increased sharply in the final stages of 2013 (on November 3, 2013 at 7:30 the closing price was 213 USD and just 26 days after at 5:40 it rocketed to 1239.8 USD). The price on November 29, 2013 at 5:40 has also been the highest price of bitcoin throughout its existence. The lowest observed closing price in our dataset is the price on December 31, 2011 at 3:10 and is equal to 4.01 USD. The average price of bitcoin in our dataset is 137.96 USD.

Just for the sake of completeness, in Figure 5.2, we present the development of bitcoin price from January 31, 2014 at 0:00 to March 14, 2015 at 0:00[1]. Again, we downloaded data from bitcoincharts.com, but now we used the closing prices on the Bitstamp exchange which became the largest BTC/USD exchange after the Mt. Gox closure[2]. This dataset is not used for further analysis of volatility of price of bitcoin.

Figure 5.2: Price of bitcoin (Bitstamp): 31/1/14 – 14/3/15



This figure presents the development of the closing price of bitcoin in USD on the Bitstamp exchange from January 31, 2014 to March 14, 2015.

Source: author and `bitcoincharts.com`.

As can be seen, the closing price of bitcoin differs in Figure 5.1 and Figure 5.2 in January 2014. The reason is that the price is taken from two different exchanges and, generally, price of bitcoin varies between exchanges (for example, on January 31, 2014 at 0:00 the closing price of bitcoin on Mt. Gox was 945 USD while at the same time the price of bitcoin on Bitstamp was 800 USD). It

[1] It is visible in Figure 5.2 that there are missing data on closing prices in the beginning of January 2015. As noted by Bitstamp website, bitstamp.net, Bitstamp exchange was attacked by hackers on January 4, 2015 so Bitstamp temporarily suspended their services on January 5, 2015 (data are missing from January 5, 9:15 to January 9, 21:00).

[2] See, for example, the article "*The Bitcoin Economy's 'Backbone' Is Bitstamp, An Exchange Run By Two Young Slovenians*" at forbes.com.

may therefore seem that there is an opportunity of arbitrage. However, there are several reasons for which the arbitrage becomes unprofitable (e.g. exchange fees). Moreover, it takes some time to get sufficient amount of confirmations (i.e. sufficient amount of subsequent blocks that has to be mined) after a transaction is broadcasted to the network so the price may change while waiting for the time when the bitcoin can be traded again[3].

5.1.1 Log-returns

In Table 5.3, we provide a summary of descriptive statistics of intraday returns and daily returns of closing prices of bitcoin in USD. We also provide characterization of the distribution of daily returns when standardized by realized volatility and by square root of realized bipower variation. It presents mean value, standard deviation, minimum and maximum return, skewness and kurtosis and number of observations.

Table 5.3: Descriptive statistics of BTC/USD intraday returns, daily returns and standardized returns

	$r_{t,i}$	r_t	r_t/RVO_t	$r_t/\sqrt{RBV_t}$
Mean	$2.396 \cdot 10^{-5}$	0.0069	0.0916	0.1048
Std. dev.	0.0207	0.065	0.4029	0.4543
Min	-0.4937	-0.3724	-1.228	-1.591
Max	0.3569	0.3966	1.983	2.177
Skewness	-0.1124	-0.26	0.7883	0.6658
Kurtosis	38.877	10.925	6.086	5.623
Obs.	220,029	764	764	764

Source: author's computations and `bitcoincharts.com`.

The kurtosis of 38.877 indicates that the distribution of intraday returns is highly leptokurtic, with thicker tails and values concentrated around the mean. The skewness of -0.1124 suggests that the distribution is more or less symmetrical. The highest intraday negative return is 49.37%, which is roughly 23.9 standard deviations away from the mean. The highest intraday positive return is 35.69%, which is approximately 17.2 standard deviations away from the mean. Intraday returns are depicted in Figure A.1.

On the basis of the skewness and kurtosis, the non-standardized daily returns do not seem to be normally distributed. The kurtosis of 10.925 suggests again leptokurtic distribution and skewness of -0.26 again indicates that the

[3]More about arbitrage opportunity can be found, for example, at digiconomist.net.

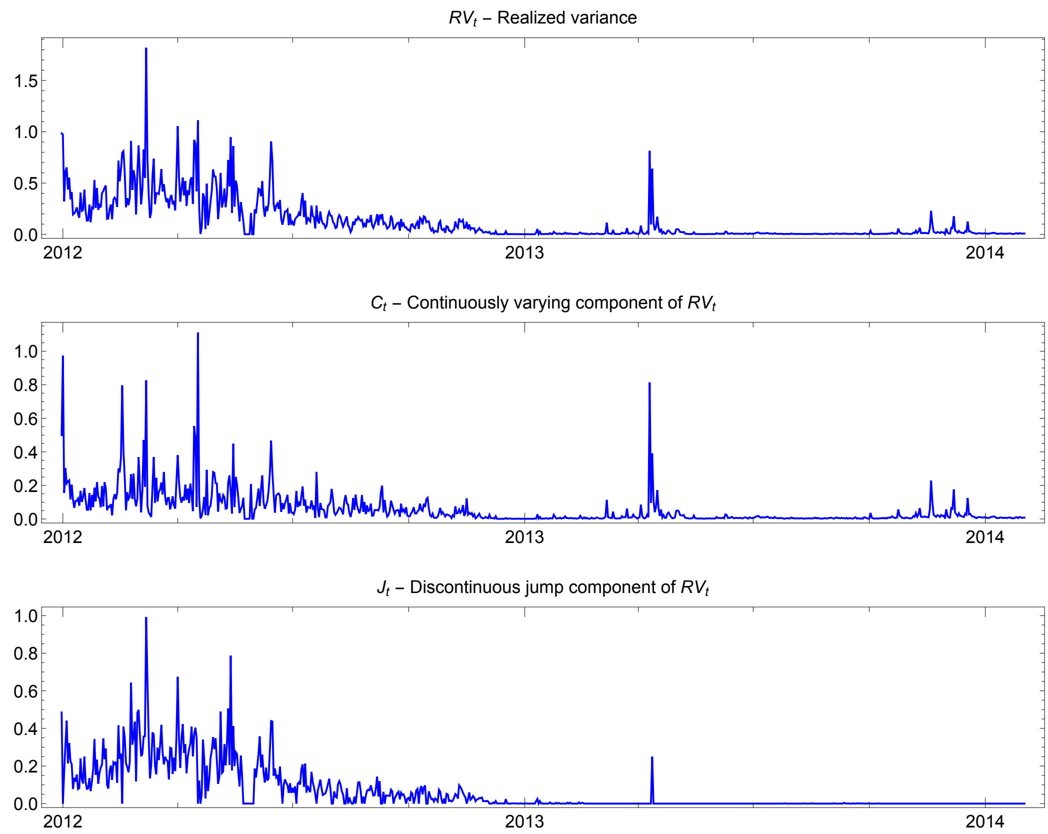
distribution is more or less symmetrical. The highest daily negative return is 37.24% and the highest daily positive return is 39.66%. We also used the Jarque–Bera ALM goodness-of-fit test as proposed by Urzúa (1996) with the null hypothesis that data was drawn from a normal distribution and the alternative hypothesis that it was not. This test effectively compares the skewness and kurtosis of our data to a normal distribution. The test was performed at the 0.05 significance level. We present the result in Table A.1. A small p-value (< 0.0001) implies that it is highly unlikely that non-standardized daily returns came from a normal distribution (null hypothesis is rejected at any reasonable level). We present the graph of daily returns in Figure A.2 and a quantile-quantile plot for daily returns in Figure A.3.

When we standardized daily returns by daily realized volatility, we can see that the distribution of these standardized returns is right skewed (skewness of 0.7883) which means that more values are concentrated on the left of the mean, with extreme values to the right. The kurtosis of 6.086 indicates leptokurtic distribution. Skewness and kurtosis changed only slightly after standardizing by the square root of realized bipower variation instead of standardizing by realized volatility. Skewness and kurtosis both changed in a way that the distribution of daily returns standardized by $\sqrt{RBV_t}$ is closer to the normal distribution. We performed the Jarque–Bera ALM test to analyze whether the standardized daily returns are normally distributed. Results of the test are presented in Table A.1 and again suggest that it is not probable that the returns (standardized either by RVO_t or by $\sqrt{RBV_t}$) are normally distributed (p-value < 0.0001 implies that the null hypothesis that the returns are normally distributed is strongly rejected at any reasonable level). Figure A.4 presents histograms of standardized daily returns from which can be seen that the distributions are sharper than a normal distribution but do not have fatter tails than the normal distribution. That means that the probability that standardized returns will take "extreme" values is smaller than in case when it followed the normal distribution.

5.1.2 Realized variance

Figure 5.3 shows the daily realized variance of BTC/USD exchange rate and its decomposition into continuous sample path variation and sum of the within day squared jumps. We can see that jumps tend to be smaller in time and appear less often. More specifically, October 8, 2013 was the last day during the monitored period when we detected the presence of jumps. This result suggests that the behavior of bitcoin price stabilized over time. The last jump contribution to the daily return variation larger than 0.004 was on April 12, 2013 and was equal to 0.2496. The largest jump contribution during the monitored period (equal to 0.9923) was on March 7, 2012. That was only 6 days after 46,703 BTC worth 228,845 USD (at that time) were stolen from the web host Linode. It was also the largest theft of bitcoins by that day[4].

Figure 5.3: Realized variance and variation components for BTC/USD exchange rate



We used intraday log-returns of BTC/USD exchange rate from December 31, 2011 to February 1, 2014 to compute daily realized variance which is depicted on the first of the three graphs. Then, following the approach described in the previous chapter, we decomposed realized variance into jump and continuous component. These are depicted on the remaining two graphs.

Source: author's computations and `bitcoincharts.com`.

We also present a summary of descriptive statistics of daily realized variance along with its components in Table 5.4. We found out that during the moni-

[4]See, for example, en.bitcoin.it.

tored period, there were 410 days without jumps. That corresponds to 53.7% of the total number of days. Figures A.5 and A.6 present jump components of the daily realized variance using 99$^{\text{th}}$ and 90$^{\text{th}}$ percentile of the standard normal distribution, respectively.

Table 5.4: Descriptive statistics of RV_t, C_t and J_t (BTC/USD)

	RV_t	C_t	J_t
Mean	0.1242	0.0589	0.0653
Std. dev.	0.2022	0.1056	0.1204
Min	$6.293 \cdot 10^{-4}$	$1.887 \cdot 10^{-4}$	0
Max	1.819	1.112	0.9923
Skewness	2.678	4.612	2.632
Kurtosis	13.177	34.104	12.254
Obs.	764	764	764

Source: author's computations and `bitcoincharts.com`.

5.2 Euro to US Dollar exchange rate

Another dataset which we use in this thesis is the dataset for Euro (EUR) to USD exchange rate. The dataset was obtained from the website of a Swiss online bank Dukascopy Bank (Dukascopy Bank 2014). Following the work written by Hansen & Lunde (2005), we use the arithmetic average of bid and ask closing exchange rate for our analysis. The monitored period remains the same as for BTC/USD exchange rate: from December 31, 2011 at 0:00 to February 1, 2014 at 23:55. However, twelve observations are missing on October 28, 2012 and another twelve observations are missing on October 27, 2013. It means that the original dataset consisted of 220,008 observations of EUR/USD exchange rate from 764 days (288 data points per day).

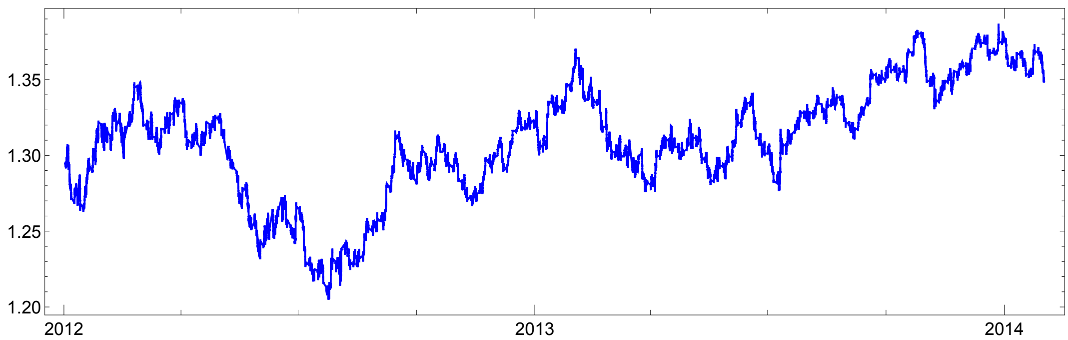
Market for EUR/USD currency pair is open from Sunday at 21:00 GMT during Summer Time (22:00 GMT during Winter Time) to Friday at 21:00 GMT during Summer Time (22:00 GMT during Winter Time)[5]. According to the Dukascopy web page, the change from Summer Time to Winter Time (and consequently the change in Dukascopy opening hours) is based on the change to Daylight Saving Time in the US eastern time zone (which happens in different time than the change to Daylight Saving Time in Europe). We downloaded the dataset along with the observations corresponding to the time when the

[5]See general features of Dukascopy Bank Forex trading at dukascopy.com.

market is closed. Exchange rate remains constant during these periods (the exception may be the change between the last observation on Friday and the first observation on Saturday). As mentioned at the beginning of this chapter, we dropped all the observations that did not fall into the period of trading hours and also discarded days with less than five hours of trading. That means that we created a dataset of 153,528 data points from 545 days observed when the market was open.

We firstly present in Figure 5.4 how the EUR/USD exchange rate was developing in the monitored period. EUR/USD exchange rate reached its minimum value equal to 1.2048 on July 24, 2012 at 17:00 and its maximum value equal to 1.3868 on December 27, 2013 at 11:40. The average value of EUR/USD exchange rate in our dataset is equal to 1.3094.

Figure 5.4: EUR/USD exchange rate: 31/12/11 – 1/2/14



The development of EUR/USD exchange rate from December 31, 2011 to February 1, 2014.

Source: author and `dukascopy.com`.

5.2.1 Log-returns

Again, we calculated the five-minute log-returns and in Table 5.5, we present the descriptive statistics of intraday returns of EUR/USD exchange rate along with the descriptive statistics of daily returns, returns standardized by realized volatility and returns standardized by square root of realized bipower variation. Looking at the kurtosis of the distribution of intraday returns which is equal to 64.915, the distribution is highly leptokurtic indicating thicker tails with values concentrated around the mean. Based on the skewness (-0.5892), the distribution is moderately left-skewed which means that the left tail is longer and most of the distribution is on the right. The highest intraday negative return is 1.4%, which is almost 45.6 standard deviations away from the mean. The highest intraday positive return is 0.89%, which is approximately 29 standard deviations away from the mean. The graph of intraday returns is depicted in Figure B.1.

Judging from the values of kurtosis and skewness, daily returns seem not to be normally distributed. Kurtosis of 4.508 implies that the distribution is leptokurtic. Skewness of 0.3356 suggests approximately symmetrical distribution. The highest daily negative return is 1.21% and the highest daily positive return is 2.62%. As in the case of BTC/USD exchange rate, we performed the Jarque–Bera ALM goodness-of-fit test with the null hypothesis that daily returns are normally distributed and the alternative hypothesis that they are not. The test was performed at the 0.05 significance level. We present the result of the test in Table B.1. A small p-value (< 0.0001) implies that it is not likely that non-standardized daily returns are normally distributed. The graph of daily returns is presented in Figure B.2 and a quantile-quantile plot for daily returns is depicted in Figure B.3.

After standardizing daily returns by daily realized volatility, the distribution of these standardized returns is on the basis of skewness (0.0018) and kurtosis (2.769) very close to the normal distribution. The same applies for standardization by square root of realized bipower variation. We performed the Jarque–Bera ALM test even for the standardized daily returns with the null hypothesis that they are normally distributed and the alternative that they are not. Results of the test are presented in Table B.1. The test was again performed at the 0.05 significance level. On the basis of the reported p-values (0.556 and 0.479), we clearly cannot reject the null hypothesis. Figure B.4 depicts histograms of standardized daily returns.

Table 5.5: Descriptive statistics of EUR/USD intraday returns, daily returns and standardized returns

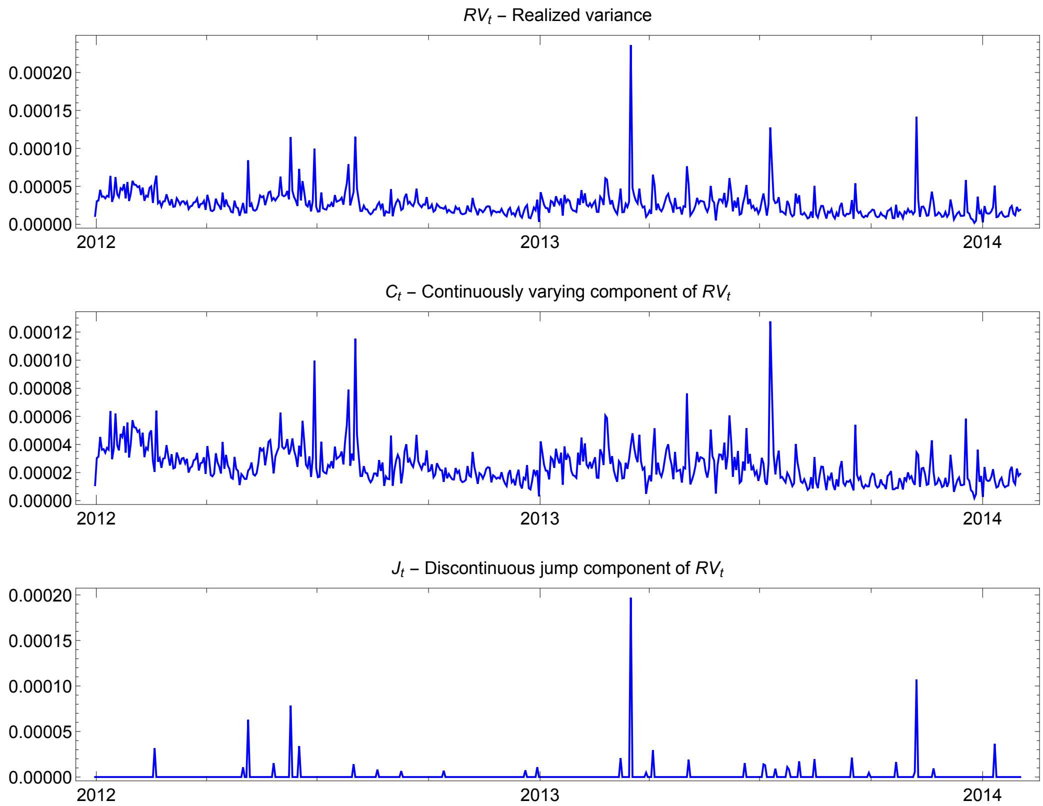
	$r_{t,i}$	r_t	r_t/RVO_t	$r_t/\sqrt{RBV_t}$
Mean	$2.671 \cdot 10^{-7}$	$7.524 \cdot 10^{-5}$	0.0159	0.0166
Std. dev.	$3.068 \cdot 10^{-4}$	0.005	0.9255	1.002
Min	-0.014	-0.0121	-2.3613	-2.517
Max	0.0089	0.0262	3	3.027
Skewness	-0.5892	0.3356	0.0018	0.0019
Kurtosis	64.915	4.508	2.769	2.744
Obs.	153,527	545	545	545

Source: author's computations and `dukascopy.com`.

5.2.2 Realized variance

Realized variance with its components for EUR/USD exchange rate is depicted in Figure 5.5. The largest jump contribution to the daily return variation was on March 18, 2013 and was equal to $1.969 \cdot 10^{-4}$. March 2013 was the time of banking crisis on Cyprus as mentioned in Chapter 3.

Figure 5.5: Realized variance and variation components for EUR/USD exchange rate



We used intraday log-returns of EUR/USD exchange rate from December 31, 2011 to February 1, 2014 to compute daily realized variance which is depicted on the first of the three graphs. Then, we decomposed realized variance into jump and continuous component. These are depicted on the remaining two graphs.

Source: author's computations and `dukascopy.com`.

As Alderman (2013) noted, it was on March 18, 2013 "for the first time since the onset of the euro zone sovereign debt crisis and the bailouts of Greece, Portugal and Ireland, ordinary depositors (including those with insured accounts) were being called on to bear part of the cost, 5.8 billion EUR."[6] Financial au-

[6] http://www.nytimes.com/2013/03/19/business/global/asian-markets-drop-on-latest-euro-concerns.html

thorities of Eurozone announced tax on bank deposits which potentially could lead to a run on money in other parts of the Eurozone. As pointed out by Davies (2013), that could result in "a new financial crisis in the 17 nation Eurozone, which is a major trading partner for many US businesses."[7] That might be the possible explanation of such a jump in EUR/USD exchange rate.

We present a summary of descriptive statistics of daily realized variance along with its components in Table 5.6. Our analysis also showed that during the monitored period there were 513 days without jumps corresponding to 94.1% of the total number of days. Figures B.5 and B.6 show jump components of the daily realized variance using 99^{th} and 90^{th} percentile of the standard normal distribution, respectively.

Table 5.6: Descriptive statistics of RV_t, C_t and J_t (EUR/USD)

	RV_t	C_t	J_t
Mean	$2.652 \cdot 10^{-5}$	$2.493 \cdot 10^{-5}$	$1.59 \cdot 10^{-6}$
Std. dev.	$1.815 \cdot 10^{-5}$	$1.389 \cdot 10^{-5}$	$1.114 \cdot 10^{-5}$
Min	$1.795 \cdot 10^{-6}$	$1.795 \cdot 10^{-6}$	0
Max	$2.362 \cdot 10^{-4}$	$1.276 \cdot 10^{-4}$	$1.969 \cdot 10^{-4}$
Skewness	4.513	2.24	12.636
Kurtosis	41.396	13.198	195.604
Obs.	545	545	545

Source: author's computations and `dukascopy.com`.

5.3 Price of gold in US Dollars

We also analyze the volatility of price of troy ounce of gold in US dollars (XAU/USD exchange rate). The dataset was, as in the case of EUR/USD exchange rate dataset, obtained from the web page of Dukascopy Bank. Moreover, the same observations as in the case of previous dataset are missing (twelve observations on October 28, 2012 and another twelve observations on October 27, 2013) so we have 220,008 observations of XAU/USD exchange rate.

Start of the trading week for XAU/USD pair is on Sunday at 21:00 GMT during Summer Time (22:00 GMT during Winter Time) and market closes on Friday at 21:00 GMT during Summer Time (22:00 GMT during Winter Time). There are also one-hour trading breaks during the trading session (21:00–22:00 during Summer Time and 22:00–23:00 during Winter Time). Moreover, XAU/USD pair

[7] http://abcnews.go.com/blogs/business/2013/03/new-euro-banking-crisis-ahead

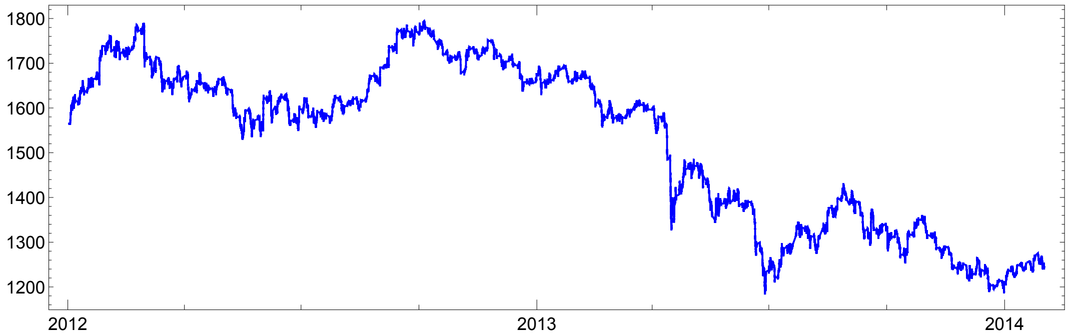
cannot be traded on US national holidays from 17:00 to 21:00 during Summer Time and from 18:00 to 22:00 during Winter Time. According to the web page of Dukascopy Bank, the change from Summer Time to Winter Time is again based on the change to Daylight Saving Time in the US eastern time zone.

As in the case of dataset mentioned in Section 5.2, this dataset contains also observations corresponding to the time when the market was closed. Exchange rate during these periods remains constant (the exception may be the change between the last observation on Friday and the first observation on Saturday and also the change between the last observation on Saturday and the first observation on Sunday) and corresponds to the closing exchange rate of the last observation during which the market was open. Similarly as Hansen & Lunde (2005) did in their work, we dropped all the data points observed during periods when XAU/USD pair cannot be traded (either because they do not fall in the period of trading hours, or they were observed during trading breaks, or they were observed during the inactive part of the day of US national holiday). We proceeded from the list of US national holidays at dukascopy.com and from the web page of US Office of Personnel Management (US Office of Personnel Management 2015) for corresponding US national holidays in 2011 and 2012 for which there were no available data on the web page of Dukascopy Bank. Moreover, we again keep only days with at least five hours of trading. We use the arithmetic average of bid and ask closing exchange rate for our analysis.

We had to drop additional 252 observations on March 29, 2013 and additional 276 observations on December 25, 2013 due to the fact that XAU/USD exchange rate remained constant during these periods and these observations were the only observations on that day which we, for above-mentioned reasons, intended to use for our analysis. The reason for dropping these observations is that if the exchange rate remains constant during the whole day, the daily realized variance is equal to zero which causes problems when calculating jump detection statistic defined in Equation 4.13. We believe that these additionally 528 dropped observations will not influence the results of our analysis to a substantial extent. Finally, we created a dataset consisting of 147,096 observations of closing XAU/USD exchange rate from 543 days.

We firstly present in Figure 5.6 the development of XAU/USD exchange rate. XAU/USD exchange rate reached its minimum (1182.995) on June 28, 2013 at 0:50 and its maximum (1795.796) on October 5, 2012 at 2:35. The average value of XAU/USD exchange rate in our dataset is equal to 1528.25.

Figure 5.6: XAU/USD exchange rate: 31/12/11 – 1/2/14



The development of XAU/USD exchange rate from December 31, 2011 to February 1, 2014.

Source: author and `dukascopy.com`.

5.3.1 Log-returns

The descriptive statistics of five-minute log-returns of XAU/USD exchange rate along with the descriptive statistics of daily returns and standardized returns (r_t/RVO_t and $r_t/\sqrt{RBV_t}$) are summarized in Table 5.7. As in case of intraday returns related to previous datasets, the distribution of XAU/USD intraday returns is highly leptokurtic and the skewness of 0.2598 implies approximately symmetric distribution. The highest intraday negative return is 2.05%, which is roughly 31.5 standard deviations away from the mean. The highest intraday positive return is 2.02%, which is approximately 31 standard deviations away from the mean. Intraday returns are depicted in Figure C.1.

Table 5.7: Descriptive statistics of XAU/USD intraday returns, daily returns and standardized returns

	$r_{t,i}$	r_t	r_t/RVO_t	$r_t/\sqrt{RBV_t}$
Mean	$-1.559 \cdot 10^{-6}$	$4.222 \cdot 10^{-4}$	-0.0168	-0.0130
Std. dev.	$6.509 \cdot 10^{-4}$	0.0113	0.9267	1.040
Min	-0.0205	-0.0927	-3.009	-3.054
Max	0.0202	0.0458	2.706	2.919
Skewness	0.2598	-1.289	-0.1049	-0.0740
Kurtosis	62.685	13.25	2.803	2.850
Obs.	147,095	543	543	543

Source: author's computations and `dukascopy.com`.

On the basis of the values of kurtosis and skewness, daily returns seem not to be normally distributed. The kurtosis of 13.25 implies leptokurtic distribution and skewness of -1.289 suggests asymmetric distribution with most values concentrated on the right of the mean, with extreme values to the left. The highest daily negative return is 9.27% and the highest positive return is 4.58%. Again, we use Jarque–Bera ALM test for normality performed at the 0.05 significance level. We reject the null hypothesis that daily returns are normally distributed (p-value < 0.0001) and therefore conclude that the distribution of non-standardized daily returns is not normal. We present daily returns in Figure C.2 and a quantile-quantile plot for daily returns in Figure C.3.

Distribution of daily returns standardized by realized volatility is according to the values of skewness (-0.1049) and kurtosis (2.803) very close to the normal distribution. Distribution of returns standardized by the square root of realized bipower variation is, based on the values of skewness and kurtosis, even closer to the normal distribution. We perform Jarque–Bera ALM test for normality (for both standardizations) at the 5 percent level. In both cases, we clearly do not have sufficient evidence to reject the null hypothesis that the distribution of standardized daily returns is normal. Results of Jarque–Bera ALM test for standardized and non-standardized daily returns are presented in Table C.1. Figure C.4 presents histograms of standardized daily returns.

5.3.2 Realized variance

In Table 5.8, we present a summary of descriptive statistics of daily realized variance along with its components. We found out that during the monitored period, there were 501 days without jumps. That corresponds to 92.3% of the total number of days. In Figures C.5 and C.6, jump components of the daily realized variance are depicted using 99^{th} and 90^{th} percentile of the standard normal distribution for the detection of jumps.

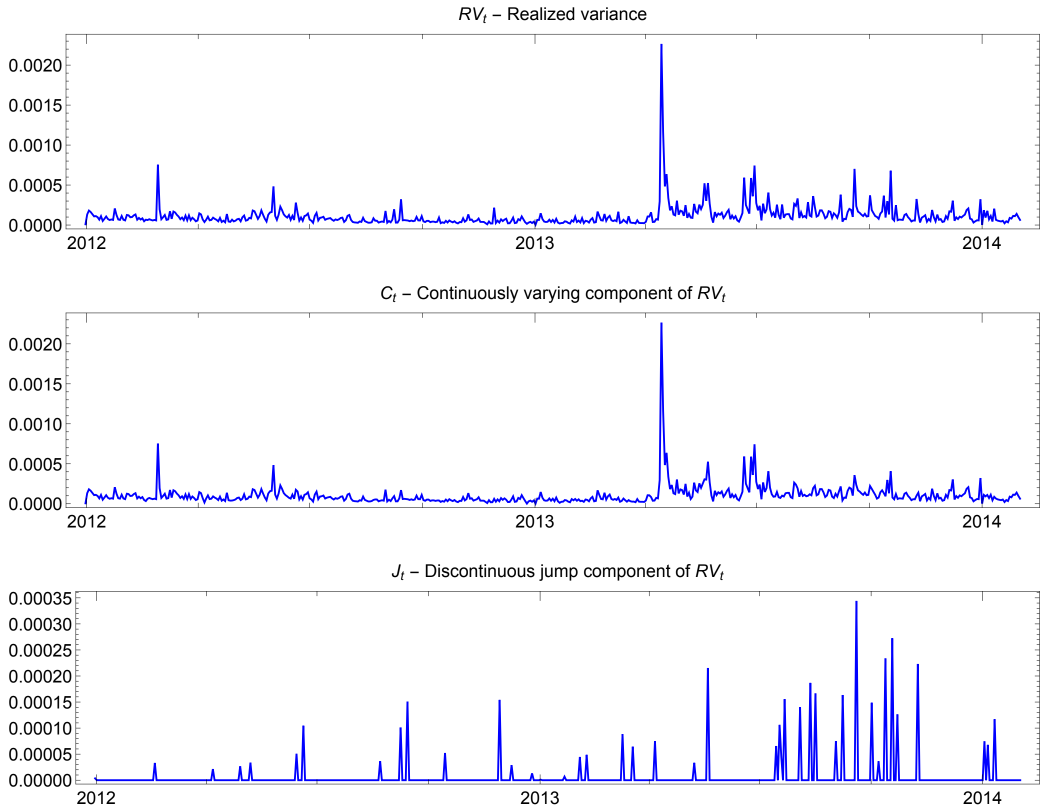
Table 5.8: Descriptive statistics of RV_t, C_t and J_t (XAU/USD)

	RV_t	C_t	J_t
Mean	$1.148 \cdot 10^{-4}$	$1.071 \cdot 10^{-4}$	$7.622 \cdot 10^{-6}$
Std. dev.	$1.44 \cdot 10^{-4}$	$1.366 \cdot 10^{-4}$	$3.42 \cdot 10^{-5}$
Min	$2.219 \cdot 10^{-6}$	$1.398 \cdot 10^{-6}$	0
Max	0.0023	0.0023	$3.441 \cdot 10^{-4}$
Skewness	7.865	9.007	5.691
Kurtosis	100.564	124.848	39.834
Obs.	543	543	543

Source: author's computations and `dukascopy.com`.

Figure 5.7 depicts the development of realized variance along with its variation components for XAU/USD exchange rate. We can see that jumps tend to be higher in the second half of the monitored period. The largest jump contribution to the daily return variation was on September 18, 2013 and was equal to $3.441 \cdot 10^{-4}$. On that day, US Federal Reserve decided to maintain its economic stimulus program and as a result gold jumped 4 percent (Terazono 2013) which as The Guardian (2013) noted was the biggest one-day jump of gold in four years since traders anticipated that the decision of US Federal Reserve would result in inflation.

Figure 5.7: Realized variance and variation components for XAU/USD exchange rate



We used intraday log-returns of XAU/USD exchange rate from December 31, 2011 to February 1, 2014 to compute daily realized variance which is depicted on the first of the three graphs. Then, we decomposed realized variance into jump and continuous component. These are depicted on the remaining two graphs.

Source: author's computations and `dukascopy.com`.

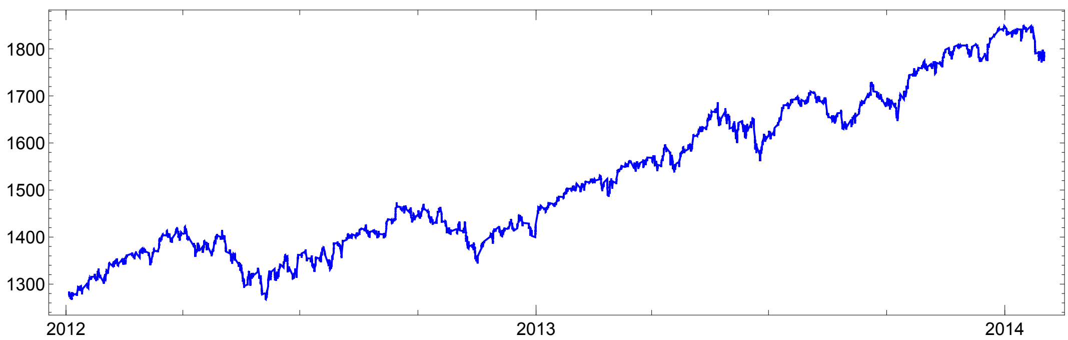
5.4 S&P 500 index

This dataset contains the closing values of the Standard & Poor's 500 index with five-minute frequency. The S&P 500 is a capitalization-weighted index including 500 leading American companies such as Apple Inc., JP Morgan Chase & Co, Procter & Gamble and Berkshire Hathaway B. It represents around 80% of the total US market capitalization[8].

We obtained this dataset from provider of online trading services, finam.ru. The monitored period is shorter compared to previous datasets. The beginning of the monitored period is on January 3, 2012, because it was not possible to obtain data for the first few missing days.

On the vast majority of days, observations were recorded in time interval beginning at 9:35 and ending at 16:05 which results in 79 observations per day. For some days, there are also observations recorded after 16:05. However, these observations appear only sporadically without any regular pattern. For other days, few observations between 9:35 and 16:05 may not be recorded. Observations on Saturdays and Sundays are not available at all. Since we were not able to obtain additional information related to this dataset, we assume that the time interval from 9:35 to 16:05 corresponds to the time when the market is open. Consequently, we keep only observations that were recorded during this time interval representing 97% of the original dataset. Moreover, we again discarded days with less than five hours of trading. Finally, we have 40,830 observations of values of the S&P 500 index which are used for our analysis.

Figure 5.8: S&P 500 index: 3/1/12 – 1/2/14



The development of the value of the S&P 500 index from January 3, 2012 to February 1, 2014.

Source: author and `finam.ru`.

[8]See, for instance, us.spindices.com.

Figure 5.8 depicts the development of the value of the S&P 500 index. The S&P 500 index reached during the monitored period its minimum value equal to 1266.61 on January 5, 2012 at 9:45 and its maximum value equal to 1850.31 on January 15, 2014 at 11:10. The average value of the S&P 500 index in our dataset is equal to 1523.7.

5.4.1 Log-returns

In Table 5.9, we present descriptive statistics of intraday returns, daily returns and of standardized daily returns of the S&P 500 index. The kurtosis of 64.551 indicates that the distribution of intraday returns is highly leptokurtic. The skewness of -0.165 suggests approximately symmetric distribution. The highest intraday negative return is 1.56%, which is approximately 19.4 standard deviations away from the mean and the highest positive return is 1.85% which is roughly 23 standard deviations away from the mean. Intraday returns are depicted in Figure D.1.

On the basis of the skewness and kurtosis, non-standardized daily returns do not seem to be normally distributed. The kurtosis of 4.047 implies again leptokurtic distribution and skewness of -0.1646 indicates that the distribution is more or less symmetrical. The highest daily negative return is 2.53% and the highest daily positive return is 2.51%. We test the normality of non-standardized daily returns with Jarque–Bera ALM test with the null hypothesis that the returns are normally distributed and the alternative that they are not (at the 5 percent level). On the basis of the resulting p-value (0.0005), we reject the null hypothesis and conclude that non-standardized daily returns are not normally distributed. We present daily returns in Figure D.2. Quantile-quantile plot for daily returns is depicted in Figure D.3.

The distribution of daily returns standardized by realized volatility is on the basis of skewness (-0.0772) and kurtosis (2.606) close to the normal distribution. The same can be stated even for returns standardized by $\sqrt{RBV_t}$. The kurtosis of 2.915 indicates roughly mesokurtic distribution and skewness of 0.1954 relatively symmetrical distribution. Using Jarque–Bera ALM test, we analyzed the normality of standardized returns again with the null hypothesis that the distribution of standardized returns is normal and the alternative that it is not. Resulting p-values indicate that the null hypothesis is not rejected at the 5 percent level (and would not be rejected even at the 10 percent level) in both cases. Results of Jarque–Bera ALM test for non-standardized and standardized daily returns are summarized in Table D.1. Figure D.4 depicts histograms of standardized daily returns.

Table 5.9: Descriptive statistics of the S&P 500 intraday returns, daily returns and standardized returns

	$r_{t,i}$	r_t	r_t/RVO_t	$r_t/\sqrt{RBV_t}$
Mean	$8.123 \cdot 10^{-6}$	$6.415 \cdot 10^{-4}$	0.1588	0.2445
Std. dev.	$8.025 \cdot 10^{-4}$	0.0076	1.0291	1.362
Min	-0.0156	-0.0253	-2.724	-2.986
Max	0.0185	0.0251	2.915	4.235
Skewness	-0.165	-0.1646	-0.0772	0.1954
Kurtosis	64.551	4.047	2.606	2.915
Obs.	40,829	517	517	517

Source: author's computations and `finam.ru`.

5.4.2 Realized variance

In Table 5.10, we present a summary of descriptive statistics of daily realized variance along with its components. We found out that during the monitored period, there were 349 days without jumps. That corresponds to 67.5% of the total number of days. Figures D.5 and D.6 present jump components of the daily realized variance using 99^{th} and 90^{th} percentile of the standard normal distribution for the detection of jumps.

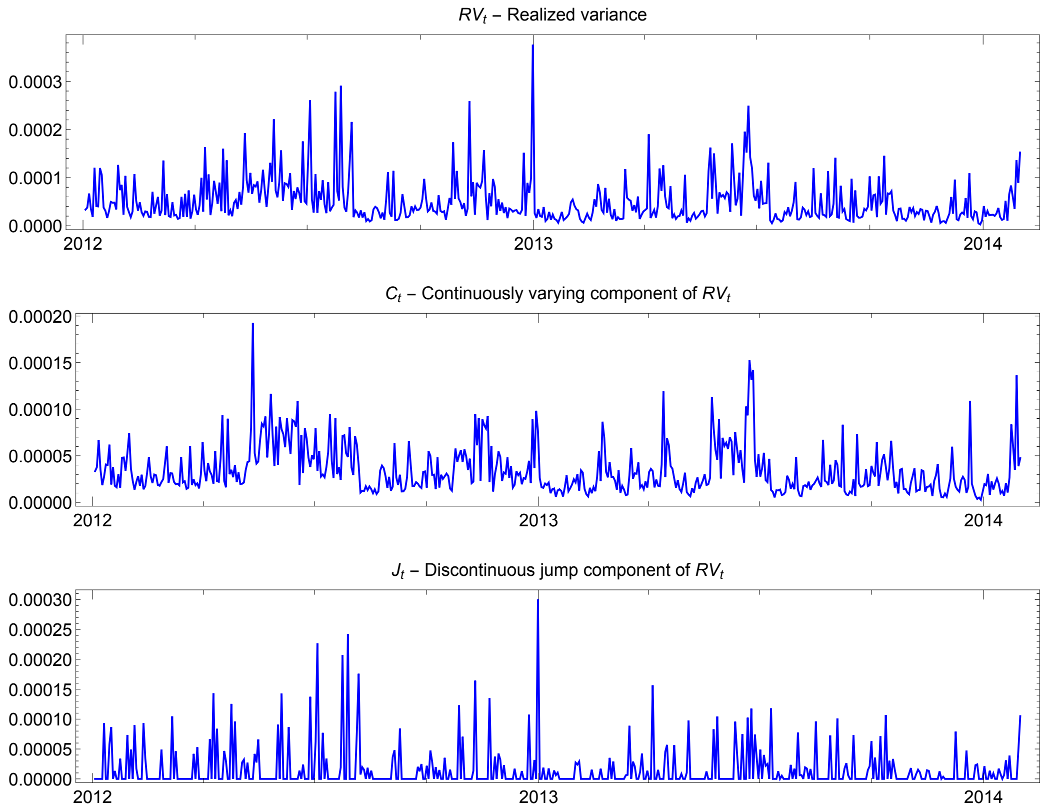
Table 5.10: Descriptive statistics of RV_t, C_t and J_t (S&P 500 index)

	RV_t	C_t	J_t
Mean	$5.086 \cdot 10^{-5}$	$3.454 \cdot 10^{-5}$	$1.633 \cdot 10^{-5}$
Std. dev.	$4.641 \cdot 10^{-5}$	$2.575 \cdot 10^{-5}$	$3.643 \cdot 10^{-5}$
Min	$2.722 \cdot 10^{-6}$	$2.722 \cdot 10^{-6}$	0
Max	$3.766 \cdot 10^{-4}$	$1.926 \cdot 10^{-4}$	$3 \cdot 10^{-4}$
Skewness	2.485	1.821	3.423
Kurtosis	11.982	7.581	18.036
Obs.	517	517	517

Source: author's computations and `finam.ru`.

Figure 5.9 depicts the development of realized variance along with its variation components for the S&P 500 index. The largest jump contribution to the daily return variation was on January 2, 2013 and was equal to $3.0 \cdot 10^{-4}$. On that day, president of the United States, Barack Obama, signed into law the American Taxpayer Relief Act of 2012 that the House and Senate had passed the previous day in order to avert the fiscal cliff. That meant, for example[9], preserved tax cuts for most American households and increased taxes on the wealthiest Americans (Smith 2013). As pointed out by Hwang (2013), "US stocks rallied, giving the Standard & Poor's 500 Index its biggest gain in more than a year"[10] and that "all 10 groups in the S&P 500 rose at least 1.8 percent"[11].

Figure 5.9: Realized variance and variation components for the S&P 500 index



We used intraday log-returns of the S&P 500 index from January 3, 2012 to February 1, 2014 to compute daily realized variance which is depicted on the first of the three graphs. Then, we decomposed realized variance into jump and continuous component. These are depicted on the remaining two graphs.

Source: author's computations and `finam.ru`.

[9] Summary of provisions in the American Taxpayer Relief Act of 2012 can be found at finance.senate.gov.

[10] http://www.bloomberg.com/news/articles/2013-01-02/u-s-stock-index-futures-rally-as-fiscal-cliff-averted

[11] More information about the Fiscal Cliff issue can be found, for example, at finance.yahoo.com.

5.5 Comparison

In this section, we compare the results reported in this chapter, particularly jumps in the Bitcoin total variation with jumps in the total variation of other instruments. We also compare the variance of Bitcoin with the variance of other instruments as given by the continuous components of daily realized variance. We also aim to evaluate the development of jumps in case of BTC/USD exchange rate in time. In addition, we discuss the comparison of the distribution of standardized daily returns for all analyzed instruments and also the microstructure noise.

5.5.1 Distribution of standardized returns and the noise contamination

We follow the work written by Fleming & Paye (2011) in which authors analyze whether the jump component causes the distribution of standardized daily returns to be non-normal. Since the realized bipower variation is an estimator of the integrated variance which is robust to the presence of jumps, we generally expect that using square root of realized bipower variation instead of realized volatility for the standardization of daily returns would lead to the distribution of returns which would be closer to the standard normal (we expect RVO_t to be biased by microstructure noise and jumps). If we compare the descriptive statistics of daily returns standardized by realized volatility and descriptive statistics of returns standardized by square root of realized bipower variation, we could see that in case of BTC/USD exchange rate, price of gold and the S&P 500 index, the coefficients of kurtosis are closer to three in case of standardization by $\sqrt{RBV_t}$. Moreover, in case of BTC/USD exchange rate and price of gold, the coefficients of skewness are closer to zero for $r_t/\sqrt{RBV_t}$. Therefore, the results partially confirmed our expectations. Moreover, they are consistent with the results in empirical studies written by Fleming & Paye (2011) and Baruník & Vacha (2012).

As we could see, in case of EUR/USD exchange rate, price of gold and (partially) in case of the S&P 500 index, the results of Jarque–Bera ALM goodness-of-fit test along with the histograms of standardized daily returns indicate that a normal distribution may be a close approximation to distributions of standardized returns ($r_t/\sqrt{RBV_t}$). The descriptive statistics reported in Tables 5.5, 5.7 and 5.9 only support this conjecture. The distributions of standardized returns are very close to a standard normal distribution. The standard deviations are roughly equal to one, the means are close to zero, the kurtosis is in all three cases close to three and the coefficients of skewness are near zero. However, in case of BTC/USD exchange rate, the null hypothesis that the standardized returns are normally distributed was strongly rejected at any reasonable level (using Jarque–Bera ALM test) as can be seen in Table A.1. That is also confirmed by the histogram presented in Figure A.4 and the descriptive statistics in Table 5.3. It could be caused by microstructure noise which can play a significant role in the price process and which cannot be to a sufficient degree treated by realized bipower variation (and also not by realized variance) used in this thesis[12] (see e.g. Hansen & Lunde 2005; Bandi & Russell 2006; Barndorff-Nielsen *et al.* 2008). That can lead to biased estimators and consequently, as already mentioned, the standard deviation of standardized returns can take values different from one (and the same applies to distorted values of kurtosis). We have already mentioned the potential sources of microstructure noise in the previous chapter: discreteness of prices (see e.g. Harris 1990), bid-ask bounce or asynchronous trading.

A possible resolution of this problem can be to use the wavelet-based methodology for estimation of quadratic variation as proposed by Baruník & Vacha (2012). Authors in this work compare the estimator proposed by them with realized variance, realized bipower variation, realized kernels introduced by Barndorff-Nielsen *et al.* (2008) and two-scale realized volatility proposed by Lan Zhang & Aït-Sahalia (2005). It was found that the wavelet-based estimator has the lowest bias in comparison with the other estimators in the jump-diffusion model with stochastic volatility and that the distribution of daily returns standardized by wavelet-based estimator is very close to the normal distribution. Fleming & Paye (2011) use above-mentioned realized kernel estimator "that is (nearly) conditionally unbiased in the presence of microstructure noise" (Fleming & Paye 2011, pp. 119–120) and find that when daily returns are standardized by this estimator instead of realized variance, the distribution of daily returns is platykurtic and not close to the standard normal distribution. Realized kernel estimator consists of two parts: realized variance defined in Equation 4.7 and the sum of weighted (weights are given by the kernel function) realized autocovariances which corrects the realized variance. However, we will not go into further details in this thesis. In the beginnings of modelling

[12]However, as already mentioned, the definition of realized bipower variation used in this thesis should make the estimator robust to certain type of microstructure noise.

realized volatility, filtering techniques such as moving average filter proposed by Andersen *et al.* (1999) or autoregressive filter proposed by Bollen & Inder (2002) were common solutions to microstructure noise. Review of most of the methods mentioned above can be found, for example, in the work written by McAleer & Medeiros (2008).

We do not employ the theory proposed by Baruník & Vacha (2012) nor the above-mentioned methods proposed by Barndorff-Nielsen *et al.* (2008), Lan Zhang & Aït-Sahalia (2005) or Fleming & Paye (2011) as solutions to the noise contamination since it is beyond the scope of this thesis. Nevertheless, using the same datasets as we use in this thesis, the comparison of different estimators and different filtering techniques can be an interesting extension of this thesis.

5.5.2 High variance of Bitcoin?

Using unequal variance t-test, we want to assess whether the variance of Bitcoin is higher than the variance of other analyzed instruments. For easier comparison, we firstly provide in Table 5.11 the mean size of the continuous components (multiplied by 10,000) for all instruments.

Table 5.11: Mean size of continuous components ($\times 10,000$)

BTC/USD	EUR/USD	XAU/USD	S&P 500
589	0.2493	1.071	0.3454

Source: author's computations.

Before using an unequal variance t-test, we verify assumptions that must be met. We already stated in the previous chapter that based on the works written by Yermack (2013) and Briere *et al.* (2013), we assume that the two samples are in all three cases independent. We test whether the continuous components are normally distributed using the Cramér-von Mises test[13] as suggested by Wolfram Mathematica software as the most powerful test of normality that applies to our data. The null hypothesis is that they are normally distributed and the alternative hypothesis that they are not. The test was performed at the 0.05 significance level. Resulting p-values are reported in Table 5.12.

The null hypothesis is thus rejected at any reasonable level. As a result, we have to rely on the central limit theorem which says that the distribution of the

[13] We do not mention the theory behind this test and rather refer to the article written by Darling (1957).

Table 5.12: Cramér-von Mises goodness-of-fit test (continuous components)

	Statistic	p-value
BTC/USD	17.958	< 0.0001
EUR/USD	1.969	< 0.0001
XAU/USD	11.815	< 0.0001
S&P 500	4.335	< 0.0001

Source: author's computations.

two sample means that we are testing approaches normal distribution as the sample sizes get large, regardless of the distributions of the underlying data. This means that with sufficiently large sample size, the normality assumption underlying a t-test is satisfied. Some authors, such as Ruxton (2006), state that central limit theorem requires a combined sample size of at least 30. However, it depends on the amounts of skewness or heavy-tailedness of the data we have at hand. We have the following number of observations of continuous components of realized variance: 764 (BTC/USD), 545 (EUR/USD), 543 (XAU/USD) and 517 (S&P 500). We consider the sample size to be sufficient to assert that the distributions generally tend to be very close to normal despite the fact that at least in case of XAU/USD exchange rate, the skewness is very large as can be seen in Table 5.8. We thus consider the normality assumption to be asymptotically satisfied.

As already mentioned in the previous chapter, we use directly the unequal variance t-test without performing preliminary tests for equality of variances. Just to see whether the preliminary tests would hypothetically support our choice of the t-test, we present in Tables A.2 and A.3 results of Conover test (as suggested by Wolfram Mathematica software as the most powerful test) and Levene test of variances[14] (which is for the two-sample case much less sensitive to the normality assumption than the F-test[15]). The null hypothesis is that the true population variances are equal and the alternative is that they are not. Both tests are performed at the 0.05 significance level. Small p-values suggest that it is unlikely that the null hypothesis is true in all three cases for both tests. That only supports our choice of the t-test.

[14]Due to the fact that our choice of the t-test does not depend on these preliminary tests, we do not introduce the theory behind them and rather refer to the textbook written by Conover (1999) and the article written by Brown & Forsythe (1974).

[15]See, for instance, wolfram.com.

Table 5.13: Unequal variance t-test: continuous part of realized variance

	Statistic	*df*	p-value
BTC/USD, EUR/USD	15.419	763	< 0.0001
BTC/USD, XAU/USD	15.397	763	< 0.0001
BTC/USD, S&P 500	15.416	763	< 0.0001

Source: author's computations.

We present the results of the unequal variance t-test in Table 5.13. On the basis of p-values, we can see that the null hypothesis that the difference in true population means of continuous part of realized variance is equal to 0 is strongly rejected at practically any reasonable level for all pairs of instruments. Therefore, we conclude that the alternative hypothesis, as stated in the previous chapter, is true. Consequently, we deduce that the variance of price of bitcoin is higher than the variance of EUR/USD exchange rate, price of gold and the value of the S&P 500 index.

5.5.3 Jump components of daily realized variance

In Table 5.14, we provide the mean size of the jump components (multiplied by 10,000) for all instruments to make it easier to compare them. We can see that the mean of the jump process of BTC/USD exchange rate, which is equal to 0.0653, is much higher in comparison with other instruments:

- it is almost 41,082 times higher than the mean of the jump process of EUR/USD exchange rate;
- it is approximately 8,570 times higher than the mean of the jump process of price of gold; and
- it is circa 4,000 times higher than the mean of the jump process of the S&P 500 index.

Table 5.14: Mean size of jump components ($\times 10,000$)

BTC/USD	EUR/USD	XAU/USD	S&P 500
653.194	0.0159	0.0762	0.1633

Source: author's computations.

Similarly as in the case of continuous components of realized variance, we perform the t-test with the null hypothesis that the difference in true population means of jump components of realized variance is equal to 0 and the alternative that the difference is larger than 0. Since we have the same number of observations as in the case of continuous part of realized variance, we again rely on central limit theorem and we also directly perform the unequal variance t-test (Welch t-test). Just to see what the results of the preliminary tests would be, we present in Tables A.4, A.5 and A.6 the results of Cramér-von Mises goodness-of-fit test of normality of the data, Conover test for equality of variances and Levene test for equality of variances. The null hypotheses and alternative hypotheses remain the same as in case of continuous components. All three tests were performed at the 0.05 significance level.

The results of the unequal variance t-test are presented in Table 5.15. We can see that in all three cases, due to very low p-values, we strongly reject the null hypothesis that the difference in true population means of jump component of realized variance is equal to 0 and conclude that the alternative is true. It indicates that jumps in the Bitcoin total variation are stronger than for other analyzed instruments.

Table 5.15: Unequal variance t-test: jump component of realized variance

	Statistic	*df*	p-value
BTC/USD, EUR/USD	14.992	763	< 0.0001
BTC/USD, XAU/USD	14.991	763	< 0.0001
BTC/USD, S&P 500	14.989	763	< 0.0001

Source: author's computations.

Following the work written by Andersen *et al.* (2010b), we also compare the relative contribution of jumps to the realized variance (JV_t/RV_t) of Bitcoin with relative contribution of jumps in case of other instruments. It is evident from Table 5.16 that the estimate of the highest mean of relative contribution of jump component is in case of BTC/USD exchange rate and is equal to 27.04% which is more than 10 times higher than the mean in case of EUR/USD exchange rate, more than 7 times higher than the mean in case of price of gold and almost 1.5 times higher than the mean in case of the S&P 500 index. We can also see that estimates of the maximum relative contribution of jumps range from 83.35% (EUR/USD) to 94.33% (BTC/USD).

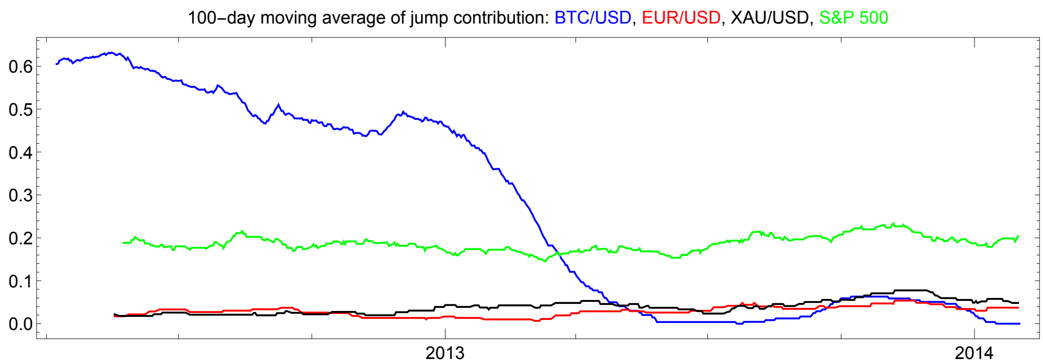
Table 5.16: Relative jump contribution (JV_t/RV_t)

	BTC/USD	EUR/USD	XAU/USD	S&P 500
Mean	0.2704	0.0270	0.0371	0.1886
Std. dev.	0.3040	0.1148	0.1320	0.2835
Min	0	0	0	0
Max	0.9433	0.8335	0.8578	0.9214
Obs.	764	545	543	517

Source: author's computations.

Figure 5.10 presents 100-day moving average of the relative jump contribution to the realized variance for all four instruments. We can see that at the beginning of the monitored period, jumps constituted a considerable part of the total daily realized variance in BTC/USD exchange rate (often over 50% of the daily realized variance).

Figure 5.10: Relative jump contribution



100-day moving average of relative jump contribution (JV_t/RV_t) for all analyzed instruments. Colors of individual lines correspond to colors used in the label of this graph: blue for BTC/USD exchange rate, red for EUR/USD exchange rate, black for XAU/USD exchange rate and green for the S&P 500 index.

Source: author's computations.

Needless to say that the considerably higher relative jump contribution at the beginning of the monitored period in case of BTC/USD exchange rate is caused primarily by the frequency of days with detected jumps. If we compute the mean of the relative jump contribution without using the data points on days without detected jumps (i.e. on days when the relative jump contribution is zero), the results are: 0.5836 (BTC/USD), 0.4611 (EUR/USD), 0.4796 (XAU/USD) and 0.5805 (S&P 500).

In the first 100 days of the monitored period, there were only 2 days for BTC/USD exchange rate without jumps whereas in case of EUR/USD exchange rate, there were 97 days without jumps, in case of XAU/USD exchange rate, there were 95 days without jumps and finally in case of the S&P 500 index, we did not detect jumps on 71 days. Nevertheless, during the last 100 days of the monitored period, no jump was detected in case of BTC/USD exchange rate, for EUR/USD exchange rate, there were 93 days without jumps, for XAU/USD exchange rate 90 days and for the S&P 500 index 64 days. Therefore, at first glance, it seems that BTC/USD exchange rate stabilized in time (while the frequency of jumps in case of other instruments seems not to change much in time).

We further analyzed this behavior by examining the duration between jumps. In Table 5.17, we present the mean, standard deviation, minimum and maximum of the duration between jumps for all instruments. The mean duration between jumps ranges from 2.2 days (BTC/USD) to 22.3 days (EUR/USD) and the maximum duration between jumps ranges from 16 days (S&P 500) to 117 days (BTC/USD). Since the longest period without jumps happened in case of BTC/USD exchange rate at the end of the monitored period, the number 117 corresponds to the number of days without jumps until February 1, 2014 (the end of the monitored period). The same applies also for EUR/USD exchange rate and XAU/USD exchange rate since there was not detected any jump on the last day of the monitored period for these exchange rates.

Table 5.17: Duration between detected jumps in days

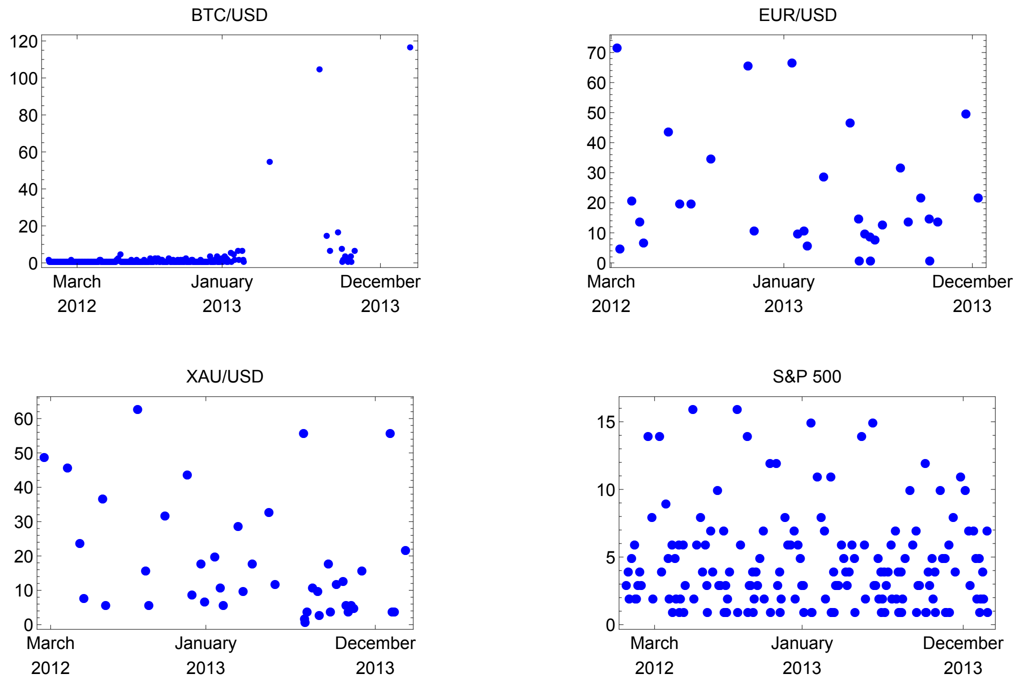
	BTC/USD	EUR/USD	XAU/USD	S&P 500
Mean	2.1581	22.25	18.1190	4.5030
Std. dev.	8.8303	19.7059	16.7946	3.5242
Min	1	1	1	1
Max	117	72	63	16

Source: author's computations.

In Figure 5.11, we also present the development of the duration between jumps for all instruments. In case of BTC/USD exchange rate, we can see that until the middle of February 2013, the duration between jumps was not longer than 7 days. After that, the duration between jumps considerably prolonged. In case of other instruments, it cannot be generally said that the duration between detected jumps significantly changed.

The relative contribution of jumps was on days with detected jumps comparable among all the instruments. However, taking into account all days, the 100-day moving average indicated that the relative jump contribution was significantly higher for Bitcoin. That was caused by the frequency of jumps in case of Bitcoin. Since the duration between jumps considerably prolonged in time and was at the end of the monitored period even longer than in case of other instruments, we can say that the characteristics of jumps for Bitcoin get closer in time to the characteristics of jumps for other instruments. This can indicate that unexpected events such as shutdown or bankruptcy of some Bitcoin services and large theft of bitcoins did not happen so frequently at the end of the monitored period (the monitored period ends before the Mt. Gox exchange bankrupted) or the Bitcoin market responded to these events more moderately.

Figure 5.11: Duration between detected jumps in days



This figure depicts the development of the duration between jumps for all four instruments. Dots represent particular daily jump contribution to the daily realized variance. Their horizontal position corresponds to the day of their occurrence and their vertical position is based on the number of days that have passed since the previous jump was detected.

Source: author's computations.

Chapter 6

Conclusion

The purpose of this thesis was to study the variance of Bitcoin and to separate the jump component from the continuous part and subsequently compare the results with the variance of other financial assets. More specifically, we focused on realized variance and its continuous and jump component for BTC/USD exchange rate, EUR/USD exchange rate, price of gold and the S&P 500 index. The main contribution of this thesis stems from the fact that it seems that no other work analyzing the Bitcoin total variation using realized variance and bipower variation to separate the jump component from the continuous part of a price process has been written so far.

At the beginning of the thesis we introduce the Bitcoin scheme, explain how Bitcoin works, describe characteristics of Bitcoin exchanges and present different types of Bitcoin clients. Then, we continue with the review of the quantitative research papers about Bitcoin, particularly works dealing with the volatility of Bitcoin exchange rate and works focusing on relationship between social media and search queries and the price of bitcoin. In the theoretical part we introduce the theory behind realized variance measures. We define realized variance, quadratic variation and integrated variance and we also mention why we assume that the price process is a jump-diffusion process. Then, we define realized bipower variation and test statistic for jump detection. Next, we describe the unequal variance t-test and present reasons for choosing this specific test.

Before comparing the results for all analyzed instruments, we describe the datasets and their modifications. Finally, we present the results of our analysis. The empirical findings suggest that jumps in the Bitcoin total variation are stronger than for EUR/USD exchange rate, price of gold and for the value of the

S&P 500 index. We found that there were only 53.6% of days without jumps for BTC/USD exchange rate which is in comparison with other analyzed instruments much lower proportion and that the mean duration between jumps ranges from 2.2 days (BTC/USD) to 22.3 days (EUR/USD). Decomposition of realized variance shows that the mean share of variance induced by jumps including days without jumps (excluding days without jumps) ranges from 2.7% (46.11%) for EUR/USD exchange rate to 27.04% (58.36%) for BTC/USD exchange rate. To sum it up, at least at the beginning of the monitored period, jumps made up a significant portion of the realized variance of BTC/USD exchange rate and jumps occurred more frequently than in case of other instruments. That indicates frequent extreme movements of price of bitcoin which could be expected since the Bitcoin market was still in early stages of its existence.

However, the results also indicate that the duration between jumps for Bitcoin considerably prolonged during the monitored period. During the last 117 days of the monitored period, there was not detected any jump for BTC/USD exchange rate. It suggests that the behavior of price of bitcoin stabilized over time and that the Bitcoin market responds to extraordinary news or other shocks to the market more moderately or that these events do not happen so frequently. It might also be a sign of increasing security measures which prevent bitcoin thefts. Another explanation can be that participants in the Bitcoin market are becoming better informed about the current state of Bitcoin and probable development in the near future.

Using unequal variance t-test, we also find out that the variance of price of bitcoin is higher than the variance of other analyzed instruments which may be caused by several reasons such as low liquidity, low Bitcoin acceptance, low market capitalization, or still a very high regulatory uncertainty.

It would be interesting for further research to use a different approach for treating the overnight data such as finding optimal weights or to scale appropriately realized variance as mentioned in Chapter 5. Another possible areas for further research are to analyze the same financial assets using data from February 2014 to present and also justify the use of data with five-minute sampling frequency using volatility signature plots as proposed by Andersen *et al.* (2010b). Another subject for additional research would be to use different jump detection procedures as noted in the works written by Andersen *et al.* (2010b) and Huang & Tauchen (2005), among others, and to compare and contrast the results to results obtained in this thesis.

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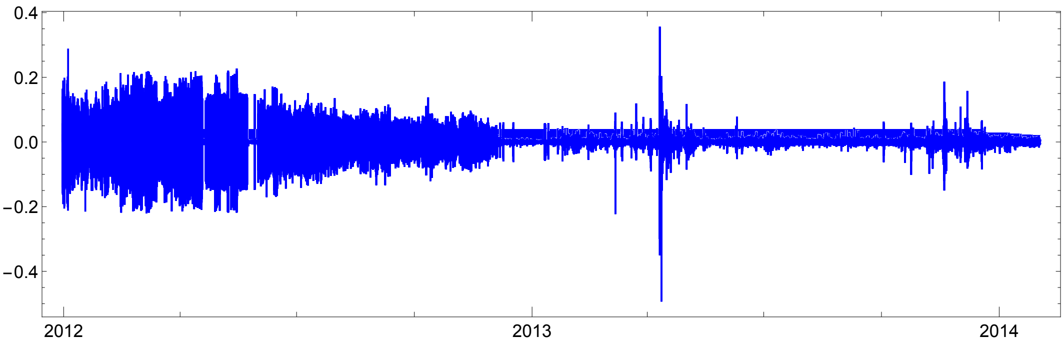
Appendix A

BTC/USD exchange rate

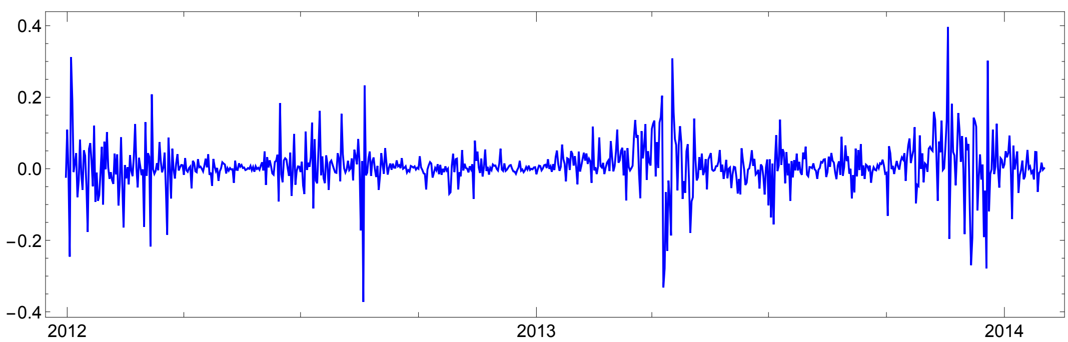
Figure A.1: Intraday returns of BTC/USD exchange rate



We used intraday closing prices of bitcoin from December 31, 2011 to February 1, 2014 to construct intraday log-returns of BTC/USD exchange rate shown in this figure.

Source: author's computations and `bitcoincharts.com`.

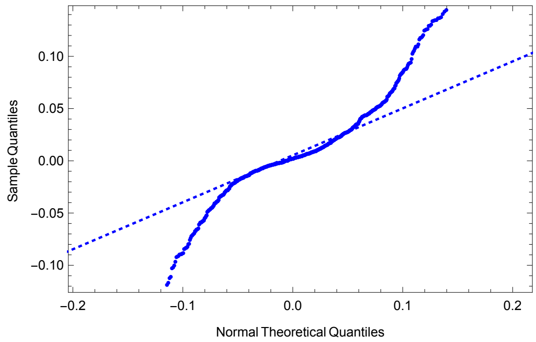
Figure A.2: Daily returns of BTC/USD exchange rate



This figure presents the daily returns of BTC/USD exchange rate computed as sum of all intraday returns of BTC/USD exchange rate depicted in Figure A.1.

Source: author's computations and `bitcoincharts.com`.

Figure A.3: Quantile-quantile plot for daily BTC/USD returns



The figure presents QQ plot of the sample quantiles of daily returns versus theoretical quantiles from a normal distribution with the mean and variance equal to the mean and variance of the daily returns. The sample period is December 31, 2011 to February 1, 2014.

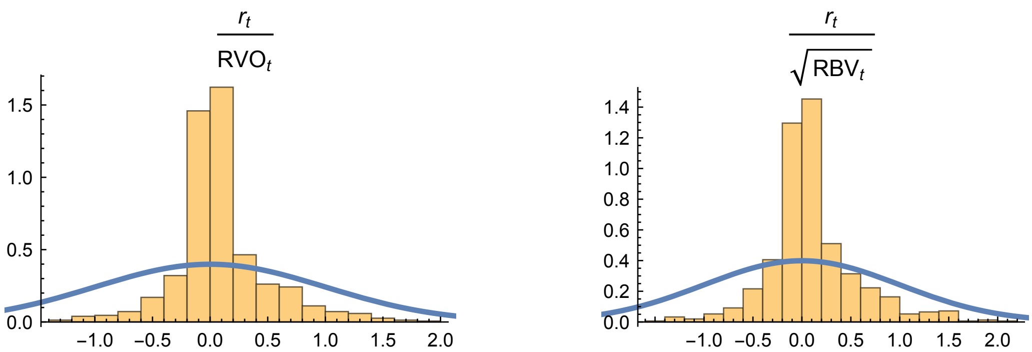
Source: author's computations and `bitcoincharts.com`.

Table A.1: Jarque–Bera ALM goodness-of-fit test (BTC/USD daily returns)

	Statistic	p-value
r_t	2051.48	< 0.0001
r_t/RVO_t	390.438	< 0.0001
$r_t/\sqrt{RBV_t}$	281.517	< 0.0001

Source: author's computations and `bitcoincharts.com`.

Figure A.4: Histogram of standardized daily returns of BTC/USD exchange rate and standard normal probability density function



The daily returns of BTC/USD exchange rate standardized by realized volatility (left) and by square root of realized bipower variation (right) are used to construct a normalized histogram (to model probability density function) shown in yellow bars (with bin widths determined automatically by Wolfram Mathematica software). Standard normal probability density function is shown using the blue line.

Source: author's computations and `bitcoincharts.com`.

Table A.2: Conover's test for equality of variances (continuous components)

	Statistic	p-value
BTC/USD, EUR/USD	28.234	< 0.0001
BTC/USD, XAU/USD	28.166	< 0.0001
BTC/USD, S&P 500	24.548	< 0.0001

Source: author's computations and `bitcoincharts.com`.

Table A.3: Levene's test for equality of variances (continuous components)

	Statistic	p-value
BTC/USD, EUR/USD	303.610	< 0.0001
BTC/USD, XAU/USD	301.992	< 0.0001
BTC/USD, S&P 500	287.918	< 0.0001

Source: author's computations and `bitcoincharts.com`.

Table A.4: Cramér-von Mises goodness-of-fit test (jump components)

	Statistic	p-value
BTC/USD	21.886	< 0.0001
EUR/USD	39.119	< 0.0001
XAU/USD	37.886	< 0.0001
S&P 500	19.022	< 0.0001

Source: author's computations and `bitcoincharts.com`.

Table A.5: Conover's test for equality of variances (jump components)

	Statistic	p-value
BTC/USD, EUR/USD	16.582	< 0.0001
BTC/USD, XAU/USD	16.131	< 0.0001
BTC/USD, S&P 500	11.330	< 0.0001

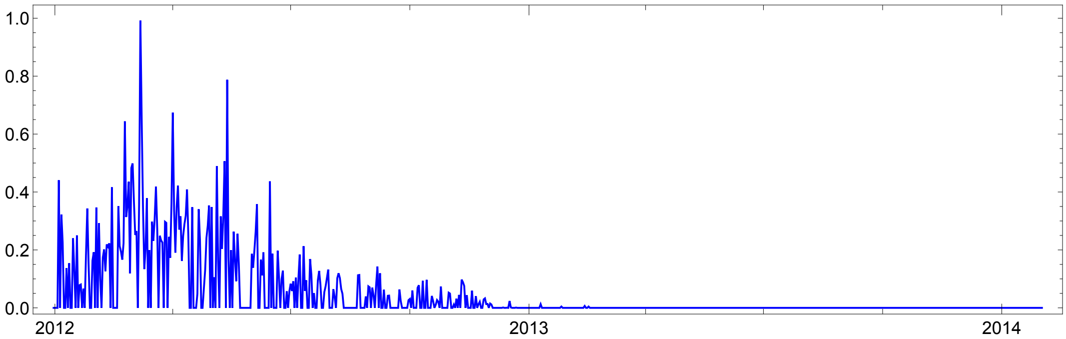
Source: author's computations and `bitcoincharts.com`.

Table A.6: Levene's test for equality of variances (jump components)

	Statistic	p-value
BTC/USD, EUR/USD	539.687	< 0.0001
BTC/USD, XAU/USD	537.564	< 0.0001
BTC/USD, S&P 500	511.703	< 0.0001

Source: author's computations and `bitcoincharts.com`.

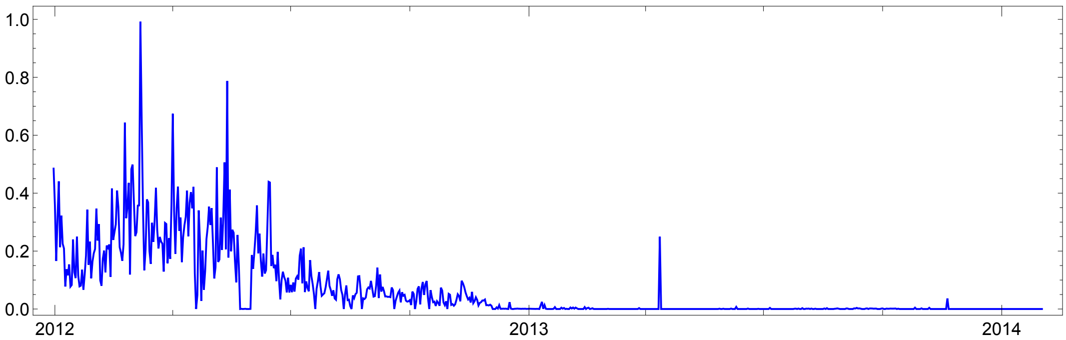
Figure A.5: Jump process in RV of BTC/USD exchange rate (99$^{\text{th}}$ perc.)



This figure presents jump components of the daily realized variance of BTC/USD exchange rate using 99$^{\text{th}}$ percentile of the standard normal distribution for their detection.

Source: author's computations and `bitcoincharts.com`.

Figure A.6: Jump process in RV of BTC/USD exchange rate (90$^{\text{th}}$ perc.)



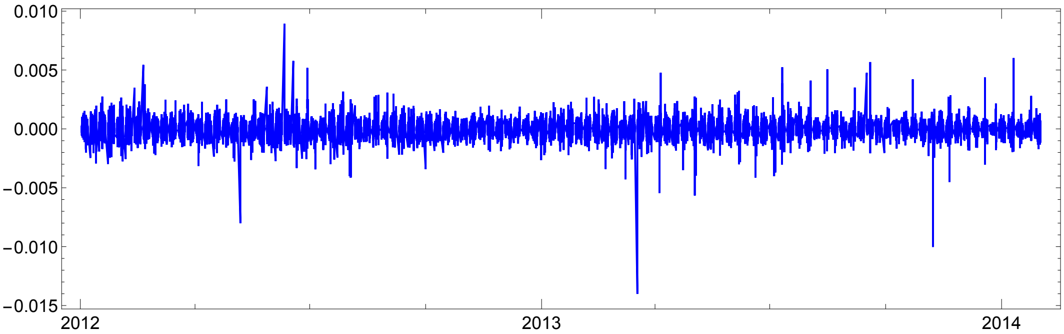
This figure depicts jump components of the daily realized variance of BTC/USD exchange rate using 90$^{\text{th}}$ percentile of the standard normal distribution for their detection.

Source: author's computations and `bitcoincharts.com`.

Appendix B

EUR/USD exchange rate

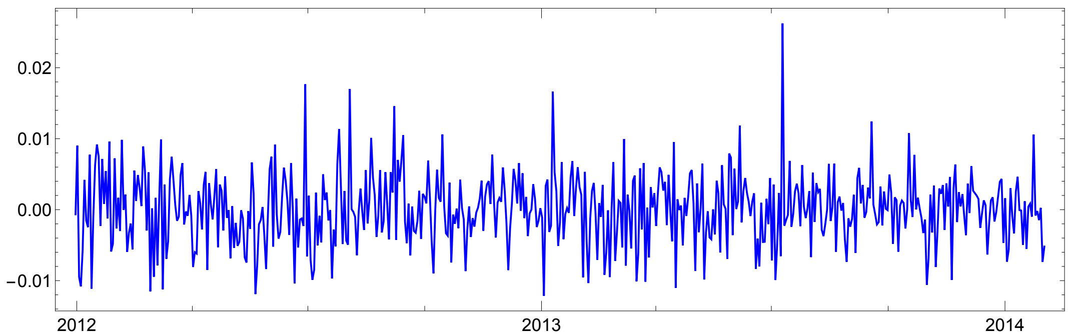
Figure B.1: Intraday returns of EUR/USD exchange rate



We used the arithmetic average of bid and ask closing intraday EUR/USD exchange rate from December 31, 2011 to February 1, 2014 to construct intraday log-returns shown in this figure.

Source: author's computations and `dukascopy.com`.

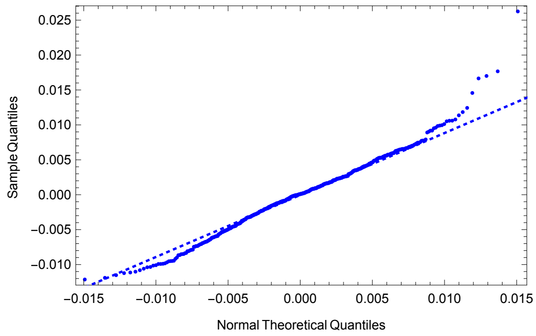
Figure B.2: Daily returns of EUR/USD exchange rate



This figure presents the daily returns of EUR/USD exchange rate computed as sum of all intraday returns of EUR/USD exchange rate depicted in Figure B.1.

Source: author's computations and `dukascopy.com`.

Figure B.3: Quantile-quantile plot for daily EUR/USD returns



The figure presents QQ plot of the sample quantiles of daily returns versus theoretical quantiles from a normal distribution with the mean and variance equal to the mean and variance of the daily returns. The sample period is December 31, 2011 to February 1, 2014.

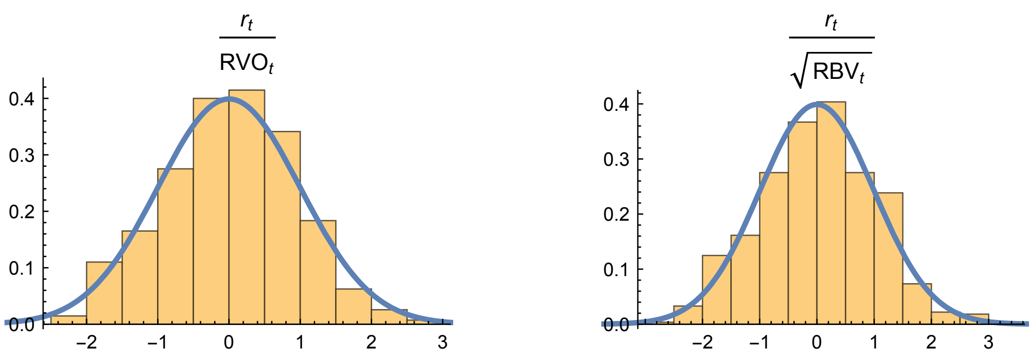
Source: author's computations and `dukascopy.com`.

Table B.1: Jarque–Bera ALM goodness-of-fit test (EUR/USD daily returns)

	Statistic	p-value
r_t	64.177	< 0.0001
r_t/RVO_t	1.128	0.556
$r_t/\sqrt{RBV_t}$	1.404	0.479

Source: author's computations and `dukascopy.com`.

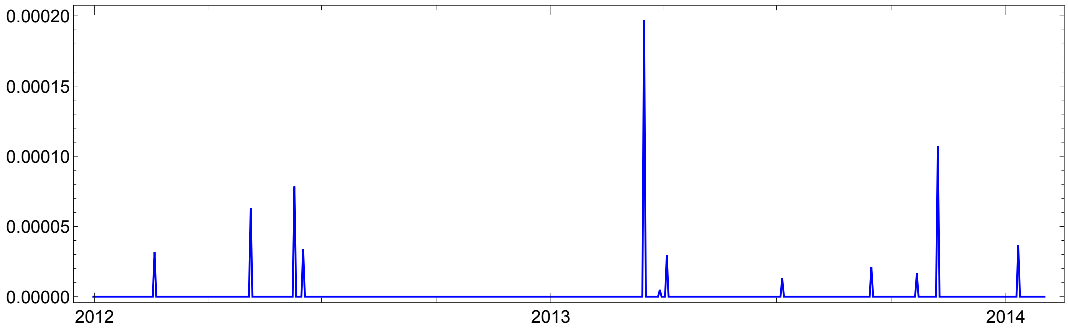
Figure B.4: Histogram of standardized daily returns of EUR/USD exchange rate and standard normal probability density function



The daily returns of EUR/USD exchange rate standardized by realized volatility (left) and by square root of realized bipower variation (right) are used to construct a normalized histogram (to model probability density function) shown in yellow bars (with bin widths determined automatically by Wolfram Mathematica software). Standard normal probability density function is shown using the blue line.

Source: author's computations and `dukascopy.com`.

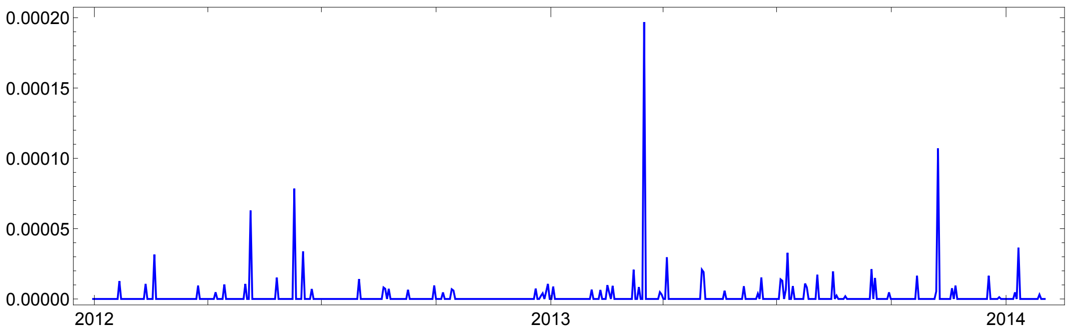
Figure B.5: Jump process in RV of EUR/USD exchange rate (99th perc.)



This figure presents jump components of the daily realized variance of EUR/USD exchange rate using 99th percentile of the standard normal distribution for their detection.

Source: author's computations and `dukascopy.com`.

Figure B.6: Jump process in RV of EUR/USD exchange rate (90th perc.)



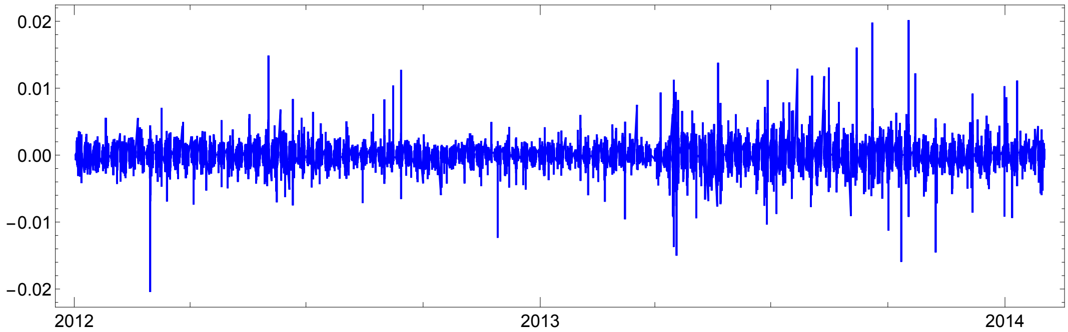
This figure depicts jump components of the daily realized variance of EUR/USD exchange rate using 90th percentile of the standard normal distribution for their detection.

Source: author's computations and `dukascopy.com`.

Appendix C

XAU/USD exchange rate

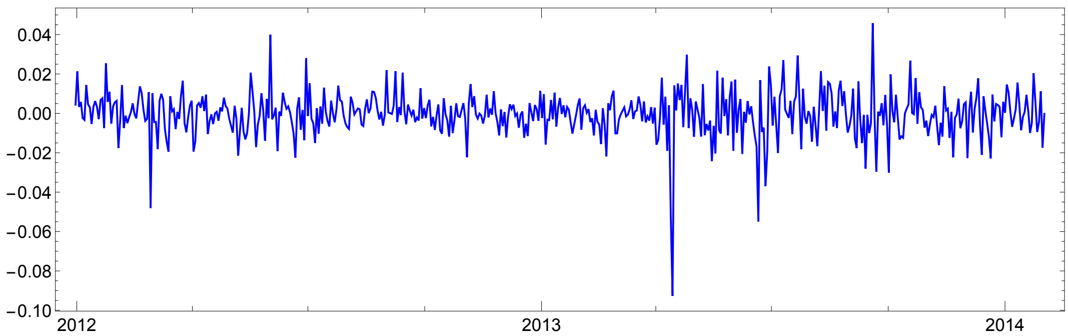
Figure C.1: Intraday returns of XAU/USD exchange rate



We used the arithmetic average of bid and ask closing intraday XAU/USD exchange rate from December 31, 2011 to February 1, 2014 to construct intraday log-returns shown in this figure.

Source: author's computations and `dukascopy.com`.

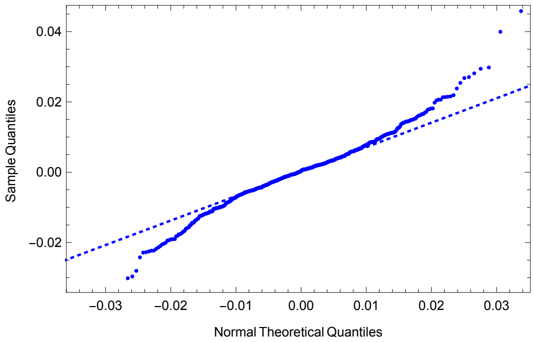
Figure C.2: Daily returns of XAU/USD exchange rate



This figure presents the daily returns of XAU/USD exchange rate computed as sum of all intraday returns of XAU/USD exchange rate depicted in Figure C.1.

Source: author's computations and `dukascopy.com`.

Figure C.3: Quantile-quantile plot for daily XAU/USD returns



The figure presents QQ plot of the sample quantiles of daily returns versus theoretical quantiles from a normal distribution with the mean and variance equal to the mean and variance of the daily returns. The sample period is December 31, 2011 to February 1, 2014.

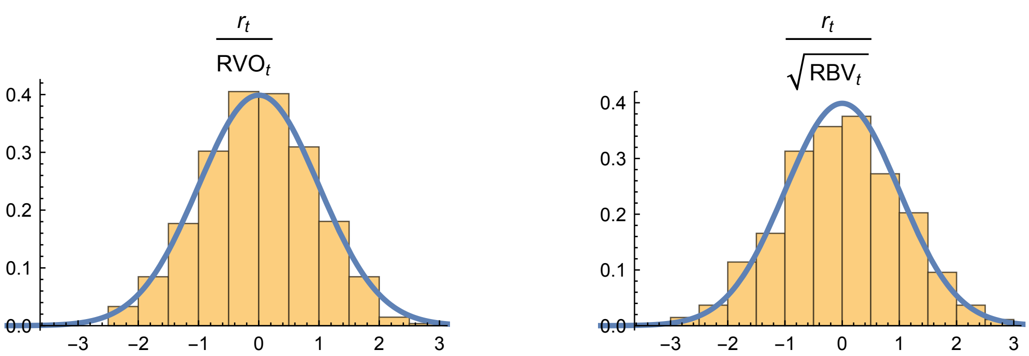
Source: author's computations and `dukascopy.com`.

Table C.1: Jarque–Bera ALM goodness-of-fit test (XAU/USD daily returns)

	Statistic	p-value
r_t	2601.01	< 0.0001
r_t/RVO_t	1.811	0.386
$r_t/\sqrt{RBV_t}$	0.950	0.609

Source: author's computations and `dukascopy.com`.

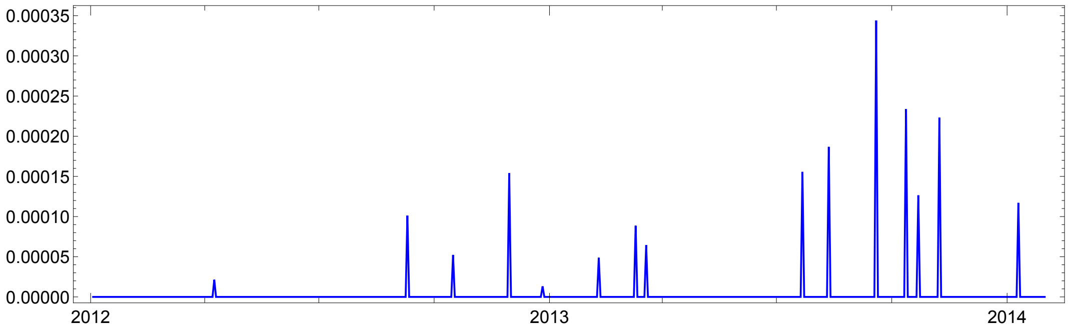
Figure C.4: Histogram of standardized daily returns of XAU/USD exchange rate and standard normal probability density function



The daily returns of XAU/USD exchange rate standardized by realized volatility (left) and by square root of realized bipower variation (right) are used to construct a normalized histogram (to model probability density function) shown in yellow bars (with bin widths determined automatically by Wolfram Mathematica software). Standard normal probability density function is shown using the blue line.

Source: author's computations and `dukascopy.com`.

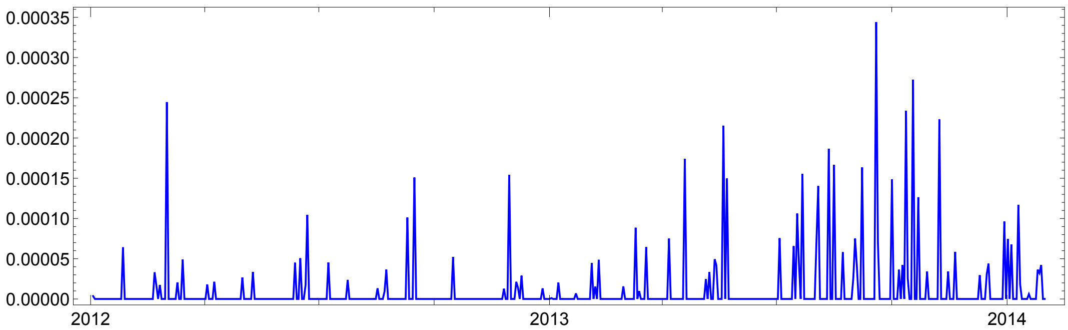
Figure C.5: Jump process in RV of XAU/USD exchange rate (99$^{\text{th}}$ perc.)



This figure presents jump components of the daily realized variance of XAU/USD exchange rate using 99$^{\text{th}}$ percentile of the standard normal distribution for their detection.

Source: author's computations and `dukascopy.com`.

Figure C.6: Jump process in RV of XAU/USD exchange rate (90$^{\text{th}}$ perc.)



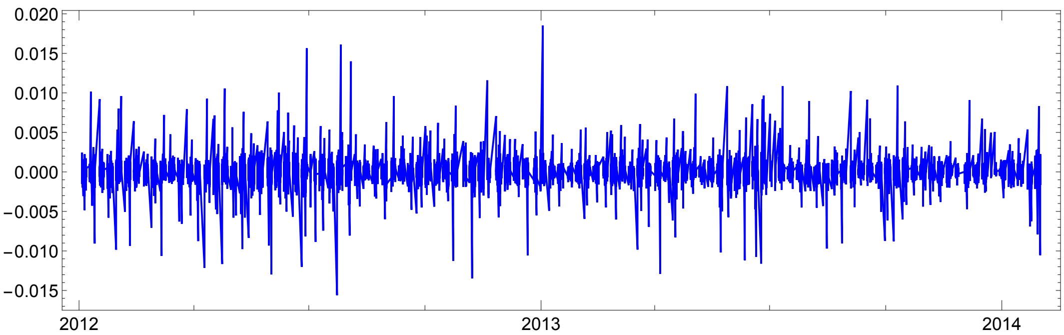
This figure depicts jump components of the daily realized variance of XAU/USD exchange rate using 90$^{\text{th}}$ percentile of the standard normal distribution for their detection.

Source: author's computations and `dukascopy.com`.

Appendix D

S&P 500 index

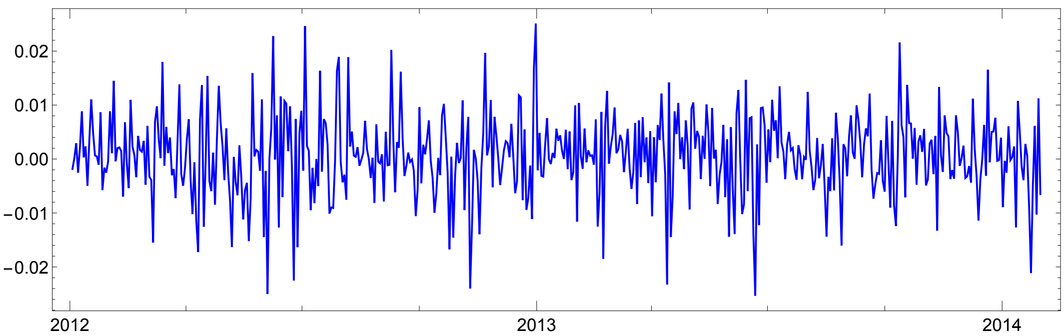
Figure D.1: Intraday returns of the S&P 500 index



We used intraday closing values of the Standard & Poor's 500 index from January 3, 2012 to February 1, 2014 to construct intraday log-returns shown in this figure.

Source: author's computations and `finam.ru`.

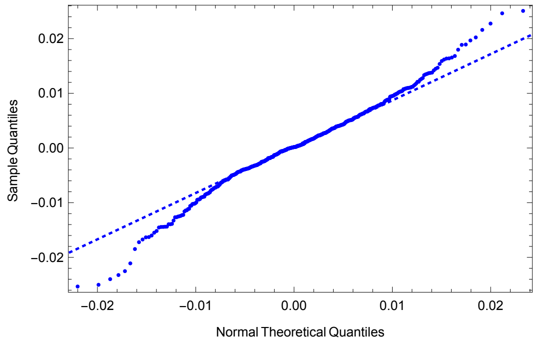
Figure D.2: Daily returns of the S&P 500 index



This figure presents the daily returns of the S&P 500 index computed as sum of all intraday returns depicted in Figure D.1.

Source: author's computations and `finam.ru`.

Figure D.3: Quantile-quantile plot for daily S&P 500 returns



The figure presents QQ plot of the sample quantiles of daily returns versus theoretical quantiles from a normal distribution with the mean and variance equal to the mean and variance of the daily returns. The sample period is January 3, 2012 to February 1, 2014.

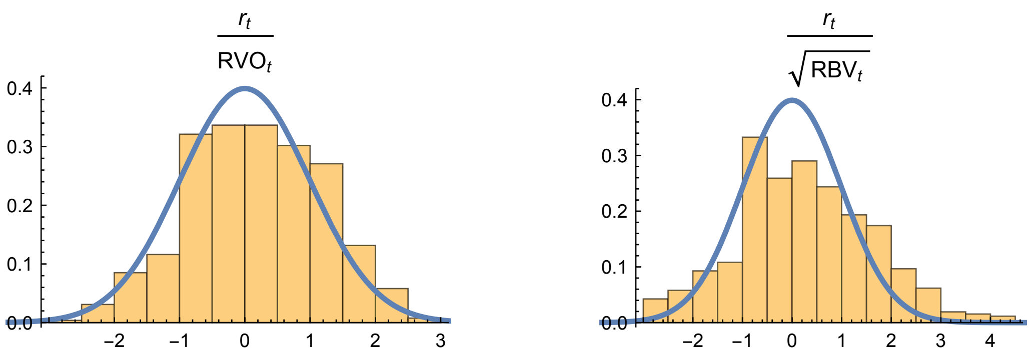
Source: author's computations and `finam.ru`.

Table D.1: Jarque–Bera ALM goodness-of-fit test (S&P 500 daily returns)

	Statistic	p-value
r_t	27.190	0.0005
r_t/RVO_t	3.763	0.140
$r_t/\sqrt{RBV_t}$	3.446	0.164

Source: author's computations `finam.ru`.

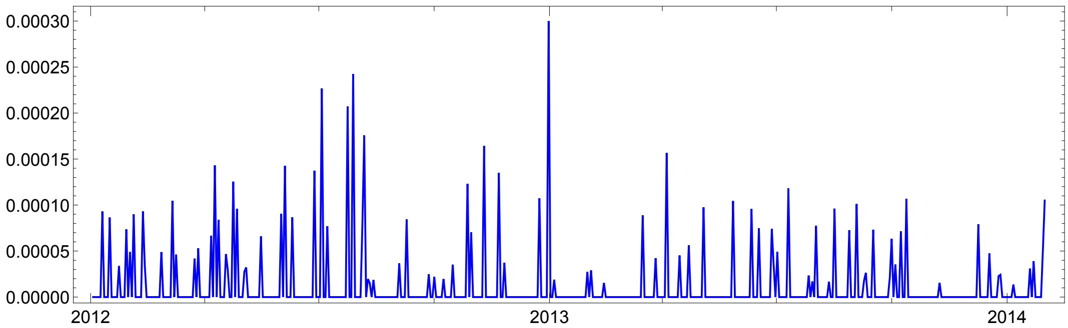
Figure D.4: Histogram of standardized daily returns of the S&P 500 index and standard normal probability density function



The daily S&P 500 index returns standardized by realized volatility (left) and by square root of realized bipower variation (right) are used to construct a normalized histogram (to model probability density function) shown in yellow bars (with bin widths determined automatically by Wolfram Mathematica software). Standard normal probability density function is shown using the blue line.

Source: author's computations and `finam.ru`.

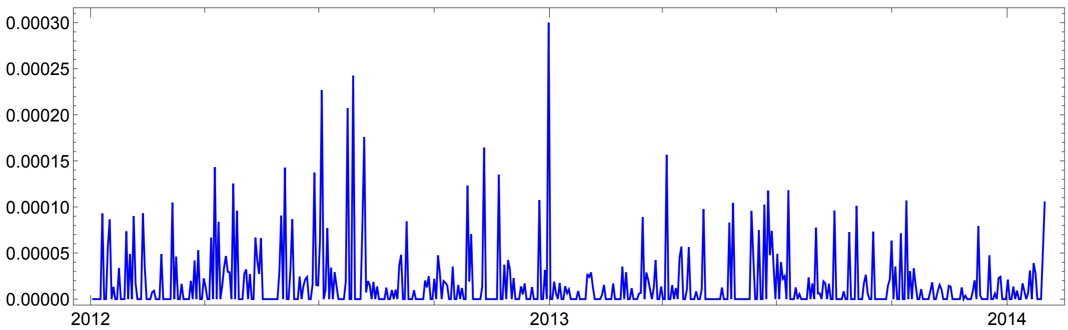
Figure D.5: Jump process in RV of the S&P 500 index (99^{th} perc.)



This figure depicts jump components of the daily realized variance of the S&P 500 index using 99^{th} percentile of the standard normal distribution for their detection.

Source: author's computations and `finam.ru`.

Figure D.6: Jump process in RV of the S&P 500 index (90^{th} perc.)



This figure presents jump components of the daily realized variance of the S&P 500 index using 90^{th} percentile of the standard normal distribution for their detection.

Source: author's computations and `finam.ru`.