

Singularities in quantum spectra - ground state and excited-state quantum phase transitions - are often connected with singularities in the classical limit of the system and have influence on other properties, such as quantum entanglement, as well. In the first part of the thesis we study quantum phase transitions within the $U(2)$ -based Lipkin model. The relation between quasistationary points of the classical potential and the respective singularities in the spectrum is shown. In the second part, a system of two-level atoms interacting with electromagnetic field in an optical cavity is studied within two simplified models (non-integrable Dicke model and its integrable approximation known as Jaynes-Cummings model). The behaviour of quantum entanglement in these models is shown with a focus on the vicinity of the singular points.