Report on "Optimization Problems under (max, min)-linear Constraints and Some Related Topics." by

Mahmoud Gad

The thesis presented by Mr. Mahmoud Gad deals with a special class of non-convex optimization problems with functions, which are not in general everywhere continuously differentiable. A special structure of the problems makes possible to find effective algorithms, which find global solutions of such problems. The problems contain both in the objective function and in the constraints functions of the form

$$g(x_1, \ldots, x_n) = \max_{j \in J}(\min(h_j, x_j)), \; J = \{1, \ldots, n\}.$$

If we introduce for any real numbers α, β notations $\alpha \oplus \beta = \max(\alpha, \beta)$, $\alpha \otimes \beta = \min(\alpha, \beta)$, the function can be written in the form

$$g(x_1, \ldots, x_n) = h_1 \otimes x_1 \oplus \ldots \oplus h_n \otimes x_n,$$

which points out the analogy with the classic linear functions. The problems solved in the present thesis contain such functions both in the objective function and in the equality or inequality constraints. Since the max-operation (or $\oplus$-operation) is only a semigroup operation the variables cannot be easily transferred form one side of the equations or inequalities to the other side and it is necessary to solve separately problems with variables on one side only (so called "one-sided relations") and with variables on both sides ("two-sided relations").

Algebraic structures of this sort appeared in the literature at the beginning of the sixities of the last century. They found successivly applications in various practical operations research probles as e.g. machine-time scheduling, reliability theory and others. Papers and monographies, which appeared in recent years proved that the subject of research chosen in the presented thesis is up to date. Among other subjects a lot of attention was devoted to the structures, in which the basic pair of operations are operations max. +. Some rent results in this direction are summarized in Chapter 2 of the thesis.

Relatively small attention found in the literature structures, in which the role of the two basic operations play operations max, min. Mr. Gad's thesis

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tries to fill in this part of the research. Chapters 2 - 7 are therefore devoted to theoretical properties of systems of max, min-linear equations and inequalities and optimization problems, the set of feasible solutions of which are sets of solutions of such systems. The objective functions are so called max-separable functions, i.e. functions of the form $f(x_1, \ldots, x_n) = \max_{j \in J}(f_j(x_j))$, where $f_j : R \longmapsto R$ are continuous nondecreasing functions. The theoretical explanations are illustrated by small numerical examples. The last chapter (Chapter 8) is devoted to a possible generalization of the results to problems with max-separable functions in equation and inequality constraints of the optimization problems.

Conclusion.

I did not find any substantial mistakes in the text of the thesis. In some parts, especially in Chapter 2, I found some formal oversights (notations, repeated assertions, which had been already mentioned before). According to my opinion, it would be better to replace the illustrations of the theory by small numerical examples by a report on a wider experience with solving larger test examples on a computer. It could be also pointed out that the proposed methods can be used not only for problems on the set of real numbers, but also for other basic sets, for instance integers or even for finite sets. The last chapter seems to me short and does not mention some possible extensions or generalizations of the results from the preceding chapters. For instance, the author did not generalized at all any results of chapter 7. It contains new up to date results partially published by the author as papers in mathematical journals. The examples presented by the author show that the results may be applied to some operations research problems in practice. The obtained results show that the author is able to continue the research in this area and further develope the subject of the thesis.

Inspite of the objections mentioned above, the presented work by Mr. Mahmoud Gad satisfies according to my opinion the requirements posed on PhD-thesis at the Faculty of Mathematics and Physics of the Charles University. I recommend therefore accepting the presented text as PhD-thesis, which is a background for granting the applicant the title PhD.

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Doc. RNDr. Libuse Grygarova, DrSc