Charles University in Prague

Faculty of Social Sciences
Institute of Economic Studies

BACHELOR THESIS

Relationship between liquidity and volatility of selected exchange rate pairs

Author: **Martin Kotek**

Supervisor: **PhDr. Ladislav Krištoufek, Ph.D.**

Academic Year: **2013/2014**

Declaration of Authorship

The author hereby declares that he compiled this thesis independently, using only the listed resources and literature.

The author grants to Charles University permission to reproduce and to distribute copies of this thesis document in whole or in part.

Prague, May 8, 2014

Signature

Acknowledgments

I am especially grateful to my supervisor PhDr. Ladislav Krištoufek, Ph.D. for valuable suggestions and comments.

Bachelor Thesis Proposal

Author	Martin Kotek
Supervisor	PhDr. Ladislav Krištoufek, Ph.D.
Proposed topic	Relationship between liquidity and volatility of selected exchange rate pairs

Topic characteristics In the thesis, I will examine how liquidity and volatility are interrelated in foreign exchange market. Forex has not been studied as extensively as equity or bond markets, mainly because of its opaque structure and data unavailability, but it has improved. I will start with construction of liquidity (some measures in Goyenko, Holden & Trzcinka will be used) and volatility measures of several exchange rate pairs. Then, time series analysis will be performed and based on the results, conclusion will be made. Hypotheses about the relationship are following: 1) liquidity is negatively related to volatility and 2) increase (or decrease) in volatility precedes decrease (increase) in liquidity, because holding large inventories is more risky in times of high volatility and market-makers, as a result, decrease provided liquidity.

Outline

1. Introduction
2. Theoretical background
3. Description of data
4. Estimation of liquidity and volatility
5. Relationship between liquidity and volatility
6. Conclusion

Core bibliography

1. Carlston, B. (2012): "Examining the Commonality in Liquidity and Volatility risk." *Duke university, working papers*, [online] [cit. 2013-05-22] http://econ.duke.edu/people/carlston/research
2. Goyenko, R. Y. & C. W. Holden & Ch. A. Trczinka (2009): "Do Measures of Liquidity Measure Liquidity?" *Journal of Financial Economics* **92(2)**: pp. 153–181.
3. Locke, P. R. & A. Sarkar (1996): "Volatility and Liquidity in Futures Markets." *Federal Reserve Bank of New York*, [online] [cit. 2013-05-22] http://www.newyorkfed.org/research/staff_reports/research_papers/9612.html
4. Mancini, L. & A. Ranaldo & J. Wrampelmeyer (2012): "Liquidity in the Foreign Exchange Market: Measurement, Commonality, and Risk Premiums." *Journal of Finance, Forthcoming*, [online] http://ssrn.com/abstract=1447869 [cit. 2013-05-22]

Author

Supervisor

Contents

List of Tables

List of Figures

Acronyms

AUD Australian dollar

BIS Bank for International Settlement

CAD Canadian dollar

CHF Swiss franc

ER Exchange rate

EUR Euro

FX Foreign Exchange

GBP Pound sterling (British pound)

JPY Japanese yen

NZD New Zealand dollar

USD United States dollar

Abstract

The thesis explores relationship between volatility and liquidity of ten selected exchange rate pairs. Several volatility and liquidity measures are computed and the relationship between volatility-liquidity pairs is tested for cointegration and Granger causality; impulse response functions are computed as well. We find that volatility measures provide similar information (are cointegrated), while volatility measures differ to a large extent. A few cointegrating relationships between volatility and liquidity are found, but they are specific to only some currency pairs. Granger causality tests give different results for different currency pairs, but in general, the relationship between volatility and liquidity is two-way (feedback). Shocks in volatility or liquidity have little impact on the other and quickly fade away, usually within one or two days.

JEL Classification	F31, G15, C10
Keywords	liquidity, volatility, foreign exchange, currency

Author's e-mail	`kotek.martin@gmail.com`
Supervisor's e-mail	`kristoufek@ies-prague.org`

Abstrakt

Bakalářská práce zkoumá vztah mezi volatilitou a likviditou deseti vybraných měnových párů. Spočítáme několik měr volatility a likvidity a následně vztah mezi páry volatilita-likvidita testujeme pomocí kointegračních testů, testujeme Grangerovu kauzalitu a počítáme funkce impulzních odezev (IRF). Zjišťujeme, že míry volatility zachycují podobné informace (jsou kointegrované), zatímco míry likvidity se do velké míry liší. Nacházíme několik kointegračních vztahů mezi volatilitou a likviditou, ale pouze u některých měnových párů. Testy Grangerovy kauzality různých měnových párů poskytují rozdílné výsledky, ale obecně nacházíme oboustranný vztah mezi volatilitou a likviditou (volatilita ovlivňuje likviditu a obráceně). Šoky do volatility (likvidity) mají malý efekt na likviditu (volatilitu) a rychle odezní, obvykle během jednoho až dvou dní.

Klasifikace JEL F31, G15, C10
Klíčová slova likvidita, volatilita, směnný kurz, měna

E-mail autora kotek.martin@gmail.com
E-mail vedoucího práce kristoufek@ies-prague.org

Chapter 1

Introduction

Foreign exchange market is the largest market in the world, daily trading averaged 5.3 trillion USD in April 2013, according to BIS (2013). Thus FX market is generally considered to be very liquid at all times, but as recent studies (Mancini *et al.* (2013) or Banti *et al.* (2012)) show, it is not always true. FX market is less studied and understood than stock markets which is caused by its size, complicated and unclear structure. First studies from 1980s studying liquidity in FX markets predominantly used quoted bid-ask spread and focused on its properties, see Glassman (1987) or Bollerslev & Melvin (1994).

Glassman (1987) study link between bid-ask spread and volatility on six exchange rate pairs between years 1975 and 1983. The author finds volatility as an important factor explaining bid-ask spread, nevertheless, volatility is not the only factor influencing spread. Market makers are setting their spreads with the use of other information – they predict long run and recent volatility and also take into account non-normality (e.g. t-distribution which has fatter tails than normal distribution) of exchange rate changes. Bollerslev & Melvin (1994) reach similar conclusions – there is a strong positive relation between bid-ask spread and volatility (estimated as MA(1) - GARCH(1,1)).

Behaviour of bid-ask spread is documented by Poskitt (2005) in NZD/USD market. As was found in the previous studies, market makers offer either constant spreads or time-varying spreads which increase with increasing volatility, but which cluster around a few values. Poskitt (2005) finds strong evidence of constant and clustered quoted bid-ask spreads, more than 40% of market makers quote constant spread and those who quote time varying spreads usually prefer one particular value. Bollerslev & Melvin (1994) find that DM/USD spreads quoted by Deutsche Bank and Morgan Guaranty were 10 basis points more than 50% of trading time and four spread values account for 97% of trading time.

Even though there is evidence that quoted spreads do not differ much among market makers (Bessembinder 1994), Grossman & Miller (1988) show that bid-ask spread has several disadvantages as a liquidity proxy. Bid-ask spread precisely measures the return to market maker only when buy and sell orders are executed exactly at the same time. In that case, however, the market maker does not provide liquidity, it only executes orders of two parties and charges them a fee (spread). Two same buy and sell orders rarely appear at the same time; if there is some time between the two orders, price may change and the market maker earns more or less, depending on the move. Thus, even in this case, bid-ask spread does not represent precise cost of liquidity provision.

Although the size of FX market increases every year and heterogeneity of institutions and participants in the market remains high, electronic trading platforms enhanced availability of data. It has had a positive impact on research in FX market, because data for longer horizon or high-frequency data can be employed. Except the positive effect on research, increasing data availability is also changing the market. Ding & Hiltrop (2010) find that quoted bid-ask spread narrowed as electronic platforms were introduced, thus reducing trading costs and increasing liquidity. On the other hand, informed dealers (e.g. market makers before release of news) quote wider spreads than they used to which is probably result of lost informational advantage. In other words, increased transparency may worsen spreads sometimes.

Liquidity was extensively studied in other markets, for example, commonality in liquidity was tested on NYSE stocks by Chordia *et al.* (2000), but liquidity in FX markets has not been explored in details until recently. Majority of liquidity proxies (Roll (1984), Corwin & Schultz (2012) and many others) were developed for stock markets and tested there (see e.g. Goyenko *et al.* (2009)), but their reliability in FX markets was not known. It has changed lately, literature explores properties of FX liquidity, too, Banti *et al.* (2012) and Mancini *et al.* (2013) construct several measures of liquidity in order to assess impact of liquidity on carry trade returns and to test commonality of liquidity across exchange rate pairs. Both studies discover strong commonality in liquidity among exchange rates, which means that liquidity is influenced by factors that have impact on the FX market as a whole. Moreover, liquidity in other markets (equity and bonds) increases and drops at the same time as FX liquidity. Liquidity risk on carry trade is also found.

Microstructure literature relates volatility to liquidity in various ways, for example, through inventory risk in Stoll (1978) or directly in Roll (1984). Locke & Sarkar (1996) study how provision of liquidity changes with volatility in futures market and find that supply of liquidity does change with volatility, but net supply of liquidity does not decrease when volatility is high. Volatility and liquidity risk is examined by Carlston (2012); the study concludes that both volatility and liquidity might be proxies for a common uncertainty risk. One of the liquidity measures used in Mancini *et al.* (2013) is price dispersion (realized volatility), but it is not compared to other liquidity measures. Mancini *et al.* (2013) note that estimation of realized volatility by standard formula in presence of market frictions and microstructure noise (usual case with high-frequency data) is not precise, as is shown in Aït-Sahalia *et al.* (2005a). Aït-Sahalia *et al.* (2005b) derive volatility estimator which does not suffer from this bias.

Recent study by Karnaukh *et al.* (2013) tests various high and low frequency estimates of liquidity (including realized volatility computed from high as well as low frequency data) and sheds more light on accuracy of these proxies in FX market. High-frequency estimates (effective cost, bid-ask spread, price impact and price dispersion) are highly correlated, correlation coefficient is always above 0.89. Volatility computed from low-frequency data is found to be one of the best low-frequency estimates, correlation with high-frequency measures is above 0.76.

This thesis explores relationship between daily volatility and liquidity of ten selected currency pairs from year 2007 to 2014. Our hypotheses are that 1) liquidity is negatively related to volatility and 2) changes in volatility (Granger) cause changes in liquidity. Two volatility and seven liquidity measures are computed for each currency pair and relationships between volatility and liquidity are tested. Cointegration is tested and VAR/VEC models are fitted which are used for tests of Granger causality and computation of impulse response functions. Evidence of cointegration between volatility and liquidity is weak with a few exceptions and Granger causality is found to be generally two-way, i.e. volatility influences liquidity and liquidity influences volatility. Positive shocks in volatility temporarily decrease liquidity, but quickly fade away.

The thesis is organized as follows: Chapter 2 introduces theoretical background of used methods, description of exchange rate data is provided in Chapter 3, results are presented and discussed in Chapter 4 and Chapter 5 concludes.

Chapter 2

Methodology

This chapter introduces theory behind methods used in empirical part. First, several volatility and liquidity measures are presented and their properties discussed. Understanding of these measures is important, because they are the time series which are the center of our interest. Then we outline basic econometric techniques used in the subsequent time series analysis. We start with (non)stationarity tests, outline concept of cointegration and introduce basic framework of vector autoregression.

2.1 Volatility measures

In finance, volatility measures price variation of an asset or financial instrument, therefore volatility can be computed as a function of returns (or prices). Let us start with definition of return in time t, denoted by r_t – simple returns and continuously compounded returns are two most common definition of returns used in empirical research. Using common notation, where p_t, p_{t-1} is price in time t and $t-1$, respectively, simple return is computed as a price difference between two periods: $r_t = \Delta p_t = p_t - p_{t-1}$. Continuously compounded return is computed similarly, as a difference between log prices, i.e. $r_t = \Delta \log(p_t) = \log(p_t) - \log(p_{t-1})$. Both simple and continuously compounded returns produce similar results, when price difference between two periods is small (as can be expected with intra-day data). Therefore, we will stick to continuously compounded returns, also referred to as log returns, in the following definitions and computations.

Volatility might be computed as a standard deviation of returns (or prices), denoting mean return as $\bar{r}$ and volatility by σ, the formula is given as:

$$\sigma = \sqrt{\frac{\sum_{i=1}^{n} r_i - \bar{r}}{n-1}} \tag{2.1}$$

However, Poon & Granger (2003) note, that sample mean is not an accurate estimate of the true mean in this case and using zero mean instead provides better results. Nevertheless, this and similar definitions are rarely used and we will not employ it in our research either.

2.1.1 Realized volatility

Widely used measure of volatility is historical realized volatility, especially because it can be easily computed from past price movements. Following Cheng *et al.* (2013), realized variance RV is computed as a sum of squared returns over a period of time, for example, daily RV is a sum of all intra-day returns:

$$RV = \sum_{i=1}^{N} r_i^2 \tag{2.2}$$

where N is the number of returns within a period over which realized volatility is computed. Then, realized volatility is equal to square root of RV. This definition of realized volatility is used in empirical section. Optimal sampling frequency is not easy to determine and the choice will always be arbitrary, because one optimal sampling frequency may not be optimal for other measures in the analysis, but it is crucial to use only one sampling frequency in the whole analysis. Even though it is (computationally) possible to use different sampling for each measure, it would mainly introduce imprecisions and errors. Aït-Sahalia *et al.* (2005a) show that sampling as often as possible is not optimal, because high-frequency data contain high amount of microstructure noise and more advanced estimators of realized volatility has to be used in order to recover the true price variation. Likewise, sampling a few values is not optimal either. Aït-Sahalia *et al.* (2005a) compute optimal sampling frequency based on standard deviation of noise term, for example, EUR/USD should be sampled every 23 minutes for computation of daily realized volatility. Cheng *et al.* (2013) use five minute intervals, when computing daily realized volatility. Generally, 5, 10, 15 or 30 minutes sampling are used by empirical researchers.

2.1.2 Realized bi-power volatility

Realized volatility introduced in the previous subsection is neither consistent estimator of volatility of high frequency data nor in the presence of jumps in log-price series. The second problem is addressed by realized bi-power variance estimator developed by Barndorff-Nielsen & Shephard (2004). It decomposes quadratic variation obtained from two sources of variation – continuous price movements and jumps in price. Realized volatility captures any price change and thus jumps bias RV upwards, while realized bi-power volatility is robust to rare jumps. Bi-power variation is computed as:

$$BPV = \frac{\pi}{2} \sum_{i=2}^{N} |r_{i-1}| \cdot |r_i| \tag{2.3}$$

Realized bi-power volatility is obtained by taking square root of BPV. Barndorff-Nielsen & Shephard (2004) show that BPV converges in probability to the same limit as realized variance and that the result do not change in presence of rare jumps. Realized bi-power volatility is used as another volatility measure in empirical part.

Many other volatility estimators that are robust to high-frequency noise and/or jumps were developed, for example, Aït-Sahalia *et al.* (2005b) present estimator robust to microstructure noise. However, such estimators are more advanced from theoretical point of view and their usage in research also requires much more computational power. Therefore, they will not be presented here nor will be used in empirical part.

2.2 Liquidity measures

Liquidity of a market or asset commonly refers to its ability to facilitate quick buy/sell order without changing the price. This broad and unclear definition makes liquidity more difficult concept than volatility. Approaches to measurement of liquidity are based on several properties of (il)liquid markets and thus liquidity measures may be divided into three groups (Mancini *et al.* 2013) – price impact, trading costs and sometimes also price dispersion (volatility). Because the aim of this thesis is to explore relationship between volatility and liquidity, we will not include volatility among liquidity proxies. Nevertheless, we can use the other two categories in the analysis, trading costs and price impact proxies. Several authors have proposed various liquidity proxies, comprehensive comparison and benchmark of many is provided by Goyenko *et al.* (2009). Karnaukh *et al.* (2013) test several liquidity measures directly on a few selected exchange rate pairs. This section presents selected liquidity proxies, starting with trading costs.

2.2.1 Trading cost and spread proxies

Effective cost

The idea behind effective cost is very simple – the best estimate of trading cost is the actual cost of trade execution. Denoting mid-price by p (average of ask and bid prices) and price at which the trade was executed by p^T, the effective cost (EC) is given as:

$$EC = \begin{cases} \frac{p^T - p}{p} & \text{for buy trades} \\ \frac{p - p^T}{p} & \text{for sell trades} \end{cases} \tag{2.4}$$

The higher the EC is, the lower liquidity. The main disadvantage of EC is that orders, their direction and execution price is rarely known and publicly available, especially in FX market. There are many similar versions relying on actual trade data (see high-frequency measures in Goyenko *et al.* (2009)), but none of them can be used in our analysis, because trade data are not available. Karnaukh *et al.* (2013) use EC as a trading cost and liquidity benchmark and find that proportional bid-ask (BA) spread computed from high-frequency data is highly correlated with effective cost (correlation 0.985) and other high-frequency liquidity estimates. Thus, proportional BA spread is the only high-frequency trading cost estimator we can use.

Proportional bid-ask spread

Simple, yet widely used, liquidity proxy is (proportional) quoted bid-ask spread. Research in FX markets in 1980s and 1990s focused on properties of quoted spread, but no comprehensive study was published recently. Denoting ask, bid and mid prices by p^A, p^B and p, respectively, proportional BA spread formula is given as follows:

$$BA = \frac{p^A - p^B}{p} \tag{2.5}$$

Daily BA is obtained by averaging intra-daily BAs. Moreover, BA's advantage over other measures is that it does not require fixed time span between intra-daily observations. On contrary, the most precise result is obtained by computation of BA as often as possible and then computing daily BA as a time-weighted average. Denoting time by t (e.g. in (mili)seconds), this notion is captured in the next formula:

$$BA = \frac{1}{\sum_{i=1}^{N-1} t_{i+1} - t_i} \sum_{i=1}^{N-1} \frac{p_i^A - p_i^B}{p_i} \cdot (t_{i+1} - t_i) \tag{2.6}$$

Similarly to EC, low liquidity is characterized by high BA. We use time weighted spread (spread in eq. 2.6 is not divided by mid price p) and proportional time weighted spread (as defined in eq. 2.6) in empirical analysis. The main advantage of BA spread its simplicity and accuracy if bid and ask prices are available and are the same for all agents, however, bid and ask prices may not be available in some markets or if they are available, bid-ask spread may not be reliable estimate of general trading costs. This is also the case of foreign exchange market, because each bank and market maker offers its spread that may differ from spreads offered by other institutions. At the same time, literature suggests that spreads do not differ much among FX brokers and banks (e.g. Poskitt (2005) documents so in NZD/USD market).

Roll

Estimator of effective bid-ask spread from past price movements is developed by Roll (1984). Assuming that a stock is traded in informationally efficient market, distribution of price changes is stationary and the true value of the stock is exactly equal to average of bid and ask prices, Roll (1984) shows that covariance of consecutive price changes $Cov(\Delta p_i, \Delta p_{i-1}) = -s^2/4$, where s is bid-ask spread. The result stems from bid-ask bounce, i.e. if no new information arrive, price changes only from ask to bid and bid to ask price (it depends on the direction of the last trade, whether it was buy or sell). Following previous literature (e.g. Goyenko *et al.* (2009)), if $Cov(r_i, r_{i-1})$ is negative and *Roll* is not defined, we set $Roll = 0$. Thus formula of *Roll* liquidity can be summarized as

$$Roll = \begin{cases} \sqrt{-4Cov(r_i, r_{i-1})} & \text{if } Cov(r_i, r_{i-1}) < 0 \\ 0 & \text{if } Cov(r_i, r_{i-1}) \geq 0 \end{cases} \tag{2.7}$$

As *Roll* is a spread estimator, the higher *Roll* is, the lower liquidity. *Roll* is also used in empirical part. It is important to note that periods of high liquidity characterized by $Roll = 0$ mean that efficient bid-ask spread is zero and thus trading costs are zero as well. This is never true in reality, even the most liquid currency pairs or stocks have positive spread between ask and bid prices.

2.2.2 Price impact proxies

Second category of liquidity measures we use are price impact proxies, which differ from trading cost proxies quantifying costs of executing a trade. Price impact proxies, as the name suggests, measure impact of a transaction on quoted price. Buying or selling of large volumes may change price (exchange rate), because sell or buy orders are not large enough and thus price must change in order to motivate other participants to sell (buy) additional quantity. This measure is of little concern for people or small companies, because they exchange (trade, hedge, etc.) only small quantities, but large banks and other large participants in FX market may face such limitations. Therefore, small price impact is associated with high liquidity.

Amihud

The notion of price impact measure introduced above is captured by a formula proposed by Amihud (2002). As many other measures, it was originally developed for stock markets and tested there (Amihud 2002), but it should perform well in FX market, too. *Amihud* is easy to understand and compute – absolute value of return between two times is divided by traded volume within the period, if volume is non-zero. Daily *Amihud* is then obtained by averaging intra daily *Amihud*s, excluding zero volume periods:

$$Amihud = \frac{1}{N} \sum_{i=1}^{N} \frac{|r_i|}{volume_i} \tag{2.8}$$

The higher *Amihud*, the lower liquidity. Computation of *Amihud* (and also *Amivest* which is introduced below) requires traded volumes, but volumes data are not available in FX market, since the structure of the FX market is decentralized. Unlike volumes in stock markets, volume data provided by banks and brokers differ from data provided by other banks and brokers. It is possible to use such volume data, but it has to be noted that they are not as representative as in stock markets and the results may be slightly different, if volumes from other sources are used. Second option is to use number of ticks as a volume proxy – it is done by e.g. Karnaukh *et al.* (2013) who find that *Amihud* in FX markets computed with the use of number-of-ticks proxy perform quite well in comparison with other liquidity measures (correlation with the best high-frequency liquidity is above 0.80). We compute *Amihud* with both volume and volume proxy number-of-ticks; comparison of volumes and number-of-ticks is presented in the following chapters.

Amivest

Similar price impact measure is *Amivest* liquidity ratio which was used e.g. by Cooper *et al.* (1985) or is included in comparison of liquidity measures provided by Goyenko *et al.* (2009). It is basically given as an inverse of *Amihud* (eq. 2.8) – volume is divided by absolute value of return, unless return is zero in which case *Amivest* is not defined. Daily *Amivest* is again obtained by averaging:

$$Amivest = \frac{1}{N} \sum_{i=1}^{N} \frac{volume_i}{|r_i|} \tag{2.9}$$

Low *Amivest* indicates low liquidity. *Amivest* computed with volume and number-of-ticks data is used in empirical part as well. The ratio is sensitive to small values in denominator, i.e. *Amivest* is inflated (biased upwards), if returns are close to zero. It also happens in our case, since we compute *Amivest* from intra-day data and some price changes are very small. Thus we use median of intra daily *Amivest*s instead of arithmetic average, because median is robust estimator of mean which allows us to compute unbiased (or less biased) daily *Amivest* liquidity ratio.

2.3 Econometric methods and tests

This section introduces econometric methods used in analysis of volatility and liquidity time series from the previous section. We start with introduction of unit root and stationarity tests, because time series econometrics usually requires stationarity and so it is crucial to test for stationarity first. Then we proceed to concept of cointegration which is useful if time series are found to be non-stationary. Both Engle-Granger and Johansen methods are presented, because the former is suitable and easier to apply in our analysis, while the latter serves mainly as an input into further econometric methods. In the end, vector autoregression (VAR) and vector error correction model (VECM) are outlined, as they are used for testing Granger causality and computation of impulse response functions. In this section we mainly use time series econometrics textbooks Lütkepohl (2005) and Tsay & Wolters (2002) and Stata reference manual by Stata Corp. (2013).

Stationarity

Many time series econometric methods require the series of our interest to be stationary, thus we start this section with definition of stationarity. Stochastic process y_t is said to be stationary, if its joint probability distribution does not change over time. More rigorously, $(y_{t1}, y_{t2}, \ldots, y_{tk})$ has the same joint probability distribution function as $(y_{t1+\tau}, y_{t2+\tau}, \ldots, y_{tk+\tau})$, for any k and τ:

$$F(y_{t1}, y_{t2}, \ldots, y_{tk}) = F(y_{t1+\tau}, y_{t2+\tau}, \ldots, y_{tk+\tau}) \tag{2.10}$$

This definition is quite strong (requires all moments to be constant) and may not be always satisfied, but weaker form, covariance stationarity, is usually satisfactory. Stochastic process y_t is covariance stationary, if its first and second moments do not change over time. That is, its mean $E(y_t) = \mu$ is constant and $Cov(y_t, y_{t+h}) = \gamma_h$ depends only on $h \geq 0$ for all t. In particular, second condition implies that variance is constant.

Order of integration

This part introduces only terminology which is used in the following parts. We say that weakly dependent stationary process is integrated of order zero $I(0)$, while nonstationary process which can be transformed into stationary process by differencing it once is said to be integrated of order one $I(1)$. Generally, a stochastic process is said to be integrated of order d, $I(d)$, if differencing the process d-times yields stationary process. We will not use process integrated of higher order than one or fractionally integrated processes, but some methods may be generalized for general order of integration.

2.3.1 Unit root tests

Unit root and stationarity tests have to be performed at the very beginning of any time series analysis, because if time series are not stationary (or at least covariance stationary), inference from econometric analysis is generally incorrect, mainly because of inflated t-statistics (and therefore lower p-values) and high R^2. Moreover, many financial and economic time series have unit roots.

Augmented Dickey-Fuller test (ADF)

Original unit root Dickey-Fuller test was developed by Dickey & Fuller (1979), but augmented version of their test is used more often nowadays, because it allows for more flexibility. We also employ augmented Dickey-Fuller test (ADF), because we suspect that higher number of lags will have to be used. Let us start with definition of autoregressive process y_t of order q, AR(q):

$$y_t = \alpha_0 + \sum_{i=1}^{q} \alpha_i y_{t-i} + e_t \tag{2.11}$$

AR(q) process has unit root if $\sum_{i=1}^{q} \alpha_i = 1$, thus equation 2.11 can be easily transformed into equation which is used for ADF test of order q. Subtracting y_{t-1} from both sides of the equation and reparametrization yields:

$$\Delta y_t = \alpha + \beta y_{t-1} + \sum_{i=1}^{q-1} \gamma_i \Delta y_{t-i} + e_t \tag{2.12}$$

Finally, ADF test statistics is obtained as t-statistics of coefficient β, i.e. $\hat{\beta}/SE(\hat{\beta})$. This test statistics, however, does not have t-distribution nor asymptotically standard normal distribution. This is the reason why critical values have to be obtained by simulations. If null hypothesis $H_0 : \beta = 0$ is rejected in favour of the alternative hypothesis $H_1 : \beta < 0$, it means that the time series does not have unit root, while if H_0 is not rejected, the time series has unit root (more precisely, there is not enough evidence that the time series does not have unit root).

An important part of ADF test is selection of lag length q, because serial correlation may bias the results if q is too low and test power decreases as lag length q increases. Therefore, application of ADF test requires careful treatment in terms of lag length selection. Schwert (1989) recommends maximum number of lags q, which is given by the following formula:

$$q_{max} = \left[12 \cdot \left(\frac{T}{100} \right)^{0.25} \right] \tag{2.13}$$

where T is a number of observations in a sample and $[x]$ is integer part of x. Furthermore, it is recommended to test down from the maximal lag.

KPSS test

Besides ADF test we use stationarity KPSS test developed by Kwiatkowski *et al.* (1992). Nature of ADF and KPSS tests significantly differ, ADF is a unit root test (its null hypothesis is unit root), while KPSS is stationarity test (null hypothesis is stationarity). This is important to keep in mind, because rejection of null hypothesis draws opposite conclusion. The idea behind KPSS test is following, let us assume that a time series of our interest y_t can be decomposed as:

$$y_t = x_t + \beta t + e_t \tag{2.14}$$

where x_t is a random walk, t is a time trend and e_t is an error term which is stationary. Random walk x_t can be written as:

$$x_t = x_{t-1} + u_t \tag{2.15}$$

where u_t is error term with zero mean and variance σ_u^2. Null hypothesis (stationarity) of the test is $\sigma_u^2 = 0$ against the alternative $\sigma_u^2 > 0$. If $\sigma_u^2 = 0$, then x_t becomes a constant term (it is not a random walk anymore) and y_t is thus stationary around its time trend. On contrary, if variance of u_t is positive, x_t is a random walk and y_t is not stationary. Test statistic is computed from model 2.14 in which x_t is replaced by a constant and is estimated by OLS. Denoting residuals from the regression by $\hat{e}_t$, estimate of error variance by $\hat{\sigma_e}^2$ and partial sum of residuals by $S_t = \sum_{i=1}^{t} \hat{e}_i$, for $t = 1, 2, \ldots, T$, the LM test statistic is computed as:

$$LM = \sum_{t=1}^{T} \frac{S_t}{\hat{\sigma_e}^2} \tag{2.16}$$

Critical values of the LM statistic can be found in Kwiatkowski *et al.* (1992).

2.3.2 Cointegration

Standard econometric analysis cannot be used whenever two or more time series are integrated. Moreover, any linear combination of series which are integrated of the same order is generally integrated and has the same order of integration as individual series (Engle & Granger 1987). That is, if $x_t \sim I(1)$ and $y_t \sim I(1)$, then generally $x_t + \beta y_t \sim I(1)$. However, it is possible that both processes are driven by the same deterministic or stochastic trend, in other words they evolve in a similar way. In such a case, linear combination $x_t + \beta y_t$ is integrated of lower order than individual time series (in particular, we are interested if the combination is $I(0)$), and we say that x_t and y_t are cointegrated. We present cointegration tests, Engle-Granger two step method and Johansen cointegration test.

Engle-Granger

Simple, but sufficient for our analysis is Engle-Granger two step method (Engle & Granger 1987). The whole testing procedure actually consist of three steps; first preliminary step is to test for unit roots in individual time series. We use ADF and KPSS tests in our analysis. Engle-Granger test is performed by estimating the following model by OLS:

$$y_t = \beta x_t + e_t \tag{2.17}$$

Even though this regression may give spurious results, coefficient estimates are consistent. The last step is to test unit root (or stationarity) of residuals $\hat{e}_t$ from the regression 2.17. ADF test is usually used – if null hypothesis of unit root is not rejected for x_t and y_t and at the same time null is rejected for $\hat{e}_t$, then x_t and y_t are cointegrated with cointegrating vector $(\beta, 1)$. Cointegrating vector is not unique, any multiple of the vector is equally good cointegrating vector, but coefficient at one variable is usually normalized at one. Furthermore, the method may be generalized to more than two variables, but its disadvantage is that it can identify only one cointegrating relationship, as opposed to Johansen method. We always test relationship between one volatility and one liquidity measure, thus it is not a serious limitation in our case.

Johansen test

Johansen (1988) develops more advanced cointegration test, because Engle-Granger method is not suitable in multivariate time series econometrics. The test itself is not described in the thesis because of its difficulty, but it can be found in Johansen (1988). We briefly present application of this test, mainly because its result is used in estimation of vector error correction model. Johansen test is based on rank of Π matrix in VECM model (see equation 2.20) which is described in the next sections. There are two versions of Johansen cointegration test, trace test and maximum eigenvalue test. Both tests have null hypothesis of cointegration rank equal to r, but alternative hypotheses differ. Alternative hypothesis of trace test is full cointegration rank k, while maximum eigenvalue test has cointegration rank $r+1$ as an alternative hypothesis. Considering VECM with k variables, the following hypotheses are considered and the test is applied sequentially:

$$
\begin{array}{lcl}
H_0 : rank(\Pi) = 0 & \text{vs} & H_1 : rank(\Pi) > 0 \\
H_0 : rank(\Pi) = 1 & \text{vs} & H_1 : rank(\Pi) > 1 \\
 & \vdots & \\
H_0 : rank(\Pi) = k-1 & \text{vs} & H_1 : rank(\Pi) = k
\end{array}
\tag{2.18}
$$

2.3.3 VAR and VEC models

Vector autoregression (VAR) and vector error correction (VEC) are two models used for multivariate autoregressive processes. The difference between them is in assumptions on input data, VAR is suitable for time series which are at least covariance stationary, while VECM is used for cointegrated unit root time series.

Vector autoregression

We start with introduction of a vector autoregression model which is a suitable model for stationary data. It can be used for time series in levels which are stationary and for first differenced unit-root series which are not cointegrated. VAR is a model in which a set of k (endogeneous) variables is explained by their own q lags as well as q lags of other $k-1$ endogenous explanatory variables and optionally by other exogenous variables. VAR(q) without exogenous variables is defined as:

$$\mathbf{y_t} = \mathbf{c} + \mathbf{A_1}\mathbf{y_{t-1}} + \ldots + \mathbf{A_q}\mathbf{y_{t-q}} + \mathbf{e_t} \tag{2.19}$$

where $\mathbf{y_t} = (y_{1t}, \ldots, y_{kt})'$ is a $k \times 1$ vector of variables, $\mathbf{A_1}$ to $\mathbf{A_q}$ are $k \times k$ matrices of parameters and $\mathbf{c}$ is a $k \times 1$ vector of constants to be estimated, $\mathbf{e_t}$ is a $k \times 1$ vector of error terms. Errors are assumed to satisfy $\mathbb{E}(\mathbf{e_t}) = 0$, $\mathbb{E}(\mathbf{e_t}\mathbf{e_t}') = \mathbf{\Omega}$ and $\mathbb{E}(\mathbf{e_t}\mathbf{e_s}') = 0$, for $t \neq s$. In other words, since $\mathbf{\Omega}$ is a covariance matrix of squared residuals, $\mathbf{e_t}$ are independent vectors with distribution $\mathbf{e_t} \sim N(0, \mathbf{\Omega})$.

Vector error correction model

Vector error correction model is a suitable model for unit-root time series which are cointegrated. The model is derived from VAR, after subtracting y_{t-1} from both sides of equation 2.19 and some algebraic manipulation we get:

$$\mathbf{\Delta y_t} = \mathbf{c} + \mathbf{\Pi y_{t-1}} + \mathbf{\Gamma_1 \Delta y_{t-1}} + \ldots + \mathbf{\Gamma_{q-1} \Delta y_{t-q+1}} + \mathbf{e_t} \tag{2.20}$$

where $\mathbf{\Pi} = \sum_{i=1}^{q} \mathbf{A_i} - \mathbf{I_k}$, $\mathbf{\Gamma_j} = -\sum_{i=j+1}^{q} \mathbf{A_i}$; $\mathbf{c}$ and $\mathbf{e_t}$ are the same as in equation 2.19. If matrix $\mathbf{\Pi}$ has a rank r and $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ are $r \times k$ matrices with rank r, matrix $\mathbf{\Pi}$ can be rewrritten as $\mathbf{\Pi} = \boldsymbol{\alpha}\boldsymbol{\beta}'$. There are five different specifications of VECM which differ in trends that are allowed in the model. Using some algebra, VECM can be rewritten as (Stata Corp. 2013):

$$\mathbf{\Delta y_t} = \boldsymbol{\alpha}(\boldsymbol{\beta}\mathbf{y_{t-1}} + \boldsymbol{\mu} + \boldsymbol{\rho} t) + \mathbf{\Gamma_1 \Delta y_{t-1}} + \ldots + \mathbf{\Gamma_{q-1} \Delta y_{t-q+1}} + \boldsymbol{\gamma} + \boldsymbol{\tau} t + \mathbf{e_t} \tag{2.21}$$

We use model with unrestricted constant, that is $\boldsymbol{\rho} = 0$ and $\boldsymbol{\tau} = 0$. This specification is compatible with linear trend in level data and a constant in cointegration relationship. Furthermore, estimation of VECM requires number of cointegrating relationships, the number is determined based on results of Johansen test. Whenever $\mathbf{\Pi}$ is found to have zero or full rank, we use VAR in first differences.

Both VAR and VEC models require selection of appropriate lag length q. The length is typically selected such that it equals the lag length in a model whose information criteria are minimized, however, there may arise a conflict, because information criteria give different results. The most used information criteria are Akaike information criterion (AIC) by Akaike (1974), Bayesian information criterion (BIC) or Hannan–Quinn information criterion (HQC). In our analysis, information criteria give very different results concerning lag length, therefore we stick to one information criterion, AIC, which is defined as:

$$AIC = -2\log(\text{likelihood}) + 2k \tag{2.22}$$

where k is a number of independently adjusted parameters in the model.

Granger causality

Causality is defined as a relationship between two events where one event (cause) causes something else (effect). It is important to distinguish causality from (statistical) correlation, because correlation only measures whether and how strongly two variables tend to move in the same (or opposite) direction; in other words, none of the variables has to cause the other one. Therefore, correlation is not a suitable method for assessment of causal relationships. Granger (1969) proposed a definition of causality which is econometrically testable in time series framework; we will refer to this definition as Granger causality, because it differs from definition of causality introduced at the beginning of this paragraph. The idea behind Granger causality is following – if variable y causes x, past values of y should improve our prediction of x, because causal relationships are assumed to be such that cause precede effect and has positive/negative impact on effect.

Following Granger (1969), let us give more precise definition. We start by denoting the set of past values of $x_{t-i}, i = 1, 2, \ldots, \infty$ by $\overline{x_t}$ and estimator of x_t with the use of past values of y_t by $P_t(x|y)$; variance of residuals $e_t = x_t - P_t(x|y)$ is denoted by $\sigma^2(x|y)$ and z_t is all available information collected from time $t-1$ ($z_t - y_t$ means all information except y_t). We say that variable y Granger causes x, if $\sigma^2(x|z) < \sigma^2(x|\overline{y-z})$, because y improves our prediction of x. Furthermore, we speak about feedback if y Granger causes x and x Granger causes y, that is $\sigma^2(x|z) < \sigma^2(x|\overline{y-z})$ and $\sigma^2(y|z) < \sigma^2(x|\overline{x-z})$. Granger causality can be easily tested by F-test in VAR and VEC models. For example, in two variable model in which we regress x on lagged values of x and y, we run F-test on lagged variables y and if null hypothesis is rejected, we conclude that y Granger causes x.

Impulse response functions

VAR and VEC models are usually difficult to interpret, because description of individual variables and their lags is cumbersome and provides little insight in the model. In addition to Granger causality, impulse response functions (IRF) are another form of presentation of the models. IRF trace out how dependent variable respond to a one unit shock (increase) to one of the errors in the model (the others are kept zero), the shock appears only in one period and error return to zero in the next period. IRFs are especially useful in our analysis, because we are interested whether and how liquidity is affected by shocks in volatility and the other way round, how volatility responds to shocks in liquidity.

Chapter 3

Data

Data are one of the most important components of any empirical study, because results are directly dependent on the input, thus this chapter introduces the data used in the thesis. We present reasons behind selection of individual currency pairs, compare high-frequency data (tick) with lower intra-day frequencies and state advantages and disadvantages of both data types. Furthermore, selection of the best time zone and filtering of the data is discussed. Methods used to transform the data and compute volatility, liquidity and other measures are outlined as well.

3.1 Selected exchange rate pairs

Exchange rate data are obtained from Swiss FX broker and bank Dukascopy which offers data for wide range of exchange rate pairs and other financial instruments. We decide to select ten currency pairs, because it is a sufficient number for comparison and possible discovery of co-movements in volatilty and/or liquidty. These ten selected currency pairs are chosen from the most traded currency pairs in interbank market, as was reported by BIS (2013). The two most traded currency pairs are EUR/USD and USD/JPY which are likely to be a result of intense international trade and financial intermediation between the US, EU and Japan. Combined share of the currency pairs is above 40% of total turnover in FX market; on average, more than 2 000 billion USD is traded daily. The complete list of daily traded volume and shares of all ten currency pairs is reported below, in Table 3.1. Combined share of the ten currency pairs is 72.6%. Czech Crown (CZK) is not included in the analysis in spite of its importance for domestic exporters and importers, because the data are not provided by Dukascopy. Other exotic currencies with high local importance (Polish złoty or Hungarian forint) are not included either.

Table 3.1: Global FX market turnover, daily averages in April 2013

Currency pair	billions USD	% of total
AUD/USD	364	6.8
EUR/CHF	71	1.3
EUR/GBP	102	1.9
EUR/JPY	147	2.8
EUR/USD	1 289	24.1
GBP/USD	472	8.8
NZD/USD	82	1.5
USD/CAD	200	3.7
USD/CHF	184	3.4
USD/JPY	978	18.3

Source: BIS, Triennial central bank survey.

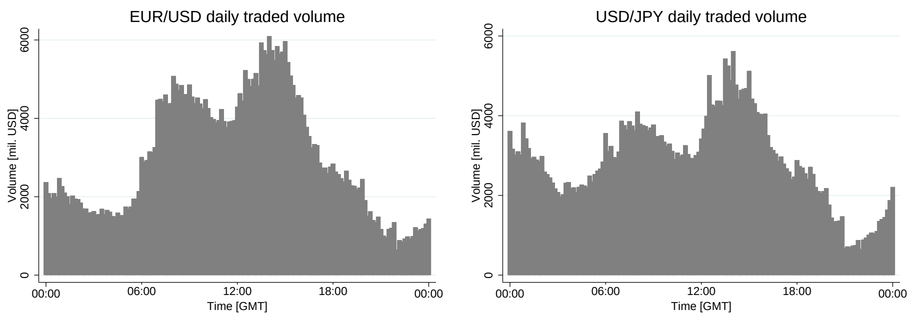
Dukascopy provides exchange rate data in frequencies ranging from very low (monthly data) to high (tick data), but tick data are available for shorter period of time than lower frequencies and only for the most traded pairs. The highest and most detailed frequency is represented by tick data, since every price change or volume traded is recorded. On contrary to lower frequencies, time span between two entries is not fixed and may be as low as one millisecond. Therefore, precision is the main advantage of tick data, because proportional BA spread (2.6) can be computed more precisely than from lower frequencies. Furthermore, any lower frequency (e.g. 621 seconds) can be easily and accurately computed from the data and also the number of ticks within a given time period can be computed. Obvious disadvantage is that tick data are more difficult to work with – tick data have high storage demands and any transformation or computation involving the data require high computational power and time. Lower than tick or second frequencies are much easier to work with, because the amount of data is greatly reduced, but such data provide less precise results, especially when there are long gaps between price changes causing that some quotes are missing. Missing quotes can be computed in the same way as from tick data, but such estimates are less precise. Moreover, quotes are offered without the number of ticks within periods between two consecutive quotes. Given these advantages and disadvantages, we use tick data as input data to our analysis.

3.2 Data processing

In the analysis we use data from 30th March 2007 (the date since when tick data are available) to 27th February 2014. Testing of relationship between volatility and liquidity will be performed on daily data, therefore we are interested in daily measures of liquidity and volatility which have to be computed from intra-day data. We decide to compute 10 minute data from ticks and use such data in a large part of the following analysis. 10 minute interval is chosen as an optimal trade-off between higher and lower frequencies. 10 minute frequency is sufficiently high, because it gives 144 quotes within one day, which is enough for statistical analysis and it is expected to be a sufficiently low frequency to avoid high-frequency noise (this assumption is based on simulations performed by Aït-Sahalia *et al.* (2005a)).

At the very beginning, the data have to be filtered and transformed. First, we had to select optimal time zone, because FX market is a truly global market – currencies are traded all over the world. Unlike stock exchanges which open at some time in the morning and close in the afternoon, currencies are traded 24 hours a day, from Monday morning in east Asia to Friday evening in the US. Using Greenwich mean time (GMT), trading in FX markets starts on Sunday at roughly 10 PM GMT in Sydney and ends on Friday, roughly at 10 PM GMT. Word "roughly" is used, because opening and closing times change during a year as countries switch between summer and winter time. To make it even more complicated, countries do not switch time on the same day. It seems that GMT time zone do not fit actual trading hours well enough and that it would be better to use GMT+2 time zone for exchange rate date.

Figure 3.1: Daily traded volume, 10 minute data



Source: Author's computations.

In order to support (or dismiss) such transformation of time stamps, distributions of average traded volumes during one day are computed. We compute average volume traded in each 10 minute interval within a day and for each currency pair and plot the volumes. We use median as an estimator of mean volume, because days with high trading activity biased arithmetic mean upwards. Distribution of EUR/USD and USD/JPY daily volume is plotted in Graph 3.1 above, volumes for all currencies are plotted in Appendix A.1. Similar analysis was done with proxy for volume, number of ticks, and we find that intra-day distributions of volumes and ticks are highly correlated, correlation is above 0.99, with the exception of USD/JPY for which the correlation is slightly below. Therefore we report only correlations in Table 3.2 and plots for two currency pairs can be found in Appendix A.2.

Table 3.2: Volume and tick correlation

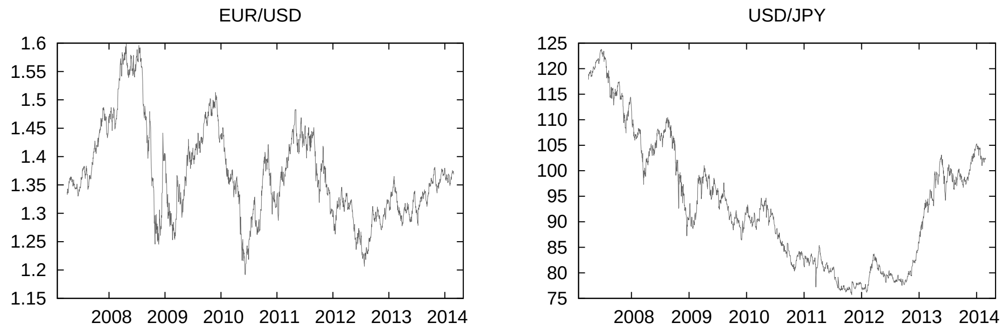
Currency pair	Intra-day	Daily
AUD/USD	0.9943	0.2054
EUR/CHF	0.9972	0.3283
EUR/GBP	0.9976	0.2266
EUR/JPY	0.9951	0.3803
EUR/USD	0.9914	0.5667
GBP/USD	0.9990	0.3111
NZD/USD	0.9986	0.5927
USD/CAD	0.9979	0.4481
USD/CHF	0.9943	0.3217
USD/JPY	0.9880	0.3330

Source: Author's computations.

Volume as well as tick data reveal intra-day seasonality, trading increases during day (GMT time) and decreases quite significantly at night. In some cases, volume traded at night may drop to only one tenth of maximum daily volumes, but usual drops in volume are not so dramatic. Even though it may seem better to restrict the analysis to hours with high trading activity, we use all 24 hours a day for further computations, because any restriction would be arbitrary. It is not possible to separate trading only in e.g. Europe, because FX market is a global market and market participants from other continents may trade whenever they want. Based on these findings, we change time zone of the data to GMT+2 and use all data available between Monday, 0:00 GMT+2 and Friday, 11:59 PM GMT+2. Besides Saturdays and Sundays data, we also disregard days with exceptionally low trading activity, such days are 1st January and 25th December. Furthermore, data for days from 15th June, 2009 to 19th

June, 2009 are missing for currency pairs AUD/USD, EUR/JPY, NZD/USD, USD/JPY. We exclude this week without further assumptions and thus we have 1789 observations for these four currency pairs and 1794 for the rest. Resulting time series are normal series of evolution of exchange rates, see two plots below.

Figure 3.2: Exchange rate, April 2007 - February 2014



Source: Author's computations.

Correlation between daily volume and daily number of ticks is rather low and varies significantly across currency pairs, as can be seen in Table 3.2. Therefore, we use both volumes and ticks as estimates of traded volume. Transformation of data and computation of volatility and liquidity measures was done with the use of Python (with NumPy library), all scripts programmed by the author of the thesis are enclosed and their list is to be found in Appendix B. Econometric tests, estimations and graph plots were done in Gretl and Stata.

Chapter 4

Empirical results

Relationship between volatility and liquidity of ten selected exchange rate pairs is empirically tested and findings are presented and discussed in this chapter. Our hypotheses are following: volatility and liquidity are negatively related, i.e. periods of high volatility are accompanied with low liquidity and vice versa, and second hypothesis is that increased volatility causes decrease in liquidity. We also compare our findings with recent study by Karnaukh *et al.* (2013) which tested liquidity (including volatility as an estimate of liquidity) in FX market. The chapter begins with brief discussion of computed volatility and liquidity time series and preliminary test. Then we proceed to the core of our study, cointegration tests and estimation of VAR and VEC models which are used for Granger causality tests and impulse response functions. We mainly focus on EUR/USD and USD/JPY exchange rate pairs, because all volatility and liquidity estimates of all ten currency pairs cannot be described in details due to the high number of series (two volatility and seven liquidity estimates).

4.1 Volatility, liquidity and preliminary tests

Exchange rate data (tick and 10 minute data) as described in the previous chapter are used for computation of two volatility and seven liquidity measures. Volatility measures are realized volatility denoted by rv in econometric tests and models and realized bi-power volatility, bpv. Liquidity measures are divided into two groups, spread and trading cost proxies and price impact estimators. There are three measures in the former group, time weighted bid-ask spread computed from tick data ($batick$), time and exchange rate weighted bid-ask spread computed from tick data (also known as proportional bid-ask spread, *bawtick*) and Roll's estimator of effective spread (*roll*). The latter group is represented by Amihud and Amivest estimators computed with both volume and number-of-ticks data; they are denoted by *amihudvol*, *amihudtick*, *amivestvol* and *amivesttick*. Because of low daily correlation between volume and number-of-ticks series, we use both approaches as we suspect them to give different results. Logarithmic transformation (denoted by l, e.g. lrv) of these series is used in the following analysis. First differenced series start with d.

Neither summary statistics nor plots of individual measures are provided, because of limited space. Therefore, let us point out some important findings before proceeding to unit root tests. All but one currency pairs in our sample are always quoted with variable spread, the exception is EUR/USD which is quoted with constant spread in two sub-periods. It is surprising to us, because we expected the most traded pairs to have variable spreads, while the less traded pairs to be offered with fixed spreads. This is important to keep in mind, because *batick* and *bawtick* are heavily influenced by the fact and is reasonable to expect them to have a different properties than spread estimates of other currency pairs. Nevertheless, we keep EUR/USD *batick* and *bawtick* and use them in further tests.

In methodology chapter, Amihud and Amivest measures were introduced as an average of intra-day Amihuds or Amivests which is common practice in applied research. As was noted, returns between 10 minute data points might be very small, causing especially Amivest to be biased upwards. We do find that some returns are very small and thus average of intra-day Amivests cannot be used at all, because series whose data points are significantly upward biased and sometimes less upward biased produce nothing else than a noise. We attempt to filter the series by censoring them from above, but such attempts proved to be unsuccessful and in the end, we opt for median of intra-day Amivests which seem to be a better choice. Amihud series are not biased because of small returns, but we also computed daily Amihud measures both as an average and median of intra-day Amihuds and find that they are highly correlated (above 0.9). Based on these findings, daily Amihud measures were obtained by averaging.

Plots of time series reveal that they are likely to be non-stationary and some series appear to be cointegrated (rigorous tests are performed and presented later), otherwise all time series except *roll* do not reveal anything unexpected at first sight. Plot of effective spread estimator *roll* is significantly different from other liquidity as well as volatility measures, because of high number of zero values (around one half, but it varies among currency pairs) in the series. This is caused by our definition of *roll*, because *roll* is substituted by zero if $Cov(r_i, r_{i-1}) \geq 0$. On the other hand, Roll (1984) does not expect $Cov(r_i, r_{i-1}) \geq 0$ to appear frequently and thus does not discuss such case in details. This result was not expected, because it implies that spread is zero (but spread is never zero in our data set). Furthermore, low *roll* means high liquidity and therefore zero *roll* should mean that the FX market could not be more liquid than it frequently is. Possible option how to deal with zero values is to use modified *roll* measure used by Bao *et al.* (2011) who use only $-Cov(r_i, r_{i-1})$ from *roll* without taking square root and multiplying it by two. We do not apply this modification and use *roll* as defined in equation 2.7, mainly because the theory behind *roll* and also because it is widely used.

The most important preliminary tests are unit root and stationarity tests, because they directly influence selection of suitable models in subsequent analysis. We suspect volatility and liquidity series to be integrated, since many financial time series have unit roots or are not stationary. Maximum lag length for ADF and KPSS tests is selected according to Schwert criterion (2.13); each time series in our analysis consist of nearly 1800 values, thus max lag length $q = 24.7$ which we round down to 24. Optimum lag is then selected automatically by Gretl, based on minimization of information criteria, so that tests of time series are performed with different lag lengths. Because of importance of the results, we report all ADF and KPSS test statistics and associated p-values in Table 4.1. Let us be rather strict in interpretation of these tests and focus on 1% significance level which is convenient for presentation as results across currency pairs are similar. At 1% significance level, majority of volatility and liquidity series of our currency pairs are integrated (more precisely, null hypothesis of unit root cannot be rejected), with the exception of *roll*.

Unit root of volatility measures *lrv* and *lbpv* cannot be rejected for eight out of ten currency pairs, the exceptions are EUR/JPY and USD/JPY. If we look at 5% significance, null cannot be rejected for AUD/USD and NZD/USD (*lrv* has p-value slightly above 0.05), too. However, stationarity of *lrv* and *lbpv* can be rejected for all ER pairs at 1% level according to KPSS test. Results of the tests at 1% level for liquidity measures can be easily summarized as well, null hypothesis of unit root can be rejected only for *lroll* and USD/CAD *lamihudvol* and *lamivestvol*. KPSS tests allows us to reject stationarity of all liquidity series except *lroll*. 5% significance slightly changes results, null hypothesis of unit root can be rejected for a few *lamihud* and *lamivest* measures, while it

Table 4.1: Unit root (ADF) and stationarity (KPSS) tests statistics

Currency pair	*Test*	lrv	lbpv	lamihudvol	lamihudtick	lamivestvol	lamivesttick	lroll	lbatick	lbawtick
AUD/USD	ADF	-2.886**	-2.927**	-3.076**	-2.378	-3.004**	-2.536	-8.566***	-1.432	-1.437
		(0.047)	(0.042)	(0.028)	(0.148)	(0.035)	(0.107)	(<0.001)	(0.568)	(0.566)
	KPSS	4.537***	4.635***	3.379***	3.788***	4.567***	3.127***	0.528**	13.715***	14.037***
EUR/CHF	ADF	-2.36	-2.33	-2.508	-2.91**	-2.545	-2.803*	-5.321***	-2.099	-2.773*
		(0.153)	(0.162)	(0.114)	(0.044)	(0.105)	(0.058)	(<0.001)	(0.245)	(0.062)
	KPSS	2.562***	2.55***	7.699***	1.208***	7.192***	1.025***	0.698**	6.134***	1.307***
EUR/GBP	ADF	-2.514	-2.496	-2.673*	-3.081**	-2.538	-3.115**	-7.31***	-1.261	-1.31
		(0.112)	(0.116)	(0.079)	(0.028)	(0.106)	(0.026)	(<0.001)	(0.65)	(0.627)
	KPSS	3.303***	3.393***	8.156***	0.935***	9.172***	1.145***	0.334	8.686***	11.645***
EUR/JPY	ADF	-3.696***	-3.87***	-2.386	-2.562	-2.452	-2.411	-8.281***	-0.774	-0.77
		(0.004)	(0.002)	(0.146)	(0.101)	(0.128)	(0.139)	(<0.001)	(0.826)	(0.827)
	KPSS	1.629***	1.706***	9.05***	1.865***	10.579***	2.147***	0.381*	11.194***	5.11***
EUR/USD	ADF	-2.257	-2.271	-2.032	-2.229	-2.027	-2.135	-7.029***	0.081	0.024
		(0.186)	(0.182)	(0.273)	(0.196)	(0.275)	(0.231)	(<0.001)	(0.964)	(0.96)
	KPSS	3.208***	3.301***	6.545***	4.042***	6.667***	3.605***	0.554**	13.789***	12.602***
GBP/USD	ADF	-2.141	-2.098	-2.902**	-2.719*	-2.882**	-2.642*	-7.115***	-0.873	-1.06
		(0.229)	(0.246)	(0.045)	(0.071)	(0.047)	(0.085)	(<0.001)	(0.797)	(0.733)
	KPSS	5.027***	5.104***	4.62***	2.005***	5.718***	1.975***	0.248	12.036***	8.337***
NZD/USD	ADF	-2.839*	-2.934**	-2.524	-2.269	-2.573*	-2.373	-7.565***	-0.949	-0.813
		(0.053)	(0.042)	(0.11)	(0.182)	(0.099)	(0.149)	(<0.001)	(0.773)	(0.815)
	KPSS	5.697***	5.665***	1.916***	5.872***	2.204***	5.363***	0.07	16.141***	15.596***
USD/CAD	ADF	-2.132	-2.141	-3.448***	-2.594*	-3.644***	-2.841*	-7.16***	-0.76	-0.601
		(0.232)	(0.229)	(0.009)	(0.094)	(0.005)	(0.053)	(<0.001)	(0.83)	(0.868)
	KPSS	9.89***	9.777***	1.304***	4.455***	1.823***	3.978***	0.09	14.842***	15.279***
USD/CHF	ADF	-2.548	-2.511	-2.317	-2.531	-2.308	-2.581*	-6.786***	-1.509	-2.263
		(0.104)	(0.113)	(0.166)	(0.108)	(0.169)	(0.097)	(<0.001)	(0.529)	(0.184)
	KPSS	3.174***	3.217***	8.818***	1.583***	9.716***	1.863***	0.215	9.752***	4.086***
USD/JPY	ADF	-3.465***	-3.525***	-2.279	-2.654*	-2.414	-2.66*	-8.594***	-0.796	-0.629
		(0.009)	(0.007)	(0.179)	(0.082)	(0.138)	(0.081)	(<0.001)	(0.82)	(0.862)
	KPSS	3.826***	4.029***	11.997***	3.419***	13.149***	4.218***	0.144	14.172***	9.301***

Significance levels: * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$; p-values in parentheses.
Source: Author's computations.

cannot be rejected for *lbatick* and *lbawtick*. Moreover, first differenced series do not have unit root (null hypotheses of ADF tests are rejected at any convenient level) and null hypothesis of KPSS stationarity tests cannot be rejected at 1% or 5% level, see Table A.1.

4.2 Cointegration

Since many volatility and liquidity series are found to be integrated, testing for cointegration is a natural step in the analysis. As we are interested in a relationship between volatility and liquidity, we take one volatility measure and one liquidity measure and test whether they are cointegrated or not. If cointegrating relationship is discovered, it will mean that volatility and liquidity is driven by the same stochastic trend and that volatility is a good proxy for liquidity, as suggested by Karnaukh *et al.* (2013). We exclude *lroll* from cointegration tests, because evidence against unit root in *lroll* is solid and thus it should not be included in cointegration tests with $I(1)$ variables. Engle-Granger cointegration test is used in the following analysis; ADF test statistics of residuals and associated p-values are reported in Table 4.2. ADF tests of individual series are reported in the previous table, Table 4.1.

We start the cointegration tests with a special pair of variables – volatility measures *lrv* and *lbpv*. This pair is tested along with volatility-liquidity pairs, because realized volatility and bi-power volatility seem to be closely related. If it is proved, analysis of VAR and VEC models can be greatly simplified, because it will be sufficient to use either *lrv* or *lbpv*. Engle-Granger test reveals that *lrv* and *lbpv* are indeed cointegrated, ADF test of residuals give much lower p-values than unit root test performed on either *lrv* or *lbpv*. The highest p-value among all currency pairs is about 10^{-7}, therefore estimation of VAR and VEC models with only *lbpv* should be sufficient.

Volatility-liquidity cointegration is less clear. We test pairs including *lrv* or *lbpv* and one of liquidity measure *lamihudvol*, *lamihudtick*, *lamivestvol*, *lamivesttick*, *lbatick* or *lbawtick*. In order to achieve some degree of consistency, we focus on 1% significance level. At this significance level, few pairs are found to be cointegrated – six currency pairs can be immediately left out. Volatility measures of EUR/JPY and USD/JPY are not integrated and none of EUR/GBP, EUR/CHF, EUR/USD and GBP/USD volatility-liquidity pair is cointegrated. On contrary, AUD/USD and NZD/USD *lamihudtick* and *lamivesttick* are cointegrated with *lbpv*. Also NZD/USD, USD/CAD and

Table 4.2: Engle-Granger cointegration, ADF test statistic

Coint. pair *Currency pair*	lrv lbpv	lrv lamihudvol	lrv lamihudtick	lrv lamivestvol	lrv lamivesttick	lrv lbatick	lrv lbawtick	lbpv lamihudvol	lbpv lamihudtick	lbpv lamivestvol	lbpv lamivesttick	lbpv lbatick	lbpv lbawtick
AUD/USD	-7.773***	-3.222*	-4.176***	-3.102*	-4.13***	-3.419**	-3.682**	-3.07***	-4.19***	-2.967	-4.151***	-3.532**	-3.798**
	(<0.001)	(0.066)	(0.004)	(0.088)	(0.005)	(0.04)	(0.019)	(0.095)	(0.004)	(0.118)	(0.004)	(0.03)	(0.014)
EUR/CHF	-7.18***	-2.22	-2.471	-2.21	-2.592	-3.351**	-3.416**	-2.2***	-2.441	-2.19	-2.566	-3.312*	-3.373**
	(<0.001)	(0.413)	(0.292)	(0.418)	(0.241)	(0.048)	(0.041)	(0.423)	(0.306)	(0.429)	(0.251)	(0.053)	(0.045)
EUR/GBP	-7.147***	-2.486	-3.019	-2.464	-3.06*	-3.484**	-3.142*	-2.467***	-3.04	-2.439	-3.085*	-3.504**	-3.169*
	(<0.001)	(0.286)	(0.106)	(0.295)	(0.097)	(0.034)	(0.08)	(0.294)	(0.101)	(0.307)	(0.092)	(0.032)	(0.075)
EUR/JPY	*lrv* and *lbpv* do not have unit root at 1% significance level												
	-8.578***	-3.867**	-4.464***	-3.778**	-4.363***	-4.457***	-4.432***	-3.882***	-4.493***	-3.795**	-4.418***	-4.69***	-4.742
	(<0.001)	(0.011)	(0.001)	(0.014)	(0.002)	(0.001)	(0.002)	(0.01)	(0.001)	(0.014)	(0.002)	(0.001)	(<0.001)
EUR/USD	-6.366***	-2.054	-2.612	-2.047	-2.64	-3.108*	-3.161*	-2.055***	-2.653	-2.018	-2.668	-3.17*	-3.229*
	(<0.001)	(0.5)	(0.233)	(0.503)	(0.222)	(0.087)	(0.077)	(0.499)	(0.217)	(0.519)	(0.211)	(0.075)	(0.065)
GBP/USD	-8.512***	-2.278	-3.351**	-2.141	-3.347**	-2.713	-2.881	-2.307***	-3.439**	-2.159	-3.428**	-2.79	-2.967
	(<0.001)	(0.384)	(0.048)	(0.454)	(0.049)	(0.195)	(0.141)	(0.369)	(0.038)	(0.445)	(0.039)	(0.169)	(0.118)
NZD/USD	-7.403***	-3.284*	-4.599***	-3.083*	-4.553***	-4.09***	-4.442***	-3.607***	-4.634***	-3.384**	-4.571***	-4.189***	-4.705***
	(<0.001)	(0.057)	(0.001)	(0.092)	(0.001)	(0.005)	(0.001)	(0.024)	(0.001)	(0.044)	(0.001)	(0.004)	(0.001)
USD/CAD	-29.522***	-2.152	-2.886	-2.082	-2.841	-4.24***	-4.05***	-2.147***	-2.899	-2.083	-2.861	-4.27***	-4.084***
	(<0.001)	(0.449)	(0.14)	(0.485)	(0.153)	(0.003)	(0.006)	(0.451)	(0.136)	(0.485)	(0.147)	(0.003)	(0.005)
USD/CHF	-28.2***	-2.458	-3.136*	-2.474	-3.109*	-3.978***	-4.407***	-2.45***	-3.128*	-2.468	-3.108*	-3.954***	-4.326***
	(<0.001)	(0.298)	(0.081)	(0.291)	(0.086)	(0.008)	(0.002)	(0.301)	(0.083)	(0.293)	(0.087)	(0.008)	(0.002)
USD/JPY	*lrv* and *lbpv* do not have unit root at 1% significance level												
	-8.298***	-3.518**	-3.439**	-3.575**	-3.402**	-3.748**	-3.524**	-3.634***	-3.786**	-3.862**	-3.47**	-3.79**	-3.554**
	(<0.001)	(0.031)	(0.038)	(0.026)	(0.042)	(0.016)	(0.03)	(0.022)	(0.014)	(0.011)	(0.035)	(0.014)	(0.028)

Significance levels: * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$; p-values in parentheses.
Source: Author's computations.

USD/CHF *lbatick* and *lbawtick* are cointegrated with *lbpv* at 1% significance level. These are the only *lbpv*-liquidity cointegrating pairs we find, results for *lrv*-liquidity cointegration are similar. Evidence of cointegration between volatility and liquidity is thus weak – only four out of ten currency pairs show some statistically significant cointegration relationships, but evidence for these cointegrating relationships is not strong either. Even though 1% significance level may seem to provide quite a strong evidence for and against hypotheses, it is important to keep in mind that we slightly did not reject unit roots of input series in the first step and then slightly rejected unit roots in residuals from regression of one volatilty measure on another liquidity measure.

4.3 Granger causality

Vector autoregression or vector error correction models have to be fitted before performing Granger causality tests. We estimate seven VAR/VEC models for each currency pair, that is seventy models in total. Every model comprises of two variables and their lags, one variable is *lbpv* and the other one is one of our liquidity measures (*amihudvol*, *amihudtick*, *amivestvol*, *amivestvol*, *lbatick*, *lbawtick* and *lroll*). First, optimal lag length for VAR/VEC model is selected based on Akaike information criterion (2.22), with maximum lag set at 24. Then number of cointegrating relationships is determined in Johansen model (except *lroll*, because we have strong evidence that *lroll* is not integrated). In our simple two variable case we are interested in one cointegrating relationship, because there cannot be more relationships between two variables. VEC model is fitted if $rank(\Pi) = 1$, if rank of Π matrix is zero or two, we use VAR model in first differences or levels (*lroll*). Complete results of Johansen test can be found in Table A.2.

Specification tests of fitted models are performed afterwards, we test residuals for heteroscedasticity (ARCH effect), autocorrelation (Ljung-box test) and normality of residuals is tested as well. Null hypothesis of homoscedasticity is rejected at 1% or 5% level in nearly all equations. Notable exception is regression of *lroll* on its lagged values and *lbpv* from $lbpv - lroll$ VAR model, however, homoscedasticity assumption is rejected for the second equation from that model. Because we find a strong evidence against homoscedasticity, heteroscedasticity robust standard errors are used. On contrary, autocorrelation is not a problem in our models, since p-values of Ljung-box tests are above 0.98. The last specification test, normality, does not give surprising results. Normality is usually rejected and so it is in our case; p-values are well below 0.1% significance level, indicating that hypothesis of normality is strongly rejected. Complete results of these tests are found in Tables A.3, A.4 and A.5.

Let us proceed to Granger causality tests between volatility and liquidity. The tests should tell us, whether volatility causes changes in liquidity or liquidity causes changes in volatility or there is no obvious pattern. Unfortunately, the results seem to be inconclusive and providing contradictory results. If we take a look at EUR/CHF, we can see that volatility very likely Granger causes changes in liquidity, joint F-test of *lbpv* lags in liquidity equations give lower p-values than the other way round. It means that *lbpv* significantly improves estimate of liquidity and according to definition of causality by Granger (1969), volatility causes liquidity. Altogether, Granger causality direction volatility $\Rightarrow$ liquidity is significant in 54 out of 70 cases at 5% level. However, EUR/GBP give opposite results, liquidity is more important in explanation of volatility than the other way round. Liquidity $\Rightarrow$ volatility direction is significant in 42 cases at 5% level. Moreover, in case of EUR/GBP both volatility and liquidity are significant in explaining the other variable, causing so called feedback. Feedback can be found in many other relationships tested, it is found in 35 cases out of 70 at 5% significance level. Therefore, feedback seems to be the best description of relationship between liquidity and volatility. In other words, volatility influences liquidity and liquidity influences volatility. Even though we can find evidence of causality in one way or the other way round, no pattern of relationships which would hold across all currency pairs can be found in our sample.

Taking a closer look at EUR/USD, we notice that there is a feedback between price impact proxies and volatility, while trading cost proxies *lbatick* and *lbawtick* Granger cause volatility. This is likely to be a result of periods with fixed spread. USD/JPY pair (and other pairs, too) is interesting, because we expected that feedback between *lamihudvol*, *lamihudtick* and *lbpv* would occur only when there is feedback between *lamivestvol*, *lamivesttick* and *lbpv*, since Amivest is basically defined as 1/Amihud. This is, however, not true, because volatility usually Granger cause Amivest, but not the other way round.

4.4 Impulse response functions

Impulse response functions allow us to asses impact of one-unit positive shock in either volatility or liquidity, so that it can be discovered whether there is

Table 4.3: Granger causality, F-test

VAR/VECM eq. *Currency pair*	lamihudvol ⇒ lbpv	lbpv ⇒ lamihudvol	lamihudtick ⇒ lbpv	lbpv ⇒ lamihudtick	lamivestvol ⇒ lbpv	lbpv ⇒ lamivestvol	lamivesttick lbpv ⇒	lbpv ⇒ lamivesttick	lbatick ⇒ lbpv	lbpv ⇒ lbatick	lbawtick ⇒ lbpv	lbpv ⇒ lbawtick	lroll ⇒ lbpv	lbpv ⇒ lroll
AUD/USD	1.169	3.066***	2.234***	3.536***	1.213	2.947***	2.07**	2.76***	1.664*	1.962**	1.598*	2.192***	1.856**	1.711**
	(0.279)	(<0.001)	(0.005)	(<0.001)	(0.241)	(<0.001)	(0.011)	(0.001)	(0.057)	(0.017)	(0.073)	(0.007)	(0.014)	(0.029)
EUR/CHF	1.176	1.832**	1.337	3.192***	1.36	1.992**	1.248	3.504***	2.26***	4.928***	2.276***	4.883***	1.761**	2.795***
	(0.287)	(0.03)	(0.178)	(<0.001)	(0.165)	(0.015)	(0.233)	(<0.001)	(0.005)	(<0.001)	(0.004)	(<0.001)	(0.028)	(<0.001)
EUR/GBP	3.44***	2.242***	3.605***	2.015***	2.543***	1.908***	2.215***	2.049***	2.464***	1.993***	2.451***	1.979***	1.247	0.996
	(<0.001)	(0.001)	(<0.001)	(0.005)	(<0.001)	(0.005)	(0.001)	(0.003)	(<0.001)	(0.007)	(0.001)	(0.007)	(0.262)	(0.441)
EUR/JPY	2.249***	2.254***	1.561**	2.785***	1.732**	1.455*	0.854	2.493***	1.371	2.444***	1.595*	2.998***	1.489	2.174**
	(0.001)	(0.001)	(0.041)	(<0.001)	(0.015)	(0.072)	(0.566)	(0.008)	(0.153)	(0.002)	(0.068)	(<0.001)	(0.146)	(0.021)
EUR/USD	3.033***	2.553***	2.586***	2.77***	1.718**	2.044**	1.59*	2.018**	2.024***	1.177	2.047***	1.296	1.552*	1.279
	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(0.046)	(0.012)	(0.075)	(0.014)	(0.002)	(0.252)	(0.002)	(0.153)	(0.086)	(0.212)
GBP/USD	2.589***	2.127***	3.134***	1.835**	1.742**	2.031***	2.369***	1.845***	1.005	1.45*	1.021	1.585**	1.786**	0.668
	(<0.001)	(0.001)	(<0.001)	(0.012)	(0.014)	(0.002)	(<0.001)	(0.009)	(0.456)	(0.073)	(0.434)	(0.036)	(0.016)	(0.868)
NZD/USD	2.932***	2.411***	2.796***	2.178***	2.409***	2.726***	2.246***	2.42***	1.988**	2.077**	2.012**	2.245***	1.4	2.015**
	(<0.001)	(0.001)	(<0.001)	(0.002)	(0.001)	(<0.001)	(0.001)	(<0.001)	(0.016)	(0.011)	(0.014)	(0.005)	(0.145)	(0.014)
USD/CAD	1.78**	2.622***	1.948**	2.613***	1.838**	1.173	1.567*	1.203	1.411	1.377	1.413	1.483	1.215	0.94
	(0.011)	(<0.001)	(0.016)	(0.001)	(0.025)	(0.286)	(0.075)	(0.262)	(0.14)	(0.156)	(0.139)	(0.109)	(0.281)	(0.489)
USD/CHF	1.622**	2.062***	1.061	2.561***	1.574**	2.329***	1.188	2.029***	1.871**	2.35***	1.812**	2.102***	1.66*	1.261
	(0.029)	(0.002)	(0.389)	(0.001)	(0.041)	(<0.001)	(0.241)	(0.002)	(0.01)	(0.001)	(0.011)	(0.002)	(0.094)	(0.253)
USD/JPY	2.508***	1.921***	2.034***	1.618**	1.288	2.892***	0.984	3.006***	3.576***	1.212	3.415***	1.396	0.997	0.749
	(<0.001)	(0.005)	(0.005)	(0.044)	(0.181)	(<0.001)	(0.477)	(<0.001)	(<0.001)	(0.255)	(<0.001)	(0.141)	(0.443)	(0.679)

Significance levels: * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$; p-values in parentheses.
Source: Author's computations.

any impact and if the impact is positive or negative. IRFs are computed from the same VAR/VEC models which were estimated in the previous section. The most important is then to select appropriate ordering of variables in Cholesky ordering (volatility is followed by liquidity or the other way round), because results are sensitive to ordering. Granger causality tests can provide a guidance how to order the variables. We have argued that there is very likely a feedback between volatility and liquidity, but this result cannot be used, as it says that there is no ordering. If p-values of volatility $\Rightarrow$ liquidity and liquidity $\Rightarrow$ volatility directions are compared, we can see that volatility $\Rightarrow$ liquidity direction has lower p-values in more cases than liquidity $\Rightarrow$ volatility. The evidence is weak, but it can be used as a general rule in Cholesky ordering. In case there is evidence against volatility $\Rightarrow$ liquidity direction, we use opposite ordering.

Plots of all IRF functions with 90% confidence band can be found in Appendix A.3 - A.12. IRF plots do not provide much evidence, either. Shocks in liquidity fluctuate around zero, but their impact is indistinguishable from zero at 90% significance in majority of cases. Notable exceptions are IRFs which were computed with opposite ordering of variables (i.e. liquidity $\Rightarrow$ volatility). On contrary, shocks in volatility have statistically significant impact on liquidity measures. The impact is positive on Amihud measures and trading cost proxies (*lbatick*, *lbawtick*, *lroll*) – i.e. increase in volatility implies increase in Amihud or trading costs which means decrease in liquidity. Also, shocks in volatility have negative impact on Amivest measures; this is not surprising, since Amivest is basically defined as an inverse of Amihud (low Amivest means low liquidity). The effects are very short-term, usually appear only in the period when shock occurs (period zero) and then fade away in the next period (become insignificant at 10% level). In some cases, return to original value takes a few more days, but such behaviour is rather rare – for example, positive shock in volatility of USD/JPY increases *lbatick* and *lbawtick* for four to six days. We can find even more persistent impacts, positive shock in volatility increases USD/CHF spread and proportional spread (*lbatick* and *lbawtick*) for more than 14 days. To sum up, volatility and liquidity are negatively related, but the evidence is weaker than was expected – positive shocks in volatility have negative impact on liquidity, in other words, increased volatility is accompanied with lower liquidity. However, positive shocks in liquidity measures (i.e. decrease of liquidity, except Amivest measures) generally do not have impact on volatility.

Chapter 5

Conclusion

We compute several volatility and liquidity measures for ten currency pairs between April 2007 and February 2014. Volatility measures, realized volatility and bi-power volatility are closely related and cointegrating relationship between them is found. Liquidity measures are more diverse, spread proxies significantly differ ($batick$ and $roll$) and price impact proxies computed with volume and number-of-ticks proxy differ, too. Almost all series except $lroll$ are found to be integrated, but only a few volatility-liquidity pairs are cointegrated which means that in general, volatility and liquidity evolve differently. Volatility-liquidity cointegration is found in AUD/USD, NZD/USD, USD/CAD and USD/CHF currency pairs, but the evidence is rather weak.

VAR and VEC models are fitted in order to examine causality and to find out if volatility and liquidity are positively or negatively related. Granger causality differs across liquidity measures and currency pairs, some pairs shows stronger evidence for volatility $\Rightarrow$ liquidity direction, while direction liquidity $\Rightarrow$ volatility is found to be more significant for other pairs. But in general, both directions are statistically significant, implying that the causality between volatility and liquidity is mutual, i.e. feedback (or equivalence). Impulse response functions indicate that volatility and liquidity evolve in opposite directions – increase in volatility is consistent with decrease in liquidity. One-unit shock in liquidity has generally small impact (insignificant) on volatility, while increase in volatility causes liquidity to decrease. The fall is generally very short term, liquidity returns to equilibrium value in the next period in most cases.

Relationship between intra-day volatility and liquidity studied on high frequency data could reveal more interesting and significant results, but it is much more difficult to perform. Moreover, such research may provide results that would be useful for banks and institutions trading large amount of currencies, but it would be of little use for exporters and importers that are more interested in long term relationships.

Bibliography

Akaike, H. (1974): "A new look at the statistical model identification." *Automatic Control, IEEE Transactions on* **19(6)**: pp. 716–723.

Amihud, Y. (2002): "Illiquidity and stock returns: cross-section and time-series effects." *Journal of Financial Markets* **5(1)**: pp. 31–56.

Aït-Sahalia, Y., P. A. Mykland, & L. Zhang (2005a): "How often to sample a continuous-time process in the presence of market microstructure noise." *Review of Financial Studies* **18(2)**: pp. 351–416.

Aït-Sahalia, Y., P. A. Mykland, & L. Zhang (2005b): "A tale of two time scales." *Journal of the American Statistical Association* **100(472)**.

Banti, C., K. Phylaktis, & L. Sarno (2012): "Global liquidity risk in the foreign exchange market." *Journal of International Money and Finance* **31(2)**: pp. 267 – 291.

Bao, J., J. Pan, & J. Wang (2011): "The illiquidity of corporate bonds." *The Journal of Finance* **66(3)**: pp. 911–946.

Barndorff-Nielsen, O. E. & N. Shephard (2004): "Power and bipower variation with stochastic volatility and jumps." *Journal of Financial Econometrics* **2(1)**: pp. 1–37.

Bessembinder, H. (1994): "Bid-ask spreads in the interbank foreign exchange markets." *Journal of Financial Economics* **35(3)**: pp. 317 – 348.

BIS (2013): "Triennial central bank survey foreign exchange turnover in april 2013: preliminary global results." *Technical report*, Bank for international settlements.

Bollerslev, T. & M. Melvin (1994): "Bid-ask spreads and volatility in the foreign exchange market: An empirical analysis." *Journal of International Economics* **36(3–4)**: pp. 355–372.

Carlston, B. (2012): "Examining the commonality in liquidity and volatility risk." *Working papers*, Duke university.

Cheng, A., K. Das, & T. Shimatani (2013): "Central bank intervention and exchange rate volatility: Evidence from japan using realized volatility." *Journal of Asian Economics* **28(0)**: pp. 87–98. ACAES-FEG Special Issue: Econometrics of Financial Markets in Asia.

Chordia, T., R. Roll, & A. Subrahmanyam (2000): "Commonality in liquidity." *Journal of Financial Economics* **56(1)**: pp. 3 – 28.

Cooper, S. K., J. C. Groth, & W. E. Avera (1985): "Liquidity, exchange listing, and common stock performance." *Journal of Economics and Business* **37(1)**: pp. 19 – 33.

Corwin, S. A. & P. Schultz (2012): "A simple way to estimate bid-ask spreads from daily high and low prices." *The Journal of Finance* **67(2)**: pp. 719–760.

Dickey, D. A. & W. A. Fuller (1979): "Distribution of the estimators for autoregressive time series with a unit root." *Journal of the American statistical association* **74(366a)**: pp. 427–431.

Ding, L. & J. Hiltrop (2010): "The electronic trading systems and bid-ask spreads in the foreign exchange market." *Journal of International Financial Markets, Institutions and Money* **20(4)**: pp. 323 – 345.

Engle, R. F. & C. W. J. Granger (1987): "Co-integration and Error Correction: Representation, Estimation, and Testing." *Econometrica* **55(2)**: pp. 251–76.

Glassman, D. (1987): "Exchange rate risk and transactions costs: Evidence from bid-ask spreads." *Journal of International Money and Finance* **6(4)**: pp. 479 – 490.

Goyenko, R. Y., C. W. Holden, & C. A. Trzcinka (2009): "Do liquidity measures measure liquidity?" *Journal of Financial Economics* **92(2)**: pp. 153–181.

Granger, C. W. (1969): "Investigating causal relations by econometric models and cross-spectral methods." *Econometrica: Journal of the Econometric Society* pp. 424–438.

GROSSMAN, S. J. & M. H. MILLER (1988): "Liquidity and market structure." *The Journal of Finance* **43(3)**: pp. 617–633.

JOHANSEN, S. (1988): "Statistical analysis of cointegration vectors." *Journal of economic dynamics and control* **12(2)**: pp. 231–254.

KARNAUKH, N., A. RANALDO, & P. SÖDERLIND (2013): "Understanding fx liquidity." *Working Papers on Finance 1315*, University of St. Gallen, School of Finance.

KWIATKOWSKI, D., P. C. PHILLIPS, P. SCHMIDT, & Y. SHIN (1992): "Testing the null hypothesis of stationarity against the alternative of a unit root: How sure are we that economic time series have a unit root?" *Journal of Econometrics* **54(1–3)**: pp. 159 – 178.

LOCKE, P. R. & A. SARKAR (1996): "Volatility and liquidity in futures markets." *Technical report*, Federal Reserve Bank of New York.

LÜTKEPOHL, H. (2005): *New introduction to multiple time series analysis.* Berlin: Springer.

MANCINI, L., A. RANALDO, & J. WRAMPELMEYER (2013): "Liquidity in the foreign exchange market: Measurement, commonality, and risk premiums." *The Journal of Finance* **68(5)**: pp. 1805–1841.

POON, S.-H. & C. W. J. GRANGER (2003): "Forecasting volatility in financial markets." *Journal of Economic Literature* **vol. 41(issue 2)**: pp. 478–539.

POSKITT, R. (2005): "Bid-ask spreads in the foreign exchange market: An alternative interpretation." *Pacific-Basin Finance Journal* **13(5)**: pp. 562 – 583.

ROLL, R. (1984): "A simple implicit measure of the effective bid-ask spread in an efficient market." *The Journal of Finance* **39(4)**: pp. 1127–1139.

SCHWERT, G. W. (1989): "Tests for Unit Roots: A Monte Carlo Investigation." *Journal of Business & Economic Statistics* **7(2)**: pp. 147–59.

STATA CORP. (2013): *Stata time-series reference manual.* Stata Press.

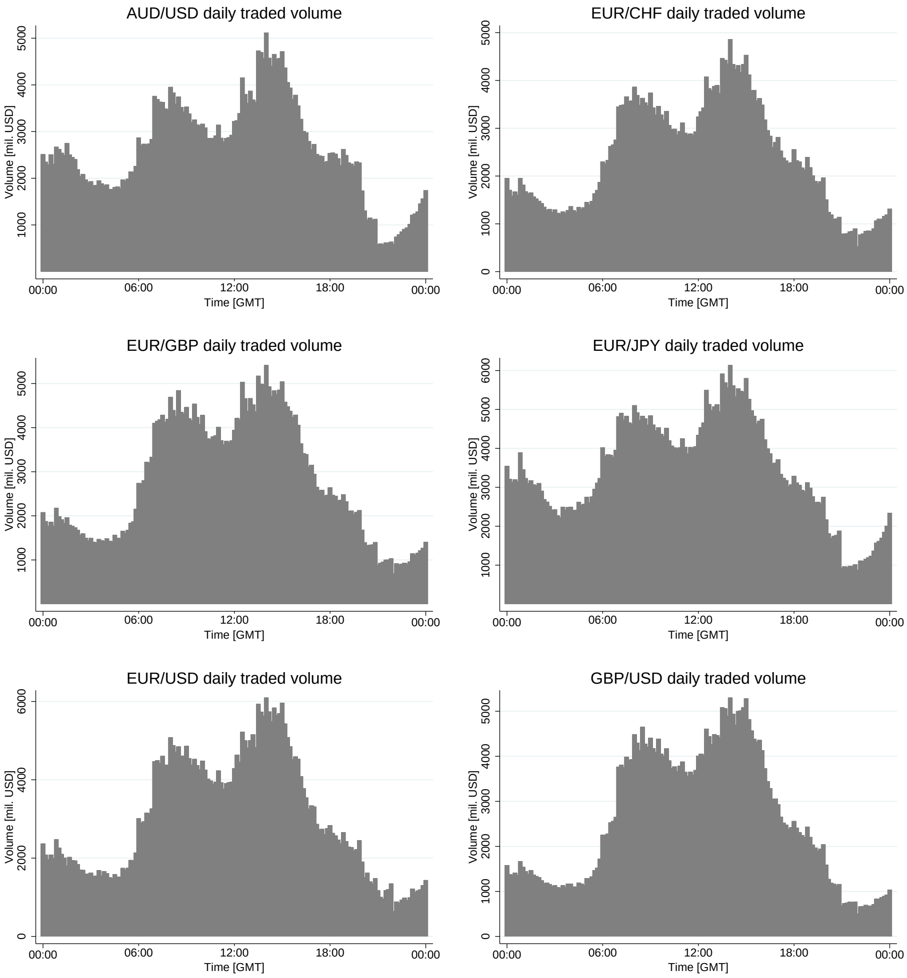
STOLL, H. R. (1978): "The supply of dealer services in securities markets." *The Journal of Finance* **33(4)**: pp. pp. 1133–1151.

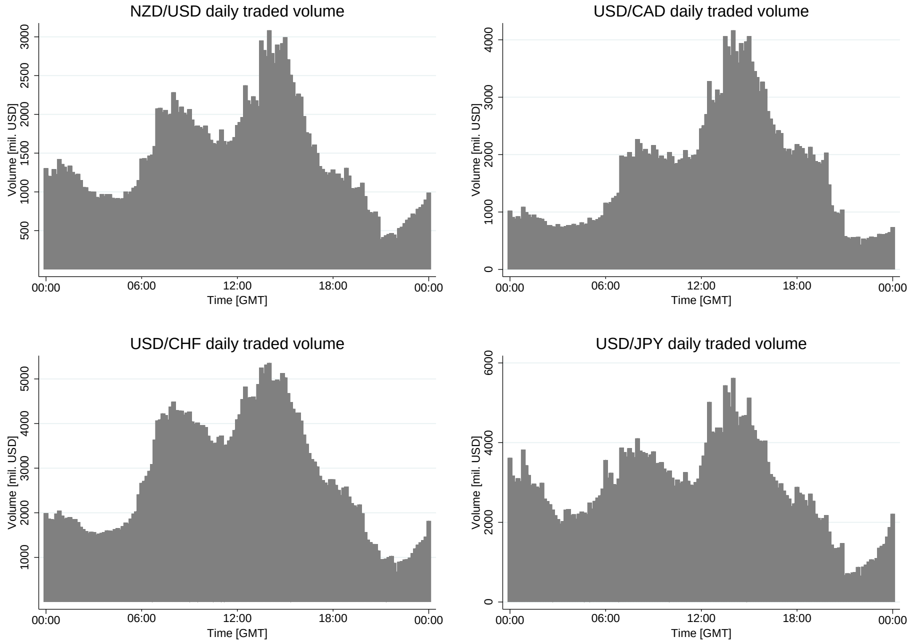
TSAY, R. S. & J. WOLTERS (2002): *Analysis of financial time series*. New York: Wiley.

Appendix A

Additional results

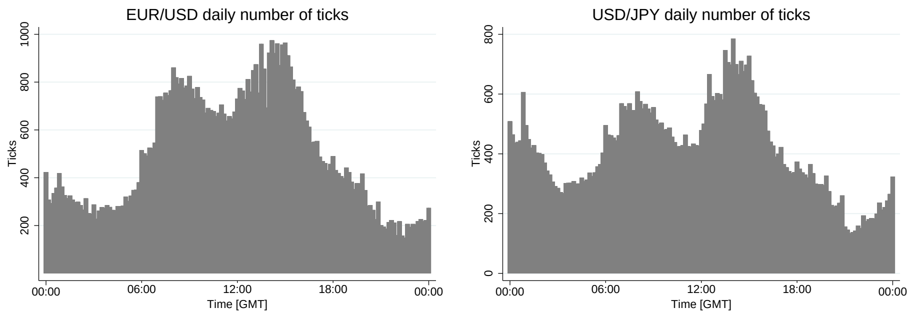
Figure A.1: Daily traded volume, 10 minute data





Source: Author's computations.

Figure A.2: Daily number of ticks, 10 minute data



Source: Author's computations.

Table A.1: Unit root and stationarity tests of differenced series

Currency pair	*Test*	lrv	lbpv	lamihudvol	lamihudtick	lamivestvol	lamivesttick	lroll	lbatick	lbawtick
AUD/USD	ADF	-41.416***	-40.658***	-41.642***	-42.894***	-39.595***	-41.321***	-50.881***	-42.787***	-42.081***
		(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)
	KPSS	0.01	0.012	0.018	0.012	0.017	0.013	0.004	0.019	0.016
EUR/CHF	ADF	-41.702***	-41.568***	-41.944***	-43.304***	-37.632***	-40.212***	-51.346***	-42.339***	-42.127***
		(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)
	KPSS	0.016	0.016	0.01	0.01	0.011	0.012	0.004	0.035	0.037
EUR/GBP	ADF	-46.812***	-45.64***	-41.829***	-43.439***	-38.555***	-40.926***	-51.242***	-43.254***	-43.468***
		(0.003)	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)
	KPSS	0.027	0.024	0.008	0.011	0.008	0.014	0.004	0.06	0.039
EUR/JPY	ADF	-42.385***	-41.905***	-47.967***	-47.747***	-38.098***	-41.679***	-50.848***	-44.532***	-44.099***
		(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)
	KPSS	0.012	0.013	0.008	0.006	0.017	0.014	0.004	0.053	0.096
EUR/USD	ADF	-44.861***	-45.846***	-45.798***	-43.25***	-43.941***	-42.828***	-52.805***	-44.412***	-44.421***
		(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)
	KPSS	0.034	0.033	0.043	0.015	0.054	0.018	0.005	0.092	0.092
GBP/USD	ADF	-45.41***	-45.046***	-41.906***	-42.378***	-37.369***	-39.942***	-51.041***	-44.917***	-44.496***
		(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)
	KPSS	0.03	0.029	0.013	0.011	0.014	0.014	0.004	0.053	0.074
NZD/USD	ADF	-43.683***	-42.809***	-40.304***	-40.142***	-37.045***	-38.659***	-52.104***	-43.07***	-42.542***
		(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)
	KPSS	0.014	0.013	0.011	0.014	0.013	0.014	0.005	0.018	0.025
USD/CAD	ADF	-44.681***	-44.631***	-39.012***	-40.609***	-38.765***	-41.165***	-52.813***	-42.961***	-43.154***
		(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)
	KPSS	0.018	0.016	0.009	0.014	0.008	0.015	0.004	0.045	0.059
USD/CHF	ADF	-46.446***	-46.272***	-40.687***	-40.944***	-37.31***	-39.989***	-50.977***	-43.785***	-43.456***
		(0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)
	KPSS	0.024	0.024	0.011	0.011	0.012	0.011	0.004	0.042	0.047
USD/JPY	ADF	-44.567***	-42.901***	-48.395***	-48.225***	-40.271***	-42.39***	-51.161***	-44.183***	-43.837***
		(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)
	KPSS	0.01	0.01	0.008	0.007	0.014	0.013	0.004	0.038	0.076

Significance levels: * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$; p-values in parentheses.
Source: Author's computations.

Table A.2: Johansen cointegration test

Model		lbpv lamihudvol		lbpv lamihudtick		lbpv lamivestvol		lbpv lamivesttick		lbpv lbatick		lbpv lbawtick	
Currency pair	*Rank*(Π)	Trace test	Max eigenvalue	Trace test	Max eigenvalue	Trace test	Max eigenvalue	Trace test	Max eigenvalue	Trace test	Max eigenvalue	Trace test	Max eigenvalue
AUD/USD	0	22.56***	13.78*	28.09***	22.33***	20.43***	10.72	27.07***	20.37***	18.51**	16.3**	21.64***	19.4***
		(0.003)	(0.058)	(<0.001)	(0.002)	(0.007)	(0.172)	(<0.001)	(0.004)	(0.016)	(0.022)	(0.004)	(0.006)
	1	8.78***	8.78***	5.76**	5.76**	9.71***	9.71***	6.7**	6.7**	2.21	2.21	2.24	2.24
		(0.003)	(0.003)	(0.016)	(0.016)	(0.002)	(0.002)	(0.01)	(0.01)	(0.137)	(0.137)	(0.134)	(0.134)
EUR/CHF	0	24.98***	15.2**	27.19***	17.39**	18.05**	9.95	20.65***	12.84*	36.64***	29.72***	43.21***	34.39***
		(0.001)	(0.033)	(<0.001)	(0.014)	(0.019)	(0.22)	(0.007)	(0.082)	(<0.001)	(<0.001)	(<0.001)	(<0.001)
	1	9.78***	9.78***	9.79***	9.79***	8.11***	8.11***	7.81***	7.81***	6.92***	6.92***	8.82***	8.82***
		(0.002)	(0.002)	(0.002)	(0.002)	(0.004)	(0.004)	(0.005)	(0.005)	(0.009)	(0.009)	(0.003)	(0.003)
EUR/GBP	0	19.54**	11.94	22.87***	15.64**	15.86**	9.18	30.56***	20.32***	15.95**	14.31**	13.4	11.71
		(0.01)	(0.113)	(0.003)	(0.028)	(0.042)	(0.278)	(<0.001)	(0.004)	(0.041)	(0.047)	(0.101)	(0.123)
	1	7.61***	7.61***	7.23***	7.23***	6.68**	6.68**	10.24***	10.24***	1.63	1.63	1.69	1.69
		(0.006)	(0.006)	(0.007)	(0.007)	(0.01)	(0.01)	(0.001)	(0.001)	(0.201)	(0.201)	(0.193)	(0.193)
EUR/JPY	1	27.05***	18.81***	31.69***	23.7***	29.34***	19.62***	38.64***	29.06***	32.26***	30.89***	33.36***	32.18***
		(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)
	0	15.12***	15.12***	15.12***	15.12***	15.12***	15.12***	15.12***	15.12***	15.12***	15.12***	15.12***	15.12***
		(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)
EUR/USD	0	10.31	6.72	14.98*	8.59	11.86	8.3	14.27*	8.15	15.5**	15.5**	15.13*	15.12**
		(0.263)	(0.531)	(0.058)	(0.33)	(0.166)	(0.357)	(0.075)	(0.372)	(0.048)	(0.03)	(0.055)	(0.034)
	1	3.59*	3.59*	6.39**	6.39**	3.56*	3.56*	6.12**	6.12**	0	0	0.01	0.01
		(0.058)	(0.058)	(0.012)	(0.012)	(0.059)	(0.059)	(0.013)	(0.013)	(0.966)	(0.966)	(0.927)	(0.927)
GBP/USD	0	19.07**	14.32**	23.43***	18.53***	14.52*	10	20.93***	16.45**	10.46	9.63	11.21	10.09
		(0.012)	(0.047)	(0.002)	(0.009)	(0.069)	(0.216)	(0.006)	(0.02)	(0.251)	(0.243)	(0.202)	(0.21)
	1	4.75**	4.75**	4.89**	4.89**	4.52**	4.52**	4.48**	4.48**	0.84	0.84	1.12	1.12
		(0.029)	(0.029)	(0.027)	(0.027)	(0.033)	(0.033)	(0.034)	(0.034)	(0.36)	(0.36)	(0.289)	(0.289)
NZD/USD	0	25.39***	19.04***	40.09***	35.54***	27.87***	20.96***	37.45***	32.42***	21.21***	19.87***	25.33***	24.31***
		(0.001)	(0.007)	(<0.001)	(<0.001)	(<0.001)	(0.003)	(<0.001)	(<0.001)	(0.005)	(0.005)	(0.001)	(0.001)
	1	6.36**	6.36**	4.55**	4.55**	6.92***	6.92***	5.03**	5.03**	1.34	1.34	1.02	1.02
		(0.012)	(0.012)	(0.033)	(0.033)	(0.009)	(0.009)	(0.025)	(0.025)	(0.248)	(0.248)	(0.312)	(0.312)
USD/CAD	0	21.75***	16.51**	23.79***	18.64***	30.49***	24.17***	26.31***	18.29***	37.93***	36.1***	21.28***	20.57***
		(0.004)	(0.02)	(0.002)	(0.008)	(<0.001)	(0.001)	(0.001)	(0.009)	(<0.001)	(<0.001)	(0.005)	(0.004)
	1	5.24**	5.24**	5.15**	5.15**	6.32**	6.32**	8.02***	8.02***	1.83	1.83	0.72	0.72
		(0.022)	(0.022)	(0.023)	(0.023)	(0.012)	(0.012)	(0.005)	(0.005)	(0.176)	(0.176)	(0.397)	(0.397)
USD/CHF	0	18.96**	10.26	26.91***	19.33***	13.13	6.99	32.23***	21.71***	21.88***	19.63***	54.94***	48.46***
		(0.013)	(0.199)	(0.001)	(0.006)	(0.11)	(0.499)	(<0.001)	(0.002)	(0.004)	(0.005)	(<0.001)	(<0.001)
	1	8.7***	8.7***	7.58***	7.58***	6.15**	6.15**	10.53***	10.53***	2.25	2.25	6.47**	6.47**
		(0.003)	(0.003)	(0.006)	(0.006)	(0.013)	(0.013)	(0.001)	(0.001)	(0.133)	(0.133)	(0.011)	(0.011)
USD/JPY	0	25.89***	18.69***	25.4***	16.49**	23.86***	17.6**	28.38***	16.58**	18.11**	17.15**	16.4**	15.67**
		(0.001)	(0.008)	(0.001)	(0.02)	(0.002)	(0.013)	(<0.001)	(0.019)	(0.018)	(0.015)	(0.035)	(0.028)
	1	7.2***	7.2***	8.91***	8.91***	6.26**	6.26**	11.81***	11.81***	0.96	0.96	0.73	0.73
		(0.007)	(0.007)	(0.003)	(0.003)	(0.012)	(0.012)	(0.001)	(0.001)	(0.328)	(0.328)	(0.392)	(0.392)

Significance levels: * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$; p-values in parentheses.
Source: Author's computations.

Table A.3: ARCH effect, VAR/VECM specification test

VAR/VECM eq. *Currency pair*	lamihudvol ⇒ lbpv	lbpv ⇒ lamihudvol	lamihudtick ⇒ lbpv	lbpv ⇒ lamihudtick	lamivestvol ⇒ lbpv	lbpv ⇒ lamivestvol	lamivesttick lbpv ⇒	lbpv ⇒ lamivesttick	lbatick ⇒ lbpv	lbpv ⇒ lbatick	lbawtick ⇒ lbpv	lbpv ⇒ lbawtick	lroll ⇒ lbpv	lbpv ⇒ lroll
AUD/USD	29.993***	158.656***	30.736***	230.533***	29.142***	107.001***	29.606***	147.379***	22.439***	153.991***	21.398***	155.505***	28.04***	2.399
	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(0.001)	(<0.001)	(<0.001)	(0.792)
EUR/CHF	48.058***	218.68***	44.727***	150.09***	45.828***	168.873***	42.083***	157.362***	46.276***	94.88***	45.396***	97.278***	49.747***	22.283***
	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)
EUR/GBP	15.86***	67.147***	15.598***	112.38***	18.108***	146.211***	13.892**	137.15***	13.259**	147.989***	12.669**	144.985***	16.004***	7.917
	(0.007)	(<0.001)	(0.008)	(<0.001)	(0.003)	(<0.001)	(0.016)	(<0.001)	(0.021)	(<0.001)	(0.027)	(<0.001)	(0.007)	(0.161)
EUR/JPY	15.527***	51.211***	18.413***	32.863***	23.406***	117.991***	22.255***	171.465***	19.697***	127.059***	18.925***	126.436***	22.61***	3.261
	(0.008)	(<0.001)	(0.002)	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(0.001)	(<0.001)	(0.002)	(<0.001)	(<0.001)	(0.66)
EUR/USD	245.776***	254.119***	253.26***	339.131***	314.48***	538.568***	292.654***	557.853***	217.573***	259.712***	220.157***	257.403***	269.355***	1.942
	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(0.857)
GBP/USD	9.06	102.548***	13.943**	93.03***	12.291**	90.23***	11.635**	204.041***	16.944***	189.368***	17.066***	196.626***	14.922**	4.596
	(0.107)	(<0.001)	(0.016)	(<0.001)	(0.031)	(<0.001)	(0.04)	(<0.001)	(0.005)	(<0.001)	(0.004)	(<0.001)	(0.011)	(0.467)
NZD/USD	18.099***	126.095***	17.567***	97.855***	17.355***	92.047***	18.257***	90.377***	12.801**	180.539***	12.178**	179.825***	14.979**	4.17
	(0.003)	(<0.001)	(0.004)	(<0.001)	(0.004)	(<0.001)	(0.003)	(<0.001)	(0.025)	(<0.001)	(0.032)	(<0.001)	(0.01)	(0.525)
USD/CAD	6.967	109.014***	9.037	179.278***	8.773	156.984***	9.133	233.435***	9.414*	324.271***	8.963	322.645***	8.804	8.19
	(0.223)	(<0.001)	(0.108)	(<0.001)	(0.118)	(<0.001)	(0.104)	(<0.001)	(0.094)	(<0.001)	(0.111)	(<0.001)	(0.117)	(0.146)
USD/CHF	32.58***	99.423***	38.263***	38.263***	32.671***	284.551***	33.872***	175.748***	30.566***	128.602***	30.022***	135.646***	28.183***	2.957
	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(0.707)
USD/JPY	14.064**	40.958***	14.479**	38.857***	15.272***	120.922***	15.365***	84.755***	17.511***	94.175***	17.422***	98.869***	19.409***	7.294
	(0.015)	(<0.001)	(0.013)	(<0.001)	(0.009)	(<0.001)	(0.009)	(<0.001)	(0.004)	(<0.001)	(0.004)	(<0.001)	(0.002)	(0.2)

Significance levels: * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$; p-values in parentheses.
Source: Author's computations.

Table A.4: Ljung box, VAR/VECM specification test

VAR/VECM eq. *Currency pair*	lamihudvol ⇒ lbpv	lbpv ⇒ lamihudvol	lamihudtick ⇒ lbpv	lbpv ⇒ lamihudtick	lamivestvol ⇒ lbpv	lbpv ⇒ lamivestvol	lamivesttick lbpv ⇒	lbpv ⇒ lamivesttick	lbatick ⇒ lbpv	lbpv ⇒ lbatick	lbawtick ⇒ lbpv	lbpv ⇒ lbawtick	lroll ⇒ lbpv	lbpv ⇒ lroll
AUD/USD	0.035	0.039	0.116	0.116	0.024	0.007	0.144	0.047	0.042	0.188	0.042	0.166	0.034	0.027
	(>0.999)	(>0.999)	(>0.999)	(>0.999)	(>0.999)	(>0.999)	(>0.999)	(>0.999)	(>0.999)	(0.999)	(>0.999)	(0.999)	(>0.999)	(>0.999)
EUR/CHF	0.019	0.1	0.027	0.057	0.014	0.278	0.021	0.114	0.01	0.278	0.012	0.235	0.007	0.183
	(>0.999)	(>0.999)	(>0.999)	(>0.999)	(>0.999)	(0.998)	(>0.999)	(>0.999)	(>0.999)	(0.998)	(>0.999)	(0.999)	(>0.999)	(0.999)
EUR/GBP	0.104	0.273	0.085	0.1	0.026	0.255	0.104	0.075	0.15	0.035	0.145	0.039	1.742	0.005
	(>0.999)	(0.998)	(>0.999)	(>0.999)	(>0.999)	(0.998)	(>0.999)	(>0.999)	(>0.999)	(>0.999)	(>0.999)	(>0.999)	(0.884)	(>0.999)
EUR/JPY	0.021	1.208	0.02	0.168	0.005	0.183	0.829	0.074	0.075	0.17	0.078	0.171	0.528	0.023
	(>0.999)	(0.944)	(>0.999)	(0.999)	(>0.999)	(0.999)	(0.975)	(>0.999)	(>0.999)	(0.999)	(>0.999)	(0.999)	(0.991)	(>0.999)
EUR/USD	0.095	0.079	0.086	0.025	0.113	0.02	0.111	0.016	0.118	0.042	0.128	0.042	0.041	0.021
	(>0.999)	(>0.999)	(>0.999)	(>0.999)	(>0.999)	(>0.999)	(>0.999)	(>0.999)	(>0.999)	(>0.999)	(>0.999)	(>0.999)	(>0.999)	(>0.999)
GBP/USD	0.083	0.209	0.198	0.011	0.1	0.144	0.173	0.008	0.061	0.05	0.06	0.052	0.124	0.022
	(>0.999)	(0.999)	(0.999)	(>0.999)	(>0.999)	(>0.999)	(0.999)	(>0.999)	(>0.999)	(>0.999)	(>0.999)	(>0.999)	(>0.999)	(>0.999)
NZD/USD	0.041	0.756	0.077	0.09	0.06	0.354	0.058	0.098	0.149	0.344	0.14	0.326	0.107	0.059
	(>0.999)	(0.98)	(>0.999)	(>0.999)	(>0.999)	(0.996)	(>0.999)	(>0.999)	(>0.999)	(0.997)	(>0.999)	(0.997)	(>0.999)	(>0.999)
USD/CAD	0.061	0.047	0.055	0.06	0.046	0.336	0.054	0.019	0.114	0.194	0.117	0.208	1.253	0.008
	(>0.999)	(>0.999)	(>0.999)	(>0.999)	(>0.999)	(0.997)	(>0.999)	(>0.999)	(>0.999)	(0.999)	(>0.999)	(0.999)	(0.94)	(>0.999)
USD/CHF	0.029	0.131	0.14	0.14	0.173	0.11	0.009	0.052	0.121	0.031	0.134	0.134	0.486	0.036
	(>0.999)	(>0.999)	(>0.999)	(>0.999)	(0.999)	(>0.999)	(>0.999)	(>0.999)	(>0.999)	(>0.999)	(>0.999)	(>0.999)	(0.993)	(>0.999)
USD/JPY	0.004	0.543	0.016	0.392	0.087	0.111	0.085	0.093	0.074	0.096	0.085	0.084	0.892	0.023
	(>0.999)	(0.99)	(>0.999)	(0.996)	(>0.999)	(>0.999)	(>0.999)	(>0.999)	(>0.999)	(>0.999)	(>0.999)	(>0.999)	(0.971)	(>0.999)

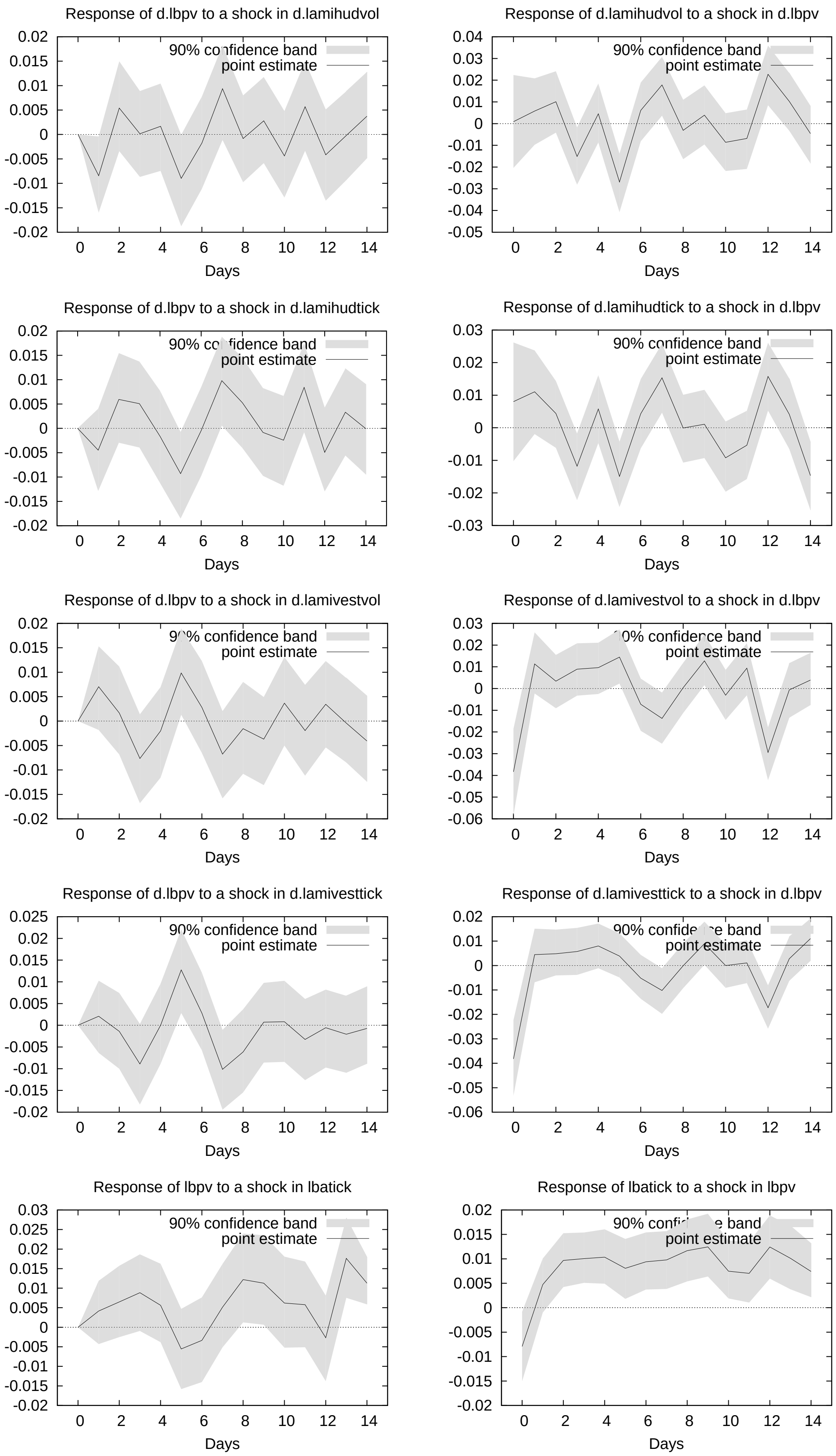
Significance levels: * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$; p-values in parentheses.
Source: Author's computations.

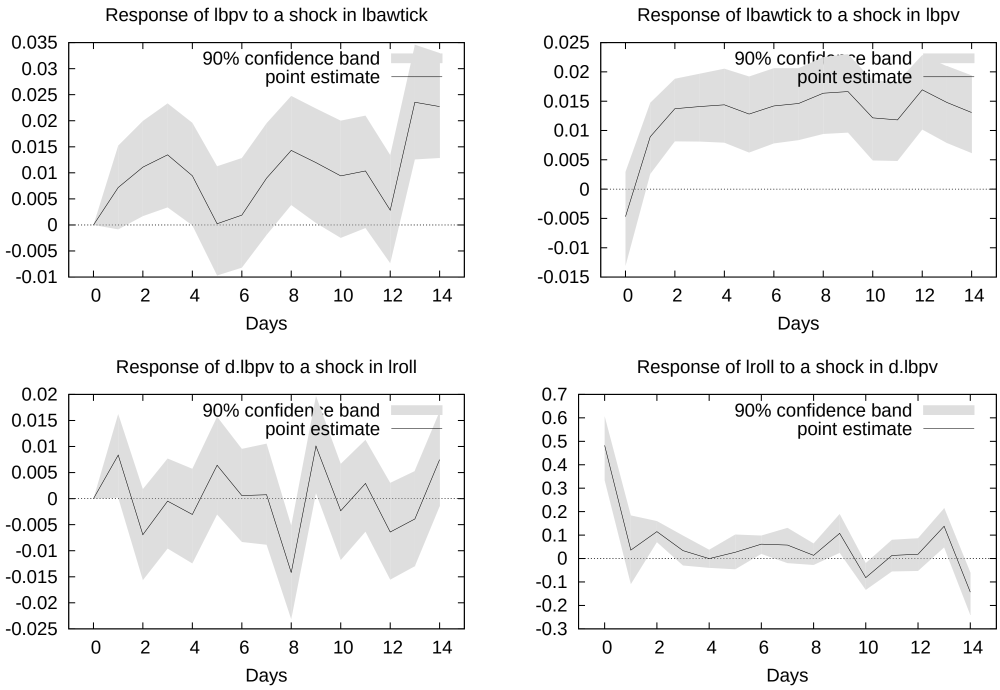
Table A.5: Normality test, VAR/VECM specification test

VAR/VECM eq. *Currency pair*	lamihudvol ⇒ lbpv	lbpv ⇒ lamihudvol	lamihudtick ⇒ lbpv	lbpv ⇒ lamihudtick	lamivestvol ⇒ lbpv	lbpv ⇒ lamivestvol	lamivesttick lbpv ⇒	lbpv ⇒ lamivesttick	lbatick ⇒ lbpv	lbpv ⇒ lbatick	lbawtick ⇒ lbpv	lbpv ⇒ lbawtick	lroll ⇒ lbpv	lbpv ⇒ lroll
AUD/USD	1891.36***	1891.36***	2255.29***	2255.29***	2492.59***	2492.59***	2148.69***	2148.69***	1086.07***	1086.07***	1070.8***	1070.8***	1667.84***	1667.84***
	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)
EUR/CHF	2335.38***	2335.38***	1336.79***	1336.79***	2520.29***	2520.29***	1023.85***	1023.85***	2032.54***	2032.54***	2011.77***	2011.77***	3142.68***	3142.68***
	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)
EUR/GBP	3273.22***	3273.22***	2129.24***	2129.24***	2258.11***	2258.11***	1076.54***	1076.54***	1186.82***	1186.82***	1207.55***	1207.55***	2295.01***	2295.01***
	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)
EUR/JPY	788.007***	788.007***	1311.65***	1311.65***	1578.28***	1578.28***	748.667***	748.667***	790.922***	790.922***	786.514***	786.514***	1679.11***	1679.11***
	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)
EUR/USD	3913.76***	3913.76***	7293.43***	7293.43***	11512.3***	11512.3***	11635.3***	11635.3***	5638.17***	5638.17***	5627.46***	5627.46***	5302.03***	5302.03***
	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)
GBP/USD	1943.88***	1943.88***	1946.32***	1946.32***	2159.18***	2159.18***	943.568***	943.568***	1073.76***	1073.76***	1086.08***	1086.08***	1745.21***	1745.21***
	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)
NZD/USD	1427.48***	1427.48***	934.388***	934.388***	1505.73***	1505.73***	1274.35***	1274.35***	1244.61***	1244.61***	1307.96***	1307.96***	1954.85***	1954.85***
	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)
USD/CAD	2850.14***	2850.14***	2012.41***	2012.41***	1452.09***	1452.09***	949.143***	949.143***	1309.95***	1309.95***	1324.57***	1324.57***	1847.82***	1847.82***
	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)
USD/CHF	3512.79***	3512.79***	1498.24***	1498.24***	2099.56***	2099.56***	952.782***	952.782***	1217.48***	1217.48***	1241.31***	1241.31***	1689.19***	1689.19***
	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)
USD/JPY	735.402***	735.402***	914.391***	914.391***	2129.61***	2129.61***	614.595***	614.595***	843.02***	843.02***	841.239***	841.239***	2033.67***	2033.67***
	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)

Significance levels: * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$; p-values in parentheses.
Source: Author's computations.

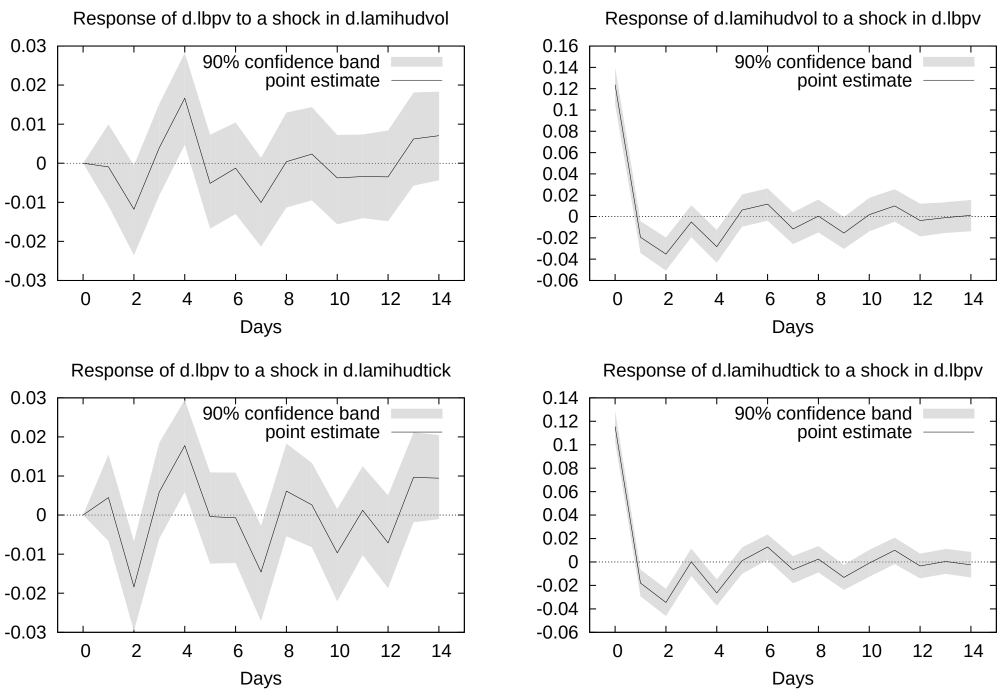
Figure A.3: Impulse response functions AUD/USD

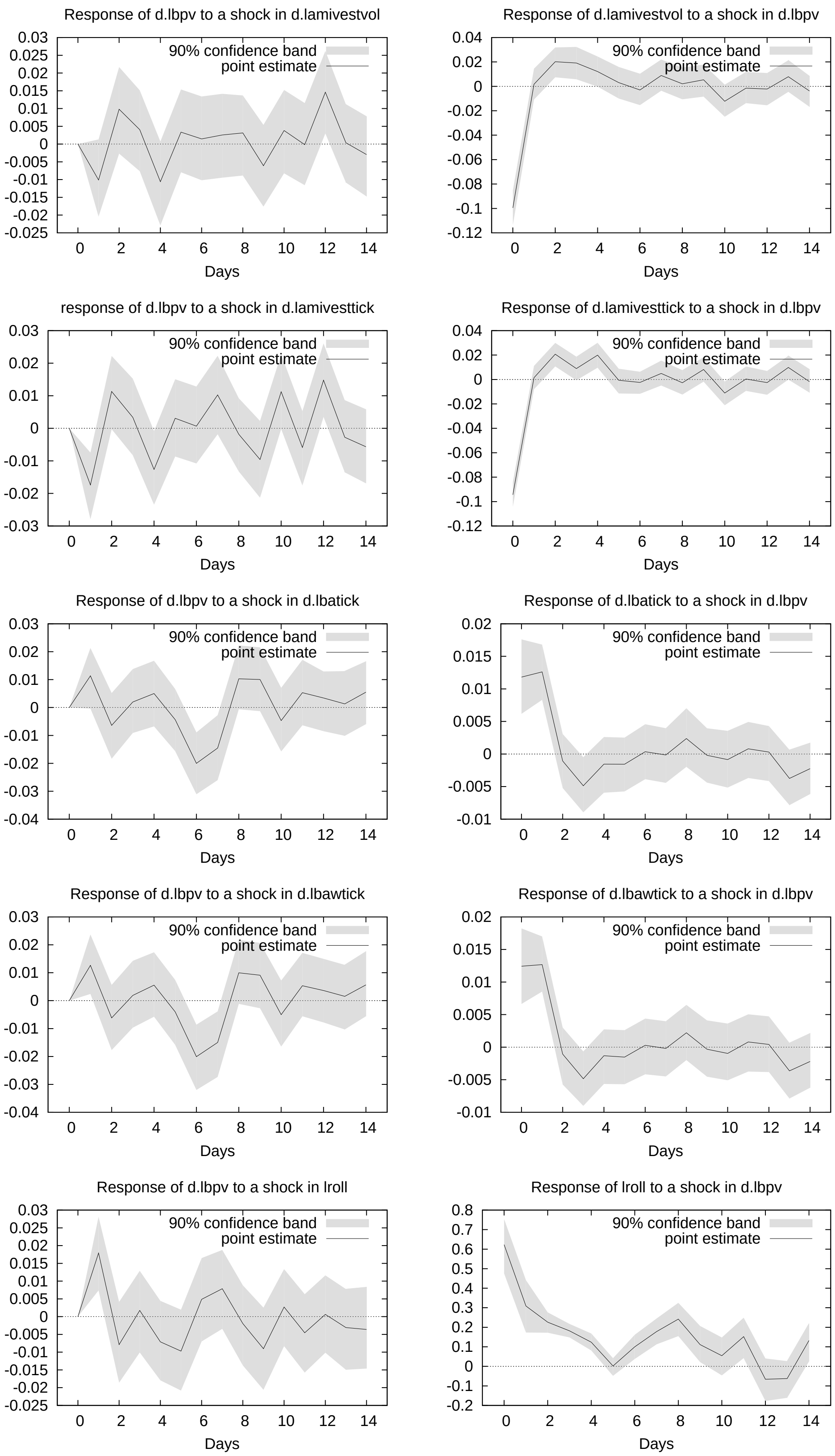




Source: Author's computations.

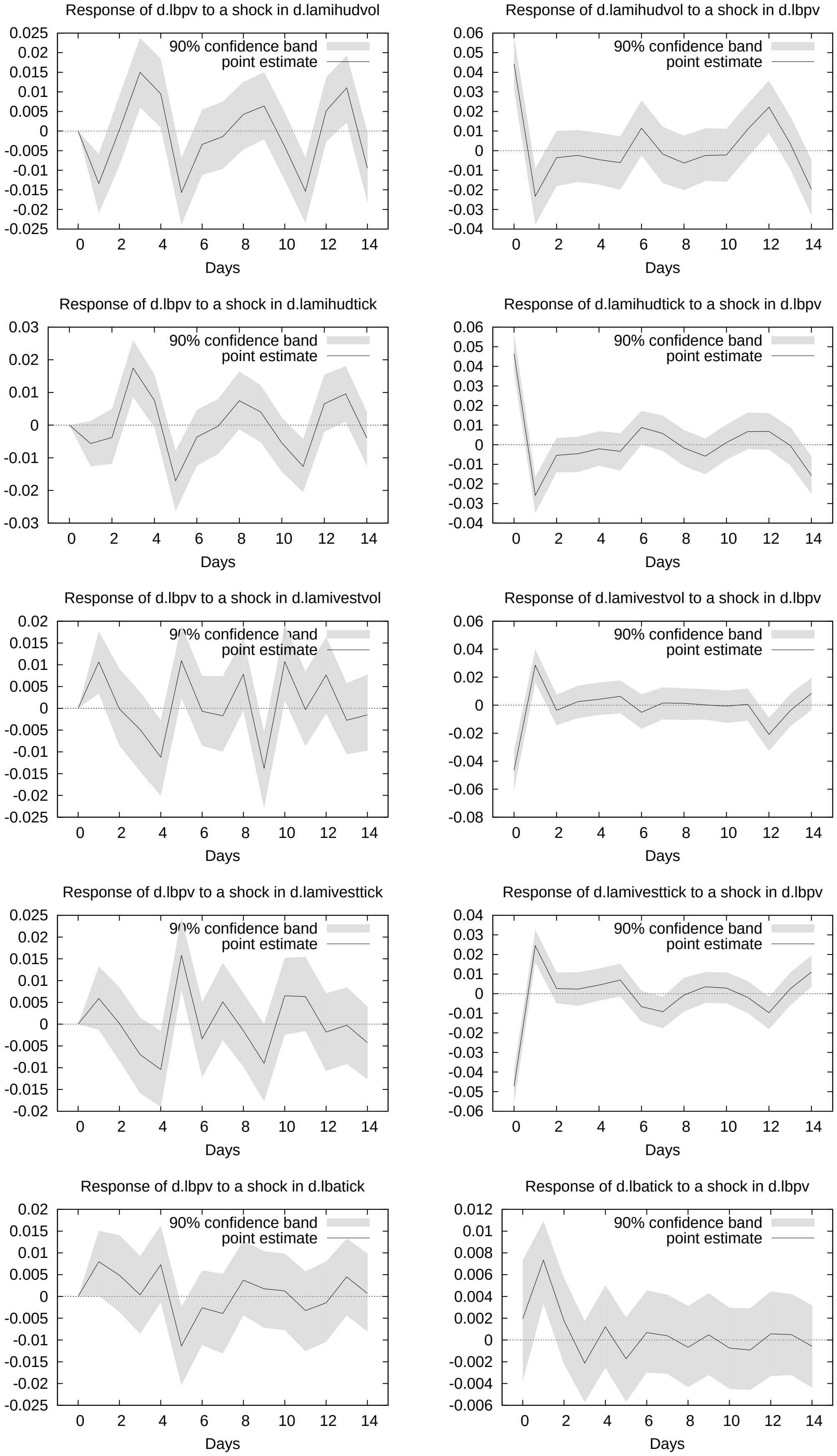
Figure A.4: Impulse response functions EUR/CHF

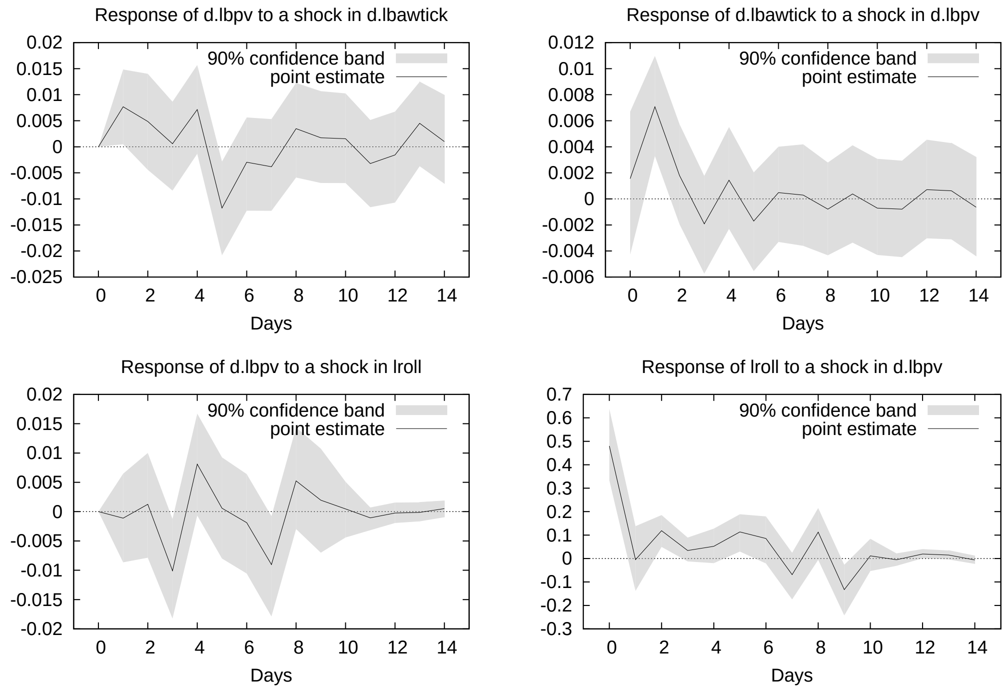




Source: Author's computations.

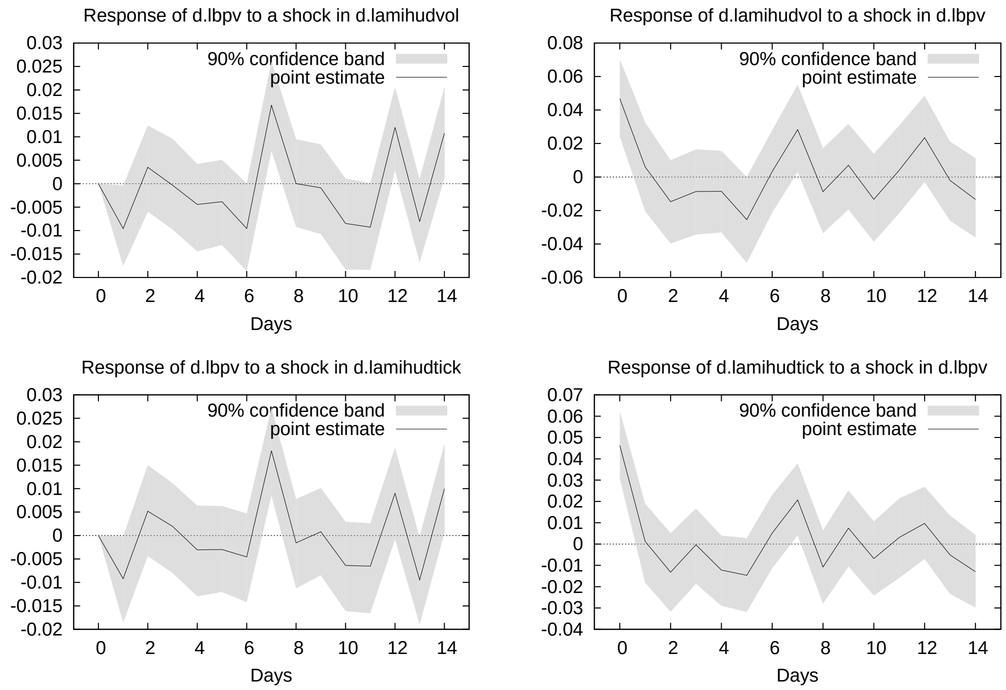
Figure A.5: Impulse response functions EUR/GBP

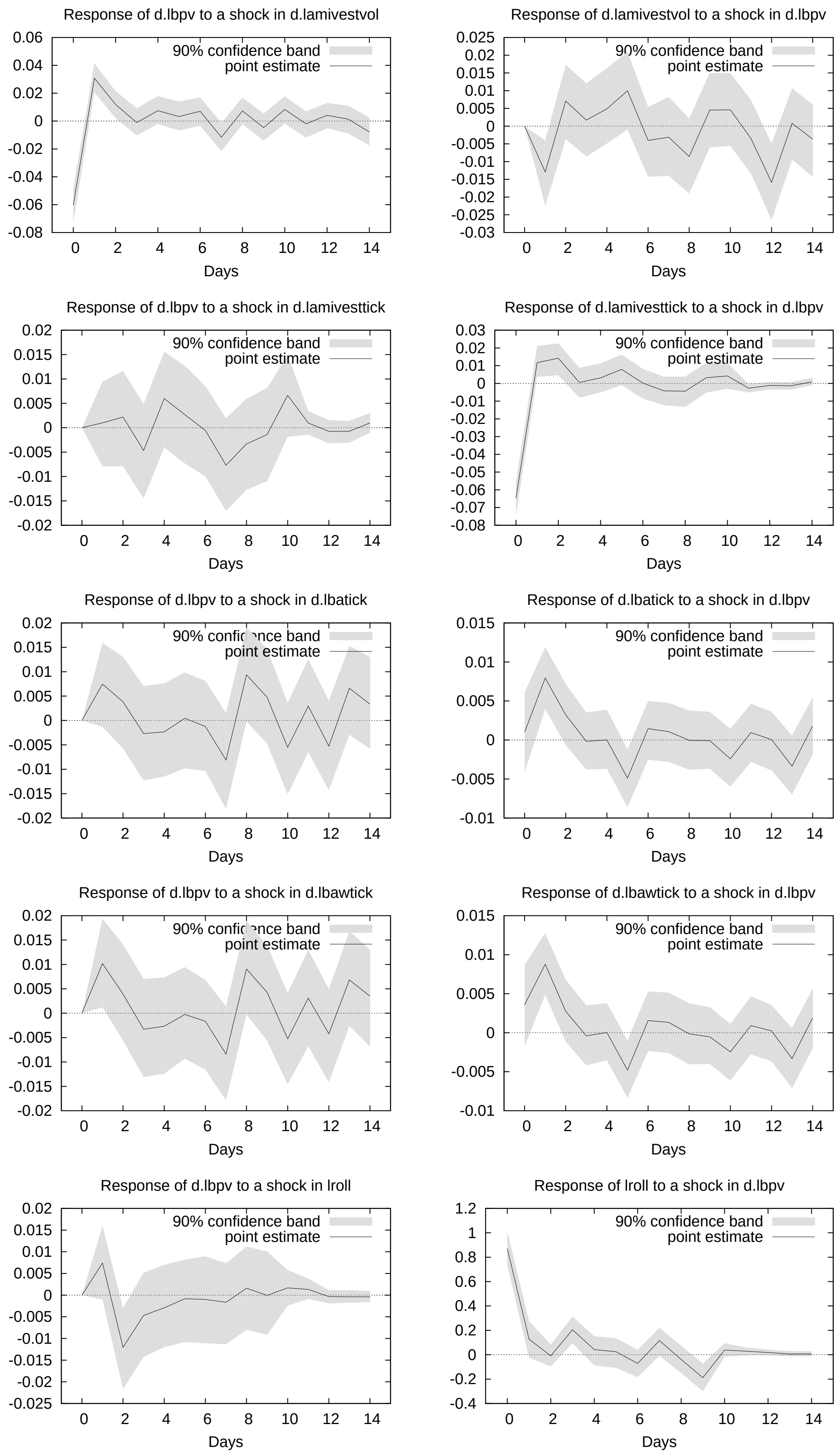




Source: Author's computations.

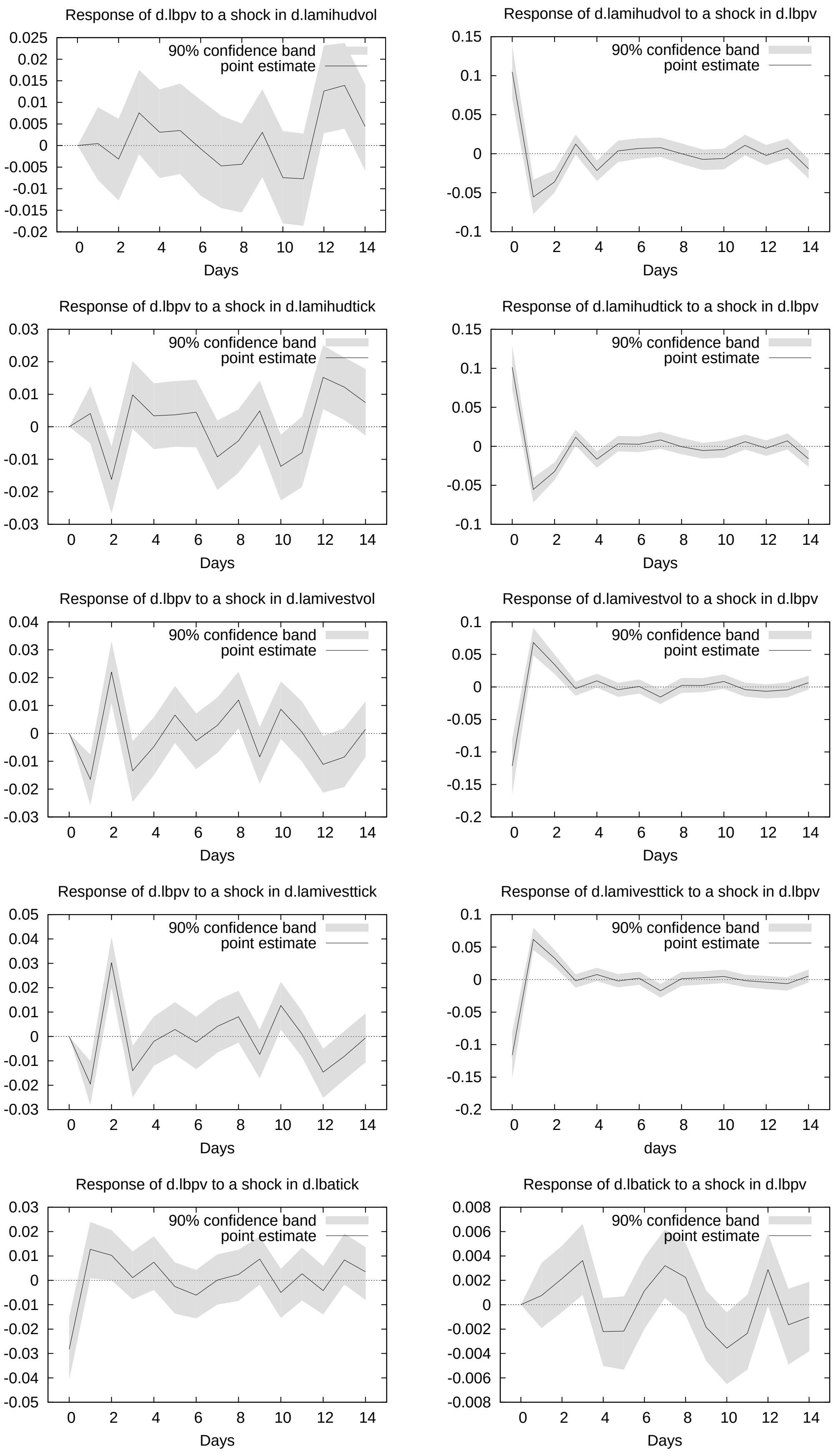
Figure A.6: Impulse response functions EUR/JPY

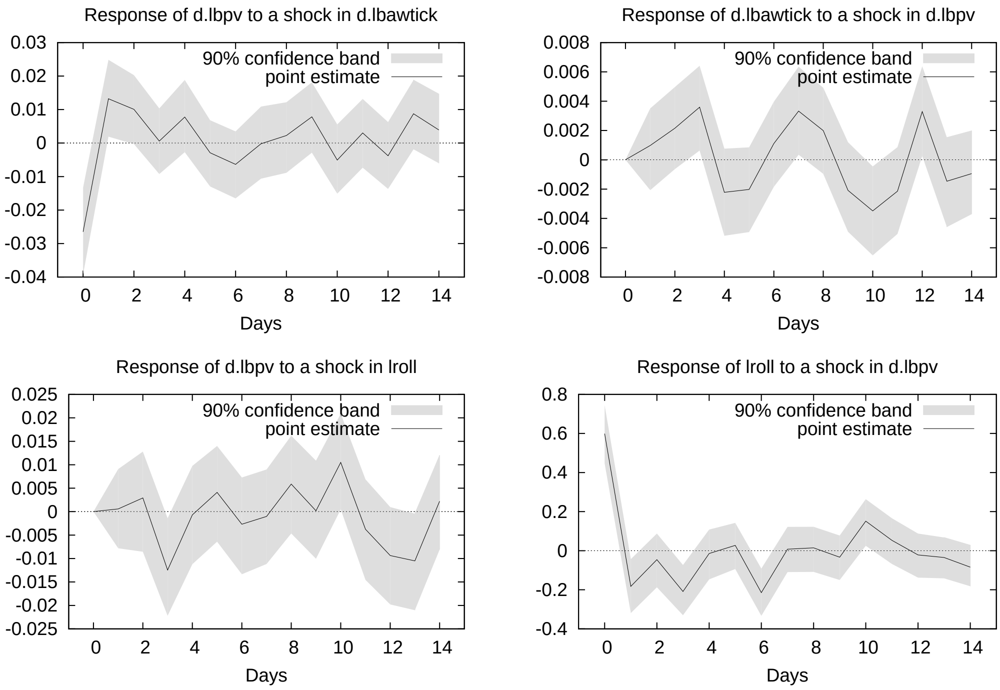




Source: Author's computations.

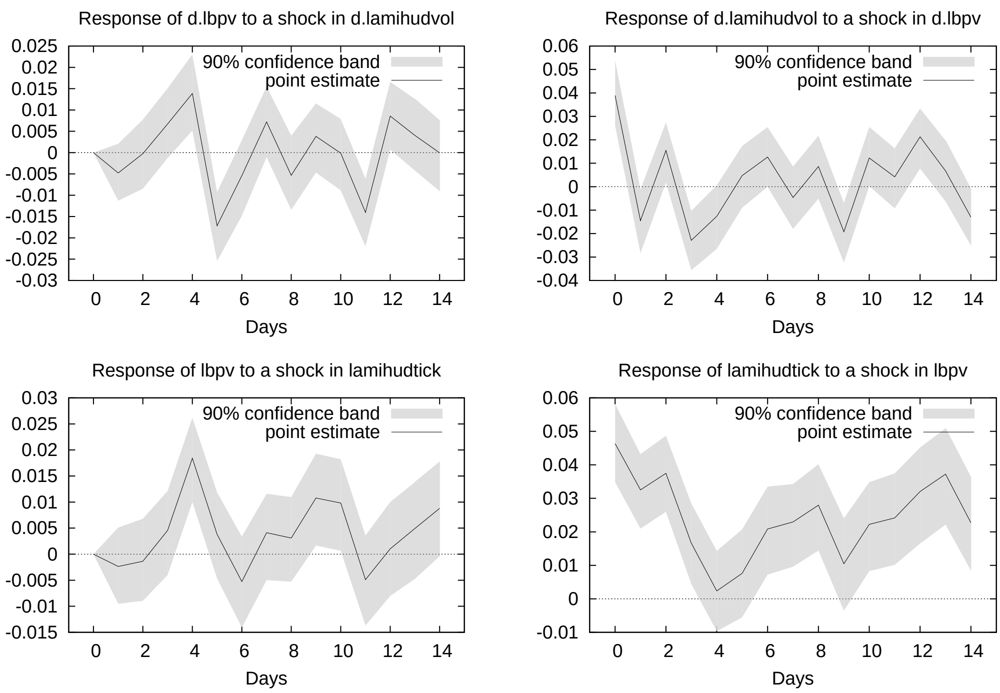
Figure A.7: Impulse response functions EUR/USD

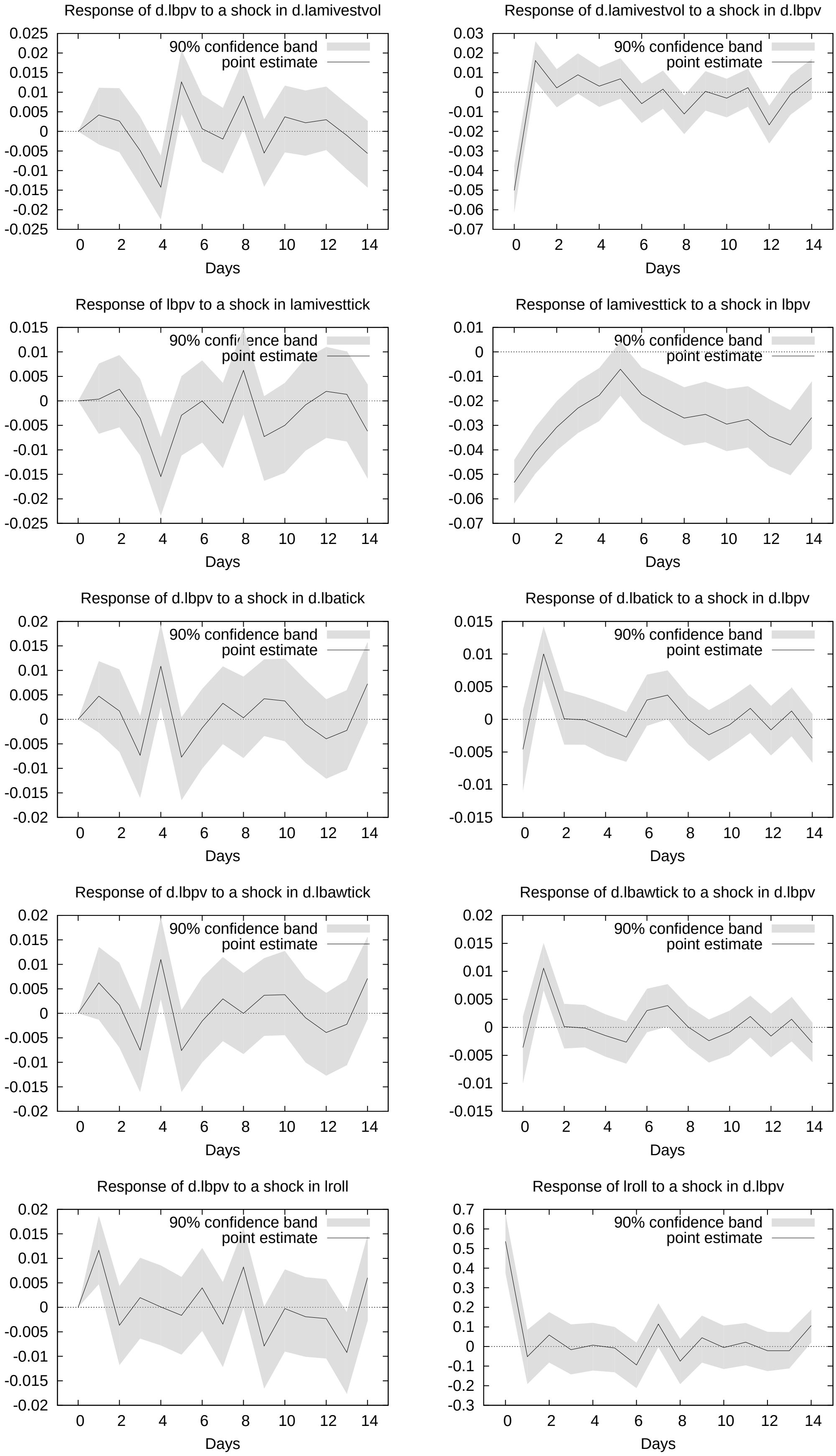




Source: Author's computations.

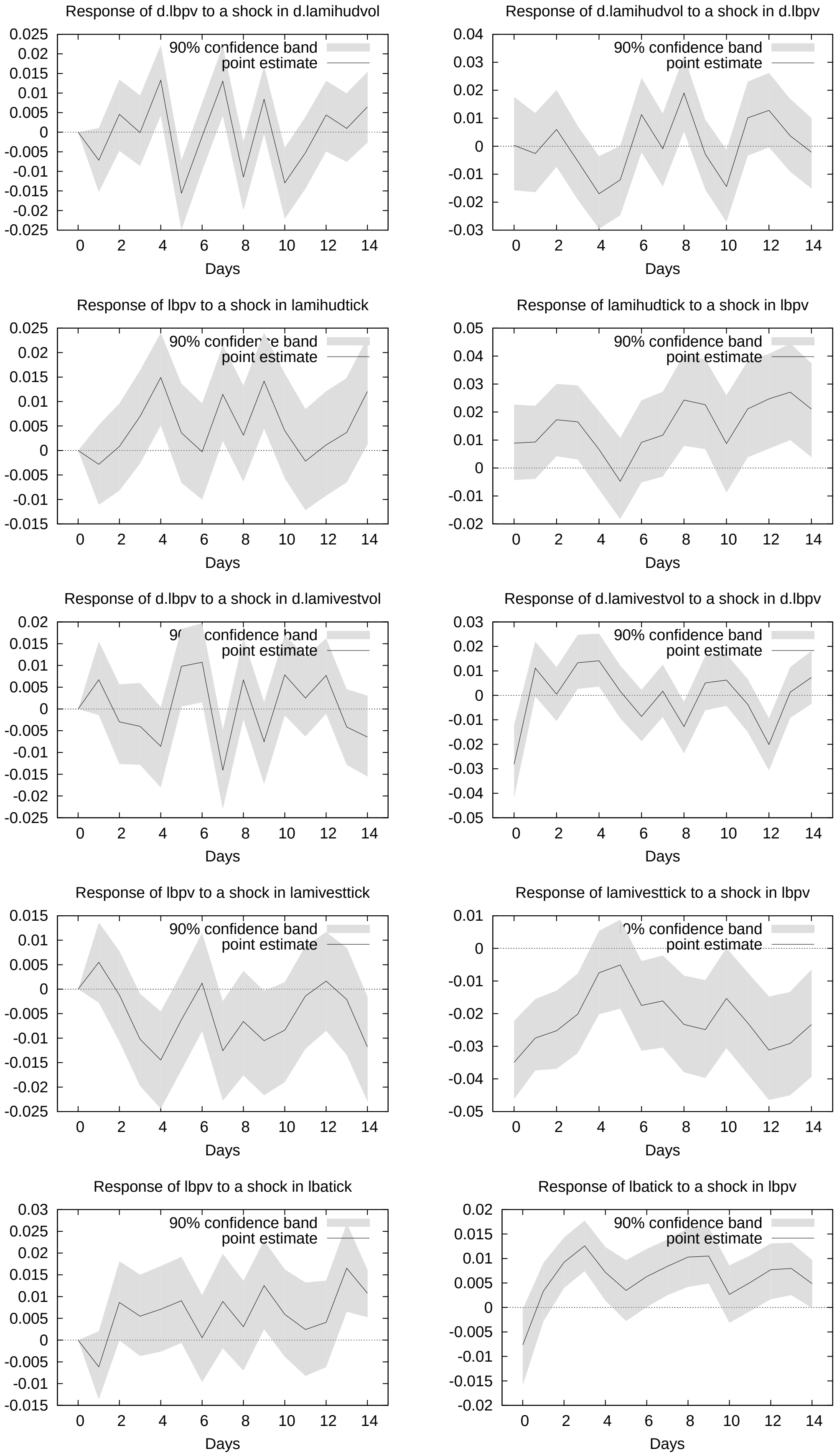
Figure A.8: Impulse response functions GBP/USD

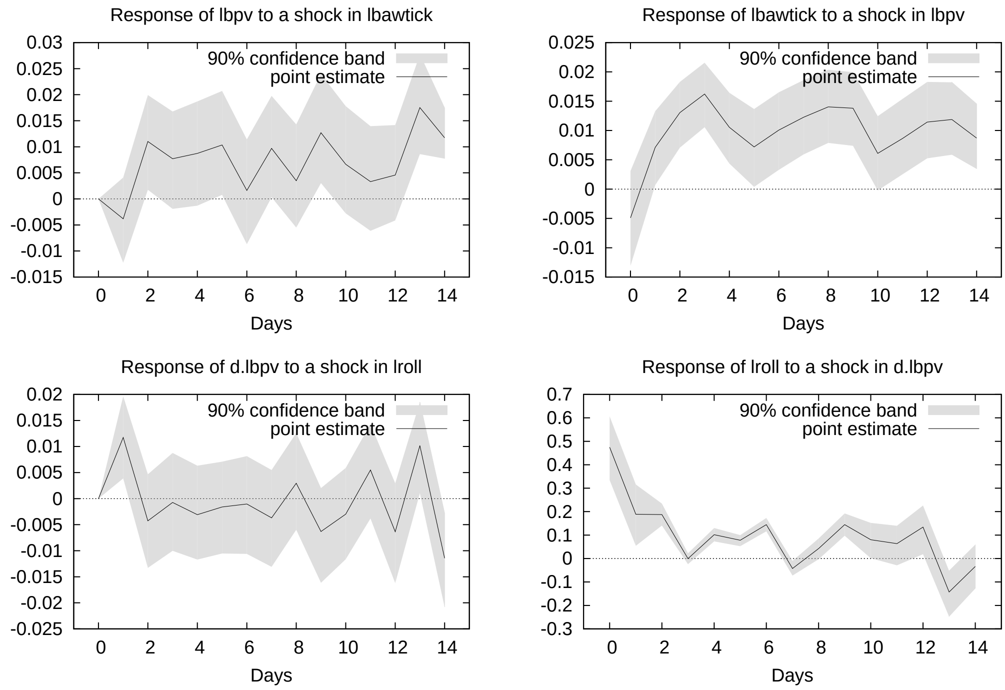




Source: Author's computations.

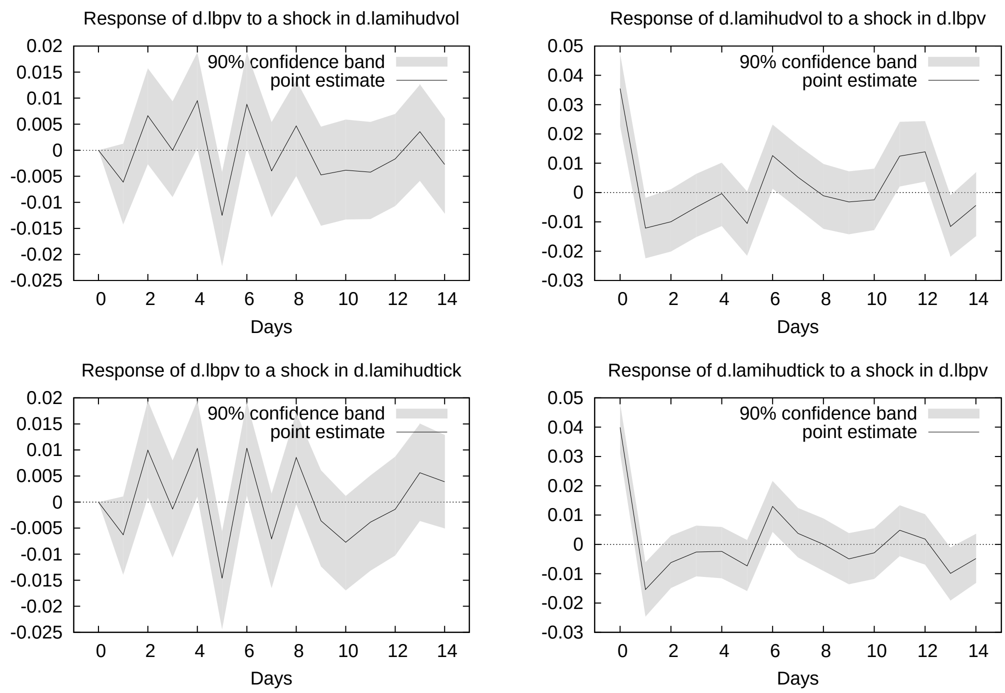
Figure A.9: Impulse response functions NZD/USD

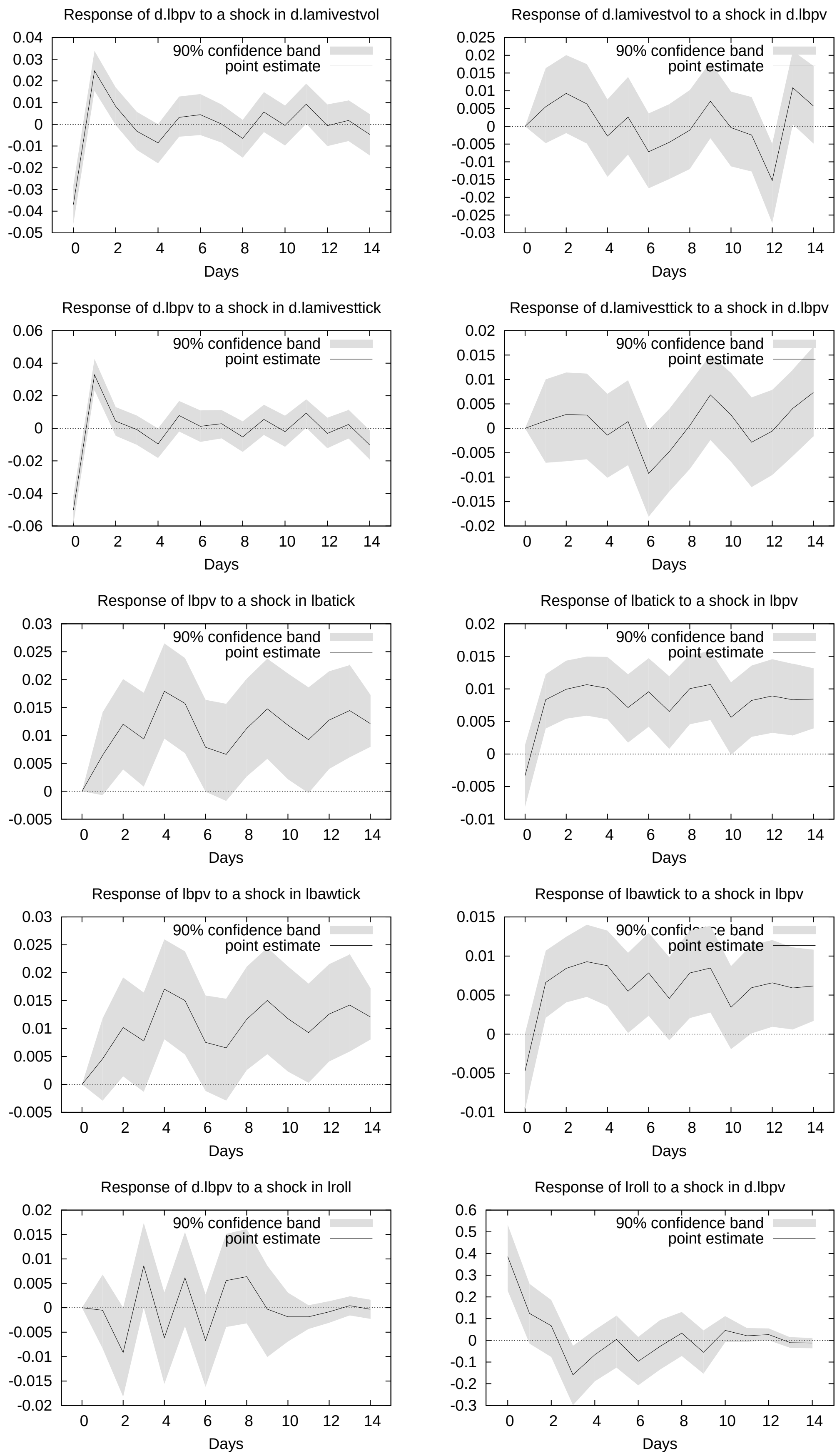




Source: Author's computations.

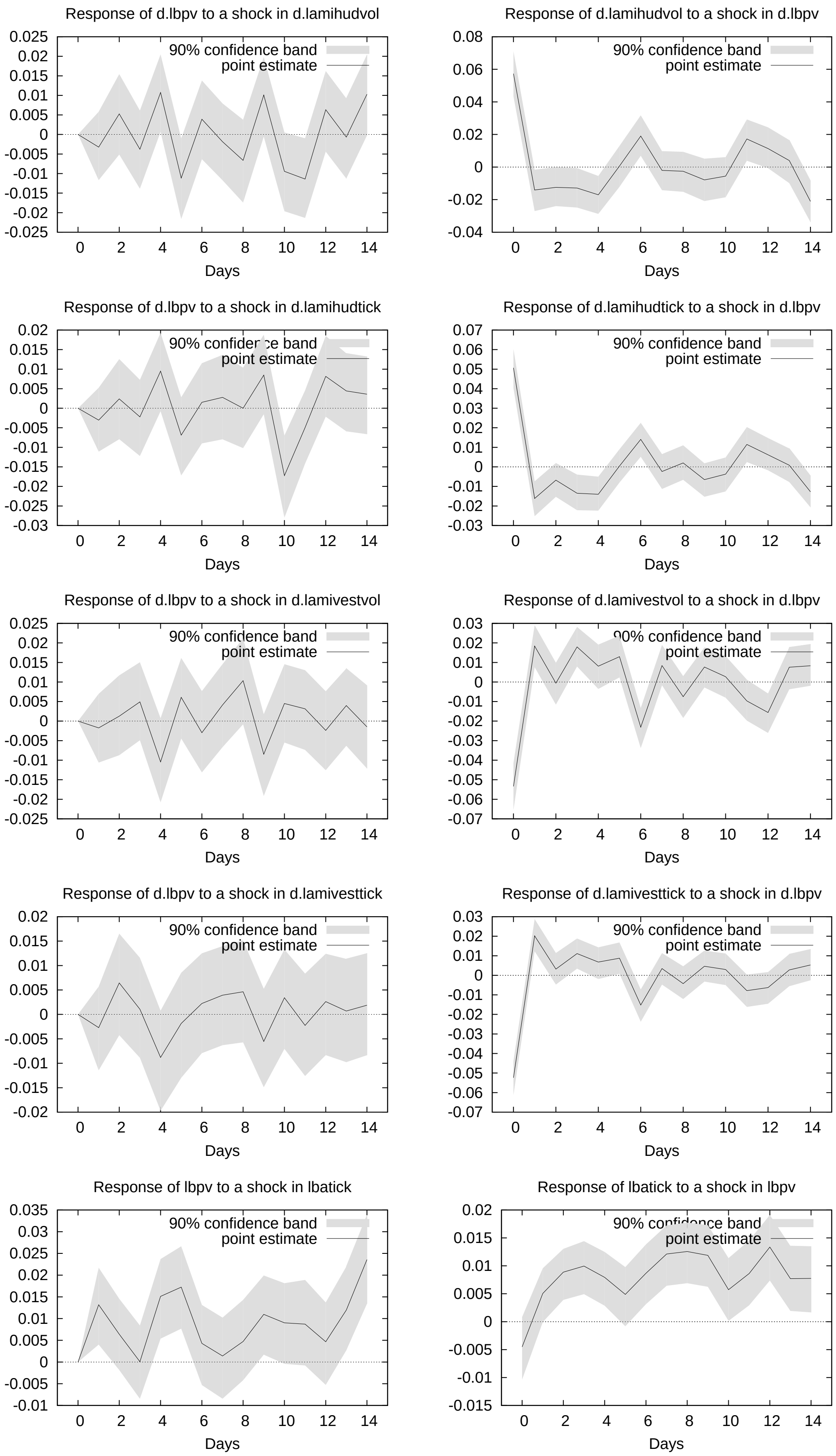
Figure A.10: Impulse response functions USD/CAD

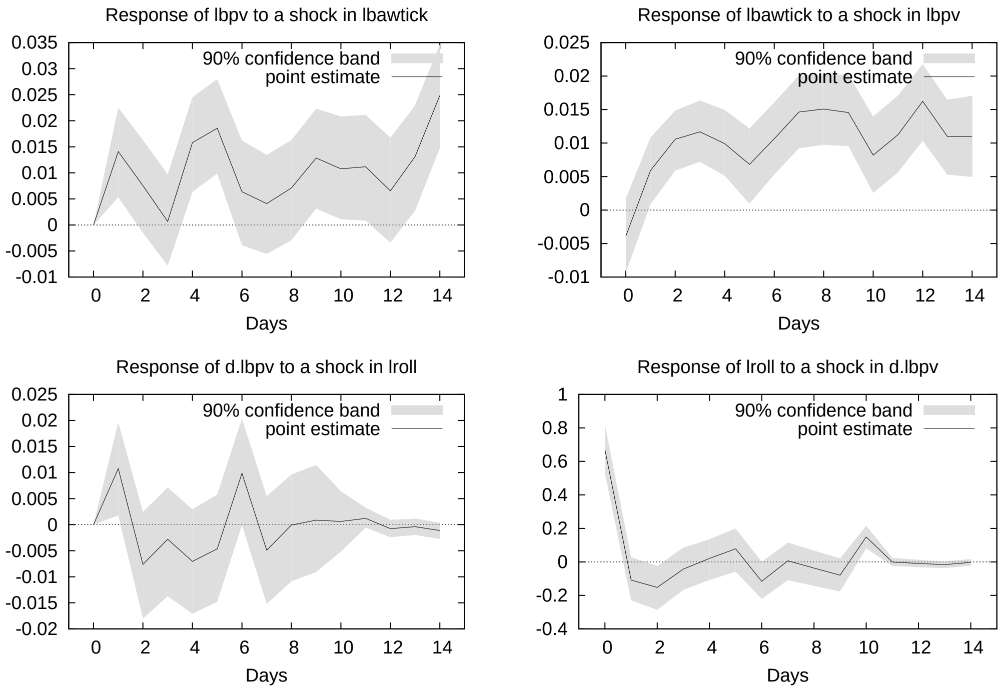




Source: Author's computations.

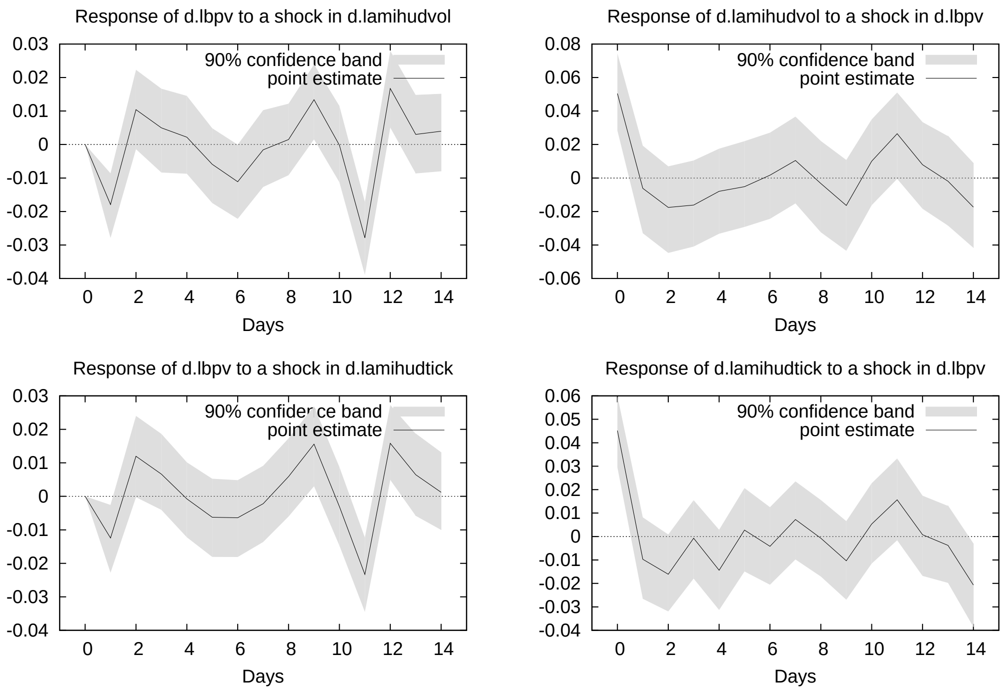
Figure A.11: Impulse response functions USD/CHF

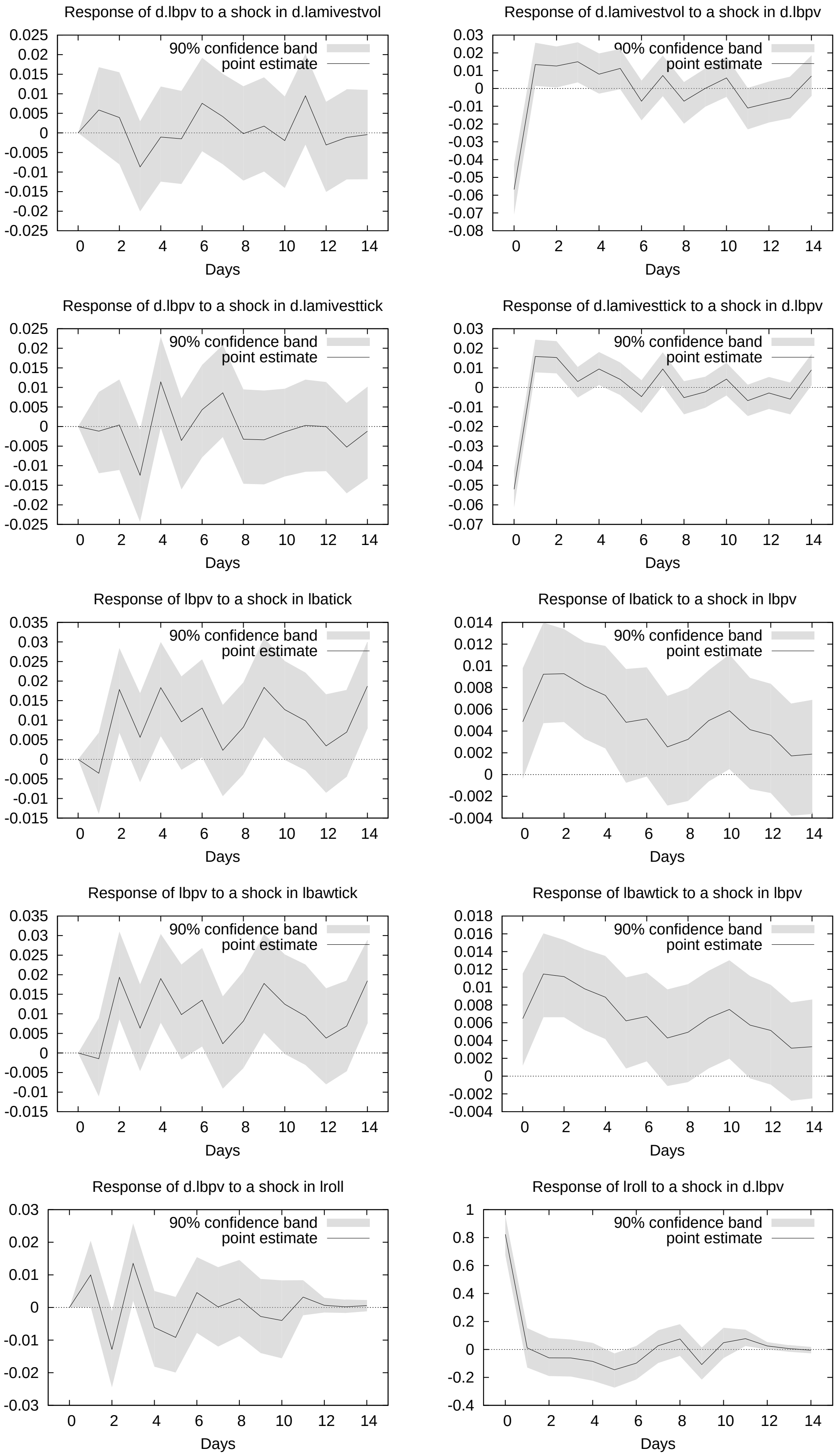




Source: Author's computations.

Figure A.12: Impulse response functions USD/JPY





Source: Author's computations.

Appendix B

List of enclosed scripts

This is a list of enclosed Python scripts which were used in processing of input data and computation of volatility and liquidity measures.

- Amihud-tick.py (uses NumPy)
- Amihud-volume.py (uses NumPy)
- Amivest-tick-median.py (uses NumPy)
- Amivest-volume-median.py (uses NumPy)
- Bipower-volatility.py
- Delete-weekend-files.py
- Merge-daily-files.py
- Proportional-ba-spread-tick.py
- Realized-volatility.py
- Roll.py (uses NumPy)
- Split-to-daily-files.py
- Split-to-daily-files-tick-data.py
- Ticks.py
- Tick-to-any.py
- Volumes.py