*Referee report on the dissertation thesis by*

*Lukáš Kotík*:Weighted Halfspace Depths and Their Properties

The thesis contributes to the theory of statistical depth functions that are popular in nonparametric multivariate analysis. The chapter is divided into five chapters (not counting Conclusion and Appendix).

Chapter 1 introduces various concepts of data depths for multivariate data and illustrate their performance on several examples. In Chapter 2 the weighted depth is introduced and its geometric properties discussed. In Chapter 3 (uniform) consistency of the weighted is proved and its influential function derived. The chapter also contains some notes on the asymptotic distribution of the weighted depth. Chapter 4 discusses how the weighted depth is tied with the kernel density estimation. Finally, Chapter 5 reviews various concepts of directional quantiles and presents a contribution to this area by introducing a new definition of a directional quantile.

The core of the thesis is in exploring the properties of the proposed weighted depth. In the thesis several possible weight functions are suggested and their geometric properties are discussed in detail. I like that many nice prepared figures are included in order to illustrate the definitions and properties of the suggested depth functions.

On the other hand I miss more care when asymptotic properties of the suggested depth functions are formulated and proved. As far as I understand there are flaws in the argumentation and some of the statements are rather vague (see the comments below). Although the thesis contain material worth publishing in impacted journals, it often fails in motivating and applying the developed methods. A notable exception is Section 4.3. When possible writing a paper the author should do a better job in convincing the reader that the presented method can be useful and thus worth studying.

In my opinion the thesis proves that the author is able to do a solid research in mathematics. Provided that all major comments (given below) are satisfactorily answered (and also reflected in an erratum), I recommend the thesis to be accepted as a dissertation thesis for the PhD degree in the study programme Mathematics in the study branch Probability and Mathematical Statistics.

10 February 2015

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**Major comments**

(1) **64**: I do not completely understand the argumentation that is used to finish the proof of Theorem 17. In particular what does the phrase *. . . it almost sure holds . . .* mean? Maybe, it would be better if the $\omega$-notation was not suppressed.

(2) **65**: I am puzzled with the definition of $\mathcal{H}_2$. How can the author guarantee that the constant $\delta$ does not depend on $\mathbf{x}$?

(3) **67**: I would like to see that the inequality

$$\sup_{\mathbf{x}\in\mathcal{H}_2} \widehat{\mathrm{RD}}_n(\mathbf{x}, \mathbf{u}_\mathbf{x}) \leq \frac{1}{\delta} \sup_{\mathbf{x}\in\mathcal{H}_2} \frac{1}{n} \sum_{i=1}^{n} w(\mathbf{X}_i - \mathbf{x}, \mathbf{u}_\mathbf{x})$$

is explained in a more detail.

(4) **75**: Can the author be more specific about which version of Donsker Theorem he refers to?

**Detailed comments**

(1) The thesis is written in English. Although the level of English is satisfactory, there are many small mistakes.

(2) **18**: It is not clear if the function $g$ in the definition of the zonoid depth function is given in advance, or if there should be $\forall g$ in the definition of $D_\alpha$.

(3) **19**: MAD stands for a median (not mean) absolute deviation.

(4) **43**: Is also the estimator $\widetilde{\mathrm{D}}_{w,n}$ affine invariant?

(5) **45**: Strictly speaking, Theorem 6 does not state that $D_{w,n}(\mathbf{x})$ is a biased estimator of $D_w(\mathbf{x})$. It would be also interesting to know the order of the bias. But such a question is probably too difficult to answer.

(6) **49**: I lack motivation for the definition of the depth function $\mathrm{RD}_w$.

(7) **66**: I find the notation $\widehat{\mathrm{RD}}(\mathbf{x}, \mathbf{u})$ rather confusing, as a very similar symbol $\widehat{\mathrm{RD}}_w(\mathbf{x})$ is used for a different quantity.

(8) **100**: I find the sentence *. . . it can be shown that under certain conditions $F_{\alpha,n}$ is an consistent estimate. . .* rather vague. Among others, $F_{\alpha,n}(r|\mathbf{s})$ is a function of two arguments ($r$, $\mathbf{s}$) and next to a point-wise consistency, also a uniform consistency in $r$ or $\mathbf{s}$ (or both) can be considered.