24/TP/ -3 -02- 2015

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Prof. Dr. Jan Kratochvil

Charles University

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Cologne,

Doctoral Thesis of Mgr. Lukas Kotik, No: 632/14

Spectability,

dear Professor Kratochvil,

thank you for your letter of December 11, 2014. I'm glad to comment on
the thesis as follows:

In multivariate analysis many concepts of data depth have been devel-
oped and applied to a broad range of statistical problems. The present
dissertation contributes to this literature. Most of the existing data
depths, as they are global notions of closeness to a center, are restrict-
ed to the analysis of unimodal distributions. Also, upper level sets are
convex (or starshaped) and, by this, not able to characterize distribu-
tions that have a non-convex (resp. non-starshaped) support. Lukas

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Kotik, in his dissertation, presents new notions of data depth that do not have these deficiencies and investigates their finite-sample and asymptotic properties.

Chapter 1 introduces into the given problem and provides a short overview of a few depth notions. In Chapter 2 the weighted halfspace depth is introduced, cases of special weights (sectors, cylinders, combined with kernels) are investigated and examples are given, where the depth is able to cope with non-convex supports. Also an extension to depth ratios is considered. Throughout the dissertation, data is regarded as an i.i.d. sample from an underlying probability distribution. In Chapter 3, strong consistency of the new weighted halfspace depth is proven and its influence function is derived. This extends a result of Romanazzi (1999), making use of rather developed mathematical machinery. Chapter 4 is about weights which have support on conic sections, parameterized by two parameters, eccentricity and 'radius'. Here, eccentricity e mainly controls the type of cone section, yielding a pure kernel density estimate ($e = 0$) and the usual halfspace depth ($e = \infty$) in the extremes. The parameter 'radius' measures the width of the support; it has to be somehow related to the degree of asymmetry in the data. Chapter 5, finally, opens the view to other depth approaches. It introduces and estimates multivariate quantiles measured on rays from a center. Further, regression depth and a notion of depth for functional data are shortly treated.

In my eyes, this dissertation contains a fine piece of mathematical inquiry. It contains deep and useful results, which merit publication in good journals. In Chapter 4 Jan Kotik manages a particularly inventive approach to bridge the gap between data depth and density estimates. However, the work raises some questions regarding the practicability of these concepts. The first open question is computability. Already the computation of the usual halfspace depth in modest dimensions (> 2) is a really hard problem; the new direction restricted and kernelized versions of halfspace depth pose even greater computational problems. Second, a geometric object (like a cone section) and a kernel have to

.be chosen, which should be done in a data-dependent way, dependent on the degree of asymmetry and multimodality, and possibly additionally dependent on the direction u. Of course, these are problems for further research, which do not diminish the scientific value of this dissertation.

The presentation is clear and, throughout the work, well understandable. But, the dissertation abounds with misprints and minor flaws in English grammar (like missing articles). I suggest that it may be carefully copy-edited before final publication.

In a nutshell: This is an impressive work in a very topical field of statistics, containing new and interesting results. I recommend it with emphasis to your faculty as a doctoral dissertation.

With kind regards,

Karl Mosler
Professor of Statistics and Econometrics