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Referee Report on Doctoral Thesis

Multi-Stage Stochastic Programming with CVaR: Modeling, Algorithms and Robustness

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The doctoral thesis “Multi-Stage Stochastic Programming with CVaR: Modeling, Algorithms and Robustness” considers three formulations of a multi-stage stochastic linear program using conditional value at risk (CVaR) to capture risk aversion. These formulations include a model with nested CVaR, a model with multi-period CVaR, and a model with a sum of single-period CVaR risk terms. In Chapter 1 the author mathematically formalizes these three models. His discussion in Chapter 1 brings out nicely that in the first model, the CVaR operator itself provides the nesting; in the second model, conditional CVaR statements are instead operated on by an expectation; and, in the third model, the CVaR terms are unconditional. Chapter 1 discusses two notions of *time consistency*, a property often regarded as desirable in a multi-stage stochastic program. And, it further discusses the notion of a polyhedral risk measure. The author then distinguishes the three CVaR-based formulations as to whether they are time consistent under both notions and whether their risk measures are polyhedral. His discussion on how the single-period model is time consistent under one notion, but not the other, is particularly valuable. The first chapter brings out computational challenges associated with these risk-averse formulations. The formulations are made concrete in terms of multi-stage asset-allocation models, without and with transaction costs, which are used later as test problems.

Chapter 2 describes the stochastic dual dynamic programming algorithm (SDDP) applied to a multi-stage stochastic linear program with interstage independence, first for the risk-neutral case and then under nested CVaR. The author’s discussion includes a path-based upper bound and a conditional (exponential) upper bound. Chapter 2 largely reviews material in the literature. That said, the author presents the chapter in a particularly attractive manner because he sets up nicely the need for new developments which follow in Chapters 3 and 4 regarding upper bounds. At the same time, he expresses the current literature in notation which lends itself to his subsequent developments.

In Chapter 3 the author describes a novel upper bound estimator for multi-stage models with nested CVaR risk measures. First, this importance sampling estimator has computational effort that grows gracefully (linearly) with the number of stages. Second, the estimator reduces variance and bias by modifying the sampling distribution to allocate about half of the samples to the tail of the distribution that contributes to the CVaR term in the objective function. The author describes two sets of assumptions under which his estimator improves upon the naive estimator of Chapter 2. Under these assumptions, he establishes consistency of the estimator and he establishes that it reduces bias. Compelling numerical results demonstrate the superiority of the new estimator on the multi-stage asset-allocation models, both without and with transaction costs.

The estimator in Chapter 3 uses importance sampling to allocate about half of the samples to the CVaR tail. While this reduces variance, the proportion is arguably *ad hoc*. In Chapter 4, the author provides a substantial improvement. First, he calculates the estimator's variance under a normality assumption. In this case, he shows the variance to be a quadratic function involving the proportion allocated to the CVaR tail. This allows for minimizing the variance by altering the proportion from Chapter 3's default value of 1/2. Chapter 4 goes on to describe how to optimize the proportion of samples allocated to the tail under any distribution (for which the required variances exist). Convincing computational results again accompany the methodological developments.

Chapter 5 studies the contamination technique in the context of the SDDP algorithm. Here, convex combinations of a nominal probability distribution and a second distribution are considered. The lower bound of the SDDP algorithm extends quite naturally under the contamination technique. However, quantities in the contamination technique's upper bound require estimation. The author develops a contamination-based upper bound estimator, using developments from Chapters 3 and 4. While it is not possible to directly store the solution (policy) associated with a specific distribution, he uses cutting planes from the SDDP algorithm to computationally form the upper bound estimate.

Chapter 6 provides a numerical study for the nested CVaR model and the multi-period CVaR model under various risk attitudes, various numbers of stages, and without and with transaction costs. The author presents interesting numerical results on the first stage allocation decision. In particular, his results distinguishing the cases without and with transaction costs as risk parameters vary, are particularly insightful.

While the estimators of Chapters 3 and 4 are described using the SDDP algorithm, they have broader application. Like most importance sampling estimators, such application will hinge on the ability to construct a good "approximation function" for specific classes of applications. The numerical results in Chapters 3 and 4 are based on examples of such approximation functions for multi-stage asset-allocation models, including models with transaction costs. In this sense, the author provides guidance for how this might be done in other settings.

I am a co-author on reference [3] from the dissertation's bibliography with the author, and the key ideas from this paper appear in Chapter 3 of the dissertation. Paper [3] is accepted in *Mathematical Programming*. The central idea in this paper concerns the variance- and bias-reducing estimators, and these ideas are fully due to Mr. Kozmík. I pointed him to some of the relevant literature on importance sampling, including the reference of Hesterberg, and we had discussions on time consistency of various multi-stage risk-averse formulations. In addition to this paper, as I mention above, Chapter 4's material significantly improves on the results of Chapter 3, and these results are accepted in a sole-authored paper in *Computational Management Science.* A third paper based on Chapter 5's material is also accepted for publication in *OR Spectrum.* These three journals are well regarded in our community. In addition, the author has two papers in proceedings and a fourth journal article which are to appear.

The author's work unambiguously proves his ability for creative scientific work. His doctoral thesis is of high quality and meets the requirements of a doctoral dissertation.

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