Report on the Doctoral Thesis

Mgr. L. Slámová, M.Sc.: Generalized Stable Distributions and Their Applications

The main aim of the Thesis is to carry out a comprehensive study of stability properties of probability distributions, especially the discrete distributions. Various concepts of stability have been introduced and studied and also illustrated by numerous examples. In the second part some models are presented where the discrete stable distributions have been utilized.

The Thesis are divided into 10 Chapters, among which the Chapters 3-7 are devoted to the general theory of (mostly) discrete stable distributions and constitute, in my understanding, the core of the Thesis, while in the remaining Chapters 8-10 applications of the general theory are dealt with. Some of the Chapters are based on published papers or forthcoming publications in most cases coauthored by L. Slámová and her advisor.

More specifically, Chapter 3 deals with discrete approximations of stable distributions (three types of approximations have been introduced). Chapter 4 contains three definitions of discrete stability for non-negative integer-valued random variables, which is the main object of the Thesis. All of them are discrete versions of corresponding concepts of stability in the usual sense. The first definition generalizes the approach by Steutel and Harn by employing a general thinning operator to normalize sums of discrete random variables. The second one uses the so called portlying operator to normalize discrete random variables. The last one combines the previous two approaches and extends both of them. Chapter 5 is a rather extensive study of analytical properties of the discrete stable distribution in the sense of the first definition. In most cases, the class of distributions considered here is formed by a modified geometric thinning operator. A variety of results on characterizations, moments, and asymptotic behaviour for positive symmetric discrete stable random variables are given. Chapter 6 is devoted to the concept of casual stable distributions, which is important e.g. in the study of tempered conterparts of positive discrete stable random variables. For instance, a limit theorem for convergence to a casual stable random variable has been proved here. Chapter 7 contains a discrete analogue of ν-stable distribution and extends for this particular case earlier results by

Klebanov and Rachev. As mentioned above, the second part of the Thesis presents the main objects of previous theoretical study in some specific models. Chapter 8 presents a model of ranking of scientific contribution (more precisely, a model for number of paper citations), Chapter 9 is concerned with the problem of estimation of parameters of a discrete stable family of laws. Since the classical maximum likelihood approach is not available here, a modification of the $\mathcal{H}$-method developed by Kagan in used, in which the maximum likelihood function is replaced by an operator called "'informant"'. A simulation study is also provided. The last application is given in Chapter 10, which is an option pricing model, in principle a generalization of the GARCH option pricing model introduced by Duan (1995).

I highly value the results obtained in the present Thesis which, in my opinion, provide an interesting and significant contribution to the theory of stable laws. It is also clearly demonstrated by publications emerging here, some of which appeared or are to appear in respected journals. In particular, some of the computations that had to be done (e.g. in Chapter 5) are fairly complicated and must have been rather time-consuming. Obviously, a lot of work has been done.

If I should point out to some parts of the Thesis which I value slightly less than the others, it would be some of the "'application"' part, in particular Chapter 10 (I am not sure if, strictly speaking, "'application"' is the right description of the second part, it is rather demonstration of some models where the main subject of study may be useful, which, admittedly, is interesting as well). The style of presentation here is quite different from the other parts and mathematically is not completely satisfactory. I do not quite agree that the popularity of models with jumps (even pure jump models) in continuous space should be an argument for discrete space models (but all of it are just models and at the end of the day it is their usefulness which in practice provides their ranking). On pages 94, 95, the author speaks about "'this paper"', which one? The reference to the previous parts of the Thesis at the beginning of 10.1 should be more explicit. The description of the model on pages 99-100 is not very detailed, I suggest that the author provides a little more detailed description and justification of the model (10.4)-(10.5) when defending the Thesis. On page 100 a risk-neutral measure is mentioned, but definitions of risk neutral measures (I mean mathematical definitions) are related to each specific market model, so I would find it suitable to give such a definition in PhD Theses even if some may think it is intuitively clear.

Some more comments:

- In the Thesis there are aplenty of definitions of various notions and concepts (some of which are rather technical). I do not think that there is enough space to study all of them into depth. It does not help clarity of exposition for a non-specialist reader. Is it necessary? For example stable distributions in the third sense are consider just on pages 36-39 and Theorems 4.24 and 4.25 could have been formulated as an interesting property of the notion of stability in the first sense. I admit that this may be a matter of taste.

- Some functional-analytic concepts could have been formulated more precisely. For example an operator (as the thinning operator) has some domain and some range. Similarly, the concept of operator semigroup should have been explained (especially because usually this notion is often used in more specific sense, when the family has some composition property with respect to an operation on parameter set - this is by the way very interesting here because in Examples 2.27 and 2.28 commutativity follows from this property with respect to the operation of multiplication, while the Example 2.29 seems to be more complex).

- on p. 64, when speaking about random normalization in the general case, the conditions on g could have been specified more explicitly.

- p. 32, 1st line: 4.2 is probably a Section, not a Subsection

- p. 6 at the beginning of 2.1: vast

Let me stress that these minor shortcomings are mostly formal and do not diminish the value of the Thesis, which is in my opinion considerably high. I strongly recommend to accept the work as Doctoral Thesis and to accept it for Defence.

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