

Referee's report on the doctoral thesis
«Generalized stable distributions and their applications»

by Mgr. Lenka Slámová

The concept of stability is of great importance in contemporary probability theory and its applications. In general, this concept incorporates both representability of the whole as an aggregate of arbitrarily many independent parts (infinite divisibility) and self-similarity, that is, the coincidence of statistical properties of the whole and its (independent) parts. These principles are very close to the principle of fractality whose manifestations are observed in the behavior of many real physical, social and financial systems.

This concept can be mathematically formalized in several ways. Each mathematical formalization of the concept of stability yields very interesting models with specific properties. In order to select a proper mathematical model for a real phenomenon, the properties of these models should be analyzed thoroughly. In the doctoral thesis of Lenka Slámová, the properties of stable models as well as some of their applications are discussed in detail.

Main attention is paid to discrete stable distributions. Three definitions are proposed for a discrete stable distribution. These definitions generalize the concept of a non-negative integer-valued stable random variable proposed by Steutel and van Harn in 1979 to random variables taking values on the set of all integers.

The thesis is written in a clear and understandable manner. New results are contained in Chapters 3 – 10. Chapter 3 contains very interesting results concerning the problem of approximation of «usual» stable laws by discrete stable distributions. Chapter 4 deals with three possible definitions of discrete stability. The relations between these definitions are discussed. In Chapters 5 and 6, the analytical properties of discrete stable distributions are presented. The discrete analogs of ν -stable distributions are introduced in Chapter 7 and their properties are described.

The rest of the thesis deals with problems related to possible applications of discrete stable distributions. In Chapter 8 the application of discrete stable distributions to modeling ratings of scientific efficiency based on citations is considered. Chapter 9 deals with methods of statistical estimation of parameters of discrete stable distributions. Chapter 10 is devoted to financial applications.

Main scientific results of Chapters 3 – 10 are new. All the statements are supplied by correct and detailed proofs.

Nevertheless, I have two minor remarks.

1. On page 1 it is written that the theorem stating that a distribution is stable if and only if it is limiting for sums of independent identically distributed random variables under appropriate linear normalization is due to Gnedenko and Kolmogorov (1949). This is not so. In the book Gnedenko and Kolmogorov (1949) this result is assigned to P. Lévy (1925) and A. Khintchin (1938).
2. The formulation of the results similar to Theorem 2.17 which deal with some kind of «distributions

with random parameters» should be supplied by comments containing strict definitions of what is actually meant by such «Bayesian» terminology. In the case of Theorem 2.17, the notion of mixed Poisson distribution might have been more appropriate.

However, these remarks can by no means diminish the excellent impression I got of the doctoral thesis of Lenka Slámová. The discrete stable models discussed in the thesis open a new promising direction in probability theory. The presented results demonstrate that the author, Lenka Slámová, is a high-qualified mathematician with excellent experience in analytical methods of probability theory, mathematical statistics and their applications, capable of creative scientific work.

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