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Faculty of Social Sciences
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RIGOROUS THESIS

The Effectiveness of Unconventional Monetary Policy Tools at the Zero Lower Bound: A DSGE Approach

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Declaration of Authorship

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Prague, September 4, 2014

Signature

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Abstract

The central bank is not able to further ease monetary conditions once it exhausts the space for managing short-term policy rate. Then it has to turn its attention to unconventional measures. The thesis provides a discussion about the suitability of different unconventional policy tools in the Czech situation while the foreign exchange (FX) interventions have proven to be the most appropriate choice. A New Keynesian small open economy DSGE model estimated for the Czech Republic is enhanced to model the FX interventions and to compare different monetary policy rules at the zero lower bound (ZLB). The thesis provides three main findings. First, the volatility of the real and nominal macroeconomic variables is magnified in the response to the domestic demand shock, the foreign financial shock and the foreign inflation shock. Second, the volatility of prices decreases significantly if the central bank adopts price-level or exchange rate targeting rule. Third, intervening to fix the nominal exchange rate on some particular target or to correct a misalignment of the real exchange rate from its fundamentals serves as a good stabilizer of prices while intervening to smooth the nominal exchange rate movements increases the overall macroeconomic volatility at the ZLB.

JEL Classification	C11, E31, E43, E52, E58, F31
Keywords	zero lower bound, monetary policy, small open economy DSGE model, Bayesian estimation, foreign exchange interventions, exchange rate and price dynamics

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Abstrakt

Jakmile centrální banka vyčerpá svůj prostor ovlivňování krátkodobých úrokových sazeb, není již dále schopna uvolňovat měnové podmínky. Poté musí svou pozornost obrátit k nekonvenčním opatřením. Tato práce diskutuje vhodnost jednotlivých nekonvenčních nástrojů měnové politiky v případě České republiky, přičemž devizové intervence se jeví jako nejvhodnější volba. Ke srovnání jednotlivých měnopolitických nástrojů v prostředí nulové dolní meze pro úrokové sazby je využit dynamický stochastický model všeobecné rovnováhy pro malou otevřenou ekonomiku odpovídající přístupům Nové keynesovské ekonomie, který je odhadnut pro Českou republiku a rozšířen o devizové intervence. Práce přináší tři hlavní poznatky. Za prvé, volatilita reálných a nominálních makroekonomických proměnných je zesílena v odezvě na domácí poptávkové, zahraniční finanční a inflační šoky. Za druhé, volatilita cen výrazně klesá, pokud centrální banka cíluje cenovou hladinu nebo měnový kurz. Za třetí, devizové intervence fixující nominální měnový kurz na stanovené hladině či intervence určené ke korekci vychýlení reálného měnového kurzu představují efektivní stabilizátor cen, přičemž intervence určené k vyhlazení výkyvů nominálního měnového kurzu zvyšují celkovou volatilitu ekonomiky v prostředí nulové dolní meze pro úrokové sazby.

Klasifikace JEL	C11, E31, E43, E52, E58, F31
Klíčová slova	nulová dolní mez, monetární politika, DSGE model malé otevřené ekonomiky, Bayesovský odhad, devizové intervence, cenová dynamika a dynamika směnných kurzů

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Acronyms

AR(1) Autoregressive Process of Order One

CES Constant Elasticity of Substitution

CNB Czech National Bank

CPI Consumer Price Index

CUMSUM Cumulative Sum Control Chart

CZK Czech Koruna

DSGE Dynamic Stochastic General Equilibrium

ER Exchange Rate

EU European Union

EUR Euro

EURIBOR Euro Interbank Offered Rate

FOMC Federal Open Market Committee

FX Foreign Exchange

GDP Gross Domestic Product

IRF Impulse Response Function

l.o.p. Law of One Price

MCMC Markov Chain Monte Carlo

MLE Maximum Likelihood Estimation

OECD Organization for Economic Co-operation and Development

PRIBOR Prague Interbank Offered Rate

SOE Small Open Economy

UIP Uncovered Interest Rate Parity

US United States

ZLB Zero Lower Bound

Rigorous Thesis Proposal

Author	Mgr. Simona Malovaná
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Proposed topic	The Effectiveness of Unconventional Monetary Policy Tools at the Zero Lower Bound: A DSGE Approach

Topic characteristics In the last a few decades, many central banks around the world tried to reduce and stabilize inflation and they were successful. However, as a consequence nominal interest rates were lowered to such a level which increased the likelihood of binding zero bound on nominal interest rate and the threat of deflation. The Japanese economy has experienced deflation and interest rate close to zero since the late 1990s. In 2009, interest rates had declined below one percent in the United States, the United Kingdom, Sweden, Switzerland, Canada and the euro area. The recent development in these countries roiled the discussion on whether monetary policy is effective when the nominal interest rate is close to zero. In such a situation, the monetary authority cannot use the conventional policy of targeting short-term nominal interest rate and the other dimensions of policy gain in importance. Therefore, the main contribution of this thesis should be to investigate and compare the effectiveness of different unconventional policy tools used by the monetary authority to avoid the liquidity trap.

Hypotheses

1. By adopting price-level targeting rule, the monetary authority can better stabilize price developments and alleviate the recession through the stabilization of private sector expectations than under inflation targeting when the zero lower bound on nominal interest rate is binding.

2. Both management of expectations and FX interventions allow the econ-

omy to avoid the liquidity trap better than introduction of quantitative easing.

Methodology In this thesis, I assess the implications of monetary policy tools when the zero lower bound is binding. The thesis attempts to address the issue of effectiveness of different unconventional policy tools. The thesis will be divided into three main parts. Firstly, the current literature on this issue will be explored and main approaches to modeling unconventional policy tools within DSGE models will be outlined. It will be focused mainly on quantitative easing, FX interventions and management of expectations.

Secondly, the benchmark DSGE model for a small open economy will be constructed and calibrated for the Czech Republic. In this situation, the monetary authority is able to pursue a policy of monetary easing by lowering nominal interest rates. The zero lower bound constraint will then be introduced as binding and the monetary authority will not be able to reduce nominal interest rates, therefore, it will not avoid the liquidity trap.

Thirdly, individual unconventional policy tools will be implemented into this model. Then the results of the enhanced model will be compared with the results of the benchmark model. Impulse response functions for various shocks as well as model implications and detailed discussion will be provided.

Outline

1. Approaches to modeling unconventional policy tools
   (a) Quantitative easing
   (b) Different targeting rules
   (c) FX market interventions
2. The benchmark NK model
   (a) Basic NK model for a small open economy
   (b) Calibration for the Czech Republic
   (c) Introducing ZLB constraint as binding
3. Implementing unconventional policy tools
4. Comparison of effectiveness of unconventional policy tools

5. Discussion

6. Conclusion

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Author

Supervisor

Chapter 1

Introduction

In the last a few decades, many central banks around the world tried to reduce and stabilize inflation, and they were successful. However, the decrease in inflation was accompanied by a decline in the nominal interest rate, in many cases to zero. The Japanese economy has experienced deflation and interest rate close to zero since the late 1990s. In the aftermath of the global financial crisis, many central banks were forced to cut their policy rates to zero. The short-term interest rates had declined below one percent in the United States, the United Kingdom, Sweden, Switzerland, Canada and the euro area, among others. The recent development in these countries roiled the discussion on whether the monetary policy is effective when the nominal interest rate is close to zero. In such a situation, the central bank cannot use the conventional policy of targeting short-term nominal interest rate and other dimensions of monetary policy gain in importance.

Once the central bank runs out the space for managing the short-term interest rate, it has to resort to unconventional methods, such as the negative interest rate, the various types of liquidity provisions or the foreign exchange (FX) interventions. The use of negative interest rate may be connected with a number of legal barriers while its effect is still quite unclear as there is not sufficient experience and empirical evidence. Similarly, there has not been the empirical research sufficiently identifying the long-term effect of quantitative easing, despite the fact that its transmission is one of the most widely discussed topics in the current monetary policy literature. Regarding FX interventions, there can be found at least some theoretical motivation suggesting a way how to avoid serious disruptions at the zero lower bound (ZLB). The core idea is based on the real exchange rate depreciation which together with future committing to a price-level target should increase inflation expectations (Svensson 2001). A wide range of studies concludes that the central bank can mitigate excessive volatility of macroeconomic variables caused in the ZLB situation by adoption of such a history-dependent monetary policy rule (e.g. Svensson 2000; Smets 2000; Reifschneider & Williams 2000; Eggertsson & Woodford 2003). It should provide a better way to control expectations and reduce variability than the pure inflation targeting.

The Czech National Bank (CNB) cut its main monetary policy rate to a technical zero in late 2012, and has decided to keep it at the lower bound over a longer horizon until inflation pressures do not increase significantly. Any serious inflation risks did not emerge over a year, so the CNB has decided to use an additional policy tool for further monetary easing. In late 2013, it started to intervene on the foreign exchange market against the appreciation of the Czech koruna. The recent Czech experience led us to ask whether the foreign exchange interventions really serve as an effective policy tool when the nominal policy rates are stuck at their lower bound, and whether the central bank should not use different unconventional measures.

In this thesis, I assess the implications of monetary policy tools when the economy is constrained with the ZLB on the short-term nominal interest rate. In particular, the effectiveness of various targeting rules is examined together with the foreign exchange interventions in a New Keynesian DSGE model for a small open economy. The main purpose of the work is to analyze the extent to which these unconventional measures might counteract deflation pressures, stabilize prices and alleviate the recession when the central bank is not further able to lower its policy rate. Three main issues are addressed and three hypotheses are set. First, the effect of a domestic supply and demand shock and a foreign financial and inflation shock is evaluated at the ZLB and compared to the response of the macroeconomic variables in normal times. The first hypothesis supposes that the effect is strengthened in the response to all foreign shocks and the domestic demand shock while the response of the economy to the positive supply shock is subdued at the ZLB. Next, the effectiveness of different monetary policy targeting rules is examined. The second hypothesis supposes that by adoption of the price-level targeting rule, the central bank is able to better anchor price expectations and alleviate the recession through the stabilization of price development than under the inflation targeting at the ZLB. Moreover, the comparison is done also for two other policy rules, a pure and dirty exchange rate targeting. Third, the effectiveness of different types of foreign exchange interventions is analyzed as this type of measure is assumed to be the best alternative among proposed unconventional monetary policy tools in the situation of the Czech Republic. Hence, the last hypothesis supposes that FX interventions are effective in further monetary easing at the ZLB.

The thesis is structured as follows. In first two chapters, an overview of the literature is presented. In particular, Chapter 2 attempts to shed a light on different views of the ZLB situation and its implications for pursuing the monetary policy. The question, whether the central bank is still able to counteract shocks when the policy rates are bounded at zero level, is discussed. Chapter 3 focuses on channels of monetary transmission through which (un)conventional measures can affect the real economy. This topic is followed by general approaches to modeling unconventional policy tools together with the empirical evidence on their (un)success. Chapter 4 proceeds with the investigation of the DSGE model used to monetary policy analysis. In Chapter 5, the Bayesian estimation of model parameters is performed for the Czech Republic. Chapter 6 implements an occasionally binding constraint into the DSGE model while its dynamic features are examined in the response to the domestic and foreign exogenous shocks. Chapter 7 characterizes the situation of the Czech economy in context of the ZLB and a choice of unconventional monetary policy instrument. Part of this chapter is devoted to the degree of exchange-rate pass through at the ZLB. Finally, Chapter 8 presents the comparison of different monetary policy tools in the response to these shocks. In the first part of this chapter, the effectiveness of different targeting rules is evaluated. The second part is dedicated to different foreign exchange interventions regimes. Both parts are followed by the discussion about the effectiveness of monetary policy at the ZLB. The core findings are summarized in the conclusion which additionally provides an outline of further research and possible extensions.

This rigorous thesis was created on the basis of the author's master thesis which was awarded by the Dean. The response to the opponent's comments is listed on page 7 in paragraph 2.

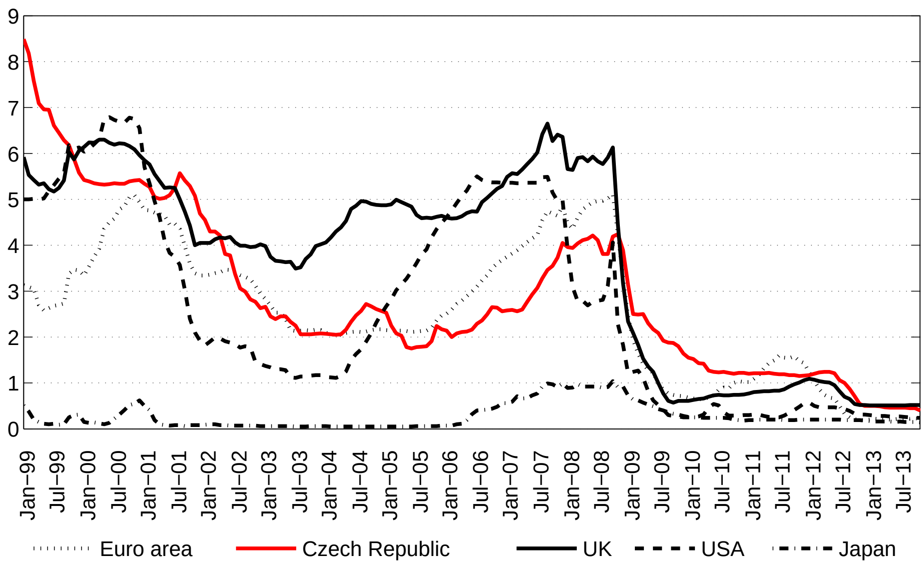
Chapter 2

Monetary Policy at the ZLB: Literature Overview

After more than a decade of inflation waves caused by the collapse of the Bretton Woods and two huge oil price shocks, the variability and the magnitude of inflation decreased in the industrialized world. This situation was very comfortable for central banks, because it seemed that a new era of stable and low inflation began (Ullersma 2002). Unfortunately, the monetary policy started facing new challenges. The decrease in inflation was accompanied by a decline in the nominal interest rate, in many cases to zero. Figure 2.1 provides a comparison of short-term interest rates in selected countries. Japan experienced the short-term interest rate close to zero since 1990s while other countries cut their policy rates in the response to the recent crisis. At that time, the Japanese case was a new situation which led to an extensive research with new implications for monetary policy (e.g. McCallum 2000; Svensson 2000; Eggertsson & Woodford 2003; Reifschneider & Williams 2000).

Three main findings should be highlighted. First, the probability that the zero lower bound starts to bind increases considerably in countries with a sufficiently low inflation target (e.g. Reifschneider & Williams 2000; Adam & Billi 2007; Coenen *et al.* 2003). Second, this research advices to cut policy rates aggressively and immediately when the deflation threats occur (e.g. Reifschneider & Williams 2000; Eggertsson & Woodford 2003). Third, unconventional monetary policies (liquidity provisions, foreign exchange interventions etc.) could help in further monetary easing (e.g. McCallum 2000; Svensson 2001; Bernanke *et al.* 2004).

Figure 2.1: Zero lower bound in selected countries



Source: EUROSTAT (2014).

In a situation of constrained short-term rates, bonds and cash become perfect substitutes. Therefore, the nominal interest rate cannot fall even more.[1] The conventional monetary policy of reducing the short-term nominal interest rate through open market operations is no longer feasible. The question is whether the central bank is still able to support the economy if an even more adverse shock occurs, or whether it can get out of the liquidity trap if the economy is already facing deflation. Two main opinion streams emerge which present the central bank either as a completely powerless institution in the ZLB situation (e.g. Summers 1991) or attempt to find alternative channels through which the central bank can still influence economic conditions (e.g. Meltzer 1995; Svensson 2001). Next section provides some motivation in a context of historical development.

2.1 Historical Journey

The issue of the ZLB on nominal interest rate is firstly mentioned by Fisher (1896). The liquidity trap problem is elaborated forty years later by Keynes (1936). Today, the Keynes' liquidity trap ceases to be just a theoretically discussed issue and gains in importance in a practical point of view. He stresses

[1]Higher cost of storing cash allows the nominal interest rate to fall slightly below zero (McCallum 2000).

the role of the liquidity preference which makes a utility of holding money always positive, even if economic agents hold more money than is required based on a transaction or precautionary motive: "There is the possibility ... that, after the rate of interest has fallen to a certain level, liquidity-preference may become virtually absolute in the sense that almost everyone prefers cash to holding a debt which yields so low a rate of interest. In this event the monetary authority would have lost effective control over the rate of interest (Keynes 1936)." Normally a marginal rate of substitution between present and future consumption, between present consumption and present money holdings and the nominal market interest rate are equal. But if the nominal interest rate is lower than the liquidity premium, the consumption falls below its equilibrium level. Then the liquidity preference dominates because of the speculative motive, mainly due to uncertainty about the future nominal interest rate. In case of very low long-term nominal interest rate, economic agents hold money with expectation about future increase in this rate, in an attempt to benefit from it. The consumption is not enough to return the market to equilibrium, and money becomes a "bottomless sink for purchasing power". The economy is caught in the liquidity trap (Keynes 1936; Ullersma 2002).

Classical economists do not agree with Keynes, especially Pigou (1943). He criticizes Keynes' General Theory for omitting the link between real balances and consumption. Defining the wealth as a ratio between money supply and current price level, a fall in prices increases real balances (real wealth), i.e. spending possibilities of consumers. It pushes up the consumption, employment and output. Pigou views the utility of money as satiable and assumes that the liquidity premium can decrease sufficiently to reach the time preference rate when the liquidity holdings increase. Hence, the monetary policy becomes neutral in the liquidity trap (Pigou 1943).

Four basic approaches of dealing with the ZLB can be proposed (Ullersma 2002), i.e. Krugman's and Meltzer's view, Gesell money and Svensson's "foolproof" way. Krugman (1998) focuses on the interest rate and expectations channel. He stresses that increased inflation expectations should lead to decline in the market real interest rate at the ZLB. But his idea seems to be difficult in practice, especially in the case when public believes the central bank's commitment to price stability or in the case of price stickiness (Orphanides & Wieland 1998). Even if Krugman would be right there is a threat of possible future inflationary spiral. The effect of higher expected inflation would be amplified at the point when the ZLB ceases to bind (Eggertsson & Woodford 2003; Woodford

2012). In such a situation a simple management of expectations is not enough, but there should be set an inflation target with highly transparent and clear reports and forecasts (Svensson 2000) and such a credible commitment which policymakers would be not able to ignore in the future (Woodford 2012). Some of the Monetarists focus on a transmission via relative price adjustment of non-monetary assets which are imperfect substitutes (Meltzer 1995), some of them emphasize the importance of credit channel because it is not hampered by the zero nominal interest rate (Bernanke & Gertler 1995). Section 3.1 focuses on particular transmission mechanisms in more details.

Another possibility how to deal with constrained short-term rates is the use of so-called Gesell money, i.e. the tax on money, initially suggested by Buiter & Panigirtzoglou (2000) and Goodfriend (2000). The introduction of Gesell money is subjected to the availability of technologies which reduce transaction costs. Furthermore, conventional money becomes less attractive than its possible alternatives (foreign currency or e-money). Finally, Svensson (2000) stresses the role of the exchange rate channel when the ZLB is a serious restriction. His "foolproof" way of escaping from the liquidity trap contains the adoption of the price-level target, imposing the medium-term strategy of exchange rate devaluation and temporary commitment to the exchange rate peg. The price-level targeting is preferred over the inflation targeting because in the inflation targeting regime the central bank does not respond to unexpected price-level variations while under the price-level targeting these changes are reversed. It means that with the price-level target, the shock that causes the fall in prices decreases inflation below its long-run level which compels the central bank to take such actions which brings prices back on the target. It increases the inflation above the long-run level for some periods to accommodate the effect of the shock. This overshooting will increase inflation expectations and therefore also the inflation.

In recent years, there have been a growing number of studies focusing on the impact of the ZLB constraint on the optimal monetary policy, mostly set in the New Keynesian DSGE framework. Probably the most important studies have been conducted by Eggertsson & Woodford (2003), Jung *et al.* (2005), Adam & Billi (2007), Nakov (2008), Woodford (2012) among others. The core research is explored in more details in Chapter 3.

Chapter 3

(Un)conventional Monetary Policies

3.1 Channels of Monetary Transmission

The main purpose of the following chapter is to provide a survey on various transmission channels through which (un)conventional monetary policy tools can affect the real economy. The monetary transmission mechanism can be described as a way how the monetary policy decisions are transmitted to the change in output and price level through the particular channel (Taylor 1995).Two basic types of channels are usually referred, i.e. neoclassical channels with perfect financial markets and non-neoclassical channels with financial market imperfections (Boivin *et al.* 2010). Through different channels, the central bank affects different variables at different speed and with different final effect. Hence, the identification of particular channels serves as a very important part of the central banks' analysis since it is crucial for proper timing of policy changes and selection of measures (Loayza & Schmidt-Hebbel 2002).

Next sections are devoted to discussion about three main types of transmission channels, (i) the interest rate channel, (ii) the asset price channel and (iii) the credit channel. Moreover, the expectations channel should be mentioned as it presents a fast and powerful way of monetary policy transmission. Monetary policy changes may affect the real economic activity through public expectations about the future development of prices or labor market conditions. However, this channel is very uncertain as it highly depends on the public interpretation of central bank actions and the credibility of central bank commitments. Moreover, the confidence of the central bank plays a crucial role in both, the intensity and the direction of final effect. For instance, a decline in policy rates might be viewed in two different ways - as an indicator of future higher economic growth (which increases public confidence, consumption and investment), or as a signal of impending downturn (which decreases public confidence, consumption and investment) (George *et al.* 1999).

3.1.1 Interest Rate Channel

The interest rate channel is the standard and core transmission mechanism of monetary policy in a basic Keynesian IS-LM framework, and can be simply described by the following relationship (Mishkin 1996)

$$M \uparrow \Rightarrow r \downarrow \Rightarrow I \uparrow \Rightarrow Y \uparrow \tag{3.1}$$

The monetary expansion (M) leads to lower real interest rate (r), which decreases the borrowing cost and increases the investment (I). It causes a rise in the aggregate demand and then a rise in the output (Y). If consumers' decisions about housing and other durable expenditures are also classified as the investment, lower real interest rate increases the consumption. Other way round, higher interest rate causes a jump in yields across the yield curve through an arbitrage in financial markets. The cost of capital increases and the investment together with consumption decreases due to a tighter liquidity constraint and due to expectations about lower demand and higher cost of capital. Lower consumption is induced through an intertemporal substitution effect and lower investment through a pressure on Tobin's q (Bernanke 1983).

The interest rate transmission channel is based on two underlying assumptions - (i) all non-monetary assets are bonds, (ii) all markets are perfect (Azali 2001). According to the first assumption, the credit market is not included in this mechanism. Therefore, banks do not distinguish between loans and government bonds in their portfolio, and borrowers do not distinguish between credit from auction markets and from banks. According to the second assumption, only price can clear the market because borrowers are homogenous and banks are not able to distinguish between them. The credit rationing is also not possible (Bernanke & Blinder 1988).

3.1.2 Asset Price Channels

The asset price channels present other very important way of monetary policy transmission. The attention is given to the equity, the real estate and the foreign exchange.

Exchange Rate Channel

The underlying mechanism can be described by the following relation (Mishkin 1996)

$$M \uparrow \Rightarrow r \downarrow \Rightarrow E \downarrow \Rightarrow NX \uparrow \Rightarrow Y \uparrow \tag{3.2}$$

The monetary expansion leads to lower interest rate which depreciates the domestic currency (E). The domestic production becomes cheaper for foreign consumers because of the higher competitiveness. Higher net export (NX) increases output. The depreciated currency also increases prices of imported goods which leads to higher firms cost and retail prices of goods and services. The relative value of foreign-denominated assets and liabilities increases. The net wealth of all agents is affected based on the composition of their balance sheets. Considering net debtors, their wealth is reduced. One of the most famous examples can be found in Hungary.

Equity and Real Estate Price Channel

The equity and real estate price channel works on the basis of Tobin's q model described in equation (3.3) and wealth effect described in equation (3.4) (Mishkin 1996).

$$M \uparrow \Rightarrow P_e \uparrow \Rightarrow q \uparrow \Rightarrow I \uparrow \Rightarrow Y \uparrow \tag{3.3}$$

$$M \uparrow \Rightarrow P_e \uparrow \Rightarrow W \uparrow \Rightarrow C \uparrow \Rightarrow Y \uparrow \tag{3.4}$$

The monetary expansion leads to increase in the equity prices (P_e)[1] which affects the Tobin's q[2] and consumer wealth[3]. According to Tobin (1969), the firm should invest in new capital if the market value of the project exceeds its cost. Higher consumer wealth (W) induces higher consumption and output.

[1] The increase in prices of equity is a consequence of its higher attractiveness (the new project becomes relatively cheaper to finance than before the decline in the interest rate).

[2] Tobin's q is a ratio between a market value of particular firm and a replacement cost of firm's assets (Tobin 1969).

[3] In this case the consumption is determined by the consumer lifetime wealth consisting of human and financial capital.

Also higher real estate prices are associated with higher consumption because of increased availability of the collateral and reduction of credit constraints.

3.1.3 Credit Channel

Despite previous discussion, financial markets are not perfect all the time. Using the standard IS-LM framework, the monetary policy is transmitted just through the interest rate channel while firms and households can freely borrow at set interest rate. In reality, most of economic agents can borrow just from banks which are not always willing to lend everyone. Such a principal agent problem is caused by asymmetric information and costly enforcement of contracts. The effect of monetary policy decisions on costs and the availability of credit is much higher than just the simple effect of risk-free interest rate. Therefore, also the credit channel is the important part of transmission mechanism of monetary policy actions. There are two basic types of this channel - a balance sheet channel (broad credit channel), and a bank lending channel (Bernanke & Gertler 1995).

Balance Sheet Channel

The emphasis is again given to the asset prices which are important determinants of the collateral value in applying for a loan. In the market with financial frictions lower value of collateral increases the risk premium which borrowers have to pay for external finance. Hence, the investment and consumption decline. With lower net wealth of firms applying for external funding there is more severe adverse selection and moral hazard problem. The collateral is not high enough and firms are more willing to engage in risky projects because of lower equity stake of firm owners. The effect of monetary policy actions in such a situation might look as follows (Mishkin 1996)

$$M \uparrow \Rightarrow P_e \uparrow \Rightarrow \text{A.S. \& M.H} \downarrow \Rightarrow \text{lending} \uparrow \Rightarrow I \uparrow \Rightarrow Y \uparrow \tag{3.5}$$

The monetary expansion leads to higher equity prices, and therefore, also to reduction of adverse selection (A.S.) and moral hazard (M.H.) costs. Financial intermediaries are willing to borrow more and investment spending rises which pushes up the output. Focusing on the empirical evidence, Bernanke (1983) finds a serious deterioration in firms' balance sheets during the Great

Depression which increases credit rationing and borrowing cost. The imposed external finance premium is a result of information asymmetries.

Bank Lending Channel

Reserves are important source of banks' funds. The monetary expansion leads increase in these reserves, and therefore, in overall availability of bank loans. It induces higher investment and consumption with rise in aggregate spending (Mishkin 1996).

$$M \uparrow \Rightarrow \text{bank deposits} \uparrow \Rightarrow \text{loans} \uparrow \Rightarrow I \uparrow \Rightarrow Y \uparrow \tag{3.6}$$

Through this channel the central bank is able to affect just smaller firms which have not the direct access to the stock and bond markets where they would be able to get external finance without financial intermediaries (Mishkin 1996).

3.2 Quantitative Easing

First, a theoretical justification for the use of quantitative easing (QE) as unconventional measure is presented. Then it is confronted with empirical evidence.

The quantitative easing is proposed to accommodate shocks by changing a size and/or a composition of the central bank balance sheet, i.e. by using both, the asset and liability side. Operations conducted on the asset price are usually called credit easing. While the quantitative easing refers to expanding the size of the central bank balance sheet regardless of its composition, the credit easing focuses "on the mix of loans securities" which the central bank holds, and "on how this composition of assets affects credit conditions for households and businesses (Bernanke 2009)".

Using quantitative easing, the central bank can stimulate aggregate demand through the credible commitment to a permanent increase in the money supply. Higher inflation expectations together with lower long-term interest rate decrease the real interest rate (Auerbach & Obstfeld 2005). As for any other monetary action, the final effect is strongly dependent on public expectations, i.e. on the transmission through expectations channel. Hansen (2012) summarizes further different possible channels through which the quantitative easing can be effective. In addition to the expectations channel he stresses (i) the duration risk channel, (ii) the liquidity channel, (iii) the safety channel, and

(iv) the default risk channel. Decreased duration risk leads to lower interest rate. Through the liquidity channel the relative price between highly liquid and less liquid assets can be revised. The safety channel is connected with highly risk-averse agents. Purchases of safe assets decrease the supply and reduce the yield on these assets relative to less safe assets. If the quantitative easing is successful in the stimulation of the economy, the default risk decreases and the interest rate paid by riskier corporations is lower.

Bernanke *et al.* (2004) find that the central bank commitment to quantitative easing has to be credible to influence the shape of expectations and that the financial frictions can significantly contribute to the final change in the interest rate. Opposite results are presented by Eggertsson & Woodford (2003). Authors conclude that the quantitative easing fails to prevent the deflationary trap at the ZLB. It does not depend either on the size neither on the type of asset which the central bank uses to open market interventions. This holds if the expansion of the central bank's monetary base is made independently on the specification of the reaction function, and if we do not assume a permanent shift in the monetary base. The quantitative easing could be potentially effective due to the change in expectations about the future path of interest rate policy. Such a shift in public expectations requires again the credible commitment.[4]

However, the model suffers from serious limitations. First, the base-supply rule differs from monetary policy reality. Most of central banks do not use money targeting as the policy instrument but rather purchase some quantity of assets to influence yields across the yield curve. Because of the lack of financial frictions in the model, the expectations channel becomes the only channel of monetary transmission (Doh 2010). But current literature has included financial imperfections quite commonly, especially in modeling of QE. Disrupted relationships within the financial market and/or between the economy and financial market should be considered (Kamada & Sugo 2006).

Smets & Wouters (2007), Gilchrist *et al.* (2009) and others incorporate financial imperfections within the New Keynesian DSGE framework. Smets & Wouters (2007) have been criticized for omitting credit channel which seems to be crucial in quantitative transmission (Mishkin 2009). Monetary policy actions might be amplified by endogenous changes in the external finance premium.[5] This extension can be found in Bernanke *et al.* (1998) or Gilchrist *et al.* (2009).

[4] This result is based on the assumption that households have a satiation level for real money balances.

[5] The external finance premium is the difference in cost between internally and externally available funds.

Iacoviello (2005) proposes an alternative way by incorporating the real estate in the Bernanke *et al.* (1998) model which can be used as the collateral for loans. The model is in line with the character of the recent global financial crisis where the real estate market played an important role in overheating the economy. Cúrdia & Woodford (2011) and Gertler & Karadi (2011) analyze the quantitative easing within slightly different frameworks. Cúrdia & Woodford (2011) include borrowing frictions and heterogeneous households while Gertler & Karadi (2011) add financial intermediaries. Gertler & Karadi (2011) find that credit policy[6] can be beneficial even if the ZLB on nominal interest rate does not bind and Cúrdia & Woodford (2011) argue that purchases of illiquid assets can be effective at the ZLB when financial markets are sufficiently disrupted.

To sum up, the theory suggests that the effectiveness of quantitative easing depends on both, incorporated financial frictions and the ability of such kind of monetary policy to change the shape of public expectations. Therefore, it is important to examine empirical evidence. The Japanese experience has served as a good basis for such analysis. Individual studies differ in their results quite a lot, but the core overall conclusion can be found (Ugai 2007). The quantitative easing has just a moderate effect on asset yields (including the nominal interest rate) through the portfolio channel and the credible commitment of the Bank of Japan. Further, it contributes to the stabilization of the financial system by providing the liquidity (Ueda & Oda 2007). The research identifies some important features, mainly a certain effect on the interest rate and an uncertain effect on the inflation and output. This is caused most likely by low asset prices and the non-performing loan problem which leads to households' and entrepreneurs' balance-sheets deterioration, and by the serious disruption in economic relationships (Kamada & Sugo 2006).

An extensive research have also focused on the quantitative easing proposed by the Bank of England (e.g. Joyce *et al.* 2011; Bridges & Thomas 2012; Gagnon *et al.* 2011; Kapetanios *et al.* 2012). These studies usually confirm the effectiveness of large scale purchases. Joyce *et al.* (2011) identify a significant decrease in corporate bond yields while Gagnon *et al.* (2011) find that the quantitative easing leads to a sharp decline in the term premium of government bonds which results in higher inflation expectations and much deeper fall in the real interest rate. Kapetanios *et al.* (2012) conclude that without large purchases the real

[6]Expansion of the central bank credit intermediation in order to offset a disruption of private financial intermediation.

output would have fallen by even more during 2009 and the inflation would have turned into the deflation.

While the transmission mechanism for quantitative easing is one of the most widely discussed topics in the current monetary policy literature, there has not been the empirical research sufficiently identifying the long-term effects of this type of unconventional policy on the core macroeconomic variables and allowing the incorporation of the appropriate transmission mechanism into the New Keynesian framework.

3.3 Foreign Exchange Interventions

Foreign exchange interventions are today common in many countries. Moreover, they become even more frequent in recent years in both, emerging markets and industrial economies. FX interventions are used in periods of large capital inflows and outflows when the central bank attempts to prevent strong currency appreciation and depreciation (Montoro & Ortiz 2013). There arise some questions which should be addressed in the context of interventions, in particular, which transmission channels are affected and how, and whether interventions can be effectively used with the inflation targeting since many central banks operate in this regime today, including the Czech National Bank.[7]

A theory does not provide much information about the compatibility of inflation targeting regime with FX interventions. It assumes that by intervening the central bank can affect the economy just in the short-run (3-month horizon) through expectations channel or portfolio balance channel while the long-term effect is clearly confirmed just in a few studies (Cavusoglu 2010).[8] Despite the recommendation not to use FX interventions in the inflation targeting regime, many central banks intervene at least occasionally even though they formally follow a pure floating (Calvo & Reinhart 2000) with increasing frequency during the recent crisis (Lízal & Schwarz 2013).

Currently, the FX interventions become one of the potential (un)conventional measures which can the central bank use if the short-term interest rate is bounded at zero level. Several authors propose the exchange rate management as an effective way how to overcome the ZLB. Starting with McCallum

[7]CNB has adopted inflation target in 1998.

[8]The intervention mechanism incorporated in the DSGE model described in Chapter 4 and used for simulations in Chapter 6 and Chapter 8 operates also through the expectations and portfolio balance channel which is in line with IT theory.

(2000), he suggests that the central bank should adopt the following Taylor-type policy rule with the exchange rate as an instrumental variable to stabilize the economy in response to the shock at the ZLB

$$s_t - s_{t-1} = \mu_0 - \mu_1(\Delta p_t - \pi^*) - \mu_2 E_{t-1}\tilde{y}_t + e_t \tag{3.7}$$

where s_t is the nominal exchange rate, p_t is the price level, π^* is the inflation target, $\tilde{y}_t$ is the output gap and e_t is the exogenous shock. With low inflation or output below its potential, the depreciation rate increases which is accompanied via central bank purchases of foreign exchange. It should bring the inflation back on its target and output gap to zero (McCallum 2000). Moreover, Svensson (2001) proposes to temporary peg the currency at the substantially devalued exchange rate together with the future committing to the price-level target. Again, the inflation expectations may be increased via unsterilized interventions. The real depreciation reduces import in favor of export which boosts the output. Both, McCallum (2000) and Svensson (2001), drop the uncovered interest rate parity (UIP) in their analysis which is quite strict assumption despite the fact that an empirical evidence on the UIP condition might not be clear. Following Christiano (2000), the UIP condition serves as an underlying relation in the model specification proposed by (McCallum 2000). Missing UIP can break the intervention mechanism as the purchase of foreign exchange might not lead to the exchange rate depreciation at the ZLB. Since the theoretical analysis of FX interventions at the ZLB is limited, we need to turn our attention at this point to their modeling in frameworks without ZLB.

Despite the empirical evidence that most of the short-term and medium-term exchange rate fluctuations are caused by order flows, a majority of current models relates the exchange rate to macroeconomic fundamentals. In recent years, several studies have addressed this issue. First, Bacchetta & van Wincoop (2006) provide a model where the exchange rate is closely related to order flows which represent a private information component of foreign exchange orders. Vitale (2011) extends the model to allow for analysis of interventions effectiveness through the portfolio balance or expectations channel. Montoro & Ortiz (2013) adopt this approach and incorporate it into the standard New Keynesian DSGE model which allows to analyze the FX interventions together with the monetary policy. The introduction of risk-averse foreign exchange dealers results in modification of the UIP condition in form of added risk premium which depends on the capital order flows and the central bank interventions.

The modification of standard UIP condition is proposed also by Benes *et al.* (2013) who incorporate the financial sector into the DSGE model to allow for the portfolio balance channel. Within this framework authors analyze the effect of different exchange rate regimes in combination with hybrid inflation targets. Their results suggest that FX interventions can be helpful in the presence of foreign shocks, but may also hamper some necessary exchange rate adjustments. The effect of interventions through the portfolio balance channel is confirmed also by Herrera *et al.* (2013).

Besides the theory, some empirical evidence should be provided. So far, three countries used FX interventions at the ZLB while the last one, the Czech Republic, does not seem to be suitable for the analysis as it starts to intervene quite recently. Moreover, the Czech situation is examined separately in Chapter 7. The other two countries are Switzerland which central bank starts to intervene in August 2011 and Israel with the start in March 2008. The detailed discussion is provided in Lízal & Schwarz (2013). To sum up, the FX interventions have proven effective if the central bank is willing to clearly and credibly commit to unlimited purchases.

3.4 Forward Guidance

The forward guidance can be viewed from two different perspectives - as a simple conventional policy tool which explains the central bank reaction function and is applied usually in normal times (non-crisis periods), and as an unconventional measure representing the central bank commitment to change in its future policy path which differs from the setting implied by the standard reaction function (Holub 2013). Constrained short-term rates encourage the central bank to experiment with a precision and a length of announcement about the future policy (Woodford 2012). In such a case the central bank usually commits to keep the short-term policy rate at the zero level for a longer period to decrease the nominal long-term interest rate, aiming at higher inflation expectations and lower real long-term interest rate (Summers 1991).[9]

The long-term interest rate becomes much more relevant for economic decisions during the crisis because of the increased uncertainty about the refinancing short-term loans. With the announcement of low expected short-term interest rate for sufficiently long time the long-term interest rate should also

[9]In the case of inflation expectations nearby zero the constraint is more likely to bind.

decrease (if the expectations hypothesis of the term structure holds). Hence, the central bank should be able to change the shape of public expectations about the future path of short-term interest rate (Hansen 2012)

$$i_{nt} = \frac{i_{1t} + i^e_{1t+1} + ... + i^e_{1t+n}}{n} \tag{3.8}$$

The forward guidance should be of a concern also when there is just a threat that the ZLB may start to bind in the near future because such expectations may considerably affect aggregate variables. Considering the simple New Keynesian model for monetary policy, the effect of higher expected inflation and income on the current price level and expenditures is alleviated via the standard Taylor rule. In the situation of expected binding constraint on the short-term rate the effect of inflation expectations is magnified. The effect is even stronger when the lower bound is expected to bind for more periods. Assuming that such expectations can shift far in the future, there would be a large effect on the economy at the point when the lower bound ceases to bind (Woodford 2012). Eggertsson & Woodford (2003) provide similar results in their numerical experiment where just a slight increase in inflation expectations creates an output boom and high inflation once the lower bound ceases to bind. Therefore, the central bank should follow a medium-term strategy which reduces the uncertainty about the future development. The short-term policy rates should be held at the zero level for much longer period than the standard forward-looking Taylor rule implies. Later on, Woodford (2012) suggests a nominal GDP targeting to be better exit strategy than the inflation targeting considering large losses in the GDP relative to the state of the economy in the absence of financial crisis.

The forward guidance is very powerful but not simple task. Its success depends entirely on the ability of the central bank to effectively communicate its decisions. In this context, there are two main reasons why it should announce the future policy path at the ZLB. First, the public might not fully understand reasons and consequences of undertaken actions which is very important for effective transmission. More important issue is a time inconsistency (Kydland & Prescott 1977). The central bank attempts to convince the public that short-term rates remain significantly low for a longer period. Once the inflation expectations increase, there is no incentive to stay with this commitment. The central bank would like to increase its policy rate to avoid high inflation. If economic agents realize this inability to respect the commitment, expectations might not be managed sufficiently (Hansen 2012; Eggertsson & Woodford 2003; Werning 2011).

The empirical evidence on this topic is still modest, but growing. Gürkaynak *et al.* (2005) and Campbell *et al.* (2012) use a principal component analysis on the US data which identifies that announcing the future path of monetary policy by the Federal Open Market Committee (FOMC) has a large impact on long-term Treasury yields. The FOMC statements significantly affect financial market expectations about the future policy actions in both, the pre-crisis and crisis period. This type of policy explains a large portion of variation in the funds rate forecast. Anyway, this methodology provides no information about particular aspects of the FOMC statements. There is a question whether the policy rate changes are based on the statement or on the information about the future economic development which would otherwise determine its policy (Woodford 2012). Bernanke *et al.* (2004) provide some useful implications of the forward guidance at the ZLB. They identify two effects of the Fed monetary policy on asset prices - (i) the change in the current level of the fund rate and (ii) the change in expectations about its future path over the next year which corresponds to FOMC statements. Hence, policymakers are able to influence the long-term rate even if the short-term rate remains unchanged. Such a commitment has to be publicly understandable to have some real effect.

3.5 Monetary Policy Rules

The analysis of the monetary policy at the ZLB still suffers from the lack of studies concerned with different monetary policy rules. Taylor-type rules become commonly used in the DSGE analysis in normal times and remain also a part of analysis under constrained short-term rates. But there is paid surprisingly little attention to both, their detailed description and possible alternatives and modifications. The policy rule proposed by Taylor (1993) was initially suggested just as a normative tool but very quickly became famous because of its ability to precisely mimic the behavior of Fed, mostly in the Greenspan era. Later on, the adoption of Taylor rule was encouraged by strong empirical evidence (e.g. Clarida & Gertler 1999; Coibion & Gorodnichenko 2011).

Nakov (2008) points out that there have been ignored some core limitations at the ZLB. For instance, there is often assumed to be a perfect foresight or a forcing certainty equivalence (e.g. Jung *et al.* 2005; Coenen *et al.* 2003). But the certainty equivalence assumption does not hold in a presence of a non-linear constraint. As mentioned above, the price-level targeting might be more effective than the inflation targeting at the ZLB (e.g. Svensson 2000; Smets 2000; Nakov 2008; Amanoy & Ambler 2008; Merola 2010; Kaufmann & Bäurle 2013). Better performance strongly depends on the central bank's credibility and public expectations. Nakov (2008) finds that under the discretionary policy the welfare losses are ten times higher than with the optimal rule commitment when the short-term interest rate is constrained at the zero level. Under the constant price-level targeting the losses are significantly lower than under the truncated Taylor rule. Amanoy & Ambler (2008), Merola (2010) and Kaufmann & Bäurle (2013) provide similar results. They show that the adoption of price-level target decreases the probability of hitting lower bound on nominal rates. The variability of inflation and short-term rates decline because public expectations are stabilized.

Eggertsson & Woodford (2003) suggest that an effective targeting rule for monetary policy should involve the natural rate of interest which is based on the evolution of real factors. Trehan & Wu (2007) propose the real interest rate as a time-varying intercept in the policy rule and Cúrdia *et al.* (2011) find that such a policy rule leads to improvement of the model compared to other alternatives.

Chapter 4

The DSGE Model

4.1 Domestic Households

The households' problem is quite standard. The world economy is populated by a continuum of households of mass 1. We assume that on average the households act as representative agents while the deviation from such a behavior is normally distributed with zero mean (and σ^2). Therefore, it can be ignored. Moreover, households are assumed to be rational. Any deviation from rational expectations is assumed again to be normally distributed with zero mean, and therefore, it is not relevant for our purpose. Each household enjoys a utility from the consumption of various goods produced by domestic and foreign firms, and receives disutility from working for domestic firms. Therefore, each household maximizes the lifetime utility subject to her budget constraint. In general, household preferences can be expressed by the following utility function

$$U_t = E_t \sum_{t=0}^{\infty} \beta^t U\left(C_t, N_t\right) \tag{4.1}$$

where U_t is the utility at time t, E_t are rational expectations at time t, $U(.)$ denotes the utility function which is assumed to be continuous and twice differentiable in both, consumption and labor, with $U' > 0$ and $U'' < 0$, $\beta \in \langle 0, 1)$ is a discount factor, C_t is the consumption at time t and N_t is the labor supplied by representative agent at time t.

The specific utility function with constant relative risk aversion preferences is assumed while the labor and consumption enters the utility function separately

$$U_t = E_0 \sum_{t=0}^{\infty} \beta^t \xi_{g,t} \left[\frac{(C_t - H_t)^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi} \right] \tag{4.2}$$

where $H_t = hC_{t-1}$ is a habit formation term which is given exogenously, $\xi_{g,t}$ is a preference shock, σ is a coefficient of relative risk aversion (measured as an inverse of the intertemporal elasticity of substitution (IES) in consumption) and φ is an inverse Frisch elasticity of labor supply. For $0 \leq \sigma < 1$, the IES is high and agents are quite willing to substitute consumption between periods. For $\sigma > 1$, the IES is low and agents do not like shifting their consumption in time. The Frisch elasticity captures the substitution effect between wage rate and labor supply, holding the marginal utility of wealth constant (Heer & Maussner 2009).

For simplicity, households are able to save only in domestic bonds. Then the budget constraint of each representative agent can be expressed as follows

$$P_tC_t + D_t = D_{t-1}(1+i_{t-1}) + \frac{\psi}{2}(D_t - \bar{D})^2 + W_tN_t + \Pi_{M,t} + \Pi_{H,t} + \Pi_{FX,t} + T_t \tag{4.3}$$

where D_t represents the quantity of one-period bonds denominated in domestic currency, P_t is a price level, W_t is a nominal wage, $\Pi_{X,t}$ denotes profits which are distributed from firms and foreign exchange dealers to households and T_t is a transfer from government. Moreover, the household faces portfolio adjustment costs $(\psi/2)(D_t - \bar{D})^2$ which provide a mechanism for closing the model, i.e. to determine a steady state (see Schmitt-Grohe & Uribe 2003).

The households' consumption basket is composed of non-traded, domestically produced goods and imported, foreign goods. It follows a form of the Dixit-Stiglitz function

$$C_t = \left[(1-\alpha)^{\frac{1}{\eta}} C_{H,t}^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}} M_t^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}} \tag{4.4}$$

where $C_{H,t}$ and M_t are indices of domestic and foreign goods consumption, respectively. Parameter η denotes the elasticity of substitution between these goods and α is the weight of foreign goods relative to the total consumption. It describes the trade openness of the economy. Both indices, $C_{H,t}$ and M_t, are given by the constant elasticity of substitution (CES) functions

$$C_{H,t} = \left(\int_0^1 C_{H,t}(j) dj^{\frac{\epsilon-1}{\epsilon}} \right)^{\frac{\epsilon}{\epsilon-1}} \tag{4.5}$$

$$M_t = \left(\int_0^1 C_{i,t} di^{\frac{\gamma-1}{\gamma}} \right)^{\frac{\gamma}{\gamma-1}} \tag{4.6}$$

$$C_{i,t} = \left(\int_0^1 C_{i,t}(j) dj^{\frac{\epsilon-1}{\epsilon}} \right)^{\frac{\epsilon}{\epsilon-1}} \tag{4.7}$$

where $C_{i,t}$ is an index of consumption of goods imported from country i, ϵ is the elasticity of substitution between various goods produced domestically and γ is the elasticity of substitution between goods produced in different foreign countries.

At this step we should briefly explain the idea behind the Dixit-Stiglitz function which uses the CES aggregator for the analysis of product heterogeneity in a monopolistic competition. Such a market consists of finite number of firms producing differentiated substitutable goods. Each firm is a monopolistic producer of its own unique product but still has to compete with other firms on the market. The elasticity of substitution is assumed to be the same for all pairs of products. Goods are perfect substitutes when the elasticity of substitution approaches infinity and perfect complements when it approaches zero (Dixit & Stiglitz 1977).

First, a within basket optimization is performed. Households make decisions about the allocation of their consumption expenditures among different goods and try to spend within the basket at least as possible. The optimization results in the optimal allocation of expenditures within each bundle of goods. The minimization of consumption expenditures P_tC_t subjected to the consumption index (4.4) yields demand functions for non-traded goods (4.8) and import (4.9). Moreover, we assume that a demand function for export 4.10 is of the same form as the demand function for import, just with different parameters

$$C_{H,t} = (1-\alpha) \left(\frac{P_{H,t}}{P_t} \right)^{-\eta} C_t \tag{4.8}$$

$$M_t = \alpha \left(\frac{P_{M,t}}{P_t} \right)^{-\eta} C_t \tag{4.9}$$

$$X_t = \zeta \left(\frac{P^*_{H,t}}{P^*_t} \right)^{-\eta} Y^*_t \tag{4.10}$$

where $P_{H,t}$, $P_{M,t}$ and $P^*_{H,t}$ are price levels of non-traded, imported and exported goods, respectively, and P^*_t is the world price. The aggregate price index P_t follows the form of the Dixit-Stiglitz function

$$P_t = \left[(1-\alpha)P_{H,t}^{1-\eta} + \alpha P_{M,t}^{1-\eta}\right]^{\frac{1}{1-\eta}} \tag{4.11}$$

As shown in equations (4.8) and (4.9), the consumption of domestically produced and imported goods increases in the total consumption level, and decreases in the corresponding relative prices. The consumption of export (4.10) increases in the foreign output and decreases in the relative foreign price level. Given these demand functions, households maximize their lifetime utility (4.2) subject to the budget constraint (4.3) by choosing the optimal amount of consumption, labor and investment. The combination of first order conditions yields following optimality conditions

$$\frac{\xi_{g,t}}{(C_t - hC_{t-1})^\sigma} = \beta E_t \left[\frac{\xi_{g,t+1}}{(C_{t+1} - hC_t)^\sigma}(1+i_t)\frac{P_t}{P_{t+1}}\right] \tag{4.12}$$

$$(C_t - hC_{t-1})^\sigma N_t^\varphi = \frac{W_t}{N_t} \tag{4.13}$$

Equation (4.12) represents the standard Euler equation and equation 4.13 describes the supply of labor.

4.2 Domestic Producers

There exists a continuum of domestic firms producing non-traded goods (goods sold and consumed at the domestic market) and goods for export (goods sold and consumed at the foreign market) of mass 1 which operates on the monopolistically competitive market. Each firm produces differentiated good using the following linear technology with labor as a single input

$$y_t(i) = \xi_{a,t} N_t(i) \tag{4.14}$$

where $\xi_{a,t}$ represents technological innovation. The aggregate production function has the form of the Dixit-Stiglitz function

$$Y_t = \left[\int_0^1 y_t(i)^{\frac{\tau-1}{\tau}} di\right]^{\frac{\tau}{\tau-1}} \tag{4.15}$$

where τ is the elasticity of substitution between different varieties.

Again, firms are on average rational and exist over infinite time horizon. Hence, they can be described and modeled as representative. Each firm faces the downward-sloping demand function given in equation (4.19) and has some monopolistic power in price setting. In particular, firms face each period an exogenous probability $(1-\theta_H)$ of re-optimizing their prices. This probability is independent of the time when the firm last changes its price (Calvo 1983). Following Bäuerle & Menz (2008), firms that cannot reset their prices at least adjust for inflation according to the following rule

$$P_{H,t}(i) = P_{H,t-1}(i)\left(\frac{P_{H,t-1}}{P_{H,t-2}}\right)^{\delta_H} \tag{4.16}$$

where δ_H describes to which extent the firm reacts to the past inflation. All firms which re-optimize their prices in period t set the same price $P'_{H,t}$. The overall price index of domestic goods is as follows

$$P_{H,t} = \left\{(1-\theta_H){P'_{H,t}}^{1-\epsilon} + \theta_H\left[P_{H,t-1}\left(\frac{P_{H,t-1}}{P_{H,t-2}}\right)^{\delta_H}\right]^{1-\epsilon}\right\}^{\frac{1}{1-\epsilon}} \tag{4.17}$$

Firms take into account the probability that they will not be able to re-optimize prices in the future. From the households' problem the form of the domestic demand for variety is known

$$C_{H,t}(i) = \left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{-\epsilon} C_{H,t} \tag{4.18}$$

We assume that the foreign demand follows the same functional form. Hence, we can construct a demand curve for firm which resets its price in the period t and adjusts its price according to (4.15) in periods $t+k$

$$C_{H,t+\tau|t} = \left[\frac{P'_{H,t}}{P_{H,t+\tau}}\left(\frac{P_{H,t+\tau-1}}{P_{H,t-1}}\right)^{\delta_H}\right]^{-\epsilon}(C_{H,t+\tau}+X_{t+\tau}) \tag{4.19}$$

Domestic firms that can reset their prices in period t maximize the expected discounted profit given by equation (4.20) subjected to equation (4.19).

$$E_t\sum_{\tau=0}^{\infty}\theta_H^{\tau}Q_{t,t+\tau}C_{H,t+\tau|t}\left[P'_{H,t}\left(\frac{P_{H,t+\tau-1}}{P_{H,t-1}}\right)^{\delta_H} - P_{H,t+\tau}MC_{t+\tau}\right] \tag{4.20}$$

where $MC_t = W_t/(P_{H,t}\xi_{a,t})$ are real marginal cost, $Q_{t,t+\tau} = \beta^\tau \Lambda_{t+\tau}/\Lambda_t$ is a stochastic discount factor (pricing kernel) with $\Lambda_t = \xi_{g,t}(C_t - hC_{t-1})/P_t$. The stochastic discount factor measures the marginal rate of substitution between consumption at t and $t+1$.

The first order condition with respect to $P'_{H,t}$ represents the optimality condition for the price level of domestically produced and consumed goods

$$P'_{H,t} = \frac{\epsilon}{\epsilon - 1} \frac{\sum_{\tau=0}^{\infty} \theta_H E_t[Q_{t,t+\tau} C_{H,t+\tau|t} P_{H,t+\tau} MC_{t+\tau}]}{\sum_{\tau=0}^{\infty} \theta_H E_t[Q_{t,t+\tau} C_{H,t+\tau|t} (P_{H,t+\tau-1}/P_{H,t-1})^{\delta_H}]} \tag{4.21}$$

Domestic goods are assumed to be sold at the law of one price at the foreign market $P_{H,t} = e_t P^*_{H,t}$. Hence, exporters face basically the same optimality condition where prices are just set in foreign currency.

4.3 Domestic Retailers

Following Monacelli (2003), the incomplete exchange rate pass-through is introduced into the model. Domestic retailers import foreign goods for which the law of one price holds at the docks. Due to the monopolistic competition these retailers have some degree of price setting power. Hence, the price of import can deviates from the law of one price in the short run. Again, we assume the Calvo price setting with the probability θ_M and the indexation rule for firms which cannot reset their prices with parameter δ_M. Domestic retailers maximize the expected discounted profit subjected to the demand for this good

$$\max \quad E_t \sum_{\tau=0}^{\infty} \theta_M^\tau Q_{t,t+\tau} M_{t+\tau|t} \left[P'_{M,t} \left(\frac{P_{M,t+\tau-1}}{P_{M,t-1}} \right)^{\delta_M} - e_{t+\tau} P^*_{M,t+\tau} \right] \tag{4.22}$$

$$s.t. \quad M_{t+\tau|t} = \left[\frac{P'_{M,t}}{P_{M,t+\tau}} \left(\frac{P_{M,t+\tau-1}}{P_{M,t-1}} \right)^{\delta_M} \right]^{-\epsilon} M_{t+\tau} \tag{4.23}$$

The maximization results in the following optimality condition for the price level of imported goods

$$P'_{M,t} = \frac{\epsilon}{\epsilon - 1} \frac{\sum_{\tau=0}^{\infty} \theta_M E_t[Q_{t,t+\tau} M_{t+\tau|t} e_{t+\tau} P^*_{M,t+\tau}]}{\sum_{\tau=0}^{\infty} \theta_M E_t[Q_{t,t+\tau} M_{t+\tau|t} (P_{M,t+\tau-1}/P_{M,t-1})^{\delta_M}]} \tag{4.24}$$

4.4 Foreign Exchange Dealers

In this section we follow an idea of Montoro & Ortiz (2013). The domestic economy is populated by a continuum of foreign exchange dealers d such that $d \in \langle 0, 1 \rangle$. Similarly to the households' and firms' problem we assume that dealers are rational and they act on average as representative agents. Any deviation from this behavior is again normally distributed with zero mean. Hence, it can be ignored. Each dealer d receives the same amount of sale and purchase orders

- ω_t^d from households in domestic bonds,
- $\omega_{CB,t}^d$ from the central bank in domestic bonds,
- ω_t^{d*} from foreign investors in foreign bonds,
- $\omega_{CB,t}^{d*}$ from the central bank in foreign bonds.

There is an exchange of these orders among dealers such that following condition is satisfied

$$\omega_t^d - \omega_{CB,t}^d + e_t(\omega_t^{d*} + \omega_{CB,t}^{d*}) = B_t^d + e_t B_t^{d*} \tag{4.25}$$

where B_t^d, B_t^{d*} are ex-post holdings of bonds in domestic and foreign currency by each dealer d.

FX dealers are assumed to be risk averse and maximize their CARA utility function which takes the following form

$$max - E_t^d e^{-\gamma \Omega_{t+1}^d} \tag{4.26}$$

The optimal portfolio allocation problem should be solved with Ω_{t+1}^d representing total dealer's investment after returns

$$\begin{aligned}\Omega_{t+1}^d =&(1+i_t)B_t^d + (1+i_t^*)E_t^d e_{t+1} B_t^{d*} \\ =&(1+i_t)[\omega_t^d - \omega_{CB,t}^d + e_t(\omega_t^{d*} + \omega_{CB,t}^{d*})] \\ &+ (i_t^* - i_t + E_t^d e_{t+1} - e_t)B_t^{d*}\end{aligned} \tag{4.27}$$

where e_t in the last equation is a logarithm of nominal exchange rate.

The optimization with respect to B_t^{d*} yields

$$0 = -\gamma(i_t^* - i_t + E_t^d e_{t+1} - e_t) + \gamma^2 B_t^{d*} \sigma_e^2$$
$$B_t^{d*} = \frac{i_t^* - i_t + E_t^d e_{t+1} - e_t}{\gamma \sigma_e^2} \tag{4.28}$$

where $\sigma_e^2 = var(\Delta e_{t+1})$.

A market clearing condition for foreign bonds in the domestic market equals the sum of sale and purchase orders from the central bank and foreign investors in foreign bonds

$$\int_0^1 B_t^{d*} dd = \int_0^1 (\omega_t^{d*} + \omega_{CB,t}^{d*}) dd = \omega_t^* + \omega_{CB,t}^* \tag{4.29}$$

After substituting for (4.28) the modified UIP condition is computed

$$\bar{E}_t e_{t+1} - e_t = i_t - i_t^* + \gamma \sigma_e^2 (\omega_t^* + \omega_{CB,t}^*) \tag{4.30}$$

where $\bar{E}_t$ is an average rational expectation across all dealers about the future nominal exchange rate. Assuming the information homogeneity, this expectation is the same for all dealers, i.e. $\bar{E}_t = E_t$.

The shortsighted and risk-averse FX dealers modify the standard UIP condition by adding a time-variant risk premium which depends on FX interventions and capital inflows/outflows. This intervention mechanism has a stabilizing effect. If the exchange rate deviates from its long-run level, the central bank starts intervening to bring it back to the steady state (or to the target). The risk premium term changes which works against the exchange rate movement. Interventions may influence the exchange rate through two transmission channels - the portfolio balance channel and the expectations channel. The effect of the portfolio balance channel is defined by the term $\gamma \sigma_e^2 (\omega_t^* + \omega_{CB,t}^*)$ while the expectations channel works through expected future exchange rate $E_t e_{t+1}$. By intervening (change in $\omega_{CB,t}^*$) the central bank affects the ratio between domestic and foreign assets held by dealers, and hence, also the risk premium which these dealers require. The change in risk premium results in the exchange rate appreciation or depreciation.

4.5 Monetary Authority

The monetary authority uses two instruments, the short-term interest rate and foreign exchange interventions. Next two subsections summarize all possible monetary policy regimes.

4.5.1 Foreign Exchange Interventions

Each period the central bank intervenes in the foreign exchange market through FX dealers which receive their purchase or sale orders in foreign bonds in exchange for domestic bonds. We assume that the central bank can always perform fully sterilized interventions. There is another important assumption that the central bank is not limited in its interventions and that it is not obliged to distribute its profits to the households. The central bank decides on the level of foreign exchange interventions according to the following rule

$$\omega^*_{CB,t} = \chi_e(e_t - e_{t-1}) + \chi_{e^T}(e_t - e^T) + \chi_q q_t + \varepsilon^*_{cb,t} \tag{4.31}$$

where e^T is an operational exchange rate target and q_t is the deviation of the real exchange rate from its steady state level. The central bank can choose from four different intervention strategies. If all parameters in (4.31) are equal to zero ($\chi_e = 0, \chi_{e^T} = 0, \chi_q = 0$) it performs unanticipated interventions. In the case of non-zero χ_e coefficient, the central bank intervenes to prevent the nominal exchange rate fluctuation, i.e. sells foreign bonds to prevent the depreciation and purchases foreign bonds to prevent the appreciation. The value of coefficient captures the intensity of the response. This kind of strategy is called "leaning-against-the-wind" and can be easily used to model a managed float. Similarly, χ_{e^T} describes to which extent the central bank intervenes in order to achieve a particular nominal exchange rate target. In the extreme case ($\chi_{e^T} \to \infty$) the central bank uses an unlimited amount of interventions to mitigate any (negligible) deviation from this target. In the last case ($\chi_q \neq 0$) the central bank attempts to correct for a misalignment of the real exchange rate from its long-run (fundamental) values.

4.5.2 Interest Rate and Exchange Rate Rules

This section describes four different monetary policy rules. We examine the case of the pure inflation targeting, the price-level targeting, the modified inflation targeting (where the central bank assigns some weight also to the deviation of the nominal exchange rate from its target), and the exchange rate targeting. All rules are summarized in equations (4.32) - (4.35).

$$
\begin{aligned}
i_t &= \rho_i i_{t-1} + (1-\rho_i)\left[\bar{i} + \psi_\pi(\pi_t - \pi^T)\right] + \varepsilon_{i,t} && (4.32)\\
i_t &= \rho_i i_{t-1} + (1-\rho_i)\left[\bar{i} + \psi_p(p_t - p^T)\right] + \varepsilon_{i,t} && (4.33)\\
i_t &= \rho_i i_{t-1} + (1-\rho_i)\left[\bar{i} + \psi_\pi(\pi_t - \pi^T) + \psi_e(e_t - e^T)\right] + \varepsilon_{i,t} && (4.34)\\
i_t &= \rho_i i_{t-1} + (1-\rho_i)\left[\bar{i} + \psi_e(e_t - e^T)\right] + \varepsilon_{i,t} && (4.35)
\end{aligned}
$$

The reason for use of the price-level targeting at the ZLB is described in Chapter 3. The exchange rate target (either in the form of dirty or pure exchange rate targeting rule) is also assumed to anchor the price-level expectations better than the pure inflation targeting at the ZLB.

4.6 Market Clearing

The market clearing condition equalizes the total domestic production of goods Y_t with the sum of non-traded goods $C_{H,t}$ and exported goods X_t.

$$Y_t = C_{H,t} + X_t \tag{4.36}$$

4.7 Foreign Economy

All foreign variables and coefficients are denoted with a * superscript. These variables are not modeled as an exogenous process. We follow Bäurle & Kaufmann (2013) who "take the model for the small open economy and - loosely speaking - solve it for the closed economy limiting case".

4.8 Additional Important SOE Relations

We need to define another three expressions to complete the model - the real exchange rate, the effective terms of trade and the law-of-one-price gap. The bilateral real exchange rate is defined as a ratio between countries' CPIs denominated in the same currency (Gali 2009)

$$\mathcal{Q}_{i,t} = \frac{\mathcal{E}_{i,t} P_t^i}{P_t} \tag{4.37}$$

where $\mathcal{E}_{i,t}$ is the nominal exchange rate of country i for $i \in \langle 0, 1\rangle$. The effective real exchange rate then follows

$$\mathcal{Q}_t = \frac{\mathcal{E}_t P_t^*}{P_t} = \left(\int_0^1 \mathcal{Q}_{i,t}^{1-\gamma} di \right)^{\frac{1}{1-\gamma}} \tag{4.38}$$

The bilateral terms of trade represents the price of country i's goods in terms of home goods for $i \in \langle 0, 1\rangle$

$$S_{i,t} = \frac{P_{i,t}}{P_{H,t}} \tag{4.39}$$

The effective terms of trade is defined as follows

$$S_t = \frac{P_{H,t}}{P_{M,t}} = \left(\int_0^1 S_{i,t}^{1-\gamma} di \right)^{\frac{1}{1-\gamma}} \tag{4.40}$$

The law-of-one-price gap refers to the ratio between world price denominated in domestic currency and the price of imported goods

$$\Psi_{M,t} = \frac{\mathcal{E}_t P_t^*}{P_{M,t}} \tag{4.41}$$

4.9 Log-linear Approximation to the Model

This section provides a set of log-linearized equilibrium conditions. Log-linearization is useful because the work with linear difference equations is much easier and the interpretation more intuitive, i.e. all terms are in percent. First, the natural logarithm of non-linear difference equations is taken. Then these expressions are approximated around the steady state using a first order Taylor series expansion. Assume a variable A_t which should be log-linearized. Its percentage change, denoted with a small letter a_t, is obtained as follows (Razin 2000)

$$A_t = \bar{A} \left(\frac{A_t}{\bar{A}} \right) = \bar{A} e^{log(A_t/\bar{A})} = \bar{A} e^{a_t}$$

After the first order Taylor approximation around the steady state $\bar{A}$ we get

$$\begin{aligned} \bar{A} e^{a_t} &\simeq \bar{A} e^0 + \bar{A} e^0 (a_t - 0) \\ &\simeq \bar{A}(1 + a_t) \end{aligned}$$

First, the Euler equation (4.12) is log-linearized. We should highlight two semi-standard things used in this process. As i_t is already in percent, we leave it in absolute deviation from the steady state $(i_t - \bar{i})$. Then we approximate the term $\bar{i}/(1+\bar{i}) = 1$ which is suitable if β is sufficiently high. Now the Euler equation can be linearized to

$$c_t - hc_{t-1} = E_t c_{t+1} - hc_t - \frac{1-h}{\sigma}\left[(i_t - E_t \pi_{t+1}) + \vartheta d_t + (\xi_{g,t} - E_t \xi_{g,t+1})\right] \tag{4.42}$$

where $\vartheta = \psi \bar{D}$.

Before the log-linearization of the budget constraint (4.12) we adjust this relation a bit. Government transfers T_t are chosen to neutralize distortions from the monopolistic competition. The aggregate profits from FX dealers are equal to zero. Moreover, we follow Bäuerle & Menz (2008) in introduction of profits from the final goods producing firms $\Pi_{FG,t} = P_t C_t - P_{M,t} M_t - P_{H,t} C_{H,t}$. Then we can write

$$\Pi_{FG,t} + \Pi_{M,t} + \Pi_{H,t} = P_t C_t - W_t N_t - P_{M,t} M_t - P_{H,t} C_{H,t} \tag{4.43}$$

Substituting the relation into the budget constraint yields

$$D_t = (1 + i_{t-1}) D_{t-1} + \frac{\psi}{2}(D_t - \bar{D})^2 - P_{M,t} M_t - P_{H,t} C_{H,t} \tag{4.44}$$

The linearization is now straightforward

$$\begin{aligned}(2\alpha - 1)[\bar{i}^{-1}\left(\beta^{-1} d_{t-1} - d_t\right) + i_{t-1}] = \\ = & \alpha(p_{M,t} + m_t) + (1-\alpha)(p_{H,t} + c_{H,t})\end{aligned} \tag{4.45}$$

Log-linearization of the market clearing condition requires some preliminaries. We assume $\bar{P} = \bar{P}_M = \bar{P}_H$ in the zero inflation steady state which implies $\bar{C}_H = (1-\alpha)\bar{C}$, $\bar{M} = \alpha \bar{C}$ and $\bar{Y} = \bar{C}$. Then the linearization of the resource constraint (4.36) proceeds as follows

$$y_t = (1-\alpha) c_{H,t} + \alpha x_t \tag{4.46}$$

The demand functions for export, import and domestic goods (4.8) - (4.10) are linearized quite easily, and they are expressed in equations (4.47) - (4.49).

$$\begin{aligned} x_t &= -\eta^*(p_t^H - p_t^* - e_t) + y_t^* && (4.47)\\ m_t &= -\eta(p_{M,t} - p_t) + c_t && (4.48)\\ c_{H,t} &= -\eta(p_{H,t} - p_t) + c_t && (4.49) \end{aligned}$$

The linearized CPI, terms of trade, law-of-one-price gap and real exchange rate are described in equations (4.54) - (4.51).

$$\begin{aligned} p_t &= (1-\alpha)p_{H,t} + \alpha p_{M,t} && (4.50)\\ s_t &= p_{M,t} - p_{H,t} && (4.51)\\ \psi_{M,t} &= e_t + p_t^* - p_{M,t} && (4.52)\\ q_t &= e_t + p_t^* - p_t && (4.53) \end{aligned}$$

By substituting the linearized terms-of-trade relation (4.51) into the aggregate price index (4.54) we get

$$p_t = p_{H,t} + \alpha s_t \qquad (4.54)$$

The CPI inflation rate is defined as $\Pi_t = P_t/P_{t-1}$, or in linearized form as

$$\pi_t = p_t - p_{t-1} \qquad (4.55)$$

The price indices for domestic goods and import follow the similar definition

$$\begin{aligned} p_{H,t} &= p_{H,t-1} + \pi_{H,t} && (4.56)\\ p_{M,t} &= p_{M,t-1} + \pi_{M,t} && (4.57) \end{aligned}$$

Now we focus on the log-linearization of optimal prices of domestic firms and domestic retailers. First, we divide both sides of equations (4.17) and (4.21) by $P_{H,t-1}$ to obtain the inflation rate

$$\Pi_{H,t}^{1-\epsilon} = \left[(1-\theta_H) \frac{P'_{H,t}}{P_{H,t-1}}^{1-\epsilon} + \theta_H \left(\Pi_{H,t-1}\right)^{\delta_H(1-\epsilon)} \right]$$

$$E_t \sum_{\tau=0}^{\infty} \theta_H Q_{t,t+\tau} C_{H,t+\tau|t} \Pi_{H,t-1,t+\tau-1}^{\delta_H} \frac{P'_{H,t}}{P_{H,t-1}} =$$
$$= \frac{\epsilon}{\epsilon-1} E_t \sum_{\tau=0}^{\infty} \theta_H Q_{t,t+\tau} C_{H,t+\tau|t} \Pi_{H,t-1,t+\tau} MC_{t+\tau}$$

Rearranging terms in the latter expression yields

$$0 = E_t \sum_{\tau=0}^{\infty} \theta_H Q_{t,t+\tau} C_{H,t+\tau|t} \left[\frac{P'_{H,t}}{P_{H,t-1}} \Pi_{H,t-1,t+\tau-1}^{\delta_H} - \frac{\epsilon}{\epsilon-1} MC_{t+\tau} \Pi_{H,t-1,t+\tau} \right] \quad (4.58)$$

In the zero inflation steady state following relationship holds $\bar{P_H} = \bar{P_H}'$. Therefore, the steady state level of real marginal cost is $\bar{MC} = (\epsilon-1)/\epsilon$. Moreover, the discount factor $Q_{t,t+\tau}$ is equal to β^τ. Now we can log-linearize equation (4.58)

$$0 = E_t \sum_{\tau=0}^{\infty} (\theta_H \beta)^\tau [p'_{H,t} - p_{H,t-1} + \delta_H (p_{H,t+\tau-} - p_{H,t-1}) - (mc_{t+\tau} + p_{H,t+\tau} - p_{H,t-1})]$$

We need to rearrange the expression to obtain a gap between the optimal price and the price from the last period. This difference is equal to the weighted sum of expected future marginal cost and domestic inflation

$$p'_{H,t} - p_{H,t-1} = (1-\theta_H \beta) E_t \sum_{\tau=0}^{\infty} (\theta_H \beta)^\tau [(1 -_H \theta_H \beta) \pi_{H,t+\tau} + (1-\theta_H \beta) mc_{t+\tau}]$$

To get rid of the summation operator we use the first difference equation

$$p'_{H,t} - p_{H,t-1} = (1 -_H \theta_H \beta) \pi_{H,t} + (1-\theta_H \beta) mc_t + (\theta_H \beta) E_t [p'_{H,t+1} - p_{H,t}] \quad (4.59)$$

In the next step we log-linearize equation (4.17) and obtain

$$p'_{H,t} - p_{H,t-1} = \frac{\pi_{H,t}}{1-\theta_H} - \frac{\delta_H \theta_H}{1-\theta_H} \pi_{H,t-1} \quad (4.60)$$

Finally, equation (4.60) is substituted into the difference equation (4.59). By rearranging we obtain the Phillips curve for domestic goods

$$\pi_{H,t} - \delta_H \pi_{H,t-1} = \beta E_t [\pi_{H,t+1} - \delta_H \pi_{H,t}] + \kappa_H mc_t \quad (4.61)$$

where $\kappa_H = (1-\theta_H)(1-\theta_H\beta)/\theta_H$. In the following step the log-linear version of marginal cost has to be found. We substitute the optimality expression for the household labor supply (4.13) into the expression for marginal cost $MC_t = W_t/(P_{H,t}\xi_{a,t})$, and derive

$$MC_t = \frac{P_t(C_t - hC_t)^\sigma N_t^\varphi}{P_{H,t}\xi_{a,t}}$$

The log-linearization yields

$$mc_t = \varphi n_t + p_t - p_{H,t} - \xi_{a,t} + \sigma(1-h)^{-1}(c_t - hc_{t-1})$$

The substitution of terms of trade and production function gives us the final expression for marginal cost

$$mc_t = \varphi y_t - (1+\varphi)\xi_{a,t} + \alpha s_t + \frac{\sigma}{1-h}(c_t - hc_{t-1}) \tag{4.62}$$

The log-linearization of the optimal price setting of domestic retailers (4.24) is similar. The steady state level of the law-of-one-price gap is $\bar{\psi}_M = (\epsilon - 1)/\epsilon$. Then we can obtain the following difference equation

$$p'_{M,t} - p_{M,t-1} = (1 - \delta_M\theta_H\beta)\pi_{M,t} + (1-\theta_H\beta)\psi_{M,t} + (\theta_M\beta)E_t[p'_{M,t+1} - p_{M,t}]$$

The linearized price index for import has the same form as the price index for non-traded goods, just with different parameters

$$p'_{M,t} - p_{M,t-1} = \frac{\pi_{M,t}}{1-\theta_M} - \frac{\delta_M\theta_M}{1-\theta_M}\pi_{M,t-1}$$

The Phillips curve for imported goods is obtained by merging these two equations

$$\pi_{M,t} - \delta_M\pi_{M,t-1} = \beta E_t[\pi_{M,t+1} - \delta_M\pi_{M,t}] + \kappa_M\psi_{M,t} \tag{4.63}$$

where $\kappa_M = (1-\theta_M)(1-\theta_M\beta)/\theta_M$.

The modified uncovered interest rate parity is already linearized

$$i_t - i_t^* = (E_t e_{t+1} - e_t) - \gamma\sigma^2(\omega_{CB,t}^* + \omega_t^*) \tag{4.64}$$

where the capital inflow follows an AR(1) process

$$\omega_t^* = \rho_\omega^* \omega_t^* + \varepsilon_{\omega,t}^* \tag{4.65}$$

As the foreign economy is of the similar structure, we present just the final set of log-linearized equations, i.e. the Euler equation, the Phillips curve, the marginal cost, the price index and the monetary policy rule

$$y_t^* - h^* y_{t-1}^* = E_t y_{t+1}^* - h^* y_t^* - \frac{1-h^*}{\sigma^*}(i_t^* - E_t \pi_{t+1}^*) + (\xi_{g,t}^* - E_t \xi_{g,t+1}^*) \tag{4.66}$$

$$\pi_t^* - \delta^* \pi_{t-1}^* = \beta^* E_t[\pi_{t+1}^* - \delta^* \pi_t^*] + \kappa^* mc_t^* \tag{4.67}$$

$$mc_t^* = \varphi^* y_t^* + \frac{\sigma^*}{1-h^*}(y_t^* - h^* y_{t-1}^*) - (1+\varphi^*)\xi_{a,t}^* \tag{4.68}$$

$$p_t^* = p_{t-1}^* + \pi_t^* \tag{4.69}$$

$$i_t^* = \rho_i^* i_{t-1}^* + (1-\rho_i^*)[\psi_\pi^*(\pi_t^* - \pi^{*T}) + \psi_y^*(y_t^* - y^{*T})] + \varepsilon_{i,t}^* \tag{4.70}$$

where $\kappa^* = (1-\theta^*)(1-\theta^*\beta^*)/\theta^*$.

The preference and technology shock follows an AR(1) process

$$\xi_{g,t} = \rho_g \xi_{g,t-1} + \varepsilon_{g,t} \tag{4.71}$$

$$\xi_{a,t} = \rho_a \xi_{a,t-1} + \varepsilon_{a,t} \tag{4.72}$$

$$\xi_{g,t}^* = \rho_g^* \xi_{g,t-1}^* + \varepsilon_{g,t}^* \tag{4.73}$$

$$\xi_{a,t}^* = \rho_a^* \xi_{a,t-1}^* + \varepsilon_{a,t}^* \tag{4.74}$$

where $\left\{\varepsilon_{g,t}, \varepsilon_{g,t}^*, \varepsilon_{a,t}, \varepsilon_{a,t}^*\right\}$ together with $\left\{\varepsilon_{cb,t}^*, \varepsilon_{\omega,t}^*, \varepsilon_{i,t}, \varepsilon_{i,t}^*\right\}$ are $iid \sim N(0, \sigma_x^2)$.

Chapter 5

Bayesian Estimation

The Bayesian technique is used to estimate the model parameters for the Czech Republic. The estimation is based on a prior information which is updated by a likelihood and results in a posterior distribution. In following sections a basic methodology is described together with a choice of the prior distribution and the data, and the final posterior distribution.

5.1 Description of the Methodology

In this section we present and discuss the basic idea of Bayesian estimation and the core methodology applied on the DSGE model. In a classical econometric analysis all parameters are treated as fixed and unknown quantities, and probabilities as limits which indicate a relative frequency of observed event. In Bayesian econometric the parameters are assumed to be random with some probability distribution. Such a probability is often based on a subjective belief of a researcher. The primary motivation lies in the fact that we cannot perform the estimation on full set of information. Each sample of the data is somehow limited and also the true model is often not available in advance. On the other hand, we have usually some theoretical concept, the set of prior information, which can be followed in the building of the model.

The Bayesian estimation provides "a bridge between calibration and maximum likelihood" (Griffoli 2013). Although the calibration is quite commonly used in most of the DSGE models, it is to a large extent based on the subjective opinion of the researcher. A wide range of parameters give a considerably large room for different values (and also interpretation), e.g. parameters of exogenous processes. The calibration has become one of the most common reasons for criticism of DSGE models. While calibrated parameters are based on some theoretical knowledge or stylized facts of the data (e.g. long run levels) maximum likelihood or Bayesian estimation provide a strong econometrical basics for precise data analysis (Beccarini & Mutschler 2012). The maximum likelihood estimation belongs to the group of classical econometric methods but the Bayesian estimation takes into the account also some uncertainty about unknown parameters. The prior information (which can be represented by calibration) together with the likelihood function (which "give more importance to certain areas of the parameter subspace") are two basic elements of the Bayesian estimation (Griffoli 2013).

At this point we should recall the Bayes' theorem which connects the degree of belief with the data. The core formula is as follows

$$p(\theta_A|Y_T, A) \propto p(Y_T|\theta_A, A)p(\theta_A|A)$$

where Y_T are the data until period T, A are hyper-parameters (parameters of the prior distribution) which describe the specific model (prior theoretical information), θ_A are parameters of specific model and $p(.)$ stands for the probability density function.

This fundamental equation is composed of three elements, the prior $p(\theta_A|A)$, the likelihood function $p(Y_T|\theta_A, A)$ and the posterior $p(\theta_A|Y_T, A)$. Following a nice example in Zellner (1971) we consider two extreme cases that provide a good picture of the link between MLE, calibration and Bayesian approach. First of all define the posterior density where a data generating process follows a Gaussian white noise[1], the prior has a Gaussian distribution with μ_0 mean and σ_μ^2 variance and the MLE is defined as $\hat{\mu}_{ML,T} = \frac{1}{T}\sum_{t=1}^{T} y_t \equiv \bar{y}$ with $V[\hat{\mu}_{ML,T}] = \frac{1}{T}$:

$$p(\mu|Y_T) \propto e^{-\frac{(\mu - E[\mu])^2}{V[\mu]}} \tag{5.1}$$

where

$$V[\mu] = \frac{1}{\left(\frac{1}{T}\right)^{-1} + \sigma_\mu^{-2}}$$

and

$$E[\mu] = \frac{\left(\frac{1}{T}\right)^{-1}\hat{\mu}_{ML,T} + \sigma_\mu^{-2}\mu_0}{\left(\frac{1}{T}\right)^{-1} + \sigma_\mu^{-2}}$$

If the variance of prior goes to infinity ($\sigma_\mu^2 \to \infty$) there is no relevant

[1] $y_t = \mu + \epsilon_t$ with $\epsilon_t \sim N(0, 1)$

prior information and just the specific model is estimated based on particular observations. In such a case the posterior is the maximum likelihood estimator. On the other hand, if $\sigma_\mu^2 \to 0$ then the parameters are completely based on our subjective belief, i.e. they are calibrated.

Using equation (5.1) the posterior distribution of model parameters can be found. The likelihood function is estimated using Kalman filter and the posterior kernel is simulated using some sampling-like procedure (Griffoli 2013).

5.1.1 State Space Representation and Kalman Filter

Kalman filter serves as a convenient tool to evaluate the likelihood of the linearized DSGE model. It is used to estimate the state of particular process by minimizing the mean of the squared error. This filtering technique is used to solve a linear state space model with normally distributed errors. Therefore, if the initial state of the model and shocks are assumed to be normally distributed, the forecasts provided by the Kalman filter are optimal against observed variables and data-generating process of states. The state space model is defined by the system of transition and measurement equation

$$X_t = FX_{t-1} + GV_t \tag{5.2}$$

$$Y_t = HX_t + W_t \tag{5.3}$$

where X_t is a vector of unobserved state variables, Y_t is a vector of observed variables, V_t and W_t are vectors of error terms where $V_t \sim N(0, \Sigma_v)$ and $W_t \sim N(0, \Sigma_w)$. The coefficient matrices F,G and H are in most cases time-invariant and (together with Σ_v and Σ_w) in principle known, but they have to be estimated in practice. Assuming that reasonable values for model parameters and both variances are available we can summarize them in the information set $\Theta = \{F, G, H, \Sigma_v, \Sigma_w\}$. The likelihood for given parameters is denoted by $f(Y_1, Y_2, ..., Y_T; \Theta)$. Now the Bayes' theorem can be used (Pichler 2007)

$$\begin{aligned} f(Y_1, Y_2, ..., Y_T; \Theta) &= f(Y_1, \Theta) f(Y_2|Y_1, \Theta) f(Y_3|Y_2, Y_1, \Theta)...f(Y_T|Y_{T-1}, ..., Y_1, \Theta) \\ &= \prod_{t=1}^{T} f(Y_t|Y^{t-1}, \Theta) \end{aligned}$$

where $Y^{t-1} = (y_1, y_2, ..., y_{t-1})$ for $t \geq 2$. The log-likelihood function is then given by

$$lnL(Y^T, \Theta) = \sum_{t=1}^{T} lnf(Y_t|Y^{t-1}, \Theta)$$

In the next step, the Kalman filter is used to construct the likelihood function which is created recursively by generating the forecasts from the system of equations (5.2) and (5.3), and their update. In principle, the procedure follows four steps - (i) the initialization, (ii) the prediction, (iii) the correction and (iv) the likelihood construction (Pichler 2007). In the initialization phase we need to provide initial values for the Kalman filter to start iterations. It can be estimated using MLE or found as steady state values of the system. Then the vector of state variables X_t is predicted based on the information set available in period *t-1* (Θ_{t-1}), i.e. before observables Y_t are known

$$\begin{aligned} X_{t|t-1} &= E(X_t|\Theta_{t-1}) = FX_{t-1|t-1} \\ \Sigma^X_{t|t-1} &= var(X_t|\Theta_{t-1}) = F\Sigma^X_{t-1|t-1}P' + \Sigma_w \end{aligned}$$

where Σ^X is the covariance matrix of the vector of state variables. Further, $X_{t|t-1}$ can be used to obtain $Y_{t|t-1} = HX_{t|t-1}$ and to compute a prediction error $\eta_{t|t-1}$ with its variance $\Sigma^\eta_{t|t-1}$

$$\begin{aligned} \eta_{t|t-1} &= Y_t - Y_{t|t-1} = Y_t - HX_{t|t-1} \\ \Sigma^\eta_{t|t-1} &= H\Sigma^X_{t|t-1}H' + \Sigma_v \end{aligned}$$

The correction of forecasted $X_{t|t-1}$ and $\Sigma^X_{t|t-1}$ is based on the observed data Y_t and the Kalman formula (Kalman 1960)

$$\begin{aligned} X_{t|t} &= X_{t|t-1} + K_t(Y_t - Y_{t|t-1}) = X_{t|t-1} + K_t(Y_t - HX_{t|t-1}) \\ \Sigma_{t|t} &= \Sigma^X_{t|t-1} - K_t(\Sigma_v + H\Sigma^X_{t|t-1}H')K'_t \end{aligned}$$

where

$$K_t = \Sigma^X_{t|t-1}H'(H\Sigma_{t|t-1}H' + \Sigma_v)^{-1}$$

Such an updating process refers simply to a linear combination of the old forecast $X_{t|t-1}$ and computed prediction error $\eta_{t|t-1}$. Following previous steps we can derive recursively densities $f(Y_t|Y^{t-1}, \Theta)$ for t = *1,2, ..., T*. Finally, the likelihood function is obtained

$$L(Y^T, \Theta) = \prod_{t=1}^{T} f(Y_t | Y^{t-1}, \Theta) \tag{5.4}$$

5.1.2 MCMC Algorithm

The log-likelihood obtained after the Kalman filter recursion together with the prior distribution gives us the log-posterior kernel

$$ln(Y^T, \Theta) = lnL(Y^T, \Theta) + lnp(\Theta) \tag{5.5}$$

To find the posterior distribution equation (5.5) is maximized with respect to Θ. Obtaining the precise posterior distribution is only rarely feasible as it is a nonlinear and complicated function of deep parameters Θ. Hence, we are not able to obtain it explicitly. Moreover, we are interested just in the moments of the posterior distribution such as mean, median, variance or quantiles which can be derived by a numerical approximation using draws from posteriors. The Metropolis-Hastings algorithm serves as a particularly efficient sampling-like method (Griffoli 2013). This algorithm "constructs a Gaussian approximation around the posterior mode and uses a scaled version of the asymptotic covariance matrix as the covariance matrix for the proposal distribution. This allows for an efficient exploration of the posterior distribution at least in the neighborhood of the mode" (An & Schorfheide 2007). The Metropolis-Hastings algorithm proceeds in four main steps (Griffoli 2013)

1. First a starting point Θ^0 needs to be defined (usually a posterior mode). Then the algorithm loops over steps 2-4.

2. In the second step a proposal Θ^* from a jumping distribution J is drawn (i.e. the algorithm draws a candidate parameter from the Normal distribution with Θ^{t-1} mean)

$$J(\Theta^* | \Theta^{t-1}) = N(\Theta^{t-1}, c\Sigma_m) \tag{5.6}$$

   where Σ_m is an inverse of the Hessian at the posterior mode and c is a scale factor. Too small scale parameter results in high acceptance ratio (see next step). Then the distribution converges very slowly to the posterior distribution and never visits tails of the distribution. Too large scale parameter implies low acceptance ratio which means that candidate parameters most likely emerge in regions of low probability density.

3. Next the acceptance ratio is computed

$$r = \frac{p(Y^T, \Theta^*)}{p(Y^T, \Theta^{t-1})} = \frac{(Y^T, \Theta^*)}{(Y^T, \Theta^{t-1})} \tag{5.7}$$

   This ratio compares the value of the posterior kernel from the mean of the drawing distribution with the posterior kernel for the candidate parameter.

4. In the last step the algorithm accepts or discards the proposal Θ^* based on the following rule

$$\Theta^t = \begin{cases} \Theta^* & \text{with probability } \mathit{min(r,1)} \\ \Theta^{t-1} & \text{otherwise} \end{cases}$$

   The algorithm discards candidate parameter if the acceptance ratio is lower than one, and goes back to the candidate from the last period.

Such a sequence of draws creates a Markov Chain with the unique stationary distribution. The algorithm should assure that the estimate is not influenced by initial draws or serial correlation. Despite this, it is recommended to discard a certain number of initial draws (Bäuerle & Menz 2008).

5.2 Calibrated Parameters

Some parameters are kept fixed throughout the estimation procedure. They can be divided into two groups - (i) parameters calibrated based on the stylized facts of the data and (ii) monetary policy parameters. The latter are not estimated as we compare the impact of the monetary policy for various values of these parameters in Chapter 8. The former are calibrated simply because we believe in the prior information here. The first group of parameters contains the discount factor for the domestic β and foreign economy β^*, the measure of openness α, the absolute risk aversion parameter for dealers γ, the elasticity of substitution for domestic η and foreign goods η^*, the foreign habit persistence parameter h^* and the standard deviation of the nominal exchange rate depreciation rate σ_e.

Using monthly data from 2000[2] to 2013 the average of the Czech Republic 10-years government bond yields is computed (ARAD 2014).[3] It equals 4.286 %

[2]Since the stabilization of inflation at the inflation target.

[3]Data available from April 2000.

(1.071 % quarterly) which is treated in our model as the steady state level of the nominal interest rate $\bar{i}$. In the steady state the discount factor corresponds to the long-run nominal interest rate as can be seen from the equation (4.12)

$$\beta = \frac{1}{1+\bar{i}} \tag{5.8}$$

After the substitution for $\bar{i}$ we get $\beta = 0.989$ which is in line with commonly used value in most of the DSGE models and also with other research studies for the Czech Republic (e.g. Štork *et al.* 2009). The discount factor for the rest of the world is set to 0.981 which is consistent with 7.78 % annual long-term interest rate (average of the long-term interest rate of 18 Euro area countries since 1970 using monthly data from OECD (2014)). Regarding the degree of openness α, Gali (2009) proposes to calibrate this parameter based on the import/GDP ratio. We follow this idea with a slight modification. Instead of using just import we sum up import and export in each period and divide it by GDP multiplied by two. Using quarterly data on GDP, export and import of goods and services in constant prices from 1996[4] to 2013 we average the ratio throughout all periods which equals 0.610 (ARAD 2014).

The standard deviation of the depreciation rate σ_e is calibrated using quarterly data on the nominal effective exchange rate index constructed by the Czech National Bank from 1996 to 2013. The parameter is set to 0.0139. In calibration of the elasticity of substitution for domestic and foreign goods we simply follow Ryšánek *et al.* (2011) who estimate $\eta = 1.870$ and $\eta^* = 1.923$. The value for the foreign habit persistence is calibrated based on estimates from Smets & Wouters (2004) and Bouakez *et al.* (2002) as 0.8. The last coefficient, the absolute risk aversion of dealers γ, is set to 500 as we follow Montoro & Ortiz (2013). All calibrated parameters are summarized in table 5.1.

5.3 Data

The calibration and estimation of model parameters for the Czech Republic is to a large extent challenge, at least because of the macroeconomic time series length. We can work just with a short period of quarterly data from Q1/1996 to Q4/2013. Moreover, these data are quiet volatile, especially the inflation and the GDP in 1990s as the Czech economy went through its transition period.

[4]Data available from March 1996.

Table 5.1: Calibrated DSGE parameters

Parameter	Description	Values
β	Domestic discount factor	0.9890
β^*	Foreign discount factor	0.9810
α	Trade openness	0.6100
h^*	Foreign habit persistence	0.8000
η	Elasticity of substitution for domestic goods	1.8700
η^*	Elasticity of substitution for foreign goods	1.9230
σ_e	Standard deviation - nominal ER depreciation rate	0.0139
γ	Absolute risk aversion of FX dealers	500
ψ_π	Domestic inflation-targeting rule (benchmark model)	1.5000
ψ_π^*	Foreign inflation-targeting rule	1.5000

Source: Author's computations.

However, the analysis of the Czech data still represents a better option than the simple calibration based on other studies.

In the data selection, Justiniano & Preston (2010) and Bäuerle & Menz (2008) are followed to some extent. We use the data on the output, the inflation, the short-term nominal interest rate and the real exchange rate for the Czech Republic, and the output, the inflation and the short-term nominal interest rate for the world economy. The data for the Czech Republic is obtained from the Czech National Bank data series system ARAD (2014), the data for the world economy comes from the OECD (2014). The Czech output is measured by the GDP in constant prices and the inflation by the GDP deflator. The real exchange rate corresponds to the index constructed by the Czech National Bank. The short-term nominal interest rate is represented by the 3-month PRIBOR.

More difficult is to choose the data on the world economy. The decision about an appropriate data source for foreign variables is based on the Czech import and export. In the last decade, more than 85 % of the Czech export was to the OECD countries. Similarly, more than 70 % of the Czech import was also from the OECD countries (CZSO 2014a). Therefore, we chose the real GDP index and the GDP deflator for the OECD countries. The foreign nominal interest rate corresponds to the 3-month EURIBOR. Logarithms and first differences of all variables are taken, apart from the short-term nominal interest rates.

5.4 Prior Distribution

The prior distribution plays a core role in the Bayesian estimation. The prior involves the initial available information which is then updated with the data via Bayes' rule to obtain the posterior. The prior can be characterized by the measure of location and spread, i.e. mode, mean, standard deviation or probability intervals. Priors are mostly based on the earlier macro and micro studies (Adolfson *et al.* 2005).

We choose a Gamma distribution with large tails for the inverse Frisch elasticity of the labor supply φ. The reason for the choice of a quite loose distribution and high variance is a diversity in estimates provided in existing research studies (e.g. Reichling & Whalen 2012; McDaniel & Balistreri 2003). The mean is set to 1.5 which refers to the standard value used in macro studies. Further, the beta distribution is assigned to most of the persistence parameters and to Calvo coefficients as they lie within 0 and 1. The prior distribution for Calvo parameters usually follows large values proposed in the literature. However, there are some studies showing that this approach may be misleading (e.g. Bils & Klenow 2004). Some prior information on Calvo parameters can be obtained using the following relation (Hristov 2011)

$$\theta = \frac{Q-1}{Q}$$

where θ is the Calvo parameter and Q is the frequency of price changes in quarters. In the Czech National Bank Inflation Report from 2011 (Collective 2011) the average frequency of price changes was set to 10.7 month which corresponds to $\theta = 0.72$. This value can be used as the prior mean for the domestic economy. Based on the fact that estimated open economy models exhibit large deviations from the law of one price we use the same (quite high) mean value for imported goods (Justiniano & Preston 2010). The prior mean of Calvo parameter for the rest of the world θ^* is set to 0.75 as advised in Smets & Wouters (2004). This value corresponds to the contract of a one-year average length. The standard deviation for all Calvo parameters is equal to 0.2. The preference and technology shock is assumed to be quite persistent with the beta distribution around 0.8 mean and the standard deviation of 0.1. Also the monetary policy persistence parameters for the Czech economy ρ_i and the rest of the world ρ_{i^*} are calibrated in the same way following Ryšánek *et al.* (2011) among others.

The distribution of remaining persistence parameters is fairly loose with the mean of 0.5 and the standard deviation of 0.2. Priors for standard deviations of all shocks have an Inverse-Gamma distribution. The prior mean for the domestic preference, productivity and monetary policy shock is based on estimates from Smets & Wouters (2004) to 0.31, 0.61 and 0.11, respectively. These values are in line also with other studies (e.g. Altig *et al.* 2010). Remaining standard deviations are calibrated around 0.02 mean with the standard deviation of 0.2. The price indexation on lagged inflation usually falls within the range of 0 and 0.5 (e.g. Ryšánek *et al.* 2011; Smets & Wouters 2004). Therefore, we set all such parameters around 0.35 mean with the standard deviation of 0.2. The habit persistence parameter is often estimated as very high (> 0.9). The opposite result is provided by Smets & Wouters (2004) who estimate this parameter for the EU as 0.59 and for the US as 0.69. We make a compromise by setting the prior mean to 0.8 with the standard deviation of 0.15. For the elasticity of substitution and the relative risk aversion we choose the Gamma distribution with 1.5 prior mean and the standard deviation of 0.5. The portfolio-adjustment-cost parameter follows also the Gamma distribution with the mean around 1 and the standard deviation of 0.75. All prior distributions are summarized together with estimated posteriors in table 5.2.

5.5 Posterior Distribution

In the next step, the Bayesian estimation can be performed. First, the Chris Sim's *csminwel* routine (incorporated in Dynare 4) is used to minimize the negative likelihood. The DSGE model is rewritten into the state-space representation and the Kalman filter is used to derive the likelihood. Second, the Metropolis-Hastings algorithm is used for the posterior sampling of parameters with two separate chains of 500 000 draws each where 150 000 initial draws are burned. The acceptance ratio is set to 25%. We control for convergence using CUMSUM (Cumulative Sum Control Chart) plots. The convergence statistics are provided in Appendix A. The result of estimation is referred in table 5.2.

Estimation results show quite high persistence of some shocks, especially foreign ones. On the other hand, the volatility of these shocks is low (except the foreign preference shock which is both, persistent and volatile). In seems that an immediate impact of foreign shocks is not so devastating but it lasts for a long time. The opposite can be viewed in the response to the domestic shocks, which are less persistent but much more volatile, notably the standard deviation of the domestic technology shock is estimated around 0.5 with the persistence parameter of 0.37. High volatility of domestic shocks can be most likely explained by the volatility of the data in the second part of 1990s (Sadeq 2008). Surprisingly, the persistence of the domestic monetary policy rule is quiet low (0.44). Hence, higher weight is given to the deviation of the inflation from its target and to the long-run value of the nominal interest rate than to the lagged value of this rate.

Regarding the structural parameters, the indexation degree of domestic non-traded goods prices, foreign goods prices and imported goods prices is substantially lower than the prior information. In the first two cases the parameter is quite close to zero (0.05 and 0.07, respectively) while imported goods prices show a little bit higher degree of indexation (0.29). Compared to Ryšánek *et al.* (2011)[5], the price indexation of import is similar but remaining two parameters are much lower. Prices in all sectors seems to be highly rigid as the Calvo parameters are estimated around 0.93 for domestic producers and importing retailers, and around 0.99 for the foreign economy, i.e. exporters. It might be a result of high inflation rates at the beginning of the transmission process (Sadeq 2008). The estimate of the Calvo parameter for prices in the non-traded sector is in line with the result from Ryšánek *et al.* (2011) while other two sectors are much less rigid.

The posterior mean of the habit persistence parameter is estimated around 0.73 which is lower value than was set in the prior information but still quite high. It seems that households care not just about the level of the consumption but also about its growth. It is logical because this parameter is higher in transition economies in comparison to developed ones (Sadeq 2008).

Regarding the precision of the estimation, the vast majority of parameters does not depend on the prior distribution. It means that the data tells us a lot about parameters because the posteriors are more informative than priors. The posterior mean is quite close to the prior information just for the domestic inverse labor supply elasticity, the standard deviation of preference shock and the persistence of the foreign monetary policy rule.

[5]Authors use Bayesian techniques to estimate parameters in the New Keynesian DSGE model with financial frictions for the Czech Republic.

Table 5.2: Prior and posterior distribution of DSGE parameters

Par.	Description	Distr.	Prior Mean	Prior StD	Posterior Mean	Posterior 90% Int.
σ	Domestic parameter of relative risk aversion	Γ	1.5	0.5	2.1392	[1.1956, 3.0443]
σ^*	Foreign parameter of relative risk aversion	Γ	1.5	0.5	3.5893	[2.6311, 4.4202]
φ^*	Domestic inverse labor supply elasticity	Γ	1.5	0.5	1.5346	[0.7898, 2.2898]
φ	Foreign inverse labor supply elasticity	Γ	1.5	0.5	2.7762	[1.8852, 3.7252]
h	Domestic habit persistence	β	0.8	0.2	0.7273	[0.5077, 0.9082]
δ_H	Indexation - domestic producers	β	0.35	0.2	0.0499	[0.0006, 0.1011]
δ_M	Indexation - importing retail firms	β	0.35	0.2	0.2916	[0.0146, 0.5451]
δ^*	Indexation - foreign economy/exporters	β	0.35	0.2	0.0714	[0.0028, 0.1440]
θ_H	Calvo-parameter - domestic producers	β	0.72	0.2	0.9332	[0.9146, 0.9530]
θ_M	Calvo-parameter - importing retail firms	β	0.72	0.2	0.9299	[0.9088, 0.9523]
θ^*	Calvo-parameter - foreign economy / exporters	β	0.75	0.2	0.9869	[0.9795, 0.9951]
ϑ	Portfolio-adjustment-cost parameter	Γ	1	0.75	2.0807	[0.3646, 4.0711]
ρ_i	Persistence - domestic monetary policy rule	N	0.8	0.1	0.4428	[0.3624, 0.5267]
ρ_i^*	Persistence - foreign monetary policy rule	N	0.8	0.1	0.8054	[0.7663, 0.8382]
ρ_g	Persistence - domestic preference shock	β	0.8	0.1	0.5890	[0.3764 , 0.8594]
ρ_a	Persistence - domestic technology shock	β	0.8	0.1	0.3662	[0.2221, 0.5016]
ρ_w^*	Persistence - capital-inflow shock	β	0.5	0.2	0.8412	[0.7887, 0.8898]
ρ_a^*	Persistence - foreign inflation shock	β	0.5	0.2	0.9838	[0.9737, 0.9948]
ρ_g^*	Persistence - foreign preference shock	β	0.5	0.2	0.9978	[0.9963, 0.9993]
σ_g	Standard deviation - domestic preference shock	Inv-Γ	0.32	0.2	0.3201	[0.1819, 0.4531]
σ_a	Standard deviation - domestic technology shock	Inv-Γ	0.61	0.2	0.5034	[0.3192, 0.6929]
σ_i	Standard deviation - domestic interest rate shock	Inv-Γ	0.11	0.2	0.0366	[0.0306, 0.0426]
σ_w^*	Standard deviation - capital-inflow shock	Inv-Γ	0.02	0.2	0.0720	[0.0446, 0.0957]
σ_i^*	Standard deviation - foreign interest rate shock	Inv-Γ	0.02	0.2	0.0058	[0.0049, 0.0067]
σ_g^*	Standard deviation - foreign preference shock	Inv-Γ	0.02	0.2	0.2540	[0.1413, 0.3801]
σ_a^*	Standard deviation - foreign inflation shock	Inv-Γ	0.02	0.2	0.0633	[0.0247, 0.1323]

Source: Author's computations.

Chapter 6

Implementation of the ZLB

Chapter 6 is dedicated to the implementation of the algorithm which can handle inequality constraints in perturbation approximation to DSGE models. The description of the algorithm is followed by the application to the DSGE model presented in Chapter 4. The response to various exogenous shocks is compared for model with and without inequality constraint.

6.1 Inequality Constraint Algorithm

We follow an algorithm introduced by Holden & Paetz (2012).[1] This method introduces anticipated news shock (called shadow price shock) to return the nominal interest rate to zero in a case when another shock can push it to negative values. The ZLB on nominal interest rate binds as long as the shock which hits the economy causes the interest rate to be negative. Basically, it ensures that:

$$i_t = max(0, i_t^T)$$

where i_t^T represents the central bank's operational target set by monetary policy rules (e.g. equations (4.32)-(4.35)).

This algorithm is consistent with rational expectations. Therefore we can explore the behavior of the economy where agents know when the ZLB on nominal interest rate will bind. This algorithm simply replaces the future ZLB constraint with anticipated (shadow price) shocks. Shocks are added to that equation which should be constrained with the ZLB, in our case to the interest

[1] For more technical details see Holden & Paetz (2012).

rate rule

$$i_t = \rho_i i_{t-1} + (1-\rho_i)(\bar{i} + \psi_\pi \pi_t + \psi_p p_t + \psi_e e_t) + \epsilon_{i,t} + \sum_{s=0}^{T^*-1} \epsilon_{s,t}^{SP}$$

where $\epsilon_{s,t}^{SP}$ is the shadow price shock in period s for $s = 0, ..., T^*$.

The shock $\epsilon_{i,t}$ is known at t but does not materialize until $t + s$. The relative impulse response functions $m_{j,s}$ of all variables to the shadow price shock are found and stacked into a matrix $M = [m_{j,0}, m_{j,1}, ..., m_{j,T^*-1}]$. Then the impulse responses to a simultaneous shock to $\epsilon_{i,t}$ of magnitude 1 and $\epsilon_{s,t}^{SP}$ of magnitude α_s are computed. The complete impulse response function to the shock $\epsilon_{i,t}$ with imposed constrained for variable x_j is given by the combination of the steady state of this variable, the impulse response to the shock $\epsilon_{i,t}$ and the multiplication between the magnitude of the news shocks and the matrix M_j, i.e. $\mu_j + v_j + M_j\alpha$ (Holden & Paetz 2012). Following authors, the main challenge is to find the value of α. It is done by solving the quadratic optimization problem

$$\alpha^* := argmin_{\alpha \geq 0_{T^*} \mu^* + v^* + M^*\alpha \geq 0_{T^*}} \left[\alpha'(\mu^* + v^*) + \frac{1}{2}\alpha'(M^* + M^{*\prime})\alpha \right]$$

This problem can be solved using MATLAB function *quadprog*.

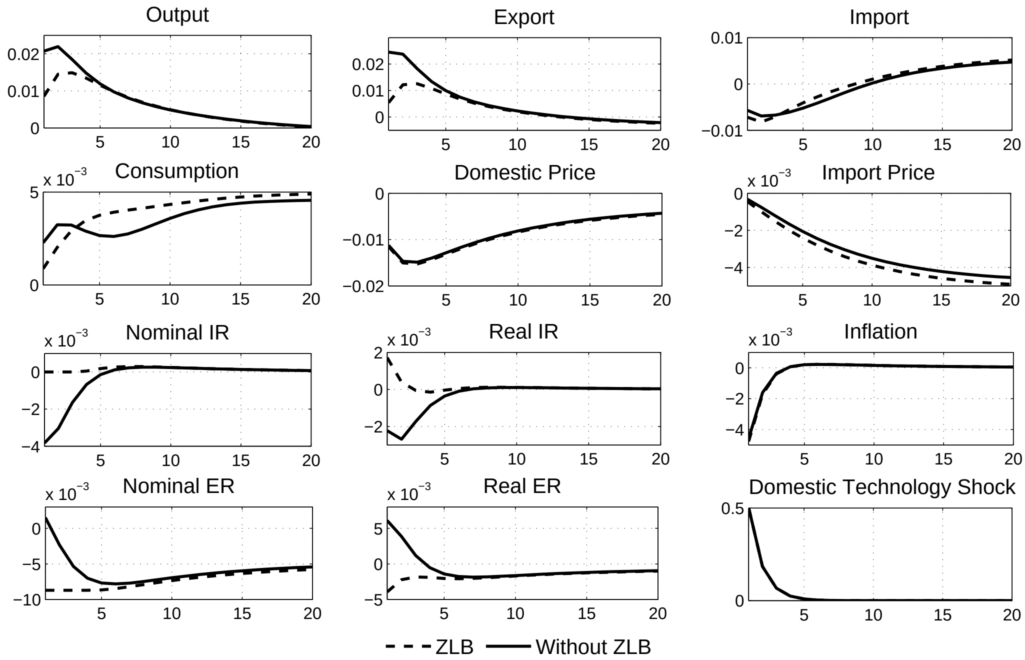
6.2 Simulations

Next the economy's equilibrium response to various temporary exogenous shocks is examined when the central bank follows the inflation targeting rule. It serves also as a benchmark in examining different policy tools in next chapter. For this purpose both, domestic and foreign, shocks were chosen. Representatives of domestic shocks are technology shock (as a supply side shock) and preference shock (as a demand side shock). Foreign shocks include foreign interest rate shock, foreign inflations shock and capital inflows shock. All variables except the interest rate are expressed in deviations from the steady state.

6.2.1 Response to the Domestic Shocks

Figures 6.1, 6.2 and 6.3 compare the impulse response functions to the positive domestic technology shock, and the positive and negative preference shock.[2]

Figure 6.1: Quarterly IRFs after a positive domestic technology shock (with and without binding ZLB)



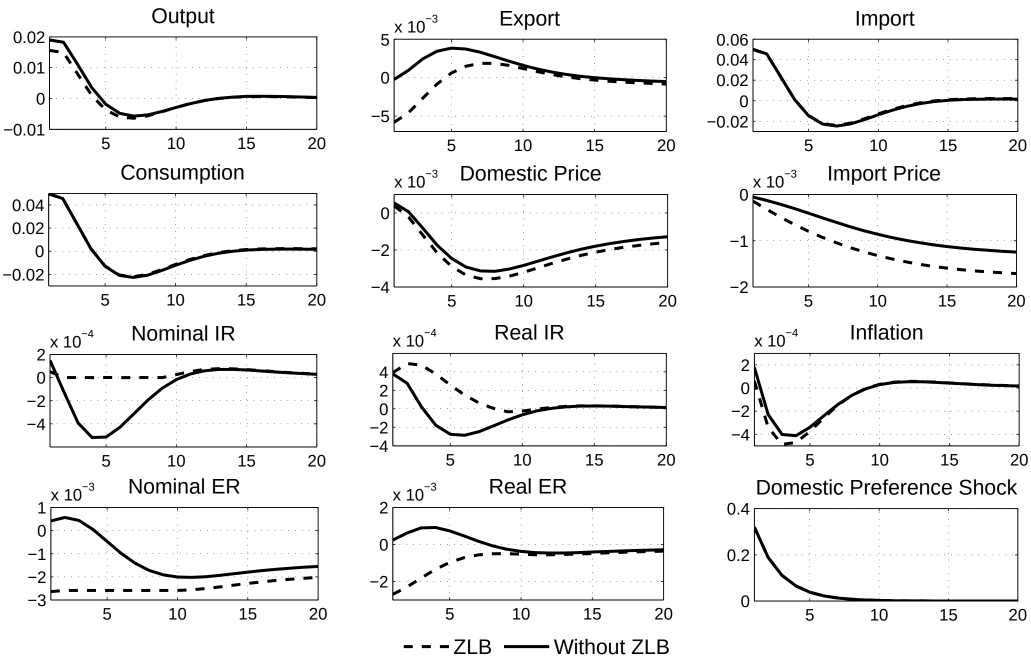
Source: Dynare output.

The improvement in technology makes production more efficient which increases output and lowers prices. It is consistent with a shift of the short-run aggregate supply curve. The nominal exchange rate tends to depreciate in first four quarters and then appreciate with the fall in import prices. It results in the temporary decline of import in first ten quarters followed by the permanent rise. At the same time the export improves due to lower prices of domestic products at foreign markets. The positive shock to innovation is accommodated by the central bank, which lowers nominal interest rate to get inflation on the target. According to the Taylor principle, for inflation to be stable, the central bank has to respond to the change in inflation with even greater change in the nominal interest rate which decreases real interest rate. As a result of permanent fall in import prices the nominal exchange rate remains stronger in the inflation targeting regime while the import increases permanently which worsen the

[2]The impulse response function presents a dynamic reaction of a variable in the model to an external change (e.g. shock or innovation).

domestic net export.[3] When the central bank is constrained with zero lower bound on nominal interest rate it is not able to counteract the shock and the real interest rate rises. The reaction of import prices is slightly stronger, the real exchange rate appreciates immediately and the change in import is more pronounced in both directions. The positive response of export is weaker and the overall economic expansion milder. The effect of increased productivity is subdued.

Figure 6.2: Quarterly IRFs after a positive domestic preference shock (with and without binding ZLB)



Source: Dynare output.

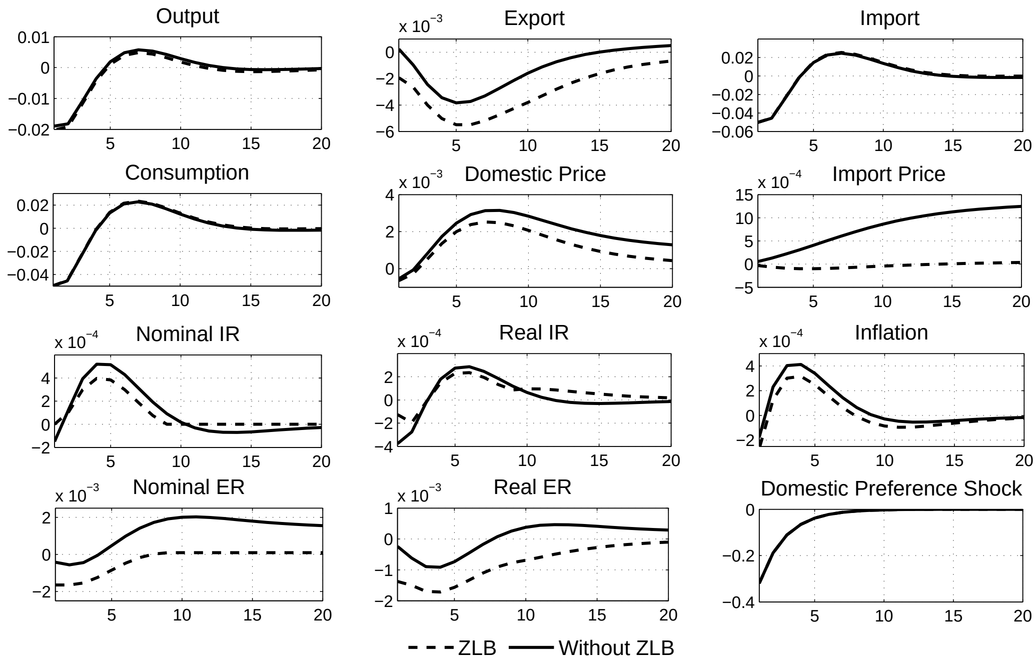
The shock on the utility side of the economy affects the intertemporal substitution of households. As we assume additively separable preferences in consumption and labor, the utility from consumption does not affect the (dis)utility from working directly but just indirectly through labor supply curve.[4] The positive (negative) preference shock shifts households' preferences towards present (future) consumption. It means that after the positive preference shock household values its future consumption less, i.e. a marginal utility of consumption is higher and consumption increases. Households need to decide on both, the amount of consumption and leisure they take. Hence, as the consumption is

[3]Changes in exchange rate and price level are permanent in response to temporary shocks under the inflation targeting (see McCallum & Nelson 2000).

[4]Cross-derivatives of utility function w.r.t. worked hours and consumption are zero.

now more valuable (in utility terms), they are willing to give up less units of consumption for a given rise in leisure, i.e. the leisure decreases and worked hours increase. The whole labor supply curve shifts to the right such that a real wage declines. It creates downward pressures on the marginal cost and the inflation. Both, domestic prices and import prices decrease. The exchange rate initially depreciates which together with lower domestic prices improves the competitiveness and contributes to higher export. The central bank lowers the nominal interest rate to bring inflation on the target. Import prices stabilize on lower level which causes the appreciation of nominal exchange rate to be permanent. With ZLB the central bank cannot mitigate deflation pressures which push up the real interest rate and results in even stronger negative response of domestic and import prices, and overall inflation. The exchange rate appreciates immediately which decreases the competitiveness of domestic export and it falls. Therefore, the increase in output is again milder.

Figure 6.3: Quarterly IRFs after a negative domestic preference shock (with and without binding ZLB)



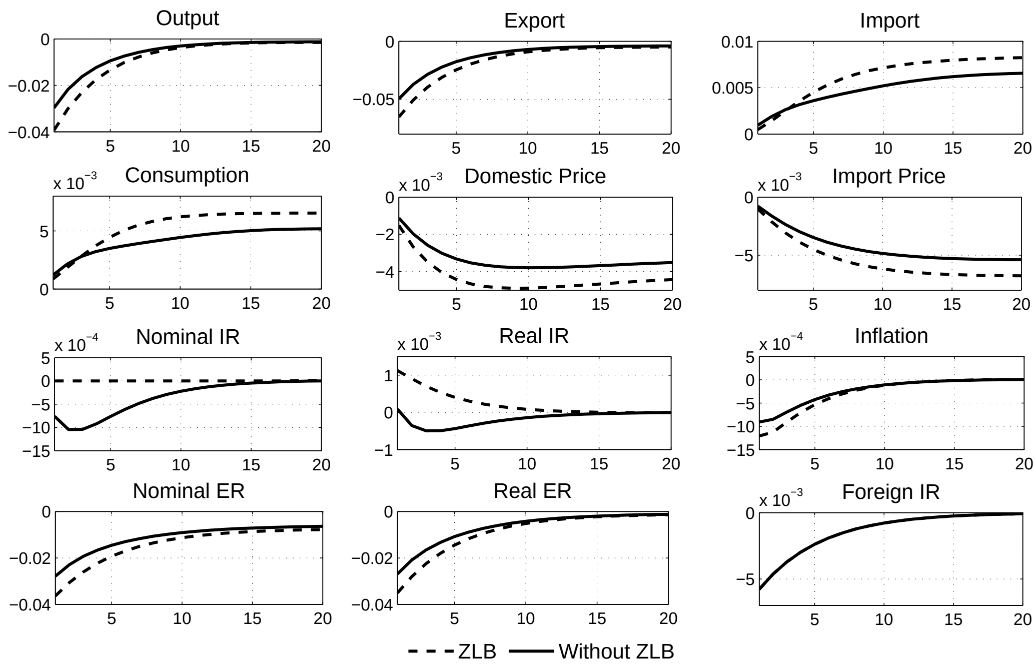
Source: Dynare output.

In the case of negative preference shock households value their future consumption more which implies the fall in both, consumption and labor. In the response, the real wage and prices increase. Export and import initially decline while the latter starts to rise after four quarters due to exchange rate depreciation. The ZLB alleviates inflation pressures which worsen export even more. The appreciation of exchange rate is stronger.

6.2.2 Response to the Foreign Shocks

Figures 6.4, 6.5 and 6.6 compare the impulse response functions to the negative foreign interest rate shock, the positive foreign inflation shock and the positive capital inflows shock. The reason for examining the foreign shocks is simple. The small open economy is affected by foreign variables and policies pursued in the rest of the world. Therefore, analyzing the effect of foreign shocks allows examining the impact of foreign economic performance on the domestic economy.

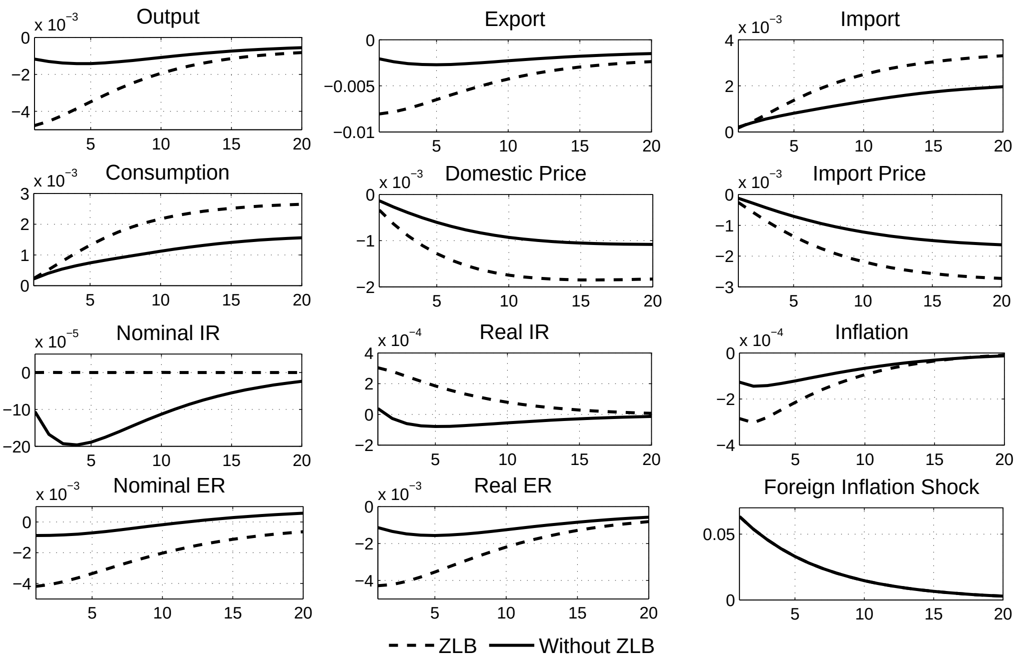
Figure 6.4: Quarterly IRFs after a negative foreign interest rate shock (with and without binding ZLB)



Source: Dynare output.

It is apparent that the dynamics changes considerably for all three shocks if we take the ZLB into account. The fall in the foreign interest rate results in the exchange rate appreciation with the decline in import prices. At the same time the export worsens due to higher prices of domestic products at foreign markets. The central bank pursues the expansionary policy to mitigate the effect of the shock by cut in the interest rate. Lower interest rate should offset the effect of the shock on income. As a result of substantial decline in import and domestic prices the nominal exchange rate remains stronger. It causes also a permanent increase in import. Domestic consumption increases, partially because of higher consumption of imported goods and partially because of higher consumption of non-traded goods. This response is pronounced at the ZLB as the decline in domestic and import prices is deeper. The fall in export together with the increase in import worsens the domestic net export. Stronger appreciation followed by deeper decline in prices, pronounced pressures on income and even worse net export. The decline in net export is much deeper than the increase in domestic consumption which results in severe downturn.

Figure 6.5: Quarterly IRFs after a positive foreign inflation shock (with and without binding ZLB)

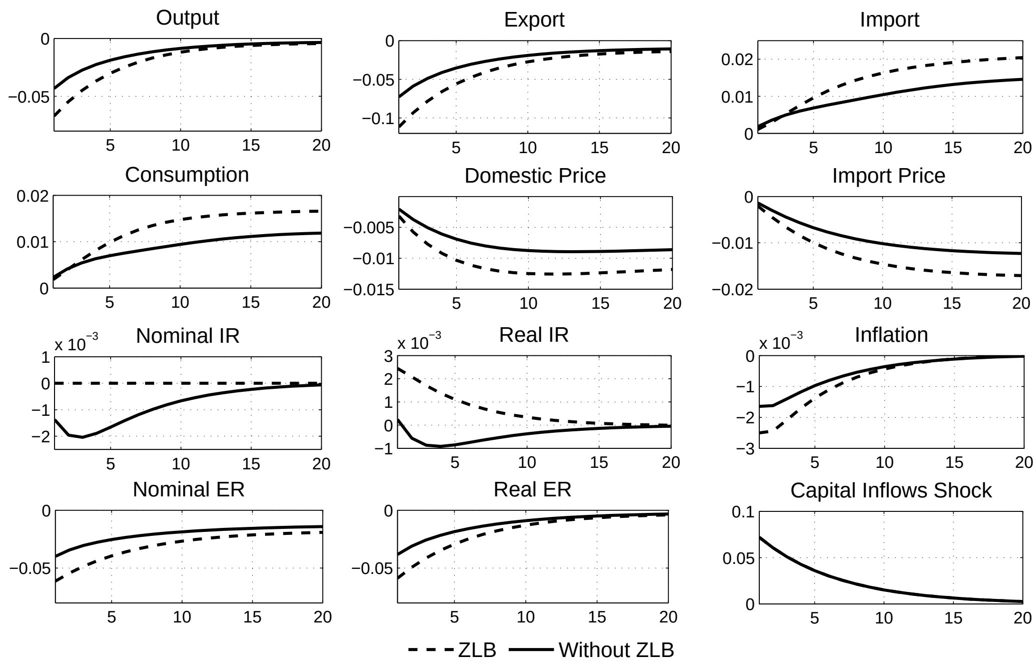


Source: Dynare output.

In the response to the downward foreign inflation shock the real interest rate differential lowers which is followed by the real exchange rate appreciation. The deterioration in net export pushes the economy into recession and generates downward pressure on prices. The central bank has to lower short-term interest rate to bring inflation back on the target and boosts the economy. If policy rates are bounded at zero, the reaction of economy is again significantly pronounced. The strength of response at the ZLB depends also on the persistence of the shock. The parameter estimated for foreign inflation shock is very high ($\rho_{a^*} = 0.9834$, see Chapter 5) which creates really large distortion. Figure 6.5 shows results for slightly lower persistence parameter ($\rho_{a^*} = 0.85$) which allows for the comparison with non-binding constraint.[5] However, the recession and deflation pressures are still much stronger at the ZLB.

The capital inflows shock affects the economy through the modified UIP condition (4.30) like the foreign interest rate shock. Hence, the reaction of all variables is similar to the foreign interest rate shock.

Figure 6.6: Quarterly IRFs after a positive capital inflows shock (with and without binding ZLB)



Source: Dynare output.

To sum up, when the central bank approaches the ZLB, it cannot counteract the shock using nominal interest rate. The positive shock to innovation is subdued. There is no significant threat of pronounced deflation pressures but the economic expansion is muted. This finding is consistent with empirical analysis provided by Gavin *et al.* (2013). They found that in the response to the positive technology shock, the consumption and output are lower at the ZLB. Similarly, in the response to the preference shock, change in inflation is slightly stronger while the main impact is on the exchange and the trade-balance. Regarding foreign exogenous shocks, the response of all variables is more volatile with pronounced downward deflation pressures.[6] Stronger appreciation of domestic

[5]Results for $\rho_{a^*} = 0.9843$ are presented in Appendix B.

[6]Pronounced volatility of real and nominal macroeconomic variables at the ZLB is in line with an extensive research including Coenen *et al.* (2003), Williams (2009), Bodenstein & Guerrieri (2009) and Amano & Shukayev (2012).

currency together with increased exchange rate pass-through (see Section 7.2) reinforces the impact on falling prices. Higher deflation pressures increase the real interest rate and magnify economic downturn. Such pronounced economic downturn is consistent with empirical analysis provided by Gust *et al.* (2012) which estimated that about 20 % of the drop of US output during the recession in 2008-2009 was caused by lower bound on interest rates. Moreover, Wu & Iwata (2004) examine the dynamic responses of the economy to macroeconomic shock causing decline in output using VAR model. They find that the recovery of economy is substantially slower and weaker if the interest rate cannot fall any more.

The severity of constrained rates is dependent on both, the size and the duration of the shock. With small or transitory shocks, the response of macroeconomic variables tends to be short and modest at the ZLB (Williams 2014). On the other hand, if the shock is quite persistent, the effect of the ZLB may be more serious which is apparent also from the analysis of foreign inflation shock with two different persistence parameters. Hence, the greatest threat comes with highly persistent shocks. Since the recovery after banking and financial crises is often very slow (e.g. Jordá *et al.* 2011; Reinhart & Rogoff 2009), it could lead to prolonged periods of binding ZLB.

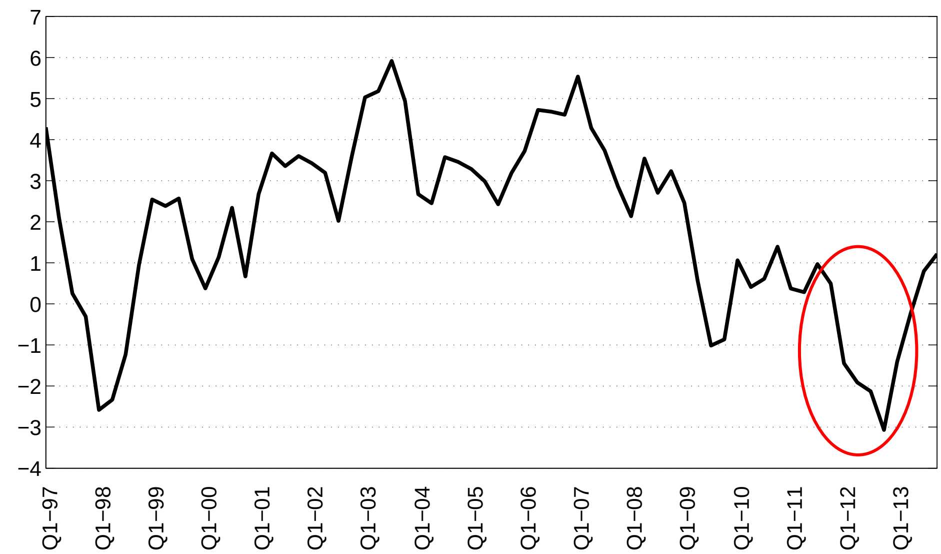
Chapter 7

The Czech Case

Conventional monetary policy tools which are based on the managing of short-term interest rate prevailed before the crisis. The prolonged period of monetary easing has pushed policy rates to zero. Depressed economic conditions require further easing in order to maintain the price and financial stability and boost the economic growth. As central banks face the limit of standard policy tools in the ZLB situation, they have resorted to unconventional methods. The most attention is given to various types of liquidity provisions and foreign exchange interventions. However, there are discussed also some less common alternatives, such as negative interest rates or "helicopter drop" of money. The Czech National Bank has also turned its attention to unconventional measures as it run out the space for regulation of short-term rates in late 2012.

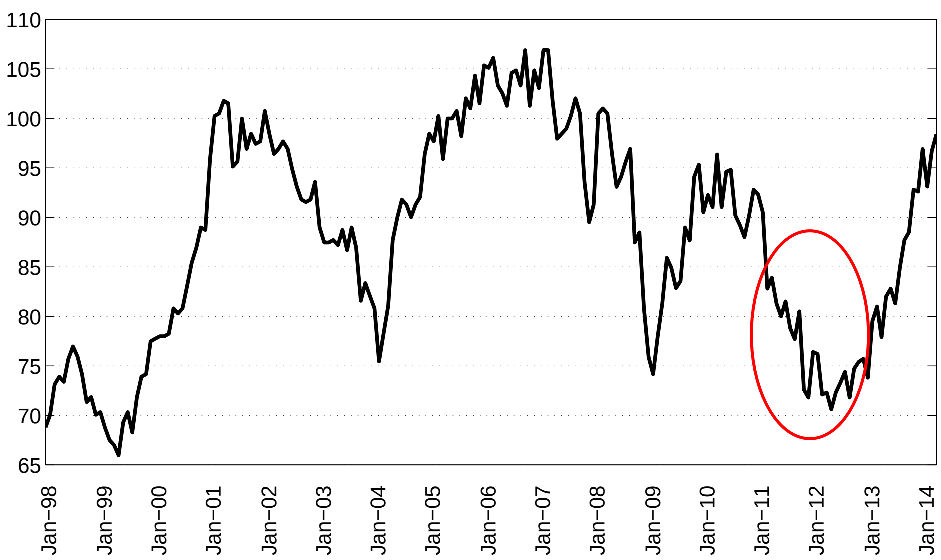
The CNB cut its main monetary policy rate (the two-weak repo rate) by 20 basis points to a technical zero (i.e. 0.05 %) on November 1, 2012. Moreover, the central bank decided to keep the interest rate at this level over the longer horizon until inflation pressures do not increase significantly. At that time, the Czech economy was already in recession for four quarters with extremely weak domestic demand (see Figure 7.1). However, the main problem was a drop in consumer confidence in the first quarter of 2012 together with decrease in real wages. The pessimism of Czech households was the highest since 1999 as shown in Figure 7.2.

Figure 7.1: Household demand (real, % yy)



Source: CZSO (2014b).

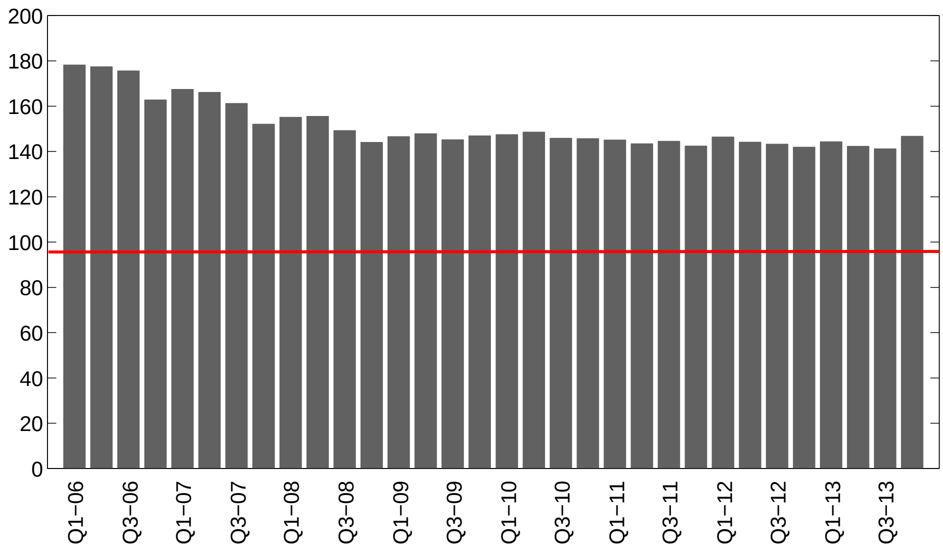
Figure 7.2: Consumer confidence (2005 = 100)



Source: CZSO (2014b).

The ongoing recession together with the threat of disinflation led the Bank Board to discuss the use of additional policy tools for monetary easing (Lízal & Schwarz 2013). The use of negative interest rate would face legal barriers. Moreover, its effect is not clear and without deeper empirical evidence. The use of "helicopter drop" of money would probably not have significant effect as pessimistic households would increase their savings instead of consumption. The money would end up on deposits with zero interest. So far, two main measures could be considered for the Czech economy - some form of liquidity provision and foreign exchange interventions. The use of quantitative easing in other economies has been supported by severe liquidity crisis and the instability of financial sectors. The Czech Republic is characterized by the opposite situation where a total amount of deposits is even higher than an amount of loans (see Figure 7.3).

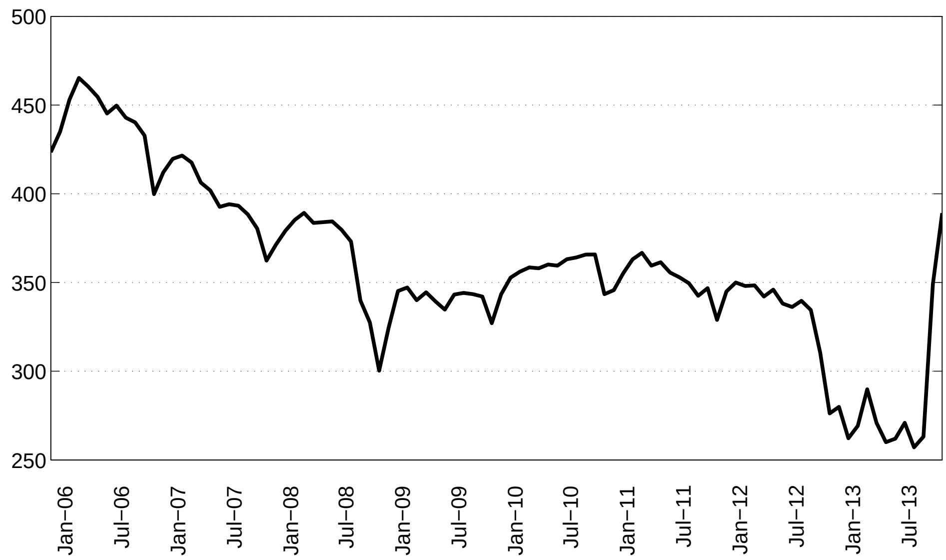
Figure 7.3: Deposits to loans ratio



Source: ARAD (2014).

Commercial banks consistently hold on average about 350 billion of excess liquidity deposited at the CNB (see Figure 7.4). Simply, the CNB would not find the counterpart demand when trying to use quantitative easing as an additional monetary policy tool. Moreover, the interest rate transmission channel may not work properly because the financial intermediaries provide a small amount of loans in comparison to their liquidity. In such a situation, quantitative easing would probably not have much impact. As the Czech economy is very open, the foreign exchange interventions seem to be the most appropriate measure at the ZLB. However, the exchange rate pass-through to the Czech inflation follows a decreasing tendency in the last few years while the estimates still vary in a wide range (see discussion in Lízal & Schwarz (2013)).

Figure 7.4: Excess liquidity deposited at the CNB



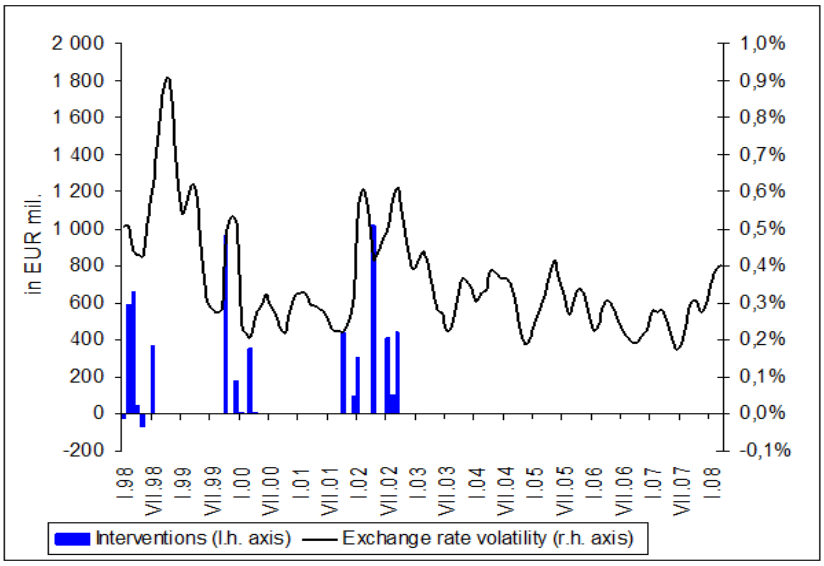
Source: ARAD (2014).

On November 11, 2013, the Bank Board has decided to use the exchange rate as an additional tool for further monetary easing. The CNB started to intervene on the FX market to lower the Czech koruna close to 27 CZK/EUR. The core question is whether FX interventions would really help to ease economic conditions as the previous experience regarding the effectiveness of interventions in the Czech economy is mixed. An additional problem could be the different response of macroeconomic variables with and without ZLB on nominal interest rate. To gain some intuition, the past Czech experience with intervening is briefly summarized in Section 7.1. Next, the exchange rate pass-through at the ZLB is examined using the DSGE model described in Chapter 4.

7.1 Past Czech Experience with FX Interventions

The floating exchange rate regime was adopted in 1997 while the CNB kept the possibility to use FX interventions in the case of "excessive volatility or unjustified exchange rate trends" (Geršl & Holub 2006). Interventions were conducted in three main periods[1] and mostly against the appreciation of currency (see Figure 7.5).

Figure 7.5: FX interventions



Source: Geršl & Holub (2006).

As mentioned above, the response of the exchange rate to interventions is not clear. Most of the empirical studies do not confirm their effectiveness or just show a short-lasting weakly significant small effect (e.g. Disyatat & Galati 2007; Geršl & Holub 2006). According to Geršl & Holub (2006), the interventions were successful in October 1999, March 2000 and July-September 2002 (see table 7.1). In the first case, the CNB purchased about 1 billion euro which depreciated the koruna by more than 3.5 %.[2] In March 2000 the amount of interventions was about 400 million euro while the depreciation peaked in May 2000 with the change of 4 %.[3] Then it gradually decreased to its initial level in horizon of 2 month. In the last successful case the amount of interventions reached again almost 1 billion euro which depreciated the koruna by more than 6 % in the horizon of 3 month after the interventions. While the immediate effect of interventions seems to be uncertain, it lasts about 3 month in some cases.

[1]February - July 1998, October 1999 - March 2000, and October 2001 - September 2002
[2]min - 5.10.1999 - 35.675 CZK/EUR, max - 15.10.1999 - 37.015 CZK/EUR
[3]min - 9.3.2000 - 35.525 CZK/EUR, max - 12.5.2000 - 37.040 CZK/EUR

Table 7.1: FX interventions - effectiveness

Start - End	Overall volume	CZK/EUR (ECU prior to 1999)						
(t - T)	(EUR mil)	t-3M avrg	t-1M avrg	Start of t	Low of [t;T]	End of T	T+1M avrg	T+3M avrg
02/98-04/98	1285	37.87	38.50	38.37	36.30	36.46	36.11	35.11
06/98-07/98	508	36.95	36.11	36.49	34.35	34.35	35.47	35.17
10/99-10/99	966	36.52	36.36	35.72	35.68	36.62	36.40	36.03
12/99-12/99	229	36.36	36.40	36.08	35.83	36.13	36.03	35.60
03/00-03/00	394	36.05	35.71	35.65	35.53	35.63	36.31	36.02
10/01-01/02	643	33.86	34.19	33.91	31.46	31.92	31.79	30.36
04/02-04/02	1009	32.08	32.08	30.62	30.06	30.63	30.56	29.75
07/02-09/02	954	30.36	30.36	29.25	28.97	30.30	30.65	31.19

Source: Geršl & Holub (2006).

7.2 Exchange Rate Pass-through at the ZLB

Standard textbooks define the exchange rate pass-through as a percentage change, in the local currency, of import prices which are the result of one percentage change in the exchange rate between two countries (Goldberg & Knetter 1997). With high exchange rate pass-through, the depreciation of currency creates high inflation pressures. Moreover, the complete pass-through implies a one-for-one response of import prices to the exchange rate. On the other hand, the incomplete pass-through creates deviation from law of one price which affects the relationship between the real exchange rate and the terms of trade (Monacelli 2003).[4] The difference is shown in equations (7.1) (complete pass-through) and (7.2) (incomplete pass-through).

$$q_t = (1-\alpha)s_t \tag{7.1}$$

$$q_t = \psi_{M,t} + (1-\alpha)s_t \tag{7.2}$$

where q_t is the real exchange rate, s_t is the terms of trade, α captures the trade openness and $\psi_{M,t} = (e_t + p_t^*) - p_{M,t}$ refers to the law-of-one-price (l.o.p.) gap which is the difference between world price denominated in domestic currency and price of imported goods. According to these relations, changes in

[4] The empirical evidence suggests that exchange rate pass-through is incomplete in the short-run while it becomes complete in the long-run (e.g. Campa & Goldberg 2002; Goldfajn & Werlang 2000).

relative prices between traded goods resulting from exchange rate movements are subdued in case of partial exchange rate pass-through.

Recall that binding constraint on some variable leads often to more volatile response of others. Under this assumption, the exchange rate pass-through should increase when the ZLB binds. Suppose that the exchange rate depreciation implies increase in import prices which pushes the inflation up. Higher inflation together with zero nominal interest rate reduces gradually the real interest rate which supports economic growth. The channel of import prices and real interest rate work together (Lízal & Schwarz 2013). Hence, we assume that exchange rate pass-through can be substantially higher at the ZLB than in normal times which supports the decision to use FX interventions for further monetary easing.

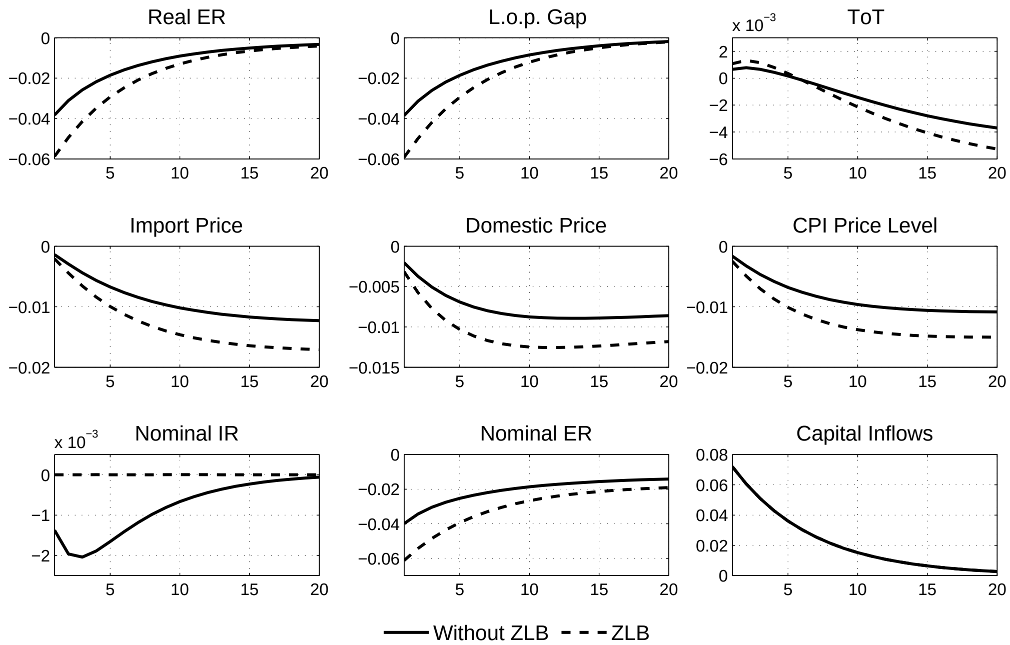
Next, using the DSGE model with incomplete pass-through described in this thesis we investigate the response of prices to the capital inflows shock with and without bounded nominal policy rates. Because we use the algorithm for occasionally binding constraint the exchange rate pass-through is examined in a case of exchange rate appreciation instead of depreciation which would be more logical.[5] However, we believe that the degree of exchange rate pass-through is similar in both cases. Results of simulation are presented in Figure 7.6.

In the response to the shock, the exchange rate becomes stronger and prices decline. The fall in l.o.p. gap is caused by the nominal appreciation together with the sluggish change in import prices denominated in domestic currency. With the ZLB, the price level together with the l.o.p. gap decreases significantly more than without constrained policy rates. Lower l.o.p. gap implies higher degree of transmission of exchange rate appreciation into relative prices. Without constraint, the central bank can influence inflation expectations and indirectly contributes to less volatile l.o.p. gap response. It is not possible when the central bank cannot lower policy rate.

To conclude, using FX interventions at the ZLB seems to be appropriate for further monetary easing in the small open economy as the exchange rate pass-through is significantly higher compared to the situation without bounded short-term rates. Moreover, the Czech financial system is characterized by the excess of liquidity where liquidity provision measures would have a small impact, if any. Hence, FX interventions can be considered as the most efficient measure among presented unconventional alternatives.

[5] The short-term nominal interest rate is bounded at its lower level while it is not restricted in its upward movement in the response to the shock.

Figure 7.6: Quarterly IRFs after a positive capital inflows shock (with and without binding ZLB)



Source: Dynare output.

Chapter 8

Monetary Policy at the ZLB

In Chapter 8 the effectiveness of monetary policy at the ZLB is examined. First part focuses on the comparison of four different targeting rules while the second part presents responses of the real and nominal macroeconomic variables if the central bank starts using foreign exchange rate as an additional policy measure. All impulse response functions are derived with inequality constraint, i.e. with bounded short-term interest rate. For the purpose of the comparison, IRFs with non-binding constraint are provided in Appendix B.

8.1 Monetary Policy Rules

We consider four monetary policy rules, (i) the inflation targeting, (ii) the price-level targeting, (iii) the modified inflation targeting with weight assigned to the exchange rate, and (iv) the exchange rate targeting (equations (4.32)-(4.35)). Table 8.1 summarizes all possible cases with their parameterization.

Table 8.1: Monetary policy rules - parameterization

	ψ_π	ψ_p	ψ_e
Inflation Targeting	1.5	0	0
Price-level Targeting	0	1.5	0
Modified Inflation Targeting	1.5	0	0.75
Exchange Rate Targeting	0	0	1.5

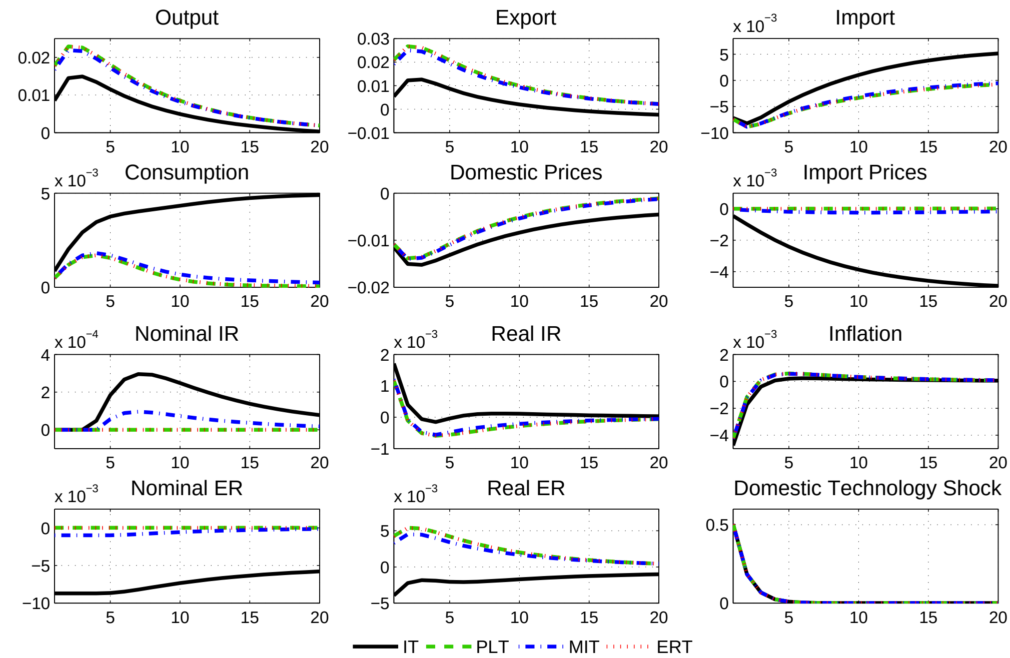
Source: Author's parameterization.

Figures 8.1 and 8.2 present the responses to the domestic shocks and Figures 8.3 and 8.4 to the foreign shocks. The length of constrained short-term nominal interest rate is different for individual regimes in response to the positive domestic preference and technology shocks. Generally, this period is prolonged for the price-level targeting rule and shortened for the exchange rate targeting rules (compared to the inflation targeting). However, there is no significant difference between price-level targeting, pure and dirty exchange rate targeting regimes in response to both domestic shocks while this group of rules performs much better than the pure inflation targeting when the ZLB is in place. The fall in prices is less pronounced and the nominal exchange rate remains on its long-run level. The price-level targeting regime leads to increased inflation expectations which alleviates the initial deflationary effect of the shock.[1] In the inflation targeting regime the central bank does not respond to unexpected price-level variations while under the price-level targeting these changes are reversed. It means that with the price-level target, the shock that causes the fall in prices decreases inflation below its long-run level which compels the central bank to take such actions which brings prices back on the target. It increases the inflation above the long-run level for some periods to accommodate the effect of the shock. The nominal interest rate remains zero for all twenty quarters while it ceases to bind after three quarters with inflation targeting rule and after four quarters with dirty exchange rate targeting rule. The reason is that we need to create stronger inflation pressures in the case of price-level target. It results in depreciation of the real exchange rate which increases the profit of exporters and the demand for export. With the pure and dirty exchange rate targeting the response is similar. As a result, the improvement of the technology feeds into the higher output for all three modified rules better than for the pure inflation targeting.

The consequences are quite similar also for the positive domestic preference shock while the response to the negative preference shock is a little bit different. With price-level target the nominal interest rate is initially constrained for two more quarters which results in stronger appreciation and decline in export. The subsequent growth in the nominal interest rate is higher and lasts about six quarters longer (compared to the inflation targeting) which allows to mitigate the shock.

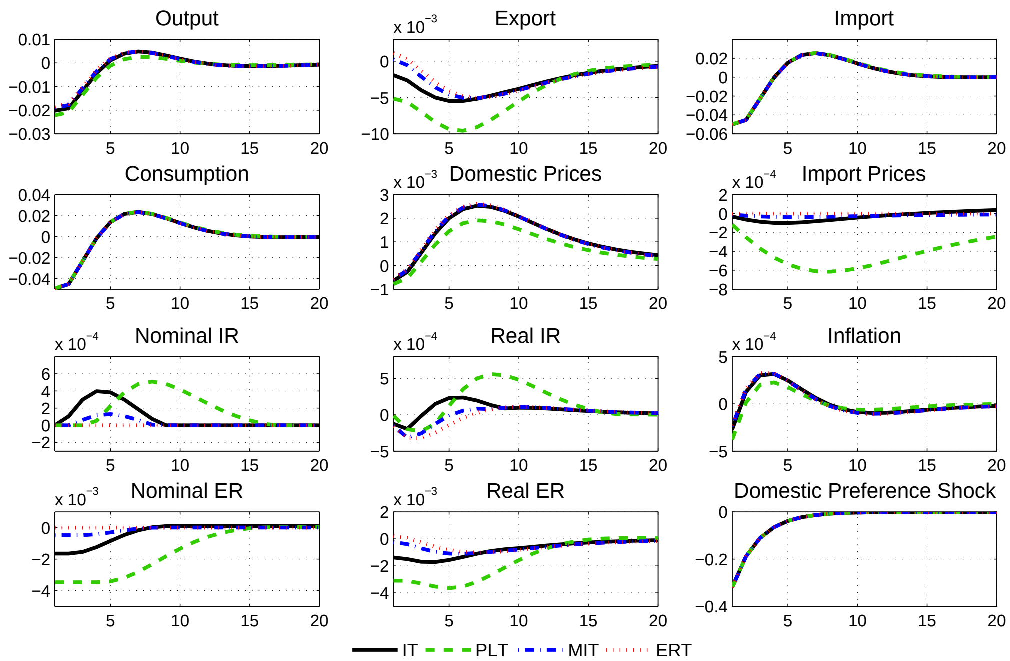
[1]Economic agents are assumed to have forward-looking expectations. Hence, after a deflationary shock and under the price-level targeting they expect a rise in inflation above its long-run level which increases inflation expectations (Merola 2010).

Figure 8.1: Quarterly IRFs after a positive domestic technology shock (binding ZLB, different policy rules)



Source: Dynare output.

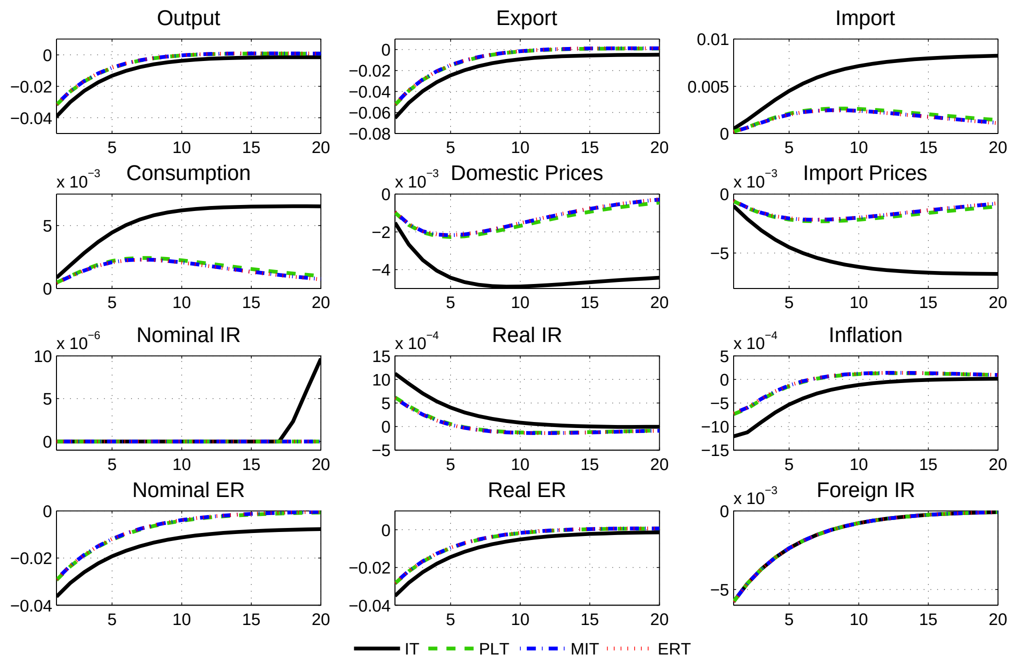
Figure 8.2: Quarterly IRFs after a negative domestic preference shock (binding ZLB, different policy rules)



Source: Dynare output.

Regarding the negative foreign interest rate shock, the response of the economy with inflation target again differs significantly from other regimes. Price-level target, dirty and pure exchange rate targets lead to more expansionary policy as the import prices do not decrease as much as for the inflation targeting. Compared to the case without constrained policy rates (see Appendix B), the effectiveness of these three alternatives is reduced at the ZLB. Nevertheless, each rule is able to mitigate the adverse effect of the shock better than standard inflation targeting. While the interest rate starts to rise after the 15th quarter in the inflation targeting regime, it remains zero for the whole period with other policy rules. The appreciation is weaker for all three alternatives as agents may anticipate the future lower interest rate. The price level of imported and domestic goods does not fall permanently and the nominal exchange rate returns gradually to its initial level.

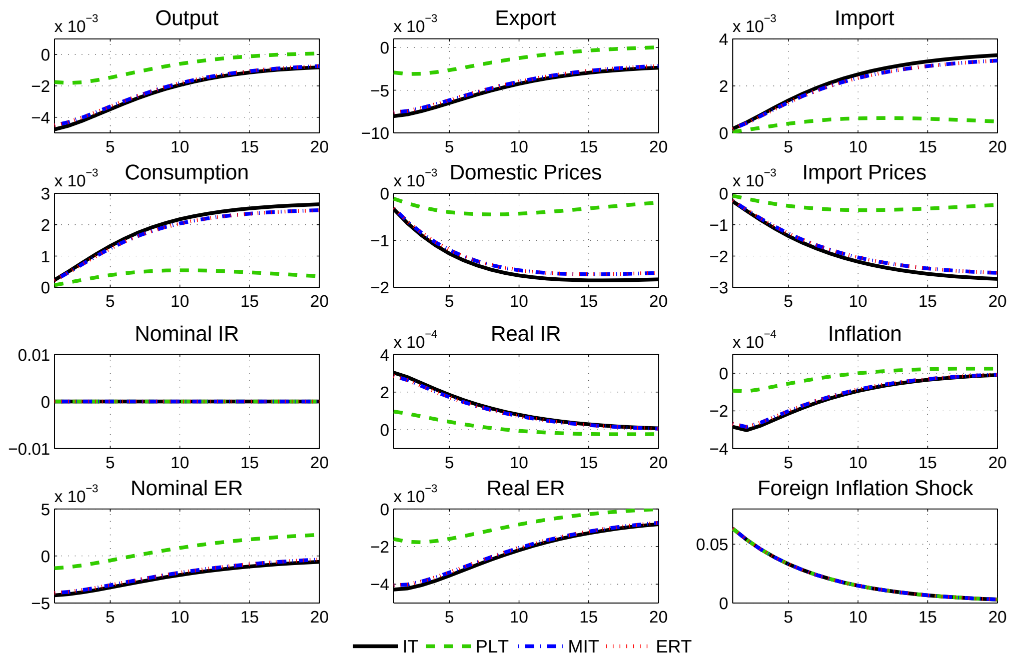
Figure 8.3: Quarterly IRFs after a negative foreign interest rate shock (binding ZLB, different policy rules)



Source: Dynare output.

The response of the economy to the foreign inflation shock is significantly different with price-level target. It is characterized by substantial decrease in volatility and faster return to equilibrium path for most of the variables. This effect can be achieved due to the depreciation of nominal exchange rate from the 7th quarter onwards. Under the price-level target, stronger inflation pressures have to be generated to bring the inflation above its long-run level and the price level on its target. Higher inflation expectations decrease domestic demand, and mitigate the fall in domestic prices and overall inflation. It causes weaker real appreciation and smaller deterioration of export. The overall economic downturn is less pronounced. Under the pure or dirty exchange rate targeting, the nominal exchange rate cannot increase above its long-run level.

Figure 8.4: Quarterly IRFs after a positive foreign inflation shock (binding ZLB, different policy rules)



Source: Dynare output.

To sum up, with all three alternative policy rules the volatility of prices decreases significantly in the response to most of the exogenous shocks at the ZLB.[2] Recall that pronounced volatility of macroeconomic variables might be avoided by adoption of the history-dependent monetary policy rule, such as the price-level targeting (Coibion *et al.* 2012). The exchange rate movements are assumed to be closely connected to the change in price level expectations. Hence, following the price-level targeting rule, the central bank can anchor price level expectations and completely counteract the effect of the shock on the nominal exchange rate. This assumption is consistent with our results. The exchange rate appreciates less implying lower deterioration of the trade-balance and milder economic downturn. Regarding the dirty and pure exchange rate

[2]The positive domestic technology shock, the negative domestic preference shock, the negative foreign interest rate shock and the positive capital inflows shock.

targeting rule, it resembles the effect of price-level targeting as the exchange rate movements are caused by the change in price-level expectations at the ZLB.

8.2 Foreign Exchange Interventions

We continue with evaluation of the FX interventions at the ZLB. As in the previous section we examine three different policy settings, (i) the leaning-against-the-wind interventions, (ii) the fixed exchange rate regime, and (iii) the correction of misalignment of the real exchange rate from its fundamentals. The central bank is assumed to follow pure inflation targeting rule which serves as a benchmark. Table 8.2 shows the parameterization of the intervention rule (4.31) in all possible cases.

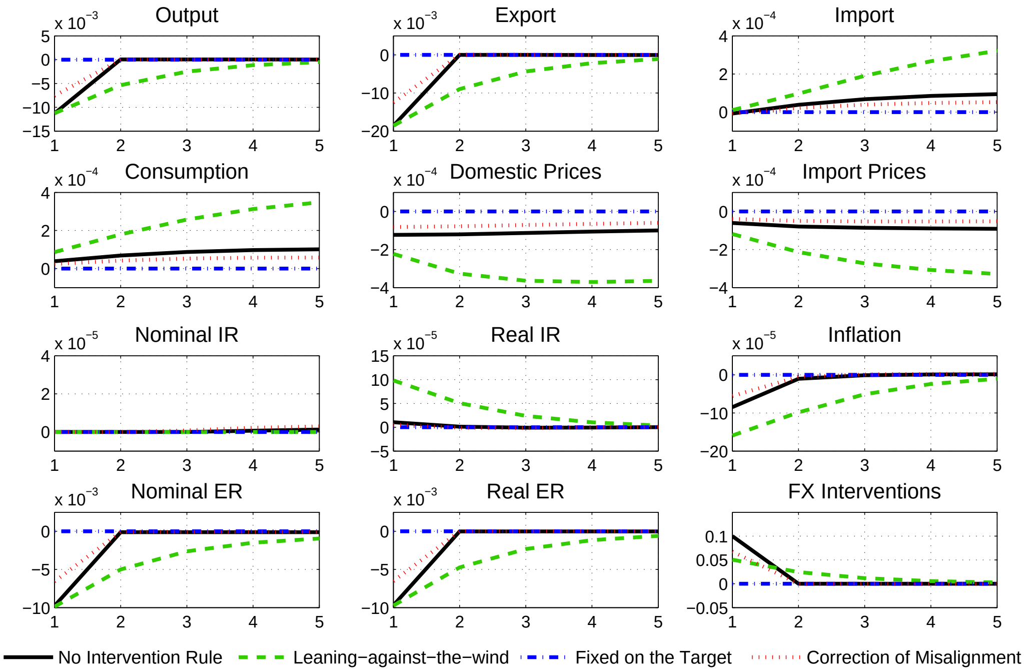
Table 8.2: Foreign exchange interventions - parameterization

	χ_e	χ_{e^T}	χ_q
Leaning-against-the-wind	5	0	0
Fixed on the Target	0	∞	0
Correction of Misalignment	0	0	5

Source: Author's parameterization.

Figure 8.5 shows the reaction of the economy to a 10% unanticipated FX intervention shock which allows for comparison of discretionary interventions with interventions performed under the rule. The strength of the response under the discretion and under the leaning-against-the-wind interventions (or managed float) is quite similar but it disappears after the first quarter for discretionary interventions while it gradually transmits through the entire economy in the horizon of five quarters for the managed float. If the central bank fixes the nominal exchange rate on its current level using an unlimited amount of interventions the shock is immediately accommodated and the economy stabilizes at the steady state. Different exchange rate targets imply different steady state values, i.e. the economy initially jumps to the different long-run levels. The comparison of particular targets is presented in Table 8.3. It is apparent that stronger devaluation boosts domestic export and decreases import more than just fixing the exchange rate the on current level. Using interventions to smooth the real exchange rate misalignments generates weaker reaction than is observed under the discretion or the managed float. As the interventions affect the exchange rate through both, the portfolio balance and expectations channel, they work as a stabilizer of the economy. Under the rule, the central bank can anchor exchange rate expectations better than under the discretion.[3] Hence, the amount of interventions is smaller under the exchange rate rules.

Figure 8.5: Quarterly IRFs after a positive FX intervention shock (binding ZLB, FX interventions)



Source: Dynare output.

Figures 8.6 and 8.7 present the response to the domestic shocks and Figures 8.8 and 8.9 to the foreign shocks. In all cases the effectiveness of different FX intervention regimes is quite similar with and without binding ZLB constraint (see Appendix B), just the reaction is pronounced with constrained policy rates. Regarding the positive domestic technology shock, the volatility of import prices is reduced for the real exchange rate rule and fixed target. Together with weaker nominal appreciation, the competitiveness of domestic export increases more than without interventions. It improves net export and contributes to higher output. On the other hand, the leaning-against-the-wind interventions have not proven effective in response to the technology shock. Basically, the response of most variables is not significantly different from the benchmark case. However, the fall in import prices is slightly deeper which together with a little bit stronger nominal appreciation increases domestic im-

[3]If economic agents know that the central bank is going to intervene in the foreign markets, the size of interventions necessary for reduction of exchange rate movements is smaller under the rules than under the discretion.

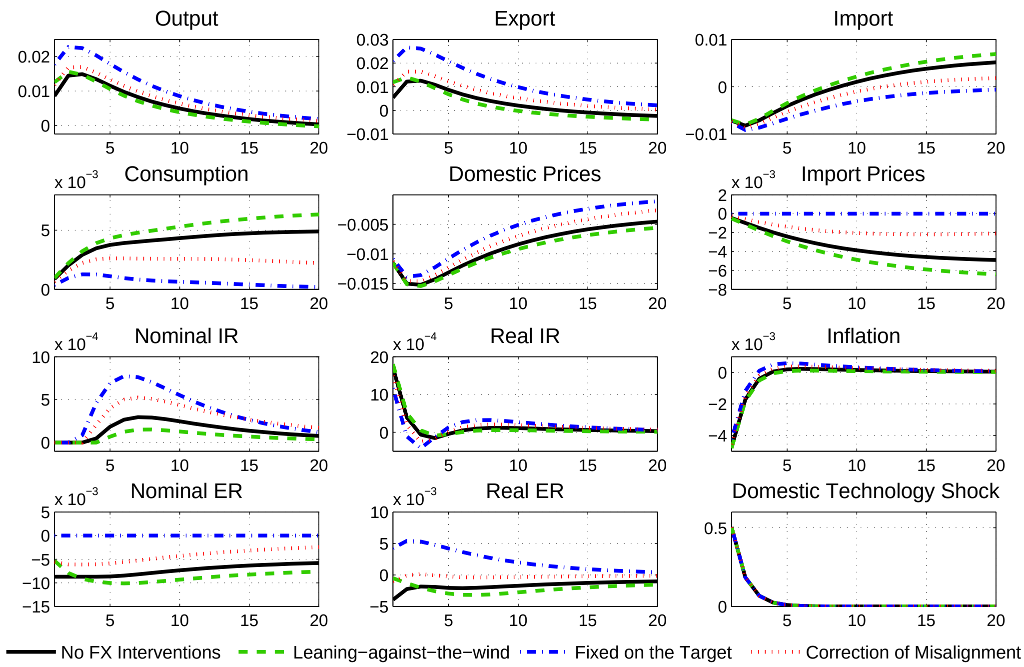
port and mitigate the rise in export. In the response to the negative domestic preference shock, any intervention rule does not significantly contribute to the accommodation of the shock. The volatility of the exchange rate is pronounced in first ten quarters for managed float which magnifies the decrease in export but does not contribute to significant change in output or inflation.

Table 8.3: Exchange rate devaluation - steady state

	$e^T = 0$	$e^T = ln(25)$	$e^T = ln(30)$	$e^T = ln(35)$
Output	-0.002	1.630	1.722	1.800
Export	-0.004	3.132	3.310	3.460
Import	0.004	-3.767	-3.981	-4.161
Consumption	0.003	-2.580	-2.730	-2.850
Domectic prices	0.002	1.590	1.680	1.760
Import prices	0.000	3.219	3.401	3.555
Inflation	0.000	0.000	0.000	0.000
Nominal IR	0.011	0.014	0.014	0.014
Real IR	0.011	0.014	0.014	0.014
Nominal ER	0.000	3.219	3.401	3.555
Real ER	-0.001	0.635	0.671	0.702

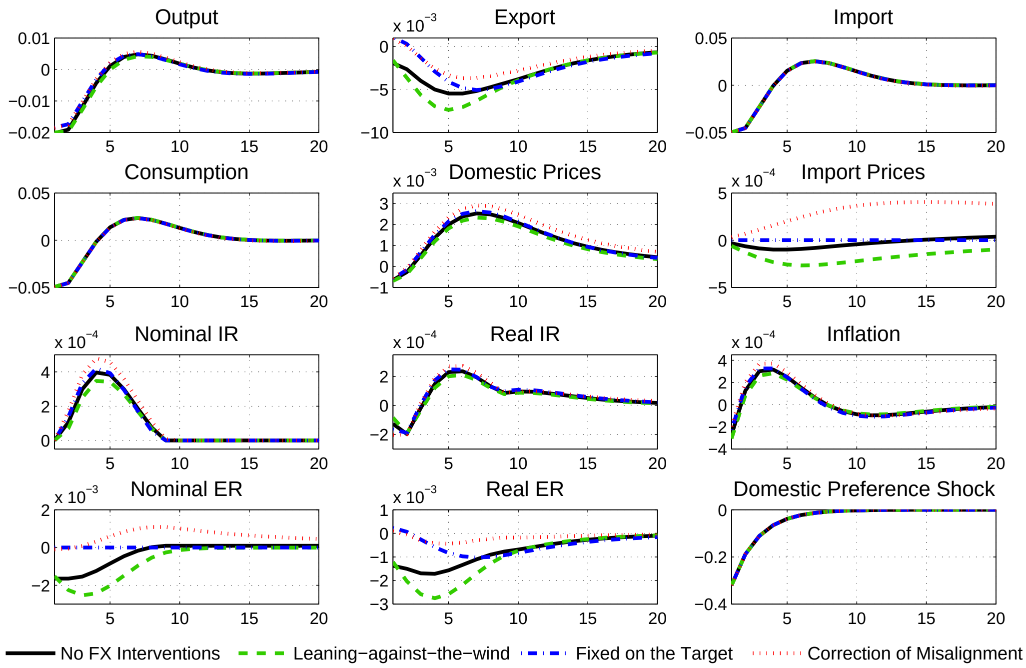
Source: Dynare output.

Figure 8.6: Quarterly IRFs after a positive domestic technology shock (binding ZLB, FX interventions)



Source: Dynare output.

Figure 8.7: Quarterly IRFs after a negative domestic preference shock (binding ZLB, FX interventions)

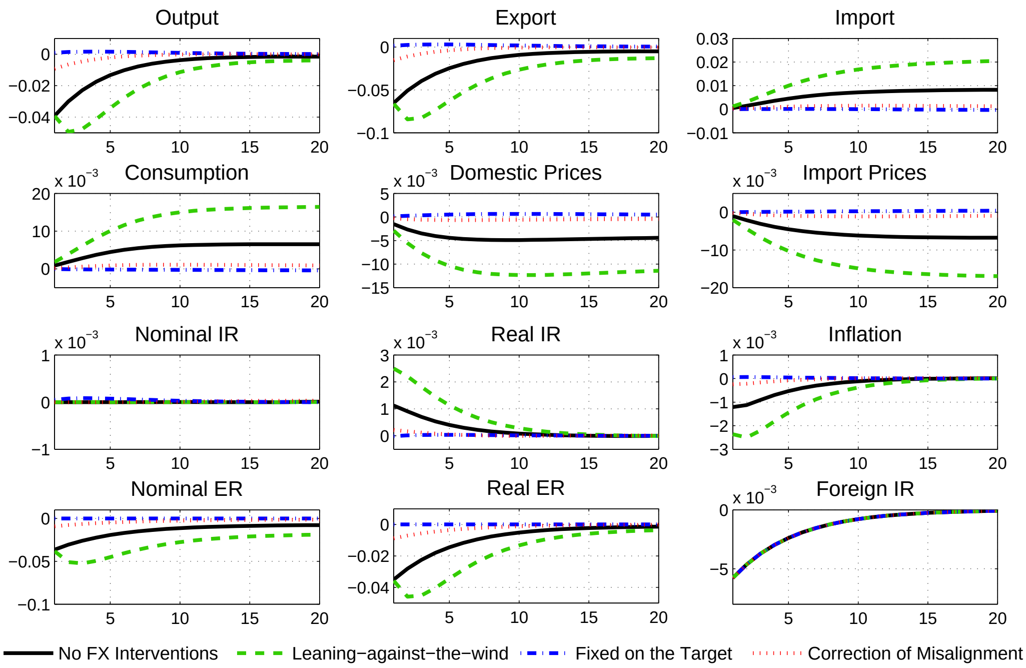


Source: Dynare output.

Regarding foreign shocks and managed float, the volatility of inflation is pronounced, the real appreciation is much stronger and the nominal exchange rate fall substantially and permanently which is mirrored in the permanent significant fall in prices and increase in import. The reason of such a strong response can be nicely seen from IRFs without ZLB (see Appendix B). The effect of the shock is mitigated by the decrease in the short-term nominal interest rate which is stronger in more periods for the managed float than for other regimes. Hence, it is useful in situation when the short-term interest rate may still decrease as the lower interest rate should help in offsetting the effect of the shock on consumption. In two other intervention settings the nominal exchange rate returns to its initial level. Although the economy still expands, these changes are negligible. It is also quite interesting to see similar effect of intervention regimes in the response to different foreign shocks. The capital inflows shock causes the exchange rate appreciation which is mitigated using interventions.[4] Similarly, interventions can dampen pressures on the exchange rate created by capital outflows in case of the negative foreign interest rate shock. Finally, FX interventions can prevent the deterioration of current account balances when the economy is hit by the foreign inflation shock.

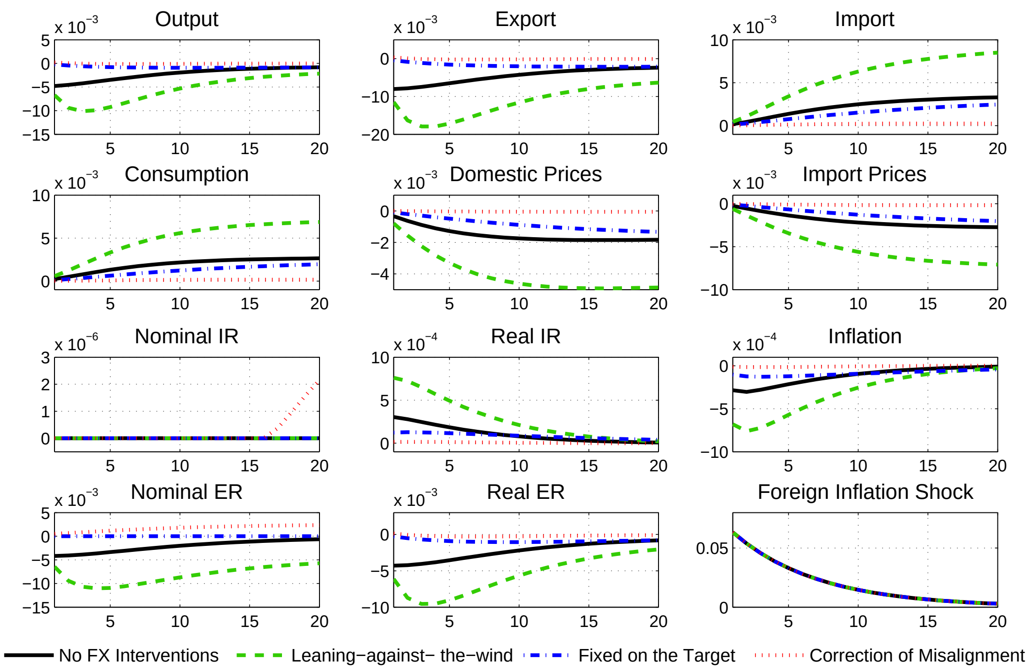
[4]IRFs for the capital inflows shock are presented in Appendix B.

Figure 8.8: Quarterly IRFs after a negative foreign interest rate shock (binding ZLB, FX interventions)



Source: Dynare output.

Figure 8.9: Quarterly IRFs after a positive foreign inflation shock (binding ZLB, FX interventions)



Source: Dynare output.

To summarize, regarding domestic shocks fixing the nominal exchange rate on some particular target serves as the best stabilizer of prices. Also the adoption of the real exchange rate rule leads to lower volatility while the effect of the leaning-against-the-wind interventions is quite similar to the benchmark case without intervening. Moreover, fixing the exchange rate restores the subdued economic expansion in the response to the positive technology shock through increased export while it significantly mitigates the response of domestic consumption. The performance of this rule together with the real exchange rate rule is even better in the response to the foreign shocks. Both policy regimes counteract the shock and lead to overall stabilization of the economy at the ZLB. On the other hand, the leaning-against-the-wind interventions increase the volatility of all macroeconomic variables. Nevertheless, it seems to be efficient to some extent in the response to the domestic shocks in normal times (without bounded policy rates) as it decreases the volatility of prices or contributes to slightly higher economic performance. Moreover, the managed float is not as radical as fixing the exchange rate on the target. The correction of misalignment seems to be quite good compromise between these two policy regimes since it is more effective than the managed float, especially at the ZLB, and does not require such strong commitment as the fixed nominal exchange rate.

8.3 Fixing ER through FX Interventions and Interest Rate

This section provides a comparison of the intermediate and operational target. The former fixes the exchange rate through interest rate rule and the latter through FX interventions. The parameterization of both cases is shown in Table 8.4.

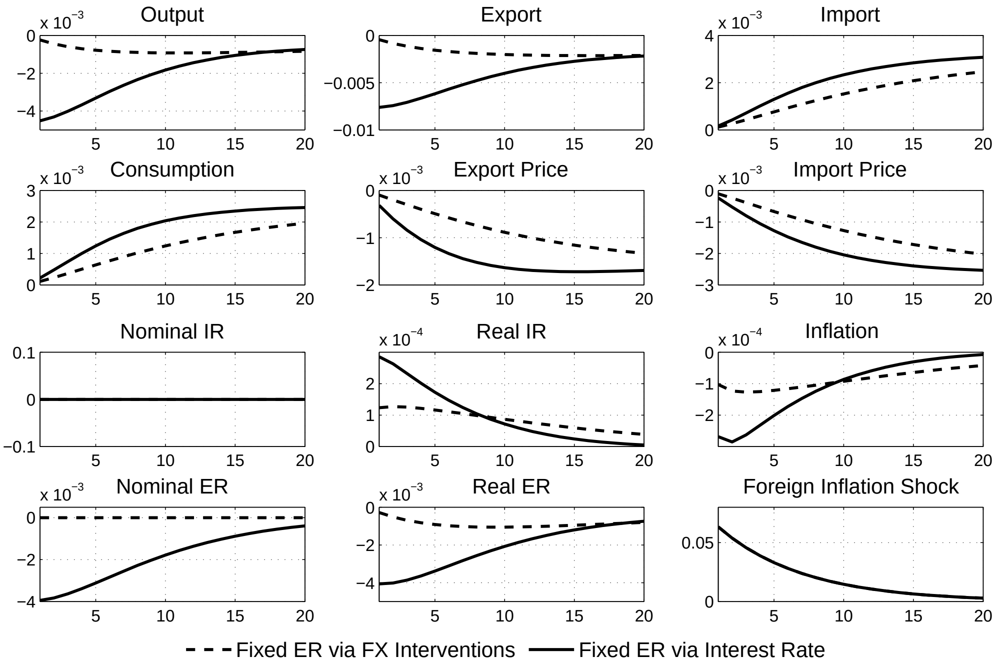
Table 8.4: Comparison of fixing ER through FX interventions and interest rate - parameterization

	χ_{e^T}	ψ_e
Fixed ER through FX Interventions	∞	0
Fixed ER through Interest Rate	0	∞

Source: Author's parameterization.

Figure 8.10 presents responses to the foreign inflation shock. On the first sight, FX interventions work as a better stabilizer than the exchange rate target at the ZLB. The volatility of domestic and import prices is lower and the recession subdued. The immediate response of inflation is pronounced for the exchange rate target but the situation turns out after the 10th quarter. Inflation is back at the steady state sooner with fixing exchange rate via the interest rate than via FX interventions.

Figure 8.10: Quarterly IRFs after a positive foreign inflation shock (binding ZLB, FX interventions, ER rule)



Source: Dynare output.

Chapter 9

Conclusion

The use of unconventional monetary policy tools may be reasonable when the central bank is not further able to lower the short-term interest rate in order to ease monetary conditions. The investigation of this situation together with the effectiveness of unconventional measures thus gains in importance as there is often a fear that economy might get on the deflation equilibrium path (into the liquidity trap). Despite a growing number of studies on the use of unconventional monetary policy tools, the analysis of their effectiveness in structural models is still limited. The main purpose of this thesis is to present the quantitative investigation of selected unconventional measures suitable for the Czech economy in the dynamic stochastic general equilibrium model.

The thesis presents three main findings. First, the volatility of the real and nominal macroeconomic variables is magnified in the response to the domestic demand shock and the foreign financial and inflation shock at the ZLB. On the other hand, the effect of the positive shock to innovation is subdued and the economic expansion is muted. Moreover, the effect of the ZLB becomes more serious in the case of highly persistent shocks which leads to stronger reaction of variables and prolong period of binding constraint. Second, the volatility of prices decreases significantly if the central bank adopts the price-level or exchange rate targeting rule. When the central bank credibly commits to the price-level target, economic agents expect that the inflation increases above its long-run level which creates higher inflation expectations and reduces deflation pressures. The exchange rate targeting rules work in similar fashion as the exchange rate movements are caused by the change in price-level expectations at the ZLB. Moreover, the degree of exchange rate pass-through increases with constrained policy rates which strengthens the transmission between the exchange rate and prices. Third, intervening to fix the nominal exchange rate on some particular target serves as the best stabilizer of prices in the response to the domestic and foreign shocks while intervening to smooth the nominal exchange rate movements (referred as the managed float) increases the overall macroeconomic volatility at the ZLB. The adoption of the real exchange rate rule (used to correct the misalignment of the exchange rate from its fundamentals) also reduces volatility while it does not require such strong commitment as fixed nominal exchange rate. The managed float is efficient to some extent in normal times (without the ZLB on the nominal interest rate) in the response to the domestic preference shock as it decreases the volatility of prices, and in the response to the domestic technology shock as it contributes to slightly higher economic performance. These findings confirm all three hypotheses stated at the beginning.

This analysis opens the door to the future research. The emphasis should be put on both, the implementation of the ZLB algorithm and the model extension. In particular, some unpleasant properties of the ZLB constraint should be investigated in more details. Attention should be given to the existence of multiple equilibria, the invisibility of equilibria connected with the linearization of equilibrium conditions and the robustness of results obtained using the linearization in comparison to nonlinear equilibrium conditions. Moreover, it seems convenient to test theoretical results empirically.

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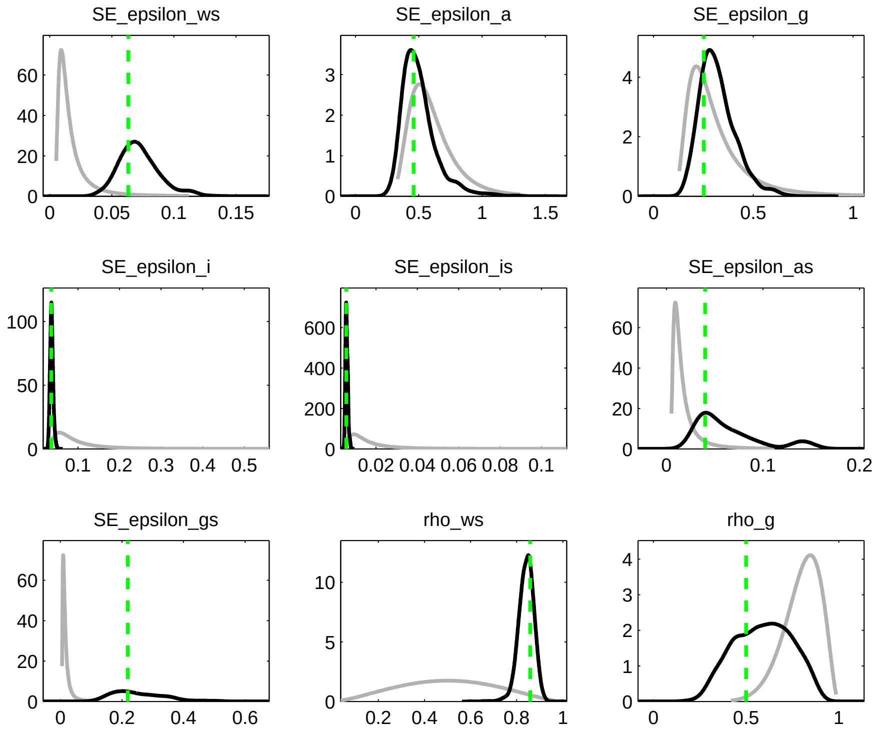
Appendix A

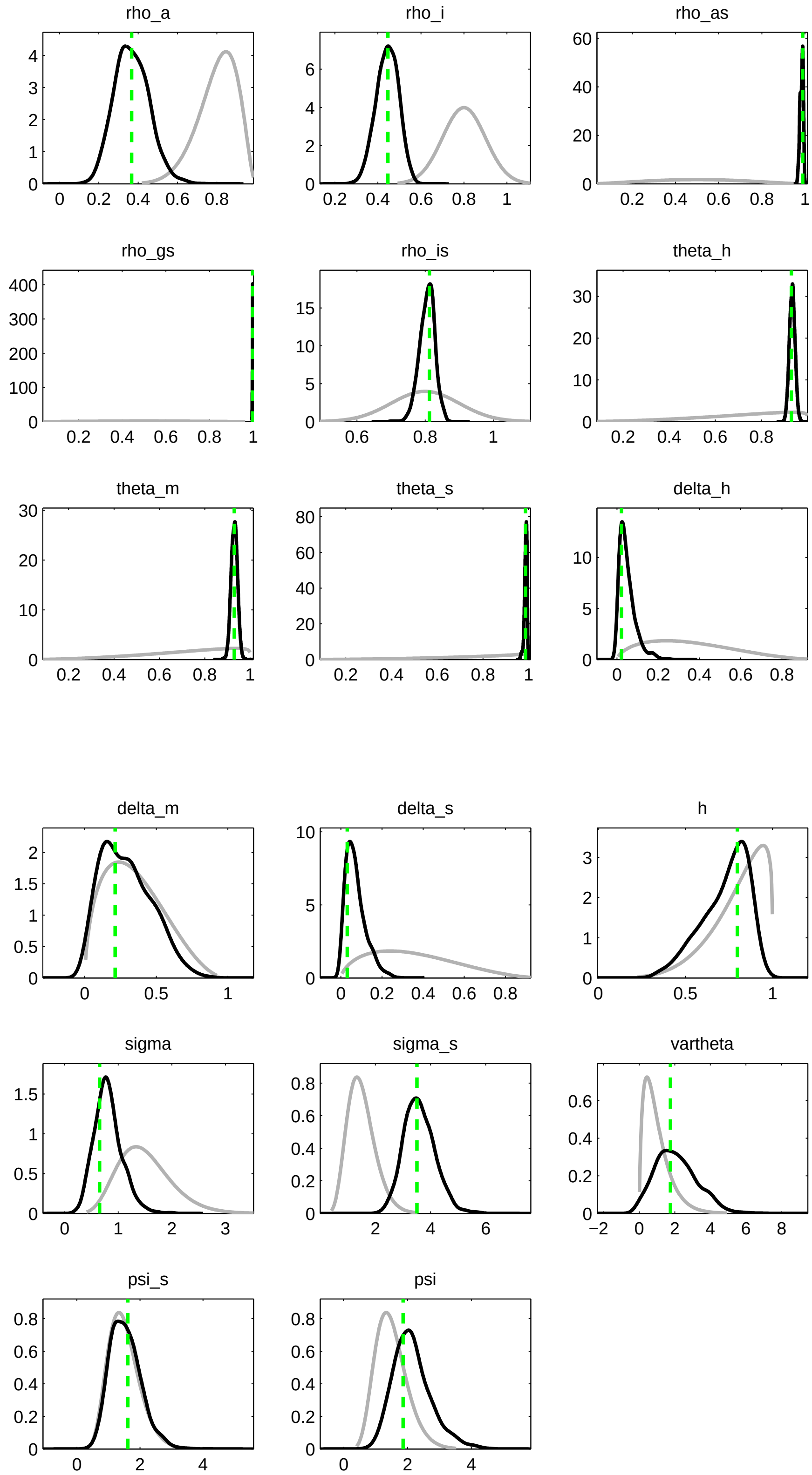
Bayesian Estimation

Figure A.1: Comparison of prior and posterior distributions



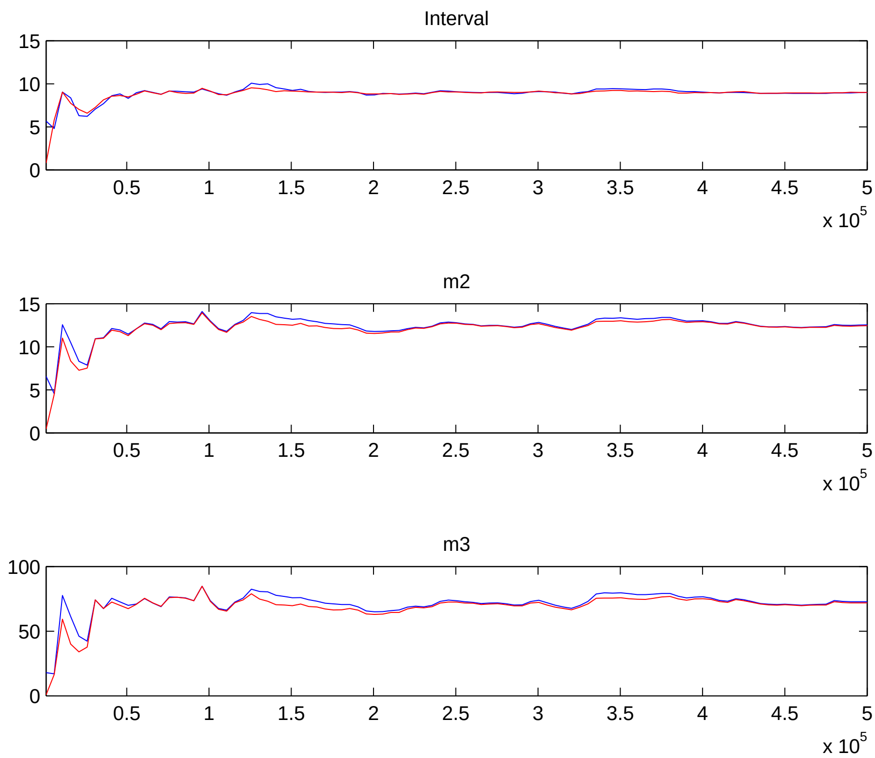
Source: Dynare output.

Figure A.2: Comparison of prior and posterior distributions



Source: Dynare output.

Figure A.3: Convergence statistics of Metropolis-Hastings algorithm (500 000 draws of 2 parallel chains)



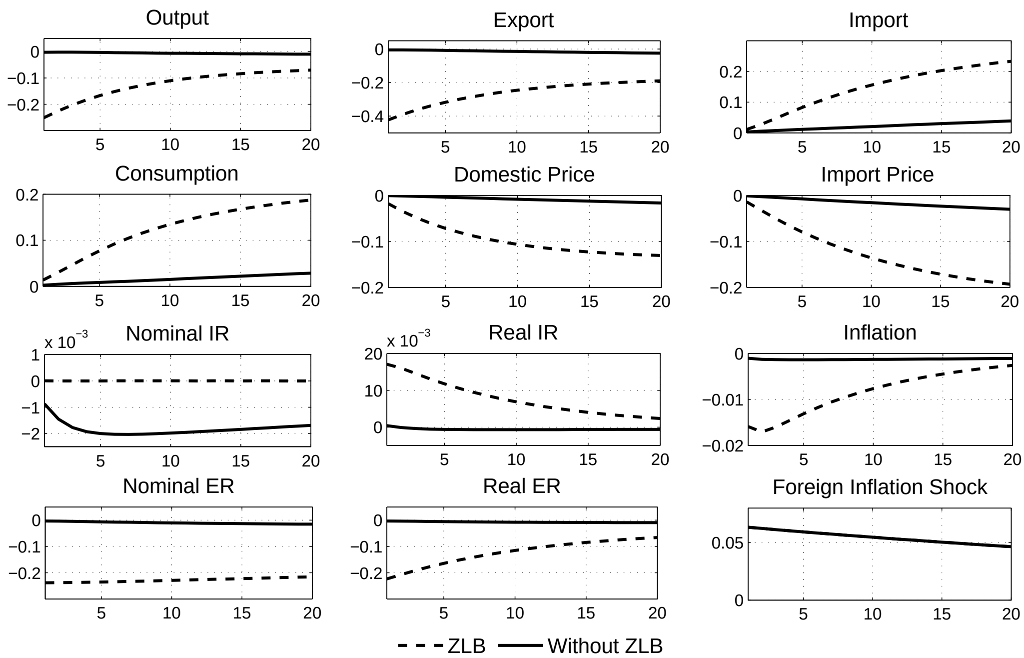
Source: Dynare output.

Appendix B

Simulations

B.1 Simulation of the ZLB (Chapter 6)

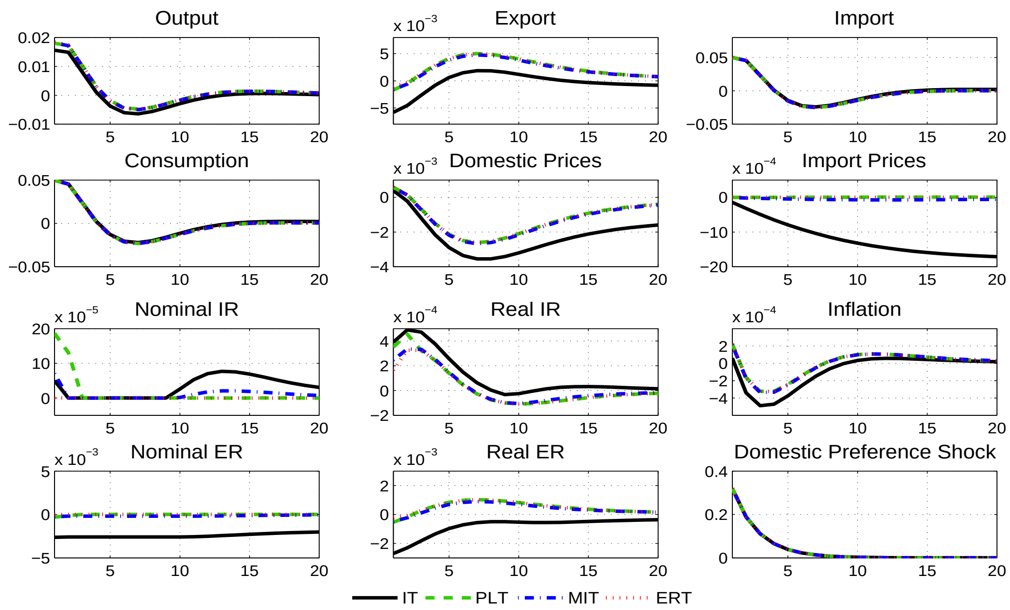
Figure B.1: Quarterly IRFs after a positive foreign inflation shock (with and without binding ZLB, higher persistence parameter)



Source: Dynare output.

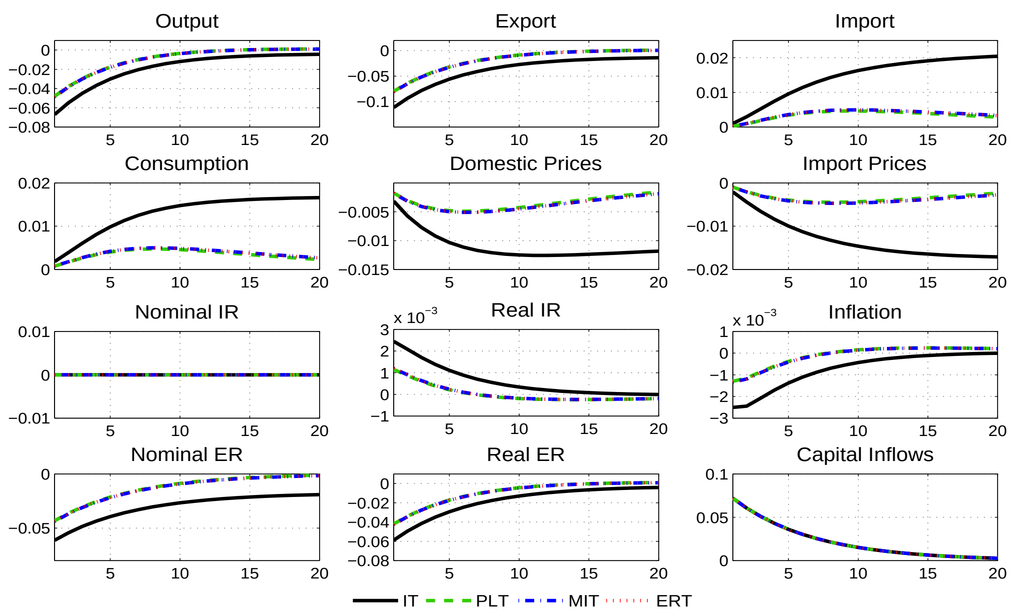
B.2 Monetary Policy Rules (Section 8.1)

Figure B.2: Quarterly IRFs after a positive domestic preference shock (binding ZLB, different policy rules)



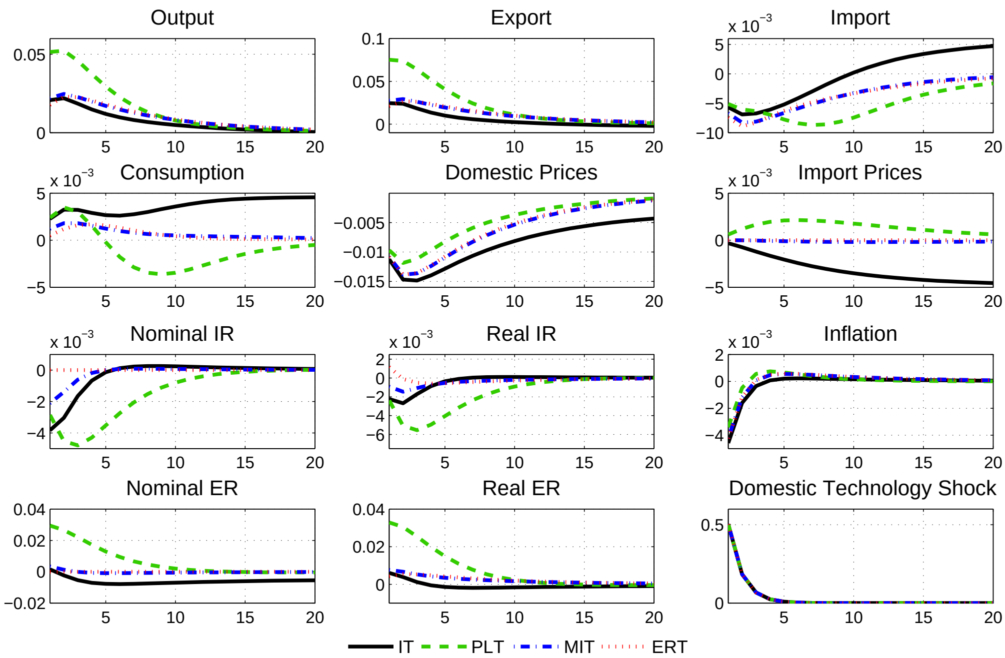
Source: Dynare output.

Figure B.3: Quarterly IRFs after a positive capital inflows shock (binding ZLB, different policy rules)



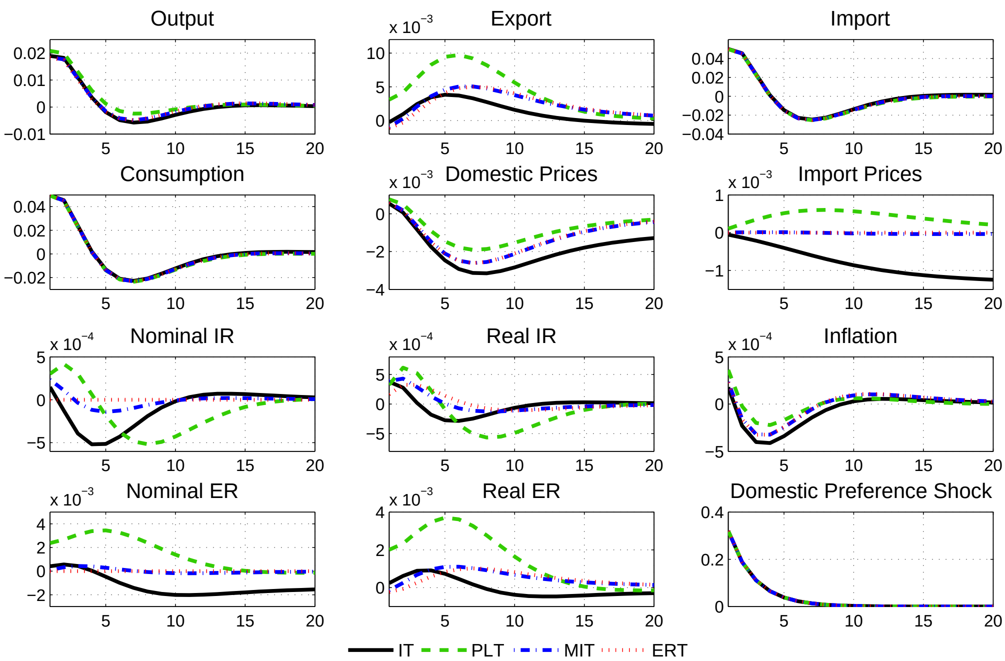
Source: Dynare output.

Figure B.4: Quarterly IRFs after a positive domestic technology shock (non-binding ZLB, different policy rules)



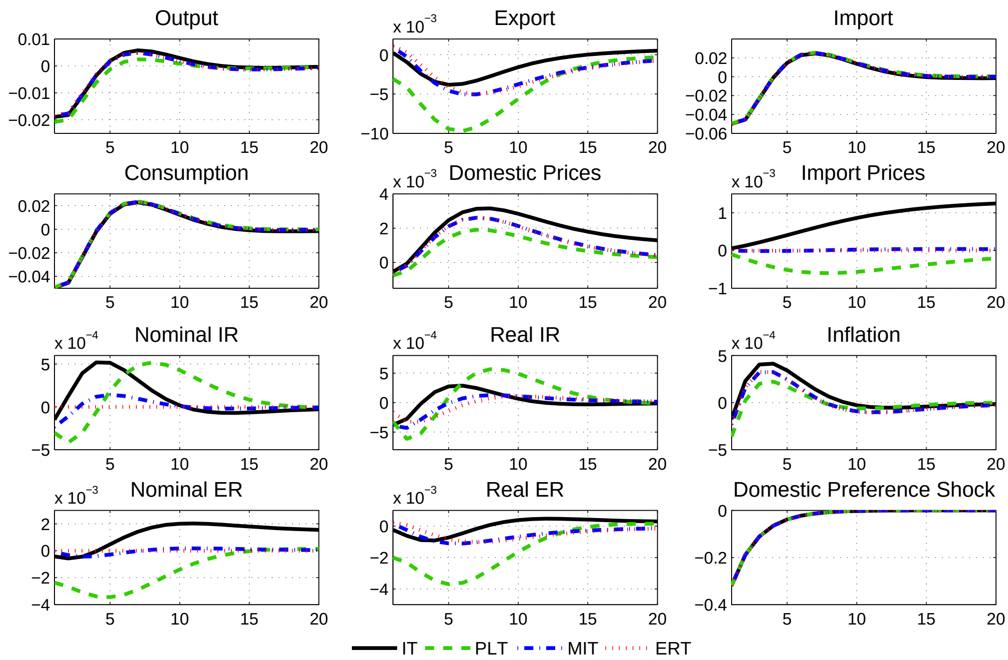
Source: Dynare output.

Figure B.5: Quarterly IRFs after a positive domestic preference shock (non-binding ZLB, different policy rules)



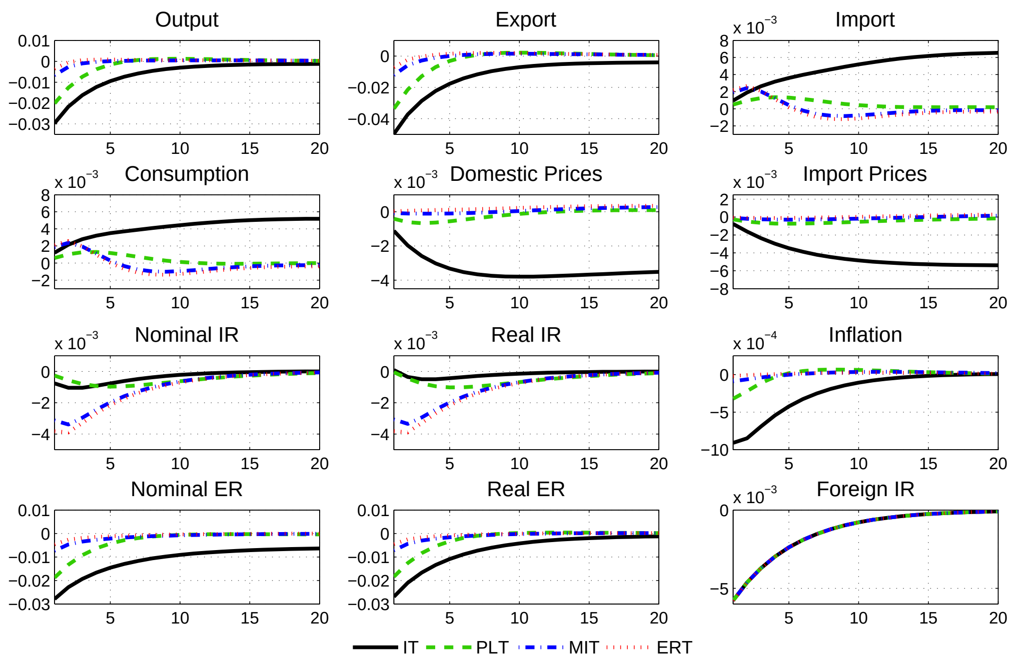
Source: Dynare output.

Figure B.6: Quarterly IRFs after a negative domestic preference shock (non-binding ZLB, different policy rules)



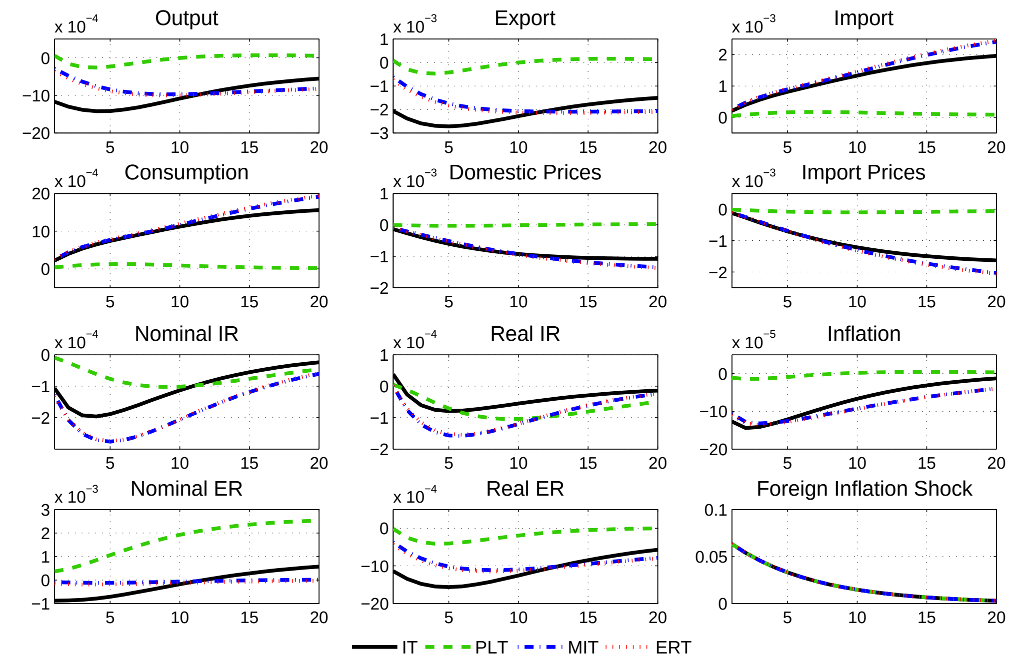
Source: Dynare output.

Figure B.7: Quarterly IRFs after a negative foreign interest rate shock (non-binding ZLB, different policy rules)



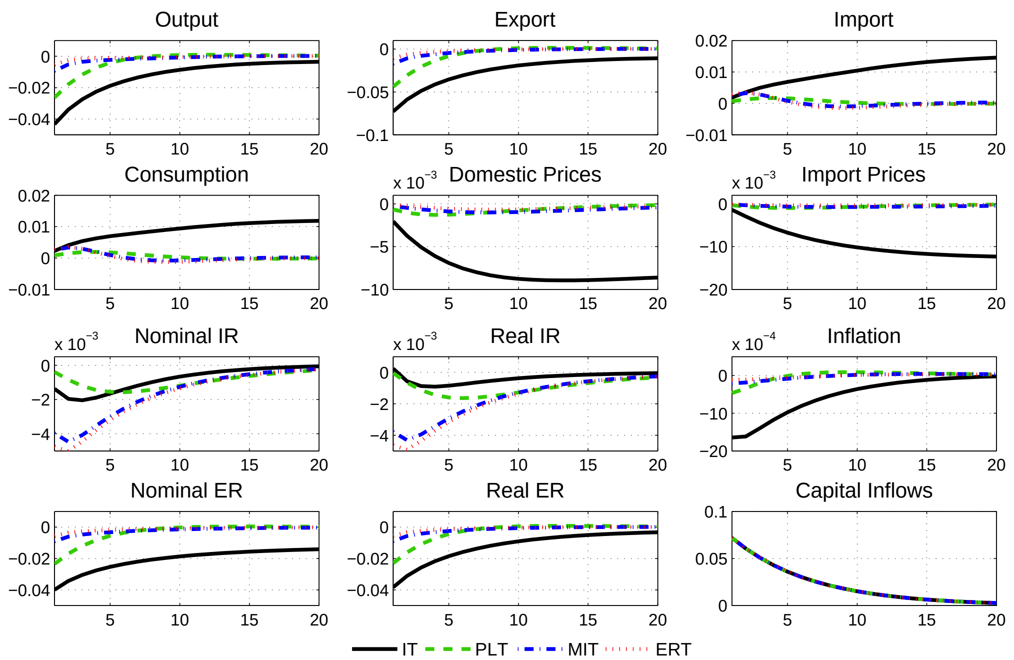
Source: Dynare output.

Figure B.8: Quarterly IRFs after a positive foreign inflation shock (non-binding ZLB, different policy rules)



Source: Dynare output.

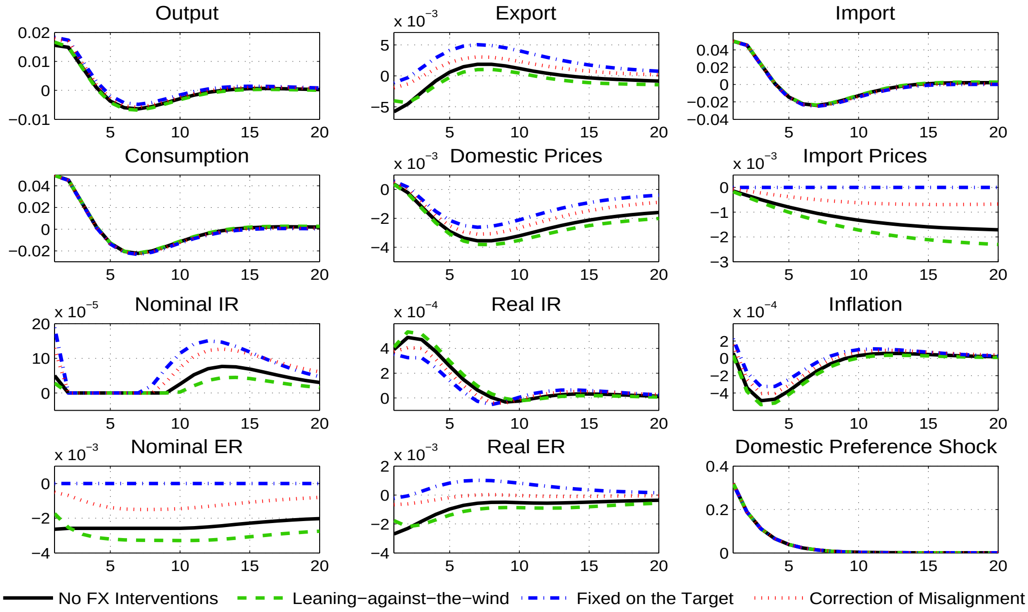
Figure B.9: Quarterly IRFs after a positive capital inflows shock (non-binding ZLB, different policy rules)



Source: Dynare output.

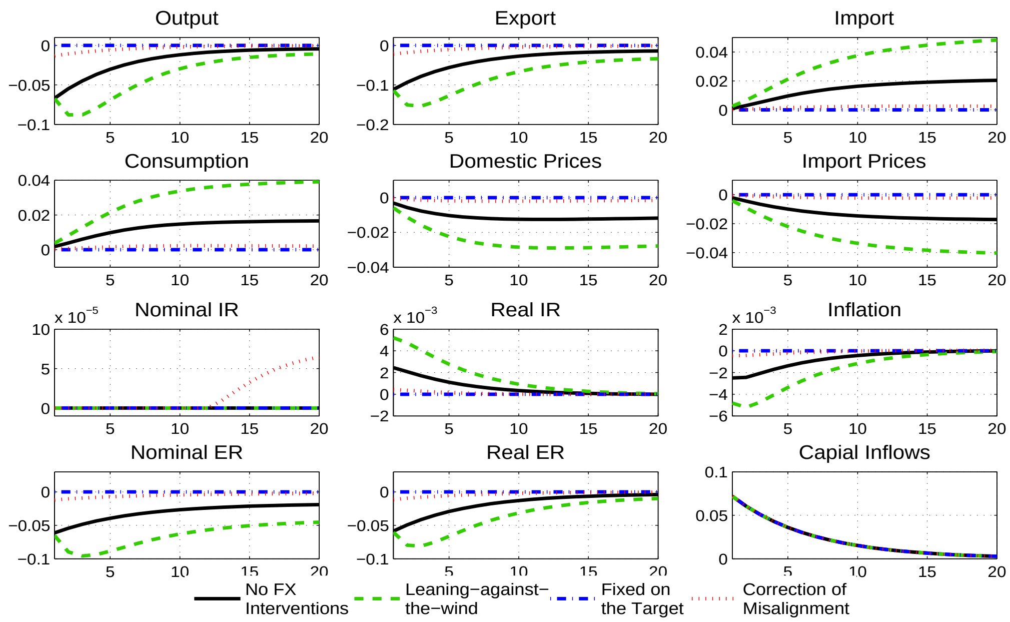
B.3 Foreign Exchange Interventions (Section 8.2)

Figure B.10: Quarterly IRFs after a positive domestic prefrenece shock (binding ZLB, FX interventions)



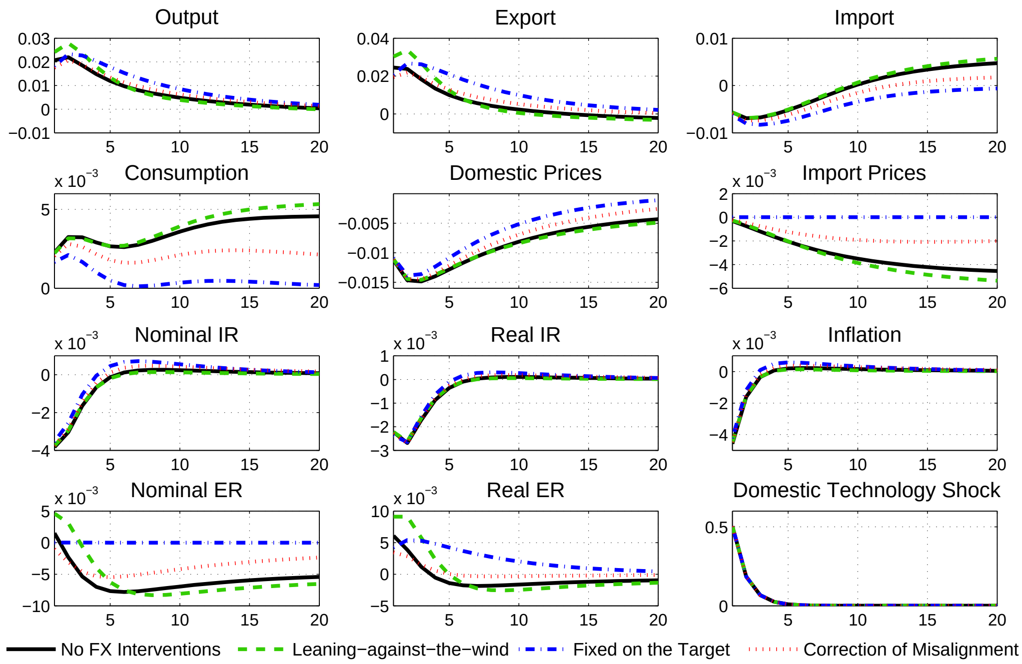
Source: Dynare output.

Figure B.11: Quarterly IRFs after a positive capital inflows shock (binding ZLB, FX interventions)



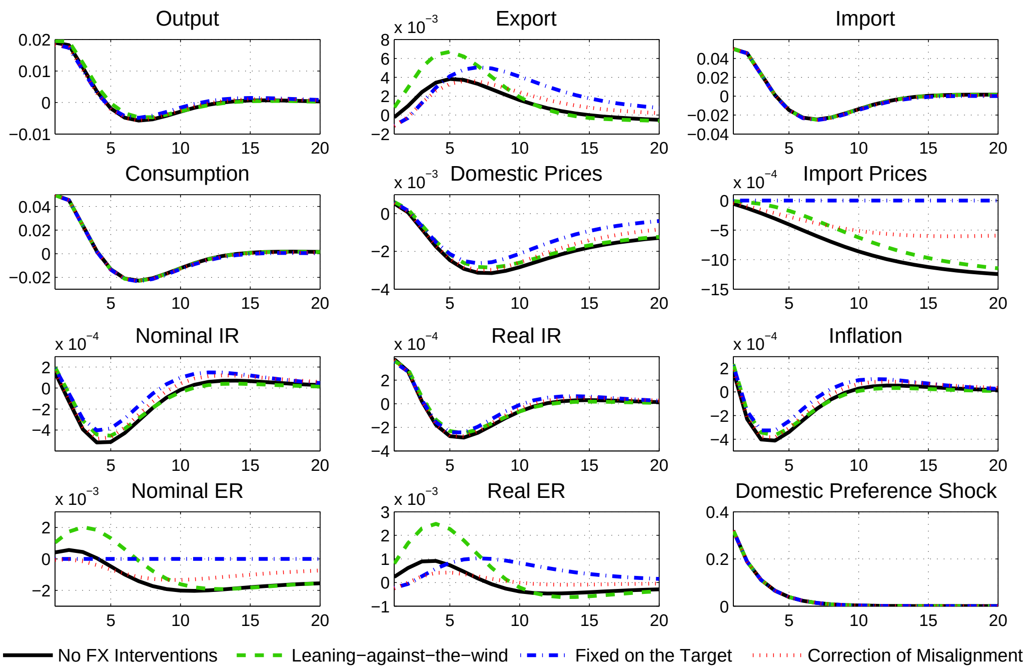
Source: Dynare output.

Figure B.12: Quarterly IRFs after a positive domestic technology shock (non-binding ZLB, FX interventions)



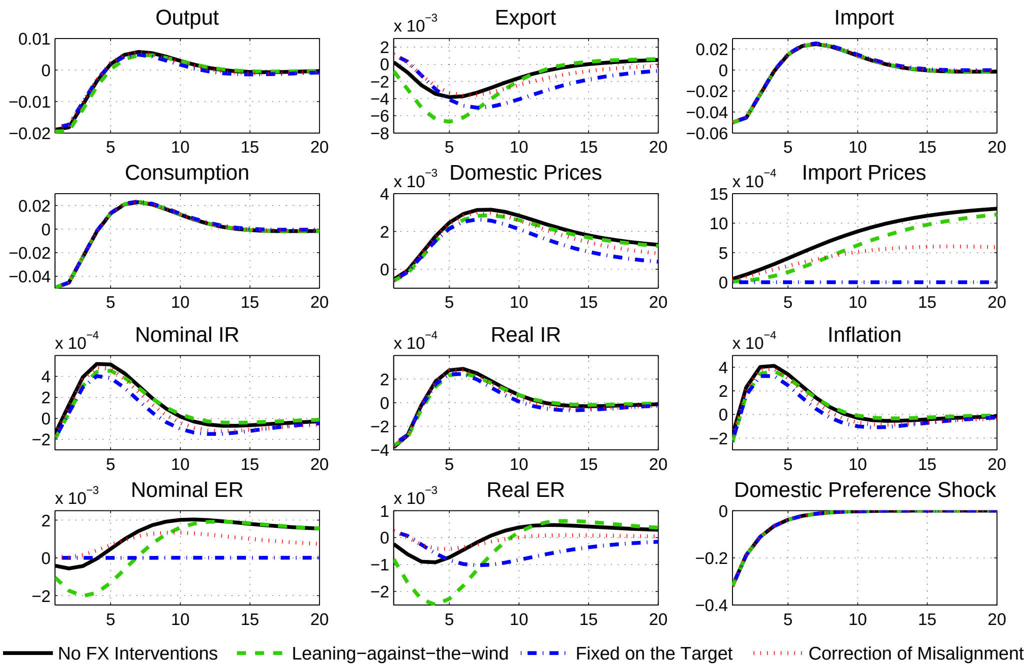
Source: Dynare output.

Figure B.13: Quarterly IRFs after a positive domestic prefrenece shock (non-binding ZLB, FX interventions)



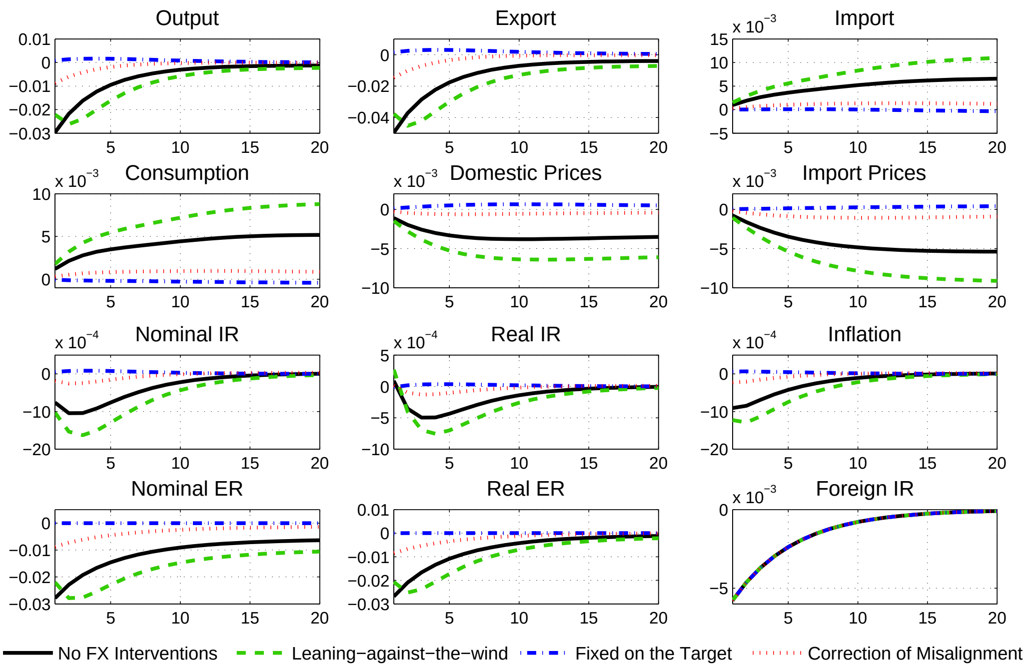
Source: Dynare output.

Figure B.14: Quarterly IRFs after a negative domestic prefrenece shock (non-binding ZLB, FX interventions)



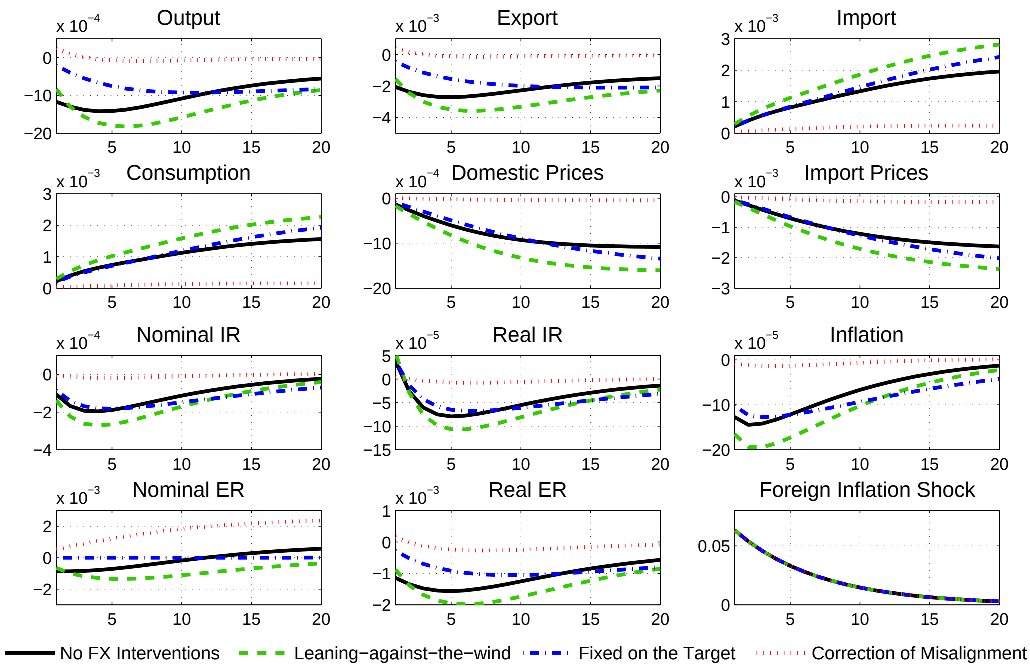
Source: Dynare output.

Figure B.15: Quarterly IRFs after a negative foreign interest rate shock (non-binding ZLB, FX interventions)



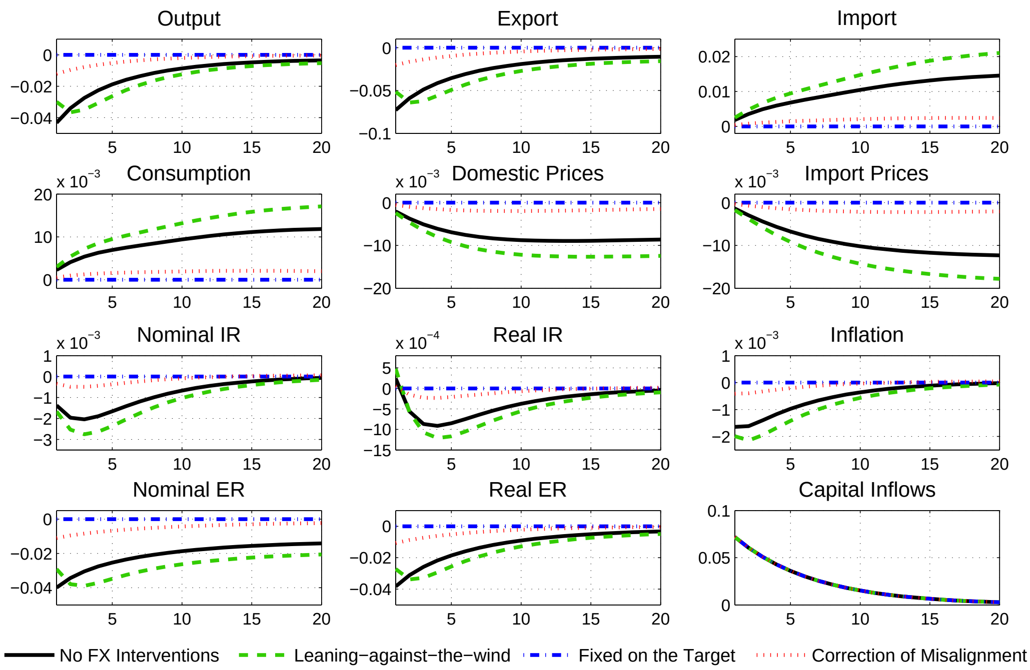
Source: Dynare output.

Figure B.16: Quarterly IRFs after a positive foreign inflation shock (non-binding ZLB, FX interventions)



Source: Dynare output.

Figure B.17: Quarterly IRFs after a positive capital inflows shock (non-binding ZLB, FX interventions)



Source: Dynare output.

Appendix C

Content of Enclosed DVD

There is a DVD enclosed to this thesis which contains empirical data and MATLAB/Dynare source codes.

- Folder 1: Source codes
- Folder 2: Empirical data