PhD Thesis

Extension properties of structures

by David Hartman

A structured object A is *homogeneous* if every (in that or other sense) structure respecting map $f : B_1 \to B_2$ between finite subobjets $B_1, B_2 \subseteq A$ (sometimes further specified) can be extended to an endomorphism $g : A \to A$. One speaks of an *ultrahomogeneous* object if each isomorphisms $f : B_1 \to B_2$ can be extended to automorphisms g. The main topic of the present thesis is the ultrahomogeneity in the context of relational structures, often related to various graph-type structures. But this concise statment does not properly describe the broad spectrum of the contents; there is a large variety of well motivated results, with potential applications.

After an interesting and useful survey of known results the author presents and analyses special classes of bicolored and L-colored graphs, and then discusses a hierarchy of related classes. The next topic is particulary inetresting (which is not to say that the previous results were not). The author uses the notion of (ultra)homogenity to introduce a concept of complexity of a relational structure. A given relational structure $(X, (R_i)_i)$, typically far from being (ultra)homogeneous, can be made (ultra)homogeneous when extended to $(X, (R_i)_i, (S_j)_j)$ with additional relations S_j. The *lift complexity* of $(X, (R_i)_i)$ is the least k such that this can be made with k-ary relations S_j. Using what the author calls invariant relations one obtains a somewhat more complicated *relational complexity* of $(X, (R_i)_i)$. These two characteristics are studied with interesting results (not-trivial results are obtained already for graphs).

A short concluding chapter summarizes the contents of the Thesis and puts it into a broader context.

In this brief report I have so far skipped a very important part, namely the chapter concerning complex network symmetries. It deserves a special praise for introducing very important motivation and for explaining how research in finite structures can contribute to solving very concrete topical problems.

It is a very good thesis, containing many interesting non-trivial results (it should be noted that some of them have been already published and I hear that more of them are prepared for publication). It is well written and can be used as a starting text for further investigation (for this purpose, the extensive bibliography can also be of use).

It certainly has the qualities required of a PhD Thesis, and unequivocally proves the defendant's abilities for independent research. I can recommend that the author be granted the PhD degree.

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June 2014

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