



Department of Logic

Supervisor's report on a doctoral thesis
Karel Chvalovský: Undecidability of Some Substructural Logics

The thesis of Karel Chvalovský consists of the following two papers, provided with an introduction:

1. K. Chvalovský: Undecidability of Consequence Relation in Full Nonassociative Lambek Calculus, *Journal of Symbolic Logic* 80 (2015), no. 2, pp. 567–586.
2. K. Chvalovský and R. Horčík: Full Lambek Calculus with Contraction is Undecidable, *Journal of Symbolic Logic*, to appear.

The first paper already appeared in a high-quality international journal – the *Journal of Symbolic Logic* –, the second paper is accepted for publication in the same journal.

The first paper concerns the complexity of the deducibility problem (i.e., the complexity of the finite consequence relation) in non-associative Full Lambek calculus. Its undecidability is proved using an encoding of a reachability problem of a 2-tag rewriting system, which is known to be undecidable. For this to work in a non-associative setting, a clever use of the join connective is employed. The encoding is shown to be sound and complete using sequent style proof theory of the logic.

The second paper deals with the complexity of deciding the set of theorems in another substructural logic – Full Lambek calculus with contraction. The deducibility problem for the same logic has been recently shown undecidable by Rostislav Horčík, using a string rewriting system. In the present paper, Karel Chvalovský and Rostislav Horčík show that the reachability problem for this string rewriting system can be actually encoded by provability in the logic FLc itself. The encoding is shown to be sound and complete using algebraic methods.

The results contained in the thesis are new, original, interesting and difficult to obtain. They answer some open questions in the area of complexity of substructural logics.

The doctoral project of Karel Chvalovský originally started on a related and broader topic of automated reasoning in non-classical logics. During his Ph.D. studies he took interest in various aspects of the topic and obtained other interesting results, concerning an independence of axioms in fuzzy logics BL and MTL, deduction theorems in contraction-free logics, and linearization of proofs in propositional Hilbert systems. But what seemed to arouse Karel's real interest were some open questions on complexity of various problems in the area of rewriting systems (like the derivational complexity of the rewriting system $\{aa \rightarrow bc, bb \rightarrow ac, cc \rightarrow ab\}$) and decidability problems in the area of substructural logics. He has managed to solve two of such open problems (open not because of being overlooked but for

their difficulty) and, in doing so, he helped to fill some gaps in our understanding of complexity of non-classical logics.

For the whole period of his Ph.D. studies Karel Chvalovský has been very much independent in obtaining his results, with a strong view on how mathematics should be done. I suspect he did not need my supervision, but on my part it has been a great pleasure to have such a student around. I believe that the cooperation with Rostislav Horčík has been substantial for a part of Karel's work contained in this thesis, and, even if it is not officially so, Rostislav Horčík should be credited as a co-advisor.

Let me finally state that in my opinion Karel Chvalovský without any doubt proved himself to be an independent researcher – his thesis contains non-trivial new results, solving some long-standing open problems in the field of complexity of substructural logics. Therefore I recommend the thesis as a basis for awarding the doctoral degree Ph.D.

Praha, 21th May 2015

Mgr. Marta Bílková, Ph.D.
supervisor