

Review report on the doctoral thesis “Triple Gravitational Microlens” written by Kamil Daněk

Contents of the thesis

The main topic of the thesis is gravitational microlensing. The thesis is divided into six chapters, starting with a short introduction (Chapter 1) where basic definitions and properties of microlensing effect are explained and ending by concise conclusions (Chapter 6) with all thesis results summarised. The core of the thesis consists of four chapters that contain original research on microlensing by n-body lens (Chapter 2), extensive study of critical curves and caustics in triple microlens in four two-parametric cases (Chapter 3) and three three-parametric cases (Chapter 4) and several examples of amplification maps including the effect of swallow tail and butterfly caustics metamorphoses (Chapter 5).

Scientific impact

The theoretical research presented in the thesis is of high quality and the main original results include:

- extensive study of critical curve topology in n-point-mass lens and its connection to Jacobian surface on the image plane of the lens with a possibility to visualise all the critical curves for differently scaled microlens,
- general n-point-mass lens definition of cusp curve useful for counting the number of cusps on critical curves,
- general n-point-mass lens definition of morph curve where cusp metamorphosis takes place with examples of beak-to-beak, swallow tail and butterfly metamorphosis types defining the conditions when these transformations happen,
- the most extensive study of triple-lens models up-to-date,
- study of amplification maps of swallow tail and butterfly caustics metamorphoses.

Chapters 2 and 3 comprise of two papers that are accepted for publication in very high impact factor journal, the *Astrophysical Journal*, which underlines the fact that very high quality scientific research is presented in the thesis. The results in Chapters 4 and 5 are not published yet, however, since they contain original interesting work with more connection to the microlensing observations, one can expect that these will be published as well before too long.

I expect that the presented results will have very high impact on the future development of microlensing studies of n-point-mass lens as well as triple lens and the works based on this thesis, either already published by the author or yet to be published in near future, will be highly cited by the astrophysical community interested in this topic.

However, highly theoretical approach of the thesis, sometimes bordering on having too many technical details in the main text, is also a bit of a drawback of the thesis. It is, at some parts, quite hard to follow all those details describing the multitude possibilities of topology changes of critical curves and shapes and number of cusps in caustics discouraging less determined reader. Putting some of these details in a dedicated appendix and expanding number of figures illustrating them (similar figures as LS model Fig. 3.6 for other triple lens examples LA, TE and TI) could be an option.

Format of the thesis

As far as the style and language is concerned, the thesis is written in a very good scientific English with few misprints (missing or redundant articles, repeating prepositions, missing or redundant characters in words or even missing a whole word, etc.). There are some misprints changing the meaning, e.g. in caption of Fig. 1.2 the colours *red* and *blue* are interchanged in the description and separation d has a wrong value (2 instead of 2.2, note that $d=2$ is a merging topology while $d=2.2$ is a wide, separated one), several occurrences of *loop* should be *cusp* on page 73.

The typesetting of mathematical formulae, tables and figures are of standard quality expected in this type of document. Some section titles and mathematical formulae, however, do not fit into the text width and stick out into the page borders (section title 3.5.6, equations (3.42), (A.9), (C.1) and (C.2)). Some of the figures might have been published in a larger size, especially the ones with caustics that can acquire quite complicated shapes while sustaining a very small scale (e.g. Fig. 3.13G), or zooming into a particular part of the figure could be used even more extensively.

I am not sure if it is a requirement of the thesis format by the university or faculty, however, I find it a bit uncommon to cite the literature by numbers only, usually it is much more convenient if the author names and publication year are explicitly mentioned.

Further comments

I would find it helpful if already at the beginning of Section 2.5.4 there was a reference to Fig. 2.5 (at the end of the first paragraph).

In Chapter 3, when describing the models LA, TE and TI, the author is using labels in the text (e.g. A_1 , A_2 , B_1 , C_1 , D_1 on page 71) not explicitly shown in the figures defining regions with different numbers of cusps. I found it hard to visualise and follow the description of transitions between these regions without helpful illustrations as was done in the preceding case for the LS model in Fig. 3.6.

It would be helpful if the topology labels T1, T2, etc. were depicted directly in Figs. 4.3, 4.5 and 4.7.

Questions to the student

1. It was not clear to me how one defines left and right side of the curve at the bottom of page 19 (Section 2.4.1) and the sign of the Jacobian on these sides, can you describe it in a little bit more detail?
2. You mention *external shear* and *convergence* but do not define it in the introduction. Can you explain it and show how does it affect critical curves and caustics on some simple example?
3. Can you discuss the definition of metamorphosis by eq. (2.36) in more detail? How does it follow from the fact that an additional cusp must appear or disappear?
4. Can you explain Chang-Refsdal limit and its validity, mentioned on page 72?
5. I was missing some nice examples of light curves that would be typical and/or special for triple microlens. Would it be possible to show some examples during the defence, e.g. crossing swallow tail, butterfly and/or caustic loop in the center of Fig. 3.13 panel D and/or G from different directions?

Further questions at general discussion

- How robust are any conclusions on number and masses of the objects forming the lens and lens geometry from fitting the light curves of lensed stars? Do we have to rely on special cases?
- What is the usual crossing time of the lensed star and change of lens geometry due to motion of stars/planets/moons forming the lens?
- The probability of lens configuration defined in Chapter 4 is different than the probability of finding a star/planet/moon obeying Keplerian laws in some configuration. Is it possible to work out a better probability description?
- Would it be possible to find a parameter space graph similar to the ternary plot in Fig. 4.3 where the change in geometry due to motion of objects forming the lens would mean just a shift in this graph?

Conclusion

I am persuaded that the research work done by Kamil Daněk that has lead to the publication of this thesis and to the two research papers accepted for publishing in the *Astrophysical Journal*, as well as the thesis itself demonstrate his capability of independent scientific research. I recommend the presented work to be accepted as a PhD thesis after its successful defence.

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