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Report on Kamil Danek's PhD thesis: "Triple Gravitational Microlens"

Dear Members of the Subject Board of the Faculty of Mathematics and Physics

It has been a great pleasure for me to read the PhD thesis by Kamil Danek about triple gravitational microlenses. As Danek says in the motivation of his work, triple systems are starting to draw more and more interest in the microlensing community, with already four examples known of triple systems discovered and many more expected as the photometric accuracy will fall down thanks to future spacecraft missions. One of the main expectations from microlensing searches of extrasolar planets is the possibility of a full characterization of a planetary system as a whole by a single microlensing light curve. If we do not develop the necessary modelling capability before these advances in the observational techniques take place, we will not be ready to interpret the main scientific results of future microlensing campaigns. In this respect, theoretical studies such as the one presented by Kamil Danek are extremely timely and important.

Apart from the importance of the motivation of Danek's thesis, I wish to stress that no researcher in microlensing has been able to go as deep as Danek did in this manuscript. Within the extreme complexity of the subject, Kamil Danek has been able to find its way through complex mathematical expressions and derive results that are as generic as possible. In my opinion, after Danek's study, the full exploration of the parameter space of triple lenses has no additional mathematical features left to discover. So, we are grateful to Danek for showing us all the possible new phenomenology we can expect from triple lenses.

All chapters of his thesis are extremely useful for microlensing modelling. All along his mathematical discussions, Danek has always been able to establish links from mathematics to possible observational consequences, something that is difficult to find in many studies. So, in definitive, I think that the thesis fully demonstrates the ability of Danek for independent scientific research, since he has been able to elaborate previous methods and develop genuine new ones in order to achieve his goals.

I have no major corrections to suggest to Danek's thesis. I just have a few comments that I report here and a few minor corrections on some typos I spotted during my reading.

Comments:

- All along the thesis, the application to Galactic microlensing is stressed. However, I believe that the results are relevant even for strong galaxy lensing or for quasar microlensing. Maybe this point might deserve some discussion.
- Section 1.4: In order to follow the discussion of Chapter 2, this section should contain more details on the binary lens case, including formulae for critical curves in the complex space. This would probably prepare the reader to the more complicated discussions of the following chapters.
- The occurrence of butterflies requires the simultaneous realization of two conditions on the lens configuration. So, their occurrence is even more exceptional than beak-to-beak and swallowtails. In fact, Schneider, Ehlers and Falco do not classify them as stable singularities. So, even if they are interesting from a mathematical point of view, their interest in practical cases is quite limited. It would be interesting to have an estimate on their relevance.
- Page 65: Metamorphoses may happen even when planets are very far from Einstein ring. This is something somewhat unexpected. I would say this is one of the most interesting outcomes of this thesis. It would be interesting to investigate the effect of such metamorphoses in the probability maps for the detection of a third body.
- I find chapter 4 extremely interesting. While there is an intuitive understanding of what one classifies as "common" or "rare", it is not easy to quantify this probability because the measure of the parameter space depends on the prior physical assumptions. The choice made by Danek of the perimeter of the triangle formed by three masses stands as a very brilliant alternative, although the final connection to observation probability is not that straightforward.
- Page 87: In Section 4.4 a planet as small as 10^-4 has been adopted. I think that this mass is too small to make the phenomenology clear. Probably a planet of 10^-2 could have allowed the calculation of the probabilities of all topologies easier.
- Section 5.3 is also very instructive. It makes the reader understand how deep Danek has gone in the code mastery and development. I note that small sources are best for contour integration. From the discussion of the errors, I understand that a grid refinement is required in order to pinpoint the source boundary. Since contour integration only deals with source boundaries, I have the impression that inverse ray shooting, in this version at least, would be much slower than a good contour integration code. So, this encourages me (and should probably encourage Danek too) to pursue the construction of a fast contour integration code for triple lenses.
- In the conclusions, (page 110) Danek says that not many more topologies can be expected in general triple lenses. Since most topologies (with the exception of T10 and T11) can just be traced to pair interactions of lenses, I have the feeling that no more topologies should arise. Can we imagine an example of how a new topology could be generated?
- I find topologies T10 and T11 very interesting, since they have a doubly nested loop generated by a planet inside one of the two secondary loops of a close binary configuration. This topology has no analogue in binary lensing. I guess that the occurrence of this topology has forced Danek to choose a very small planet mass, since the planet Einstein ring must be smaller than the size of the secondary loop. I am very curious to see a deeper characterization of this doubly nested loop, since it is something completely new in multiple lenses studies.
- Page 104: I am not convinced that the sentence "Up to this point, there is no study dedicated to the amplification maps due to caustic metamorphoses" is true. Local maps around swallowtails can be found in Schneider, Ehlers and Falco. Furthermore, in Aazami and Petters (Jour. Math. Phys. 50,

032501 (2009)) an amplification map can be found. The latter study also refers to a book by M. Golubitsky, V. Guillemin, "Stable Mappings and Their Singularities", Springer (1973) that should be checked.

Minor corrections:

- Page 3: The paper by Dominik (23) should be cited in this introduction.
- Page 4: z =\theta/\theta_E. A theta_E factor is missing here.
- Page 5: Eq. (1.8): the argument of the square root is z^2+4.
- Page 16: After Eq. (2.14) and (2.15) the same paragraph is duplicated. Reference is made to Eqs. (3.1) and (3.6), which are probably wrongly numbered.
- Page 23: Before Eq. (2.30). The condition that the tangent to the caustic vanishes is common to all cuspoids. The cusp is a cuspoid of order 1, for which the tangent vanishes but its derivative does not. The swallowtail is a cuspoid of order 2, with T and its derivative vanishing but with a non-vanishing second derivative and so on. So, I would refer to Eq. (2.30) as the equation defining the cuspoid series. It is just a matter of terminology, but I would prefer to say that swallowtail and butterflies are cuspoids rather than cusps.
- Page 28: "so that is passes" -> "so that it passes"
- Page 59: The figure for topology E has the central mass closer to the right one. For uniformity with the other panels, I would show the symmetric case with the central mass closer to the left mass.
- Page 84: the words "topologicy" and "hight" should be corrected.
- Page 88: the words "linarized", "pressent", "the the" should be corrected.
- Page 91: the word "atributed" should be corrected.
- Page 93: For an extended (the word source is missing)
- Page 98: There is a missing reference (??). "For the both" –> "For both".
- Page 108: "negative negative" appears

In conclusion, I am convinced that Kamil Danek would be an excellent candidate for post-doc positions in gravitational lensing or any other field of astrophysics that require very strong analytic and numerical capabilities.

Best wishes

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