Thesis supervisor's report on the doctoral dissertation by Mgr. Kamil Daněk

"Triple Gravitational Microlens"

Gravitational microlensing has been detected routinely since the early 1990's by the transient brightening of a distant star due to gravitational focusing of its light by another star passing in the foreground. In the majority of observed events the lens is a single star, followed by binary stars, stars with a planet (since 2003), stars with two planets (since 2006), and binary stars with a planet (since 2008). Planet hunting has become the primary focus of ongoing as well as planned microlensing projects. In comparison with other planet-detection methods, microlensing is particularly attractive due to its sensitivity to Solar-system analogs, including terrestrial planets on AU-scale orbits.

Analysis of microlensing events is based on the understanding of the lens properties, such as its critical curve and caustic. The structure of observed light curves is closely tied to the geometry of the caustic and the flux amplification in its vicinity. The properties of two-point-mass lenses have been well understood since the early 1990's. However, for the three-point-mass lens only limiting cases or special regimes have been studied so far (e.g., by Gaudi, Bozza, Han). Even though such events have already been detected, general understanding of triple-lens properties is still lacking. The main objective of this dissertation project was to systematically explore microlensing by triple lenses, concentrating primarily on their critical curves and caustics. Even if a full overview of the general triple lens wouldn't be achieved, the aim was to devise suitable methods for its analysis and make systematic inroads for further exploration.

Kamil Daněk started to work on the topic under my supervision already for his Master's thesis. He applied the Sylvester matrix method, used previously for the two-point-mass lens analysis, to three simple triple-lens models and mapped the different topologies of their critical curves. The results initiated further research that eventually led to the article which now forms chapter 3 of the doctoral thesis. In his following work Kamil developed new analytical and numerical methods for the systematic exploration of n-point-mass lenses, studied the caustics of triple lenses by tracking cusp-number changes in caustic metamorphoses, and demonstrated triple-lens amplification-map features for point-like and extended background sources.

Chapter 2 of the thesis introduces new techniques for studying critical curves and caustics of n-point-mass lenses. The approach is based on Kamil's surprising realization that all contours of the lens-equation Jacobian represent critical curves of re-scaled versions of the original lens configuration. After contemplating this fundamental result for a while it is so obvious that it is surprising no one noticed it before. This property permits the study of the full range of critical curves of a given lens configuration from the close to the wide limit from a single plot. Two new tools then enable the study of caustic changes with lens scale: the cusp curve, which pinpoints the positions of cusp images along the Jacobian contours, and the morph curve, which intersects the cusp curve at scales corresponding to caustic metamorphoses.

Chapter 3 introduces new analytical and numerical methods for mapping critical-curve topology and caustic cusp number in the parameter space of the lens. The methods, developed for a general n-point-mass lens, are then illustrated by a detailed analysis of simple two-parameter triple-lens models. Particular attention is paid to interesting behavior in the close, wide, and planetary limits. Chapter 4 maps the changes in critical-curve topology for three-parameter triple-lens models. Such models permit the analysis of lensing by three bodies with fixed masses in an arbitrary spatial configuration. The topologies are mapped using a sequence of ternary plots, and the probabilities of their occurrence are evaluated.

Chapter 5 forms a bridge between the results of the previous chapters and the analysis of observed events. Kamil developed a code for computing the amplification of an extended source with a general surface-brightness profile by an arbitrary triple lens. The code can be used for computing amplification maps or for modeling light curves of specific microlensing events. The algorithm is based on image-plane integration, using a flood-fill routine combined with inverse ray-shooting, as well as integration-grid refinement when necessary. The code is employed to demonstrate triple-lens features not occurring in simpler lenses: changes in amplification and extended-source effect maps in the vicinity of swallow tail and butterfly caustic metamorphoses.

At present Kamil has two major papers based on his thesis research accepted by The Astrophysical Journal, both coauthored by me. The first appears as Chapter 2 of the thesis. The main parts of the work including all key concepts and tools are Kamil's original results. My contribution is limited to parts of the introductory formalism, details of the specific morph-curve formulation, and the section on distinguishing butterflies from swallow tails. The second paper appears as Chapter 3 of the thesis. Even here the majority of the presented methods and the results of the parameter-space analyses are Kamil's work. Here my contributions are similarly limited: the numerical method in Section 3.3.3, the formulation of the polynomials in terms of lens-configuration moments in Section 3.5.1, and Appendixes A and B.

In the course of his doctoral studies Kamil presented his results regularly at scientific conferences, including talks at the Pasadena (2012), Santa Barbara (2014) and Annapolis (2015) International Microlensing Conferences. In 2013 he gave an Astronomy Group lunch talk at the University of St Andrews, where he then spent the spring of 2014 visiting the RoboNet and MiNDSTEp microlensing groups.

Supervising Kamil's research has been a rewarding experience. The seemingly straightforward project led to the thrill of discovering yet unknown properties of gravitational lenses, and devising new tools for studying and visualizing them. This could not be achieved without Kamil's independent, creative and enthusiastic approach to the problems solved, as well as his willingness to master topics beyond gravitational lensing, ranging from catastrophe theory, complex analysis, advanced numerical methods, coding algorithms, and visualization techniques. On closer inspection the thesis reveals it would have benefitted from more attention to final text editing and language polishing. However, this is just a minor blemish on its excellent and original scientific content. By the submitted thesis Kamil Daněk clearly demonstrated his capability of independent creative work.

Prague, 8 June 2015

David Heyrovský

Institute of Theoretical Physics, MFF UK