Gravitational microlenses are stellar- or planetary-mass objects which cause a transient amplification when passing in the foreground of a distant source. We study microlenses that consist of three point masses. Such a model can represent a triple star, a binary star with a planet, a star with two planets, or a star + planet + moon system. Up to date, four planetary systems have been discovered in triple-lens microlensing events. We aim to expand the theory of triple lenses in order to simplify the interpretation of observed data and enable correct analysis in more complex cases. We focus mainly on the classification of triple-lens models with respect to their caustics. For a given source trajectory, the caustic determines prominent features on the light curve and thus its knowledge is essential for the analysis of microlensing events. We map the topology of the critical curve (the main caustic image) and the number of cusps of the caustic in the lens-model parameter space. We introduce methods for the classification of general n-point-mass microlenses. The methods are then demonstrated on four two-parameter and three three-parameter models. Furthermore, we study amplification maps for both point sources and extended sources, with an emphasis on new features appearing in triple and general n-point-mass microlenses.