Charles University in Prague

Faculty of Social Sciences
Institute of Economic Studies

MASTER THESIS

Pension Systems in a World with Stagnant Population and Market Inefficiencies: A Comparison

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Declaration of Authorship

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Prague, May 14, 2014

Signature

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Master Thesis Proposal

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Proposed Topic:

Pension Systems in a World with Stagnant Population and Market Inefficiencies: A Comparison

Topic Characteristics:

The topic of pension system reform has become particularly important lately in Central-East Europe as most of the countries there currently cover their pension expenses from contemporary worker's taxes and contributions (so called unfunded or pay-as-you-go system). As the average human's life has been increasing, population size stagnates and the financial crisis has taken its toll, there is a dire need of either substantial change in current system's parameters or transformation of the pension system into a new better-fitting one because there is simply not enough money from current taxpayers to suffice the retirement expenses. Blake and Mayhew (2006) demonstrate the issue on the example of UK pension system that will, even in case of the most optimistic version of population and economic growth, fail to deliver the current level of pensions by 2030.

It is often argued that substituting state-run pay-as-you-go pension systems by private fully-funded schemes (where people continuously invest throughout their lives to save up money for their own retirement) could raise saving and eliminate factor market distortions, increasing long-term growth and welfare levels. Generally, such reform enables economic agents to transfer resources from formal, i.e. taxed and regulated, to unregulated informal sector while usually offering substantive tax benefits at the same time (Corsetti and Schmidt-Hebbel 1995). The advantages of a funded system against an unfunded system largely depend on its parameters, particularly on the level of state regulation and facilities offered by the financial sector.

The studies of e.g. Arrau and Schmidt-Hebbel (1993) and Auerbach and Kotlikoff (1987) suggest a funded system to increase both output and social welfare in comparison to an unfunded system, with differences growing larger depending on tax or debt financing of the transition period between systems. Corsetti and Schmidt-Hebbel (1995) find that the increase is even larger when assuming endogenous rather than exogenous long-term growth in the economic model. Supporting the previous findings, Marek (2007) shows that even accounting for the transformation costs, the long-term analysis of funded systems offer better results than any parametrical changes in unfunded systems. This is shown also in the work of Miles and Cerny (2006) who analyze the recent pension reform in Japan.

Funded systems have several drawbacks, though, which are often fully or partially omitted in the studies; first, people are heterogeneous with different degrees of myopia – while some of them understand the necessity of contributing for their own pensions, many people would prefer present over future consumption at any time. Second, effectiveness of funded pensions systems relies on stability and returnability of financial sector, primarily capital markets. The recent years have shown us that an assumption of long-term bull markets is questionable at least, and so is the efficiency of lifetime savings into them. Finally, as Murthi et al. (1999) point out, there are substantive costs connected with decentralized approach to individual accounts (i.e. individual contributions into state-run funded pension system); up to 25 percent of the contributions is lost on simple management and administrative costs, depending on the level of state regulation, 15 percent of the total wealth is lost due to individual inability to regularly save a constant amount of money, and up to 10 percent is lost by converting the amount saved into a lifetime annuity upon retirement.

Economic models of pension systems must take these and several other downsides into account to provide correct results, mainly when considering the microeconomic impact of funded systems on private and social welfare. In my thesis, I would compare the fully funded pension system with the pay-as-you-go pension system, while including the additional costs and imperfections in the analysis, unlike the preceding authors, to obtain a more precise information about their impacts on the social welfare and economic performance. I currently plan on including data from the Czech Republic and possibly from other European countries.

Hypotheses:

1. Under the additional assumptions, would fully funded scheme improve social and particularly individual welfare over a pay-as-you-go scheme in the long term, as opposite to the previous findings?

2. Under the additional assumptions, would fully funded scheme improve social and particularly individual welfare over a pay-as-you-go scheme in the long term in case there are no or little parametrical changes in the PAYGO system? (i.e. if it stays in its current state)

3. Would debt financing of the transition between pay-as-you-go and fully funded system result in better economic performance of the country in the long run when compared to tax financing?

Methodology:

I plan on using a comprehensive general equilibrium model, perhaps an overlapping generations (OLG) model of endogenous growth, assuming flat-rate tax rate on wage income of contemporary workers to finance pay-as-you-go system and forced regular savings with tax benefits for workers to save for old age in the fully funded pension system. Further, I would include the assumption of volatile capital market returns and low-probability events with excessive impacts on level of returns, administrative and other costs related to private saving via individual accounts and possible substitutes to such approach, individual displeasure from being a part of fully funded scheme (in case certain percentage of people prefers immediate consumption), and possibly other real-world deviations from the assumptions shown in the literature. To obtain simulation results, I would utilize computer programs (e.g. Matlab, Gauss, etc.). Additionally, I would include several different realizations of funded systems and a sensitivity/scenario analysis to see which model would offer the best results. Finally, I would like to propose improvements to both systems that would make people better-off than under the current conditions.

Outline:

1. Introduction
2. Literature Overview
3. Fully funded and pay-as-you-go system description
4. Administrative and other costs related to pension systems
5. Creation of theoretical models
6. Computer Analysis
7. Discussion of the Results
8. Conclusions

Core Bibliography:

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Author **Supervisor**

Abstract

Financial unsustainability of pension systems in developed economies looms large on the horizon due to increasing life expectancy and continuous drop in fertility. In spite of a broad discussion, there has been but a little consensus on appropriate remedy. Besides, the theoretical arguments supporting funded pension systems often build upon the unrealistic assumption of stable financial markets and fair transformation of saved funds into retirement benefits. This work provides an insight into performance of various pension systems in real-world conditions using large-scale simulations of an overlapping generations model based on existing pension schemes in the Czech Republic, Sweden, and Chile. Specifically, my model assumes adverse demographics, individual uncertainty, volatile financial markets' returns, and administrative costs to affect social security systems and estimates magnitude of the effects. According to the results, each pension system seems to be partially advantageous - in promotion of economic growth, level of retirement benefits, or protection against market risks - but no scheme is dominant or dominated overall.

JEL Classification	E27, C68, H55
Keywords	pension, OLG, simulation

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Abstrakt

Finanční udržitelnost penzijních systémů ve vyspělých ekonomikách je ohrožena v důsledku neustále se prodlužující průměrné délky života a poklesu porodnosti. Navzdory široké diskuzi na dané téma zatím neexistuje shoda na vhodném řešení této situace. Některé argumenty ve prospěch fondových penzijních systémů jsou navíc založeny na nereálném předpokladu stabilních finančních trhů a možnosti spravedlivého převodu našetřených prostředků na doživotní důchod. Tato práce nabízí náhled do fungování různých penzijních systémů ve skutečných podmínkách za pomoci simulace OLG modelu postaveného na existujících penzijních systémech v České republice, Švédsku a Chile. Můj model konkrétně bere v potaz vliv populačního vývoje, proměnlivosti výnosů na finančních trzích, administrativních nákladů a individuální nejistoty na penzijní systémy a zkoumá rozsah těchto vlivů. Dle výsledků je zřejmé, že žádný z daných systémů jednoznačně nepřevyšuje ostatní a žádný významně nezaostává. Spíše je každý z nich v některém ohledu - makroekonomických důsledcích, výši penzí a odolnosti vůči tržním rizikům - lepší než ostatní a v jiných horší.

JEL klasifikace	E27, C68, H55
Klíčová slova	penze, OLG, simulace

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Acronyms

PAYG Pay-as-you-go (unfunded) pension system

DB Defined-benefit pension

DC Defined-contribution pension

PA Pensions adjust

TA Taxes adjust

NA No adjustment

SA Standard assumptions

EA Extended assumptions

Chapter 1

Introduction

The topic of pension system reform has become particularly important for most of developed countries which cover pension expenses from contemporary workers' contributions in so called unfunded system.[1] Due to increasing average life expectancy, decreasing fertility, and the financial crisis taking its toll, there is a dire need of a substantial change in the current systems' parameters or their transformation into new better-fitting ones, for current taxpayers cannot contribute enough to sufficiently cover the retirement expenses.

It is often argued that substituting a private funded system - where people continuously save and invest a part of their salary for later consumption during retirement - for state-run PAYG scheme could raise savings and eliminate factor market distortions, increasing long-term growth and welfare levels (see e.g. Kotlikoff et al. 1999 or Homburg 1997). These studies have several drawbacks, which substantially reduce their explanatory and predictive power. While it is possible to rather precisely forecast future performance of an unfunded system, where the only relevant variables are demographic and economic projections (labour productivity, wages, employment, and other variables that determine tax revenues), forecasting a funded pension system requires certain assumptions about financial markets performance as well.

Looking back, each generation in the last hundred years experienced momentous stock market falls and periods of sluggish economic growth and low asset returns. Hence, conclusions based on ever-growing markets could be far from reality and need to be reconsidered. Moreover, as discussed later, returns on bonds and equities are expected to decrease in the long-run due to size disproportion between working and retired generations (Brooks 2000).

[1] Also PAYG, PAYGO, or PAY-GO, which stands for pay-as-you-go system from the USA.

Findings of Murthi et al. (2001) also suggest that incompleteness of annuity markets and administrative costs of pension funds may decrease individual account balance by more than 30 percent in total, which severely disadvantages defined-contribution schemes compared with defined-benefit ones.

Finally, while it is often mentioned in the ongoing discussion that population ageing and shrinking in the developed countries may indeed be a bane of PAYG pension scheme, there is just a little sophisticated research done on the effects of change in population age-structure on a funded pension scheme. The general belief is that funded systems are in principle protected against such adverse demographics. However, my results rather support the conclusions of Barr (2002), who argues that funded systems are in fact just as vulnerable as PAYG systems.

This work provides an insight into performance of various pension systems in real-world circumstances using large-scale simulations of an overlapping generations model based on existing pension schemes in the Czech Republic, Sweden, and Chile, and aims to answer the two following questions: can funded systems be considered better than PAYG schemes even in the light of the actual market imperfections and adverse demographics? And, if yes, would the prospective benefits outbalance the costs of transition? That is, I am primarily focused on the mounting problems of the Czech pension system (which can, of course, be generalized to other developed countries with the old-fashioned pure PAYG systems as well) and investigate the possible solutions.

The model simulates population ageing according to the newest projections, volatility of asset returns, income inequality, various levels of aggregate productivity growth, and many other aspects never implemented in a single model before. Through simulation of lifetime utility maximization for each of the 60 living generations, the computable theoretical framework renders microeconomic foundations as well as aggregate macroeconomic output in diverse scenarios. It also allows for a rich sensitivity analysis as the input parameters may be easily changed and any of the implicitly computed variables may be explicitly set up in advance. Upon completion of simulation, the program yields detailed results about retirement benefits, wages, interest rate, budget deficits, capital accumulation, and aggregate output of the economy.

The model assumes an economy to be in a steady state initially, and computes transition to a new steady state 300 periods later. During the transition, the economy faces ageing population and market volatility, and may go through a change of pension system. Based on the simulation outcomes, some pension systems provide better partial results than the others, but there is neither a dominant nor a dominated system overall. Following are the main conclusions.

First of all, as expected, the assumed population changes have negative effects on all the respective pension schemes. PAYG systems could significantly reduce income inequality (as in the current Czech implementation) as they provide high retirement benefits for low-income classes relative to the prior earnings, but the total pension transfers are lower than in the other pension schemes due to strong dependency on population development.

The fully-funded scheme, on the other hand, is better at promoting aggregate economic growth, but it is highly vulnerable to volatility on financial markets. The multipillar system seems to be the best option overall for its high degree of versatility, possibility to diversify risks, satisfactory promotion of economic growth, and reduction of market distortions. That is, it shows the highest growth-protection ratio defined, for purposes of this work, as a sum of pensions and aggregate output, divided by the difference in these parameters between the initial and the final steady states. However, the advantageousness of funded systems significantly decreases under the extended assumptions of additional administrative costs and market imperfections, and the multipillar pension scheme is still surpassed by the fully-funded scheme in promotion of economic growth.

The analysis then follows up with a simulation of a fundamental pension reform (i.e. a transition between two pension schemes). Despite the broadly positive effect on the economy in the long-run, such modification is extremely costly due to necessity to finance pensions for the current and some of the prospective pensioners from the general government budget. As to the question regarding profitability of such transition, the answered is unclear; replacing a pre-existing PAYG pension scheme with a fully-funded one is unfavourable in general, yet profitability of a transition towards a multipillar scheme would primarily depend on the actual situation. Essentially, a change towards a multipillar scheme with emphasis on the unfunded part would be nearly costless, bring positive results, and therefore be beneficial. On the contrary, change towards a scheme with emphasis mainly on the funded part could imply extreme external indebtedness with marginal positive effects.

The thesis is structured as follows: the next chapter describes the institutional background of social security systems, the selected pensions systems, demographic projections, and some important real-world facts regarding pension systems. Chapter 3 presents an overview of the relevant literature and points out its main conclusions, while Chapter 4 deals with the methodology, model description, and its calibration. Consequently, Chapter 5 constitutes the principal part of the work and contains simulation results with discussion. Finally, Chapter 6 provides main policy implications stemming from my analysis and concludes the thesis.

Chapter 2

Pension Systems and Ageing Populations

As the baby-boom generation slowly reaches retirement age, there has been a clear shift in demographic trends resulting in reduction of the working-age population share recently. In particular, the ratio of working to retired people (the old-age dependency ratio) is expected to fall from 5:1 to 2:1 in just forty years in the Czech Republic and most of the other developed countries as a result of increasing life expectancy and simultaneous decline in fertility. While population of western Europe is assumed to stagnate, predictions for central and eastern Europe are even worse: population is expected to decrease by 11 percent by 2050 due to the recent transformation processes.[2] Consequently, demographic changes are likely to decrease economic growth (Bloom et al. 2010) and imply significant increase of state expenses on pension system in the absence of meaningful offsetting parametric or fundamental changes.

As Barr (2002) puts it, there are only two ways of seeking security in old age: one can either store current production for future use or get along with a mere claim on future production. Because storing production for decades is literally impossible, having a claim on future assets is the only relevant option. Such claims can be either self-given, i.e. one may save a part of income during work-age, or come from government, or children. These two ways of organizing pensions are broadly used all around the world in the form of PAYG and funded schemes described in detail below.[3]

Besides differences in obtaining finances for pension payments, retirement plans differ also in how the benefits are determined:

[2] Eurostat (ec.europa.eu/eurostat).

[3] For a similar discussion with accent on private institutions refer to Štěpánek (2014).

- **Defined-benefit (DB) pension** - Pension transfers are determined as a function of worker's history of pensionable earnings. Actual implementation varies, but the formula is often based on the person's final wage and length of service, or on salaries over given period of time. Retirement plans in DB pensions are generally administered by institutions existing specifically for this purpose, employers, or, in case of state workers, by the government itself. Individuals who opt for an early retirement are usually sanctioned and receive lower retirement benefits. An obvious advantage is no dependency of pensions on investment returns or a risk of outliving the retirement income (in case of pension transfers being paid through a specific withdrawal scheme). Defined-benefit pensions may be either funded or unfunded.

- **Defined-contribution (DC) pension** - Benefits in DC pensions are determined by value of assets accumulated through one's life via private savings with possible contributions from employer or government. Contributions are usually invested in the stock market, albeit any interest-bearing assets are, in principle, appropriate to be invested into, depending on individual taste and settings of a pension system. At retirement age, people may be allowed to purchase an annuity and thus imitate defined-benefit pensions, take benefits as a sequence of withdrawals, or as a lump sum. DC schemes have become popular lately and are introduced worldwide as a supplement to DB pensions, or even as a dominant pension scheme in many countries. As opposite to DB pensions, investment risks and rewards are assumed by each individual rather than sponsor (government or employer). DC pensions can be funded only.

The underlying idea of a functioning pension scheme is that the retirement income should be secured by a mix of tax and transfer systems, yet also propped up by private savings, earnings, and other voluntary contributions. Many, if not all, of these ingredients are already included in the current pension systems and so the transition may not seem drastic, but their respective shares do not reflect the impossibility to further exploit the demographic dividend. Essentially, the share of private savings and funded systems will need to be raised in order to alleviate the burden put on workers.

Pension systems are also subject to other risk though, both personal and aggregate. From the personal perspective, relying solely on state funding proved to be sufficient until now thanks to the tradition of generous social states.

However, the ongoing financial and debt crises impose long-term difficulties for government budgets even without any aggravating circumstances of demographic trends, so it is appropriate for people to seek for additional pension funding elsewhere. Reliance on private funding can prove to be even more dangerous, though, as personal income is subject to many idiosyncratic shocks and depends on the aggregate state of the economy. A combination of as many sources of funding as possible is thus required. Analogously, whole pension systems face certain risks as well: PAYG schemes are principally dependent on macroeconomic performance and demographic structure and funded schemes rely heavily on financial markets performance. While demographic changes can be, unlike development on financial markets, rather reliably projected, it is virtually impossible to protect against them, whereas market risks can be diversified at least.

Another issue of pension systems is generation solidarity and burden-sharing in case of a radical pension reform. There are, in principal, two ways of financing such a reform: through tax increase or through higher government indebtedness. In the former case, the whole burden is on the current workers only. In the latter case, expenses are more equally distributed over a longer time horizon, yet it is reasonable to expect simultaneous government expenditure cuts, so the current population is worse-off anyway. Nowadays, citizens of countries with unfunded pension systems implicitly accept the obligation to finance the contemporary retirement benefits as an exchange for a promise of having similar benefits in the future. But in the case of a pension reform, all reform-related expenses would probably fall down on the current workers only. Additionally, in the event of a fundamental change to pension scheme (e.g. transition from an unfunded to a funded pension scheme), the current generation would have to finance both, retirement benefits for the current pensioners and its own future retirement. Needless to say, these issues make any reform considerably harder to be carried out.

In a sense, all pension systems are a combination of funded and unfunded schemes, and come out in a form of a pure PAYG, multipillar, or a fully-funded scheme. Hence, I have selected the three respective pension systems and their actual representations for my analysis; let us now go through their specifics.

2.1 PAYG Systems

Unfunded pension system is based on intergeneration solidarity; tax revenues from working agents are immediately used to pay pension transfers for older generations. No assets are generally set aside and all potential savings are voluntary only. Ideally, PAYG systems ought to be balanced as their expenditures should match revenues. In case of a social security budget surplus or deficit, taxes or retirement benefits (or both) can be adjusted, although taxes are rather rigid and the level of pensions significantly influence public opinion in reality. Therefore, virtually no pension system is balanced as it is easier for politicians to lead the economy through a (hopefully short) period of unfavourable economic conditions, rather than to change the parameters.

As noted earlier, PAYG systems are vulnerable to adverse demographics - changes in the old-age dependency ratio - and economic downturn, which leads to sluggish growth in nominal wages and lower growth in social security contributions. On the other hand, contributions are not invested further and unfunded systems are thus immune to volatility of financial markets returns.

Nowadays, the number of truly unfunded pension schemes has been decreasing as most of the PAYG schemes allow consensual savings into specific state-controlled institutions - pension funds - often with additional subsidy from the government. For the sake of simplicity, I consider any pension system without voluntary contributions into a funded scheme to be a pure PAYG system, which is also the case of the Czech pension system.

2.1.1 Pension System in the Czech Republic

Unlike many others, the Czech pension system did not go through a radical change following the transformation from central-planned economy and remains a PAYG scheme with no mandatory private savings, although there is possibility for inhabitants to make additional monthly contributions with limited support from the government. The system is universal and provides for all individuals working in the formal sector; the legal regulation is the same for all the insured persons and there are no industry-specific schemes or advantages for particular groups.

Pension transfers are calculated from past (taxed) earnings using replacement rates, which aim to reduce social disparities through curtailing pension transfers for high-income classes relatively to low-income agents. That is, a base amount of income is included as a whole in the retirement benefits calculation, and all income above it is included only partially. As of 2013, income up to CZK 11,389 was included fully, income between CZK 11,389 and CZK 30,026 was reduced to 27 percent, income between CZK 30,026 and CZK 103,536 is reduced to 19 percent, and any income above it was reduced to only 6 percent.

Monthly contributions are paid by employers and employees, and are currently set at 21.5 and 6.5 percent, respectively. Retirement benefits are usually annually adjusted to reflect changes in wages and prices, but since the decision to adjust pensions is not automatic and must be approved by the government, there is often a considerable time delay in the indexation. For instance, nominal pensions grew by approximately 6 percent in 2008 in spite of the global economic downturn, little growth in nominal wages, and negative growth in real GDP.

Neither the voluntary funded scheme nor the recently introduced funded opt-in scheme are included in my analysis because only a part of population utilizes them. Hence, they are not described any further. For a more detailed description of the Czech pension system, refer to Slavík (2006).

2.2 Funded Systems

Funded schemes are arrangements where workers' mandatory regular contributions are saved on individual accounts at pension funds or, rarely, used for purchase of non-financial assets, and subsequently serve as basis for pension transfers to the same people upon reaching retirement age. Savings are invested on financial and capital markets by specialized investors; the system thus resembles investment into mutual funds. Pension funds are essentially more regulated, though, as they bear a significant social risk. Although pension funds must, in principle, offer a variety of investment options with diverse risk-return combinations, the return on investment is mostly equal to conservative branches of mutual funds. Unlike PAYG, funded systems are therefore not protected against inconsistent returns on durable assets in general, but they are considerably more immune to political or legislative changes, especially if it is possible to choose a foreign pension fund or to diversify the idiosyncratic risks through investment into more funds at once.

An important difference between PAYG and funded pension systems is in contribution rate. In a PAYG system, social security contributions are subject to optimization in a pension-indebtedness space that should allow government to provide stable pension transfers at a reasonable level in comparison to previous income (i.e. to keep constant replacement rates) without causing extensive explicit or implicit indebtedness. In particular, high-income classes are expected to contribute more in relative terms or, at least, in absolute terms compared with low-income classes in order to maximize social welfare. In a funded system, on the other hand, a flat contribution rate is generally set for everyone regardless of their income; such set-up leads to greater differences in retirement benefits as it is based on actuarially fair principle.

Another contrast is in cut-off from investment. While occasional inability to contribute into the social system has a little effect on pension transfers in a PAYG scheme, it has severe impacts in funded settings, particularly if people cannot contribute in early life.

Funded schemes are mostly represented in hybrid pension systems that merge social equality of PAYG with encouragement of private savings of funded schemes (a multipillar system, see below). Nevertheless, some countries, particularly in Latin America, have been more radical in this perspective and employed virtually fully-funded pension systems in which workers save only for their own pensions. The pioneering country in this regard and also the one that has been most recognized for its pension reform is Chile, which introduced a major pension reform in 1981, transforming a previously pure PAYG scheme into a fully-funded one. Unfortunately, the approach has not proven to be desirable, as it provides very low replacement rates for people who do not remain in the labour market for long, and thus creates large social disparities. Besides, even though it is theoretically almost costless in the long-run under the assumptions of stable population and output, it is extremely expensive at first, often resulting in excessive external government indebtedness.

It is to be noted that a truly fully funded-scheme is but a myth as it is impossible to design a pension system without some form of a safety net to protect the poorest, who do not have a chance to create sufficient reserves for their retirement, and people primarily employed in informal sector (i.e. some form of illegal activities or activities not subject to tax and social deductions).

2.2.1 Pension System in Chile

As an answer to continuous population ageing and decrease in the old-age dependency ratio, Chilean government transformed the existing PAYG system, characterized by a large degree of heterogeneity in pension transfers and contribution rates dependent on prior earnings, into a fully-funded pension scheme that has later served as a basis for similar reforms in nearby countries such as Bolivia, Columbia, Costa Rica, Mexico, Panama, Peru, or Uruguay. Every affiliate working with labour contract is obliged to contribute to the system since the very first job, creating a personal account at a privately owned and managed pension fund that would invest the accumulated resources into various assets. In the current settings, workers make compulsorily monthly contributions of 10 percent of pre-tax earnings, whereas they contributed by 16-23 percent in the pre-reform pension system. The rate applies only to a certain amount of earnings; excessive earnings are not contributed to the second pillar and may be saved into the third, voluntary pillar, which works on a similar basis to the Czech one.

Besides the mandatory funded second tier, Chilean pension system has also an unfunded part (zero tier) aiming to alleviate poverty for the lowest-income inhabitants that could not accumulate satisfactory capital on their personal accounts. The zero tier was present in the original pension reform, yet it had to be substantially extended soon after because the original reform resulted in a widespread poverty. Although the system resembles the multipillar scheme described below, there is in fact an important difference in financing of the zero tier. In unfunded plans, any poverty alleviation mechanism is generally financed from social tax contributions, which implies reallocation of money in the pension system from wealthier classes to the poorest. On the contrary, the zero tier in the Chilean pension system is financed from the general government budget instead.

Nowadays, virtually every Chilean worker has a personal account at one of the selected pension funds, and the assets saved in pension funds have been growing constantly, amounting to approximately 90 percent of the Chilean GDP.[4] There are currently four options of account balance withdrawal: lifetime annuity, programmed withdrawal, temporary income with deferred lifetime annuity, and immediate annuity plus programmed withdrawals (Shelton 2012). The principle - receipt of pre-calculated monthly transfers adjusted for inflation - is equal in all of them, the difference is in their amount, legal claim on the remaining funds, and risk sharing. Account holder opting for a lifetime annuity transfers ownership of the assets to a life insurance company, which assumes both, financial and longevity risk. Choice of withdrawal scheme, on the other hand, may produce higher retirement benefits, but only at the cost of

[4]Chilean Pension Supervisor (Superintendencia de Pensiones, www.safp.cl).

facing the risks. That is, people may outlive the pre-defined payments period and receive lower pensions in case of negative asset returns.

Chilean pension funds' performance was outstanding at first, particularly in the early years after the initial reform, as the average real rate of return was approximately 11.3 percent per year. The recent results have been considerably worse; depending on the investment portfolio, pension funds offered up to 30 percent negative annual return in 2008. Importantly, pension funds' performance has an immediate impact on individual account balances. The average account balance of a retiring worker dropped by over 40 percent between 2007 and 2009 and has just partially recovered since then.[5] And since the individual account balance and life expectancy are the only parameters in calculation of retirement benefits, those have dropped accordingly.

Each worker is allowed to choose pension fund freely and, since 2002, each pension fund must provide at least four fund options differing in combinations of risk and return depending on chosen assets invested into. People may transfer their savings up to twice a year from one fund to another without any administrative costs, yet pension funds charge additional ca 2 percent of monthly contributions to cover their administrative costs and survivor risk.

For additional information about the Chilean pension scheme refer to Arenas de Mesa et al. (2006) or Rodríguez (2012).

2.3 World Bank's Three Pillar System

The World Bank's conceptual framework is presented in Holzmann et al. (2005), who build upon the principles established in Palacios (1996), and focuses on refining system design in a way that it adapts the idea of three pillars to varying socio-economic conditions to manage risks in old age. Even though there are is no universal solution to the complex array of retirement issues, the model provides a general framework that shall be adjusted according to core objectives and conditions of particular economies. First, I briefly summarize the tenets suggested by the World Bank and then I describe in detail their implementation in the Swedish pension system, which is also simulated in my model.

What is commonly called a three pillar system is in fact a five pillar framework, where the first and the fifth pillar blend with the others. The pillars are defined as following:

[5] Chilean Pension Supervisor (Superintendencia de Pensiones, www.safp.cl).

- **Non-contributory zero pillar** - Being typically financed by the government, it serves as the very basic assistance (a safety net) for the poorest or those who participate only marginally in the formal economy.

- **First pillar** - A PAYG scheme with mandatory contributions linked to prior earnings. Contributions are, relative to salary, mostly linear with a certain upper limit. In presence of a funded second pillar, the first tier primarily addresses low-income classes that did not have sufficient resources to save for retirement, but also the risk of individual myopia (adverse time preference) or financial market breakdown. It is a defined-benefit system; pension transfers can be either flat in absolute terms, flat relative to lifetime salary, or progressive. Due to its pay-as-you-go financing, the first pillar is mainly subject to demographic and political risks.

- **Second pillar** - Usually a defined-contribution funded system of individual accounts, administered by the state or private pension funds. Funds collect mandatory contributions above those into the first tier and invest them into wide range of assets using active or passive investment management. Depending on specific implementation and state regulation, pension funds may have strictly defined investment options, including list of allowed entities, and pensioners can have specific options for the withdrawal phase.

- **Third pillar** - A system of voluntary contributions into usually private pension funds with similar characteristics and risks as the second-pillar funds. Contributions into the third pillar are often tax-deductible or employer sponsored.

- **Non-financial fourth pillar** - The fourth pillar provides other formal social programs (such as health care and/or housing), includes family support or, for instance, subsidized mortgages.

Each tier targets a specific part of population and old-age risks. While the zero, third, and fourth pillar are aimed at both the formal and informal sector, the first and second pillars are only available to people employed in jobs with normal hours and regular wages that are recognized as income sources on which income tax must be paid.

2.3.1 Pension System in Sweden

The Swedish pension scheme is a social-democratic system due to its high level of pension transfers and non-discriminatory character. Pensions are paid to everyone according to a specific scheme, except for the guarantee pension (see below) and housing allowances being available only for people older than 65 years. It is primarily an unfunded scheme, yet with gradually increasing share of the funded part - the premium pension. Each worker or self-employed is required to contribute 18.5 percent of his pre-tax salary to the social system. Currently, 16 percent go into the PAYG component (a defined-contribution plan - the income pension) to secure pensions for contemporary senior citizens and the remaining 2.5 percent are stored on individual accounts. The PAYG pension is based on lifetime contributions in a degressive manner, similar to the Czech scheme. There is a ceiling on income qualifying for pension credits, all income above the threshold is free of social security contributions, but it is taxed by additional 9 percent.

Besides the standard techniques, the Swedish pension system has a balancing mechanism that adjusts the PAYG component in times of economic downturn so as not to run into debt. A part of contributions is put aside and constitute a buffer fund utilized when pension liabilities exceed contribution assets, together with an automatic adjustment of retirement benefits so that the initial balance is soon restored. Under normal conditions, the PAYG pension payments are guaranteed to grow equal to per-capita real wage growth, creating a link between pension rights and working population. In a case of low social security tax revenues, indexation is reduced for as long as necessary. In the long-term, ignoring economic fluctuations, income index is equal to pension index.

Savings on individual accounts are invested in a standard way: people can choose among more than 800 domestic and foreign funds and frequently switch among them without additional costs. Upon retirement, the total account balance is transformed into annuity using a standardized procedure - the amount is divided by an annuity divisor, which is regularly updated to reflect changes in life expectancy. Finally, the Swedish scheme provides a substantial support for low-income classes through the guarantee pension (zero tier, financed from the general government budget). Its detailed specification is described in Chapter 4.

The bottom line is that the Swedish pension system is a classic example of the three pillar pension scheme proposed by the World Bank, where the PAYG part constitutes the first tier, the second tier is funded with mandatory participation, and the third tier comprises of voluntary contributions beyond those to the second pillar. Sweden is also a good example of developed country with ongoing adverse population changes. For additional information about the Swedish pension system and its unique distribution of contributions among four equal pension funds, refer to Sunden (2004) or Kruse (2010).

To conclude this section, let us briefly discuss some of the other recent reform efforts in Europe. While most of the European countries have adopted some form of parametric changes to the pre-existing pension systems, several countries (including Sweden) have gone further and opted for a more radical reforms to enhance the role of private pension schemes, particularly as a result of transformation in 1990s. Unfortunately, not all of them were successful; Hungary, which first reformed its pension system in 1998, effectively nationalized all private owned funds in 2010 to finance its pension budget. Similarly Poland, which approved a thorough pension reform as a part of transformation in 1999, confiscated half of the private pension funds to cut sovereign debt load in 2013. While the initial objective and strategy was not completely wrong in either of the countries, the second pillar was made too attractive for people - they were offered a substantial tax relief and state contribution to monthly savings without a detailed analysis of the impact on pension budget.

Needless to say, the aforementioned pension reforms in Sweden and Chile were not costless either. In fact, in addition to the direct effect on pension budget, there are also many other implicit costs assumed by individual inhabitants. Those may be connected to changes in replacement rates, necessity to make active investment decisions, and so on. It is not a purpose of this work to discuss them further in detail and I would like to ask the reader to refer to a more specific literature on the subject (e.g. Palmer 2000 for information about the Swedish pension reform and Edwards 1998 or Berstein et al. 2006 for the Chilean pension reform).

2.4 Demographic Changes

Demographic changes have always been a natural and integral part of human evolution. Magnitude of the changes has, however, grown exponentially in the recent decades, particularly as a result of rapid innovation in medicine technology, globalization, and hospitable environment. No matter that longevity does certainly have numerous plausible effects on economic development, it also puts future generations at a considerable risk. There has been an strong change in representation of age groups within population and the trend is expected to press on. European Commission (2012) published a thorough and up-to-date summary of the issues. According to the report, there are two major factors driving the population change: longevity and declining fertility.

Since 1960, average life length of an European citizen has increased by five years and it is expected to increase by another five years until 2050. Besides Europe, most of the developed countries exhibit a similar trend. For instance, life expectancy in Japan has increased from 65 to 83 years since 1960. Indeed, several states, such as China, already face sizeable difficulties regarding its labour force and providing for pensioners. Moreover, to make things even more complicated, a simple growth in fertility would hardly provide a solution because there is a large time gap between a child birth and its involvement in work process, and increasing population would put additional burden on the future generations.

Table 2.1: Demographic projections for the Czech Republic.

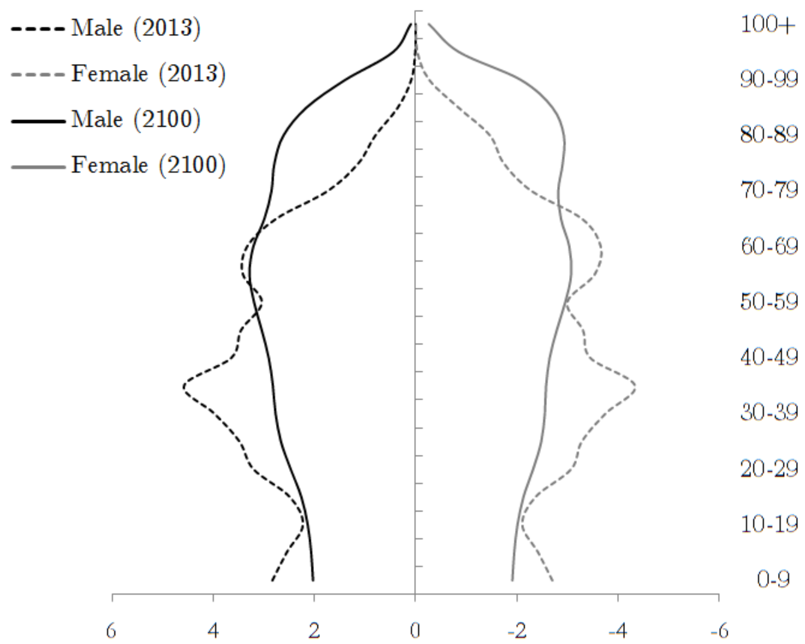
		Projection	
Year	Low	Moderate	High
2012	10 505 445	10 505 445	10 505 445
2020	10 439 628	10 532 373	10 623 359
2030	10 107 178	10 396 701	10 658 023
2040	9 618 111	10 126 418	10 580 711
2050	9 086 977	9 812 872	10 457 236
2060	8 446 121	9 388 273	10 219 438
2070	7 709 787	8 836 298	9 833 490
2080	7 071 254	8 348 483	9 478 432
2090	6 571 867	7 994 692	9 250 556
2100	6 138 552	7 712 096	9 099 041
2101	6 095 234	7 683 652	9 083 414

Source: Czech Statistical Office (www.czso.cz).

Declining birth rate is a long-term trend, too, which has commenced in 1960s. After the World War II, an average woman had nearly three children. Since 1990s, the decline has diminished, but the average birth rate in EU 27 fluctuates around just 1.5 children per woman, which is not sufficient to even keep a stable population size.[6] Although, in spite of low fertility, some developed countries have all but problems with declining population thanks to ceaseless inflow of immigrants. For the purposes of this work, the effects of immigration are not taken into consideration as it is a highly complicated topic to grasp. For an example of population shrinking in the Czech Republic, see Table 2.4.

Putting the opposite trends together, population in the EU 27 countries between age of 15 and 64 is expected to decrease by ca 48 million, while population older than 65 years shall increase by 58 million until 2050. To illustrate the change, I the present estimates from the Czech Republic, which can well serve as a classic example of the issue. Figure 2.1, exhibits the age structure in the Czech Republic in 2013 and its projection for 2100.

Figure 2.1: Czech Population Age Structure in 2013 and 2100.



Source: Czech Statistical Office.

The figure clearly exhibits the immense change in population structure: the straight-line population pyramid is much narrower at the very bottom, signalling a proportional decrease in younger age cohorts in comparison to older generations. It is also higher, which points at higher life expectancy. Most of

[6]The lowest required birth rate to prevent population decrease is assumed to be approximately 2.1 (European Commission 2012)

all, it is nearly straight up to age of 90, whereas the dashed-line pyramid shows higher proportion of working-age population to pensioners.

A similar trend is visible in virtually all developed countries, although it varies in magnitude. In Japan, which is often referred to as an example of population changes due to its fastest ageing population in the world (Miles and Cerny 2002), the old-age dependency ratio is projected to increase by 190 percent until 2050, in Italy by 170 percent, in the France, Canada, and the USA by 130 percent, and in Sweden or United Kingdom by 60 percent.[7] Generally, the old-age dependency ratio is projected to change from current 24 percent to 36 percent by 2030 in developed countries and from 9 to 15 percent in developing countries (Muto et al. 2012). These predictions are without a doubt highly uncertain and the real situation may differ, but the main message remains unchanged.

Regarding social security, population ageing would not be an issue if the effective working life prolonged accordingly so that the ratio of workers to pensioners remained unchanged; but such decision is subject to political negotiations and defiance from the public, which rule out the possibility of appropriate shifts in the retirement age. Moreover, a simple increase in retirement age would only help if the number of jobs increased as well. Otherwise, workers would still have the same amount of people to feed, just the distribution between pensioners and unemployed would change.

[7] Statistical offices in the respective countries.

Chapter 3

Literature Review

The unsustainability of unfunded pension schemes in the light of demographic changes has been widely recognized. Blake and Mayhew (2006) suggest that most of the European social security systems will not be able to secure pension transfers at their current level without an excessive burden being placed on each subsequent future generation, unless substantial changes to the systems are made. They also point out that the upcoming situation will be in a bright contrast with surplus balances in the past decades stemming from exploitation of demographic dividend - a combination of population ageing and declining fertility that lasts until the smaller newborn generations enter the labour market and the baby boom generations retire - which promoted the extensive aggregate economic growth. A thorough discussion of the prospective problems of PAYG systems, with particular focus on the Czech Republic, can be also found in Marek (2007).

The often suggested remedy is a partial or a substantial pension system reform, i.e. transformation of PAYG scheme to either of the other two systems presented in Chapter 2. While funded systems have great many supporters, Nicholas Barr, a pioneer in the field of pension systems, have long ago pointed out that funded schemes are not less vulnerable to demographic pressures than PAYG schemes (Barr 1979). In his newer works (Barr 2002; 2006; Barr and Diamond 2009), he explicitly cautions against improper use of first-best analysis assuming a world with perfect information and no distortions. Further, he argues that problem of longevity is in fact a triumph that is to be utilized rather than feared; he thinks that people simply retire too soon and that pensionable age should rise fairly steadily as life expectancy increases in order to keep the old-age dependency ratio constant.

Barr points out that social security systems face a number of serious deviations from the perfect world assumed in the theoretical concepts. Individuals are assumed to maximize their lifetime utility through selection of only optimal and competitively-priced saving and borrowing instruments that, on the aggregate level, maximize social welfare. Yet, market imperfections imply that economic agents do not choose optimal solutions as a result of incomplete rationality. In particular, even if we neglect personal disparities and imperfect information, the actual markets are unable to provide competitively-priced products and fit everyone's needs. Barr also emphasizes that the contemporary analyses ignore distributional effects; any change in pension system necessarily redistributes wealth across cohorts with different birth years, as noted in the previous chapter. Introduction of a PAYG system provides higher pension transfers for early generations, but the subsequent generations are worse-off. Analogously, introduction of a funded system favours later generations at the expense of current workers. Besides, as Barr argues, shifts in contribution rates within already existing PAYG system or similar parametric changes in funded schemes have the same effects.

The actual profitability of funded pension schemes in comparison to a PAYG schemes, or vice versa, has so far not been unambiguously proven - neither theoretically nor empirically. Some authors suggest that transition to a funded system may generate efficiency gains (e.g. Homburg 1997 or Kotlikoff et al. 1999 - see below), while others (e.g. Sinn 2001, or Brunner 1996) find it impossible. Sinn (2000) shows that funded pensions cannot offer better rate of return than PAYG system, an argument often put forward by advocates of funded schemes; even though there is no doubt that PAYG system offers lower rate of return than the capital market, it is not satisfactory to make clear conclusions. Specifically, PAYG system redistributes wealth from later generations to earlier, which results in a burden put on later generations that disables them from enjoying true efficiency gains in case of a fundamental pension reform. He argues that any pension system is in fact a zero-sum game for all participating cohorts, meaning that the present value of all contributions equals the present value of all pensions.

At the same time, though, Sinn (2000) supports a partial transition towards a multipillar system to combat the ongoing population changes. In particular, low fertility implies that people simply cannot expect their children to fully cover the prospective pension expenses, which calls for another source of funding. However, transition towards a funded system in fact only diversifies risks among future generations and capital markets, while the future generations will have hard times anyway. Essentially, they will either bear a double burden of saving for their own pensions and providing support to contemporary pensioners (in a funded system) or contributing for pension transfers significantly more as a result of change in the old-age dependency ratio (in a PAYG system).

Despite miscellaneous results regarding effectiveness of pension schemes shown in the literature, all works come to the same conclusion: the projected population changes will have a strong negative impact on social security systems, no matter what action will be taken. Besides highly improbable unexpected events, such as discovery of enormous natural resources, the only relevant factor that may alleviate or negate the effects of ageing is technological progress. Assuming the widely accepted neoclassical production function in the form

$$Y_t = A_t K_t^{\alpha} L_t^{1-\alpha}, \tag{3.1}$$

where Y_t represents total production in the economy, K_t and L_t denote aggregate level of capital and labour, respectively, t denotes time period, and A_t is the labour-augmenting technology, it is clear that a drop in L as a result of population changes may be directly offset by a proportional change in the level of technology. However, the effect of technology growth on public pensions has not been thoroughly analysed yet.

3.1 Market Imperfections

Recall, that there are various options how to transfer funds accumulated on individual accounts upon retirement. These range from a simple transfer between bank accounts to creation of lifetime annuity through specialized insurance companies and depend principally on the pension system design. For fully-funded schemes, compulsory annuity creation and strict regulation of insurance companies are a must, while in hybrid multipillar systems, people may have more freedom of choice in this respect as the unfunded pillar provides the necessary security.

In models with definite death age, annuity market is the second-best solution if economic agents are rational; they can easily plan consumption according to their utility functions and spend savings accordingly. In a world with uncertainty, retirement annuity represents a safe and easy way of insurance against longevity. Either way, healthy and fair annuity market is one of many fundamental prerequisites for a desirable funded pension scheme. Note, that people in PAYG schemes also receive a form of retirement annuity as well, yet it is the only way of organizing payment of pension benefits rather than a form of insurance as in funded pension systems.

Closely related to annuity markets - and often neglected - market imperfection affecting funded pension schemes is existence of incidental expenses related to managing of individual accounts, annuity creation, and various other administrative costs. James et al. (2001) investigate the cost-effectiveness of two alternative methods for organizing mandatory individual accounts: investment through retail market and investment through institutional market. In the former case, people do not have any bargaining power and must accept conditions set by the funds, while in the latter case, entry and price conditions are negotiated for a larger group of labour force. They find that retail market pools funds, such as those in Chile or the USA, have significantly higher administrative costs as their annual fees range from 0.8-1.5 percent of assets, while it is only about 0.3-0.6 percent at the institutional market (e.g. Sweden). This is due to higher marketing expenses as retail funds need to attract and aggregate small investments into large pools. James et al. (2001) show that the administrative costs have been decreasing over the last decades, but are still considerable. For example, in the case of Chile, expenses per unit of assets have dropped from 5.65 percent in 1983 to 1.13 in 1998, and have remained rather constant since then. There are, of course, disadvantages linked to institutional investing: especially higher risk of corruption, collusion, and lower incentive for outstanding performance. However, since a 1 percent annual fee reduces retirement savings by approximately 20 percent for a lifetime contributor, the cumulative difference is striking.

Another interesting research in this regard is done by Murthi et al. (2001), who argue that administrative costs associated with individual accounts can lower the value of an account by up to 40 percent. Based on data from the United Kingdom they differentiate between three major impacts: the accumulation ratio, the alternation ratio, and the annuitization ratio. The accumulation ratio refers to management and administrative costs used by funds to operate and make profit, while the alternation ratio measures costs of failing to contribute consistently throughout one's life, and the annuitization ratio reflects the costs of converting savings into a lifetime annuity upon retirement. According to the study, these can decrease value of the savings by 25, 15, and 10 percent, respectively.

It is to be noted that the UK pension system consists of privately managed, decentralized accounts and annuities, which have generally, as suggested by James et al. (2001), higher administrative fees. The fees do not need to be explicit only; similar to standard insurance, insuring company in a perfectly competitive market is supposed to set the annuity such that, according to the law of large numbers, it would have exactly zero gains or losses in the long run. Since virtually all markets are imperfect, insurance companies need to take especially adverse selection and transaction costs into account, resulting in lower annuity payments. That is, individuals who wish to purchase annuity must pay a premium relative to a non-charge and actuarially fair level, i.e. to pay implicit administrative costs of annuity creation. In particular, if annuity purchase is not mandatory, people who are likely to live longer than the general population tend to purchase annuities more often, resulting in skewed distribution of life expectancy. Similarly, the alternation ratio is higher for individuals who fail to consistently contribute to the individual accounts in youth due to interest in time.

3.2 Pension System Simulations

There are numerous works analysing a two-period overlapping generations model (e.g. Annicchiarico and Giammarioli 2004 or Michel et al. 2006) to investigate dynamics of pension systems in real-like economies in a simple manner. OLG models are generally the most appropriate tools of pension system exploration due to possibility to distinguish households' behaviour in respective life periods. Specifically, people work and pay income taxes in the first period and receive pension transfers in the second period. Broadly speaking, inclusion of additional periods in the model increases its explanatory power as it allows for deeper behaviour diversification, yet even a two-period model has several convenient advantages as it is very comprehensible, clear, low demanding with respect to computation, and can replicate most of the essential processes in a pension system.

3.2.1 PAYG vs Funded Systems

Drawing on their earlier work, Kotlikoff et al. (1999) investigate alternative ways to privatize the U.S. Social Security System using a modified OLG model. They find that transition towards funded system may result in long-run benefits and raise living standards; in particular, capital stock could be increased by up to 37 percent and output *per capita* by 11 percent. In the short-run, however, people are worse-off unless they are offered a chance to remain in the original unfunded scheme. In that case, the total transition costs are much lower, but at the cost of low pensions for the first generations to have funded pensions only - they must pay higher social security taxes to finance the remaining unfunded pensions and thus operate with lower disposable income.

Corsetti and Schmidt-Hebbel (1995) review some of the first simulation literature focused on macroeconomic and welfare implications of switching from PAYG to a fully-funded scheme, and conclude that the transition would lead to efficiency improvements, reduce labour market distortions, and be beneficial overall. In particular, they argue that introduction of a funded pension scheme would strengthen the position of pension funds, provide incentives to liberalize financial markets, and promote development of long-term investment instruments. Further, recall, that the implicit link between worker contributions and benefits is rather weak in the PAYG schemes - the actual retirement benefits depend primarily on replacement rates and indexation set by government - while it is strong and clear in funded schemes. Labour market is thus likely improve through resource relocation from informal (untaxed and unregulated) to formal (taxed and regulated) sector as a result of better incentives in factor and product markets.

Using an OLG model, Pecchenino and Pollard (1995) investigate the effects of annuity markets on the economy. They find that government sponsored, actuarially fair annuity market indeed leads to improvements in social welfare, yet the result does not have to be dynamically optimal in case of unintentional bequests that generate excess savings. Although these and other earlier works[8] using OLG models provide several important insights, the simulations are based on simplistic models and the strong assumptions of no uncertainty, one-sector economy, a single income class, or no demographic changes. Let us now discuss newer and more complex models that are partially improved in this regard.

Nishiyama and Smetters (2007) agree with Kotlikoff et al. (1999) that privatization of PAYG social security system, under the assumption of deterministic economic variables and inelastic labour supply, leads to reallocation of resources

[8]See e.g. Annicchiarico and Giammarioli (2004), who analyse linkages between fiscal variables and the rate of growth dynamics in economy with PAYG scheme, or Hviding and Mérette (1998), who investigate macroeconomic effects of various pension reform strategies, such as gradual abolition of the public pension system, a cut in replacement rate, and an increase in the effective retirement age.

between generations. On the other hand, assuming elastic labour supply, privatization can substantially improve labour supply incentives and lead to increase in net resources produced for future households. The analysis also shows that inclusion of idiosyncratic wage shocks and uncertainty in life expectation leads to decrease in overall efficiency of the economy because PAYG system provides a convenient source of risk sharing through its progressive benefit formula. The calculations used in the study are based on heterogeneous overlapping-generations model that assumes an economy in steady state running a PAYG social system in the first period and a sudden transition towards funded pension scheme in the second period, which is also a methodology I employ in my analysis.

The inability to grasp all mutual relationships between variables of interest is demonstrated in Buyse et al. (2013). As they argue, certain government policies may make people to postpone their retirement and work longer, which would result in increase of return on investment, education, and also in higher economic growth. In this regard, the assumption of endogenous human capital, growth, and labour supply is appropriate. In their four-period OLG model with PAYG pension system, labour supply represented by hours worked by young, middle aged, and older individuals, as well as education and retirement decisions are all endogenous. That is, the statutory retirement age in the model is exogenously given but individuals may choose to retire earlier at the cost of lower pension transfers received. They conclude, based on investigation of the effects of various parametric adjustments to the PAYG system, that a well designed PAYG scheme provides better results than a fully-funded system.

In particular, a tight link between wage and future pension and emphasis on salary at older age to determine the pension level seem to have a distinctive positive effect on employment and aggregate welfare. In such settings, young people are encouraged to pursue higher education and thus increase human capital in the economy, resulting in higher long-term growth. Analogously, older generations have stronger incentives to remain in work despite being beyond retirement age. It is to be noted, however, that such system yields better aggregate results through improving welfare for majority of population while it implies welfare losses for the current low-ability (regarding learning, job performance, etc.) workers, which leads to increase in welfare inequality as in the case of funded system without a safety net for the poorest.

Based on the previous findings, Oksanen (2009) develops a computable OLG model with uniform labour supply across generations of workers, exogenous interest rate, wage, labour productivity, and realistic changes in population. His model's main asset is simplicity of computation facilitated by explicit determination of variables, which allows easier focus on transition period between the two steady states. The simplicity is, however, a substantive downside at the same time, because the model does not allow for investigation of the effects of certain parametric changes (e.g. shift in interest rate) on the economy.

Specifically, Oksanen assumes that people always adjust their current consumption to the general income level determined by labour productivity; there is no intertemporal substitution - interest rate does not have impact on the time profile of consumption - and the interest rate itself is given exogenously rather than based on the marginal productivity of capital. Essentially, a change towards reflection of such connections, however desirable, would in fact not result in a major difference in the outcome, especially not in the basic conclusions regarding savings behaviour within the economy. The aim should always be on a balance between performance and its costs, i.e. relevance of the output and its reliability. In my study, I loosely pick up threads of Oksanen's work, especially the idea of focusing on realistic population settings, and enrich the OLG model by simulating additional micro-relationships between variables as well as various pension systems.

3.2.2 Simulation Issues

To be sure, any simulation of OLG model - no matter how precisely formulated - is a mere approximation of the real world situation. Additionally, even similar OLG models may lead to contradictory conclusions due to small parametric changes. The fundamental question to ask before designing a new model is then whether the purpose is to produce a reliable projection of future state of the world or to observe impacts of changes in identified exogenous factors on the economy. Neither of the options fits its purpose flawlessly; real-world projections are only reliable for reasonably short period of time under very specific assumptions, while investigation of impulse-response behaviour may often be very complex and depend on various (possibly unobserved) relationships.

Regarding pension systems, getting precise estimates is virtually impossible because all projections of future states of the economy are subject to numerous random variables and projections of these variables. It is therefore more reasonable to focus on the latter intent - to investigate simulation outcomes under various circumstances. In my model, the observed economy does not change (i.e. there is always the same number of workers and pensioners in each scenario, offering the same amount of labour, having the same utility function etc.) and each simulated scenario differs in government decisions regarding social security system and exogenous growth rate of technology.

Following Auerbach and Kotlikoff (1987) and Heer and Maussner (2009), I assume the economy to be in a steady state initially. It is then actually irrelevant whether the economy is fictitious or based on actual real-world data as it takes – at least – an average human lifetime for an OLG model to reach steady state equilibrium and many more decades to observe decision-making processes of the individuals. In the real world, even much shorter time spans have always contained numerous unpredictable events with a decisive impact on observed economy that spoil any reliable conclusions. It is thus more appropriate to use a hypothetical equilibrium variables that deviate from the observed data or, in fact, use any reasonable values.

Of course, the statement is true only for aggregate variables specifying the initial state of the economy, not function variables that impact these variables. In other words, it is irrelevant what is for example the initial number of people living in the assumed economy (as long as they fit certain characteristics regarding age, productivity, workers to pensioners ratio, etc.) but one must properly specify population dynamics in order to obtain relevant results.

Oksanen (2009) agrees that overlapping-generations model is the most accurate and convenient theoretical framework to assess pension systems' performance, but he is fairly critical about its empirical testing. He argues that not only there are no data on economies in steady state, but the disturbing events may also result in multicollinearity in observed variables, which can be readily traced but it is tough to cope with. The bottom line is that there are not enough observations to permit sophisticated and particularly valid statistical testing of the OLG hypotheses. However, while aggregate data are unavailable, it seems possible to run selective tests on data from individual economic agents. Some works (e.g. Attanasio et al. 1999) found that people may indeed follow a hump shape consumption profile suggested by OLG model (see Chapter 5), but the model needs to be specifically adjusted for time-varying demographic factors, such as retirement, family size, or death age.

3.3 Pension Myths

Pension systems are shrouded by certain myths, some of which have recently been uncovered and theoretically or empirically proven wrong, yet they play an important role in common decision-making, be it on individual or state level. Techniques employed in my analysis allow me to address some of these myths and to verify them. Below I specify some of the myths as described by Barr (1979) and Barr (2002). The approach to their testing and results and discussed in the next chapters, together with methodology and other simulation outcomes.

3.3.1 Funding Resolves Adverse Demographics

Barr (2002) notes that the funded pillars are now considered a desirable part of social system as they seemingly decrease negative effects of demographic shocks on the social system. The truth is, though, that a funded system does not provide any better protection against longevity and population ageing than a PAYG scheme. The only difference is in the transmission mechanism, which may be rather unclear in case of a funded system. Consider the general equation liking collected contributions to paid pension transfers in a PAYG system:

$$t \cdot W \cdot L = P \cdot R, \tag{3.2}$$

where t denotes contribution rate, W is average monthly wage, L the number of workers, P the average nominal pension, and R the number of pensioners. Clearly, a decrease in workforce must be proportionally counterbalanced by either decrease in pensions or pensioners, otherwise the whole system gets into red numbers.

On the contrary, in a funded scheme, everyone saves for his own prospective retirement and amount of pension transfers is, at first sight, wholly independent on the size of future workforce. The opposite is true as an imbalance between workers and pensioners (or better, between economically active and inactive people) creates pressure on the market equilibrium, leading to decrease in real pensions. The actual mechanism is discussed also in Brooks (2000): disproportions in supply and demand for durable assets as a result of large cohort of pensioners with assets to sell and smaller cohort of young workers wishing to save for retirement are assumed to significantly decrease asset prices in the long-run.

Importantly, a decrease in workforce can alternatively be balanced by a raise in nominal wage, W, caused by growing output. For PAYG scheme, such alternative ultimately leads to fall in the replacement rate (earnings to pension ratio) but pensioners do get the real promised pension. Growing output serves as a solution funded scheme as well. Essentially, there is no inflation in the simplistic settings because aggregate supply of goods increases, which generally implies higher wages. Hence, assets market clears without a decrease in asset prices - workers have excessive income to spend on asset purchases that meets supply created by larger cohort of pensioners.

3.3.2 Funded Schemes Facilitate Savings

As suggested by Feldstein (1997) and Orszag and Stiglitz (2001), transition from a PAYG social system to individual account based funded system would consequently lead to a substantive increase of investments into enterprises and their facilities, and eventually to increase in output, real wages, and social welfare. Such relationship has insofar not been empirically proven though, and even from a theoretical point of view social system privatization merely relocates savings rather than changing its absolute value (Barr 2002). To see that, assume that government would transfer all lifetime social contributions in a PAYG system to people's individual accounts, increasing individual savings. At the same time, public savings decrease in the same amount and hence aggregate savings do not change at all.

Nevertheless, as I argue in Chapter 5, this theoretical argument is incomplete as well. Essentially, while funded systems lead to higher pensions for wealthier income groups, low-income groups often receive retirement benefits below a certain satisfactory level and must save additional funds to maintain reasonable living standards in retirement.

Chapter 4

The Model

The overlapping generations (OLG) model was first introduced by Samuelson (1958) and Diamond (1965). Since then, OLG models have been greatly enhanced and constitute a major theoretical framework for pension system analysis. While the early works focused on simple two-period OLG models, the invention of computers has allowed analytical investigation of much more complex and sophisticated models; virtually any set of equations may be accompanied by a numerical output nowadays. Still, the computation has its price as it can take several hours or even days for a standard personal computer to reach a solution.

OLG models assume economic agents to have rational expectations about economic development and possible idiosyncratic shocks. Specifically, consumption and savings decisions are not determined by simple contemporary variables, such as the old-age dependency ratio or life expectancy, but rather by expected future wage, time spent working, and time in retirement. Even in case of a sudden change in the economy, individuals are expected to identify changes fast enough to adjust their decisions within the same time period. Importantly, rational expectations facilitate perfect foresight and embody the life-cycle hypothesis suggesting that economic agents aim to smooth their consumption throughout the life. While the assumption is often criticized for its unrealistic nature, several recent studies (e.g. Modigliani and Cao 2004 or Bloom et al. 2003) supported the hypothesis by empirical evidence. Hence, as suggested by Oksanen (2009), OLG model should not be reprobated *ex ante*, one must be only cautious when making conclusions from its outcome.

The pioneering large-scale numerical OLG model was developed by Auerbach and Kotlikoff (1987), who constructed a model with 55 generations, no uncertainty, exogenous labour supply, and perfect foresight. The basic framework of the model used in this study is inspired by Heer and Maussner (2009) and further developed to match my objectives. I assume a simple two-sector economy consisting of households and firms, with government running pension system only. Since there are no banks, firms are wholly owned by households.

4.1 Households

Households in the model are assumed not to live infinitely but rather die at a certain age to be replaced by a newborn generation. The settings may closely resemble the actual real-world age distribution and allow for diverse behaviour at different age throughout one's life - particularly regarding household asset holdings - and thus offer a better possibility for research of pension systems than other frameworks.

4.1.1 Survival

Every year, the remainder of the oldest cohort dies and a new generation of equal measure (normalized to one) is born. Households live for maximum of 60 years and children are excluded; people are born at real age of 20 and thus survive up to 80 years of real age. I assume uncertain death in my work - agents face possibility of death each year from period 35 onwards. At age 60, all the remaining population dies with certainty.[9]

The survival probabilities are taken from Krueger and Ludwig (2007) and are presented in Figure B.1 in the Appendix. Note, that the vector of survival probabilities describes the chance of living another period (conditional on surviving up to that period), the cumulative probability of death at age t is then product of all probabilities up to time t. Essentially, population changes can be partially simulated by alternation of survival probabilities; for additional information, refer to section 4.6 below.

[9]There are three notes to be made: First, simple models usually assume less than ten periods per household's life (each of them spanning over several years or even decades) to decrease computation time. Such approach would substantially diminish possibility to investigate the microeconomic behaviour, so I decided to include 60 periods. Second, only adults (i.e. workmen or pensioners) are principally appropriate for modelling as they earn money and spend it or save it for future consumption; hence, children are not assumed in the model. Finally, a model with wholly uncertain time of death (i.e. no maximum age) would be more realistic, yet it poses certain difficulties for implementation and also increases computation demands, so I do not work with that assumption.

4.1.2 Utility Maximization

The goal of all households is to maximize their lifetime utility function given by

$$E_1 = [\sum_{t=1}^{T+TR} \beta^{t-1} (\prod_{j=1}^{t} SP_j)\, u(c_t)], \tag{4.1}$$

where SP_j denotes the probability of survival another year at age j, β is a discount factor representing agent's time preference, and c_t is consumption at time t. Finally, $u(c_t)$ represents the instantaneous utility function with constant relative-risk aversion, defined as

$$u(c_t) = \frac{c^{1-\eta} - 1}{1 - \eta}, \tag{4.2}$$

with η being the coefficient of relative risk aversion.

Note, that while the model follows the standard microeconomic theory assuming that households prefer immediate (or earlier, for that matter) consumption rather than later, β need not to be smaller than one as in infinite-lifetime models so that lifetime utility is finite. Following Heer and Maussner (2009), I set $\beta = 1.011$.

4.1.3 Labour Supply and Income Inequality

For the first $T = 40$ years, agents supply inelastic amount of labour l. Albeit the labour supply is inelastic, agents are heterogeneous in income through individual labour efficiency depending on their age and idiosyncratic productivity shocks. Labour efficiency increases and decreases throughout one's life and simulates course of changes in earnings as a result of employment changes, experience gains, productivity loss at higher age etc.[10] As the model assumes generations to be of sufficient size (i.e. whole states), individual deviations from the basic scheme do not matter and the overall income allocation is normally distributed. The age-productivity profile is taken from Hanse (1993) and is in accordance with the log-normal distribution of US wage profile.

Agents are forced to stop working at age T and spend the rest of their lives (up to $TR = 20$ periods) in retirement, living of saved up capital (bond

[10]For instance, imagine a fresh graduate looking for a job; at first, he would most likely be a low-salary assistant, but as he grows older he ascends the social ladder to eventually reach his prime position. Subsequently, younger people would eventually take his place and his wage would decline again.

holdings, i.e. investment into firms) and pension transfers. Although agents cannot have perfect foresight due to uncertain death age, I assume that they leave no bequests to children - retirement benefits cannot be passed to relatives and the government confiscates all other accidental bequests.[11]

Throughout their life, agents receive income based on the age-productivity profile and personal labour-productivity shocks (see below). The overall labour-endowment process is given by $e(z,t) = e^{z_t + \overline{y}_t}$, where $\overline{y}_t$ is the mean log-normal income of respective aged workmen. Total annual salary is thus

$$Salary = (1-\tau)\, e(z,t)\, wn, \tag{4.3}$$

where τ and w are taxes and default wage, respectively, and n is the exogenously given labour supply.

Note, that neither wage nor labour supply need to have a subscript as they are uniform for all households. In other words, the model implicitly assumes that every household is employed - it supplies a given amount of labour - and obtains undifferentiated basic salary, which allows for easy change in average wages (analogy to growth in nominal wages), while income and labour differentiation is done through the labour-endowment process only.

The labour-endowment process essentially depend on labour efficiency discussed earlier and idiosyncratic productivity shocks. These are an important tool providing the model with essential characteristics of intra-generation wage distribution and include probability of unemployment, illness, or inability to work, and, most importantly, enrich the model by introduction of various income levels depending on employment position. The shocks are assumed to follow autoregressive process of order one (AR(1) process) given by

$$z_t = \rho z_{t-1} + \epsilon_t, \tag{4.4}$$

where $\epsilon_t \sim N(0, \sigma_\epsilon)$, and thus depend on past realizations.[12]

[11] Again, this is an innocent simplification without any significant impact on the results, yet it is necessary for a reasonable computational time. Indeed, most of other possible adjustments are fairly easy to model and implement, yet every stochastic variable multiplies the number of procedures to be calculated by the number of its own realizations. Any quality improvement thus comes at a cost and has to be carefully considered.

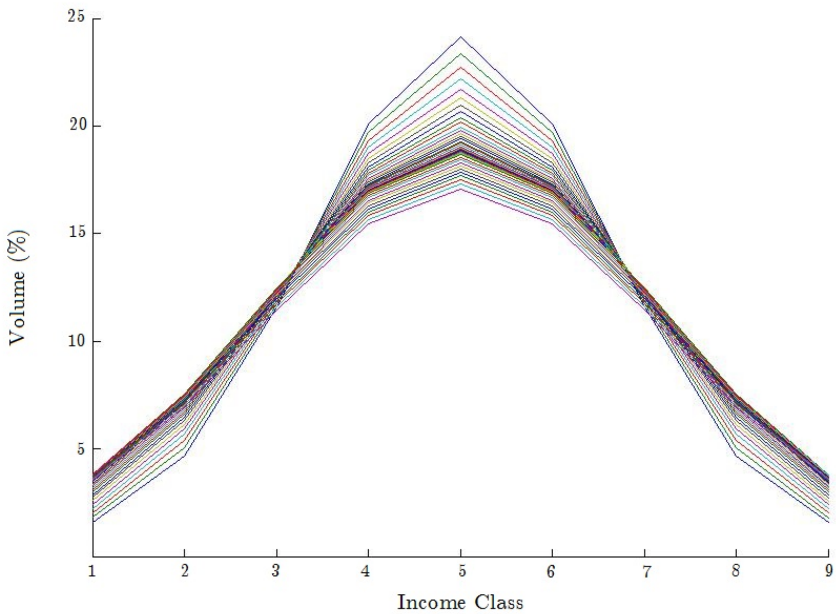
[12] In reality, the autoregressive process of order one may be understood as a low probability that a worker would be promoted or demoted, and a high probability of remaining at the original level. Similarly, unemployed agents are more likely to be unemployed in the subsequent period than employed agents, and so on.

To be precise, the next period's shock is normally distributed with mean $E(Z_{t+1}|Z_t) = (1-\rho)\overline{Z} + \rho Z_t$ and variance σ^2.

In order to approximate the autoregressive process, the continuum of all possible shocks must be limited; to do so, I follow Huggett (1996) and discretize the state space Z containing all shocks into nine realizations ranging from $-2\sigma_{y1}$ to $2\sigma_{y1}$, where σ_{y1} is standard deviation of log-normal earnings distribution of newborn agents. These realizations in fact constitute nine different income classes. The probability of having a given productivity shock can then be computed using integration over corresponding area under the normal distribution. For full specification of the AR(1) process approximation see section A.1 in the Appendix A.

Additional information in form of an initial wealth distribution for newborn agents in necessary as the autoregressive process only provides information about transition probabilities between shocks in different periods. The initial distribution may be understood as a set of personal characteristics, such as quality of education, level of unemployment in region, or personal connections determining available job positions. The choice of the initial distribution is discussed in section 4.6. The resulting distribution and its transformation during worker's life is depicted in Figure 4.1.

Figure 4.1: Distribution of idiosyncratic productivity shocks.



Note: The distribution is flatter (with heavy tails) for older workers.

4.1.4 Budget Constraints

Agents cannot borrow money so their consumption cannot exceed revenue, i.e. the sum of annual salary, pension payments (if retired), and one-year bond holdings earning risk-free interest rate r:

$$c_t = (1+r)\,k_{t-1} + (1-\tau)\,e(z,t)\,wn - k_t, \tag{4.5}$$

As I assume no intentional bequests from parents and no state transfers except for pensions, all households are born with no capital, $k_0 = 0$. Finally, given their utility function, agents buy no bonds in $t = T+TR$ and consume all their possessions. Overall per-period budget constraints for each agent are given by

$$(1-\tau)\,e(z,t)\,wn \geq c_t \qquad \text{for } t = 1 \tag{4.6}$$

$$(1+r)\,k_t + (1-\tau)\,e(z,t)\,wn \geq k_{t+1} + c_t \qquad \text{for } t = 2, ..., T \tag{4.7}$$

$$(1+r)\,k_t + pension \geq k_{t+1} + c_t \qquad \text{for } t = T+1, ..., TR-1 \tag{4.8}$$

$$(1+r)\,k_t + pension \geq c_t \qquad \text{for } t = TR \tag{4.9}$$

That is, a newborn generation cannot consume more than its first (and only) salary (Equation 4.6). Working generations have the standard budget constraint assuming that consumption and bond holdings cannot exceed the current-period salary, market value of held assets, and interest stemming from asset holdings (Equation 4.7). Retired generations essentially face the same budget constraint, salary is just substituted with current-period retirement benefits (Equation 4.8). Lastly, households of age 59 are rational and do not buy any new bonds due to certain death in the next period; hence, they consume all disposable income (Equation 4.9).

All period-related variables are reported for the upcoming period; that is, savings must be zero for a newborn generation and can be positive in the last reported period, despite the assumption of no bequests.

4.1.5 Taxation

I assume both, a semi-flat income tax rate and a progressive tax rate. The former is based on data from the Czech Republic, where people with monthly income below 4-times the average salary are taxed by a 15 percent flat rate, whereas any income above the threshold is taxed by a marginal rate of 22 percent. Progressive taxation is based on the German social system: no income tax is charged on the basic allowance and the marginal tax rate increases linearly from 14 to 24 percent and from 24 to 42 percent for taxable income within specified intervals. Finally, the highest-income households are taxed by a marginal rate of 45 percent.

Given the varied marginal rates, another obstacle is the actual income time-variance within the income groups. According the age-productivity profile, I assume that households have over two times higher salaries at age 35 than in the initial year. To avoid complicated computation of the overall income distribution across all income groups and generations, I rather use the same level of marginal taxation for all household ages. That is, instead of computing average earnings across all households alive in the economy and then assessing $Z \cdot (T + TR) = 540$ different tax rates, I assume taxation to vary only across income classes, not across generations. Because I assume normal distribution for the transition matrix, the overall result is virtually identical.

4.2 Firms

Firms produce output Y using two inputs: effective labour N and capital K. They are of unit measure and operate in a perfectly competitive environment. Production is characterized by constant returns to scale and is assumed to have the standard Cobb-Douglas form, i.e.

$$Y_t = AK_t^{\alpha} N_t^{1-\alpha}, \tag{4.10}$$

where A denotes a scaling constant representing technological advancement. Throughout my simulations, I assume it to be at a constant level in all respective periods and to depend only on the selected scenario (see further).

Note, that for reporting purposes it is irrelevant whether we observe savings or capital accumulation because the model does not assume banking sector; firms are thus owned by households and all unexpended funds are invested into firms' capital. That is, knowing individual savings decisions, we may directly compute the aggregate economy output given by Equation 4.10.

In a perfectly competitive settings with homogeneous agents and products, all firms must pay the same wage and interest rate on corporate bonds. From the profit maximization problem, firms aim to maximize

$$\Pi = K_t^{\alpha} N_t^{1-\alpha} - rK_t - wN_t; \tag{4.11}$$

first derivation of Function 4.11 with respect to both factors leads to an equilibrium where both factors are paid equal to their marginal product. That is, assuming a depreciation rate δ

$$w = AK_t^{\alpha}\,(1-\alpha)\,N_t^{-\alpha} \tag{4.12}$$

$$r = AK_t^{\alpha-1}\,\alpha N_t^{1-\alpha} - \delta \tag{4.13}$$

The production function assumes units of effective labour - wage and interest rate are then equal for agents indifferent of age and therefore have no subscripts as long as the economy is in steady state. Of course, when computing a transition path, the variables adjust each period and thus vary in time.

4.3 Pension System

All of the previously discussed pension schemes are simulated: a PAYG scheme without mandatory private savings as it is defined in the Czech Republic, the Chilean fully-funded scheme with mandatory contributions to individual account, and the Swedish scheme, a mix of the former two. Government, otherwise excluded from the model, pays yearly transfers to pensioners (in case of a PAYG system) or runs auxiliary pension funds that collect life-time savings and pay agents annuity in retirement.

A notable contribution of my model compared with the prior literature is differentiation among income groups in retirement. Income inequality has so far been only assumed for workmen, while pensioners received a constant public pension irrespective of their employment history. On the contrary, I simulate retirement benefits for each of the nine income groups separately, which allows for investigation of retirement benefits variance within the pension systems. Because the total amount of all households is normalized to unity, sizes of the respective subgroups are in fact equal to the probabilities of agents belonging to them - that is, households are normally distributed in level of retirement benefits according to Figure 4.1.

The actual implementation of retirement benefits calculation depends on the selected pension scheme. For instance, in the PAYG scheme I assume different replacement rates for each income group according to the Czech pension system, while in funded schemes the program keeps a record of savings for each income group and period, so that it can calculate respective pension upon retirement. Note, that the model does not assume inflation, nominal and real variables are then equal. It essentially means that pensions need not to be annually adjusted to remain constant in real terms.

4.3.1 The Czech Pension System

Let $mass_s$ denote size of generation aged s. I assume the pension budget to be balanced initially:

$$\sum_{s=T+1}^{T+TR} mass_s \cdot pension = \tau \cdot w \cdot N, \tag{4.14}$$

that is, pension transfers are equal to money collected from contemporary workers through the social security tax. Possible future budget deficit is financed from government debt, that is

$$\Delta D_t = T_t - TR_t - r_{d,t} D_t, \tag{4.15}$$

where D_t is the stock of outstanding debt at the beginning of year t, T_t denotes net social security tax collections, TR_t pension transfers, and $r_{d,t}D_t$ is debt service to be paid on the outstanding debt.

From the second period on, pension budget may remain balanced or not, depending on user choice. In a case of unbalanced budget, pension transfers and taxes remain on the initial steady-state level despite resulting budget deficit.[13] Such scenario reflects public unwillingness to accept any unfavourable changes in pension payments or taxation, and is thus the most realistic one.

On the contrary, in a case of balanced budget, either pension payments or taxation (or both) must be changed to reflect excessive pension expenses. Choice of balancing measure depends on the selected scenario. If pensions adjust, retirement benefits can decrease according to demographical and economic changes so that taxes remain on the initial level. Analogously, if taxes adjust, pensions remain on par with real wages while taxes increase.

Calculation of pension transfers closely matches the actual methodology used in the Czech Republic. First, a calculation base is obtained from yearly salaries using thresholds based on those presented in Chapter 2. Second, pension amount is computed as a percentage of the calculation base according to given replacement rates. Real replacement rates depend on number of employ-

[13] Specifically, retirement benefits are assumed to remain on par with real wages, which may substantially increase as a result of growth in productivity.

ment years, but as I assume everyone to work at all times, replacement rates are unique for all households in an income class, and set such that income qualified for pension (i.e. lifetime pre-tax earnings reduced using the given thresholds) is divided into $TR = 20$ equal instalments. That is, retirement benefits are equal across all generations retired within a single period, differ only among income classes, and are recalculated each period to reflect changes in real wages and other economic conditions. Finally, the social security tax is set so that pension budget is balanced in the initial steady state.

4.3.2 The Chilean Pension System

Recall, that the Chilean pension scheme is in fact not a perfectly fully-funded one in its current settings due to existence of a safety net that alleviates poverty. As I am primarily interested in observation of retirement benefits exclusive of supplementary transfers, I rather use it as a illustrative example and model the pension scheme without the zero tier. Importantly, pension budget should be void; no matter if people save through state-run or third-party pension funds, retirement benefits are arranged by specialized institutions that convert accumulated savings to annuities according to the formula described below. In a sufficiently large economy with known probability distribution of death, fair-calculated annuities ought not to, on average, require additional funding nor generate profits.

Simulation of the fully-funded scheme therefore does not include the three pension budget scenarios (i.e. unbalanced budget and balanced budget with pensions/taxes adjusting to demographic changes). On the other hand, I assume two other scenarios regarding the underlying assumptions. In the baseline scenario, the annuity market is perfectly fair, financial markets stable, and pension funds charge no fees to cover administration costs. In the second scenario with extended assumptions, account balance used for annuity calculation is decreased 30 percent to reflect findings of Murthi et al. (2001) and asset returns are volatile.

As noted above, the program keeps a record of all contributions made to the system and is thus able to calculate retirement benefits for various income classes separately. Moreover, the approach allows effortless simulation of transition between two pension systems. Unlike in the PAYG scheme, where taxes are initially set up such that pension budget is balanced, rates in the Chilean scheme are set exogenously in advance. Concretely, taxes are at 13 percent irrespective of income class affiliation, which corresponds to the average effective social security tax implicitly calculated in the Czech scheme.

4.3.3 The Swedish Pension System

The multipillar pension scheme based on the Swedish pension system is essentially similar to the fully-funded scheme in a sense that retirement benefits from the income pension (first tier) and the premium pension (second tier) are calculated as annuities from total individual account balances. There are, however, two differences. First, the income pension is financed on unfunded basis; and second, I also include the guarantee pension in the model. The guarantee pension calculation mimics the actual Swedish methodology and is as following:

$$GP = 2.13pba - Pension = 1 \qquad \text{if } Pension \leq 1.26pba \tag{4.16}$$

$$GP = 0.87pba - 0.48(Pension - 1.26pba) = 1 \quad \text{if } Pension > 1.26pba \tag{4.17}$$

where *Pension* denotes the sum of retirement benefits stemming from the first and the second pillar and *pba* is the *price base amount*, a variable used to price-index pensions, adjusted once a year in response to changes in economy. In my model, *pba* is set as 10 percent of average real wage, so that the guarantee pension values approximately correspond to real numbers in Sweden. I decided to deviate from the previous two models and set the social security tax rate equal to the actual rate of 18.5 percent in Sweden for the sake of authenticity and possibility to observe the effects of such change on the economy.

4.3.4 Annuity Creation

From the perspective of an insurance company, the average life length of their customers should precisely match the expectations (if made upon realistic assumptions) because age of death - or analogously the survival function - has a normal distribution with mean equal to the life expectancy for a given cohort, gender, and other classifying variables. The positive idiosyncratic effects promoting longer life, such as healthy lifestyle, then negate, on average, opposite influences (e.g. smoking) that would cause diseases and premature death. Insurance companies thus set annuities in a following way:

$$Annuity_{i,j} = \frac{Savings_i \cdot \prod_{s=0}^{j} r_s \cdot (1 - C)}{E(LL_i)}, \tag{4.18}$$

where $Annuity_{i,j}$ denotes pension transfer to person i at time j, $Savings$ denote the total sum of life-time contributions plus the accumulated interest on it, $C \in [0; 1)$ denotes administrative and other costs, r_s is indexing that evens up raise in real wages, and $E(LL_i)$ is individual's life expectancy.

As opposite to the PAYG system, where pension transfers change annually for all living households at that time, pension transfers in funded pension schemes remain constant for respective individuals. From the programming perspective, it is then necessary to keep a record of the annuity calculation period as well as the actual period of interest. The program saves results of each calculation in database so that households obtain constant benefits despite changes in the economy.

4.4 Steady State and Transition Path

Recall, that the economy is assumed to be in a steady state initially. Formally, steady state is an equilibrium for given government policy and the initial distribution of capital $\{k_0^s\}_{s=1}^{T+TR}$ and is characterized by a set of value functions $V^s(k_t^s, K_t, N_t)$, individual policy rules $c^s(k_t^s, K_t, N_t), n^s(k_t^s, K_t, N_t)$, and $k^{s+1}(k_t^s, K_t, N_t)$, and relative prices of labour and capital w_t, r_t, such that:

1. Individual and aggregate behaviour are consistent:

$$N_t = \sum_{s=1}^{T} \frac{n_t^s}{T + TR} \tag{4.19}$$

$$K_t = \sum_{s=1}^{T+TR} \frac{k_t^s}{T + TR} \tag{4.20}$$

2. Households' dynamic programs and firms' optimization problems are solved by satisfying 4.6-4.9, and 4.11-4.13, respectively, using the relative prices w_t, r_t, pensions, and the individual policy rules $c^s(.), n_t^s(.)$, and $k_{t+1}^s(.)$.

3. The goods market clears:

$$AK_t^{\alpha} N_t^{1-\alpha} = \sum_{s=1}^{T+TR} \frac{c_t^s}{T + TR} + K_{t+1} - (1 - \delta)K_t \tag{4.21}$$

4. The government budget is balanced.

The initial and the final steady states discussed in this work as essentially the same from this perspective - they just denote the starting and the terminal point of the analysis and differ in population settings (see below). As I assume no population growth or technology development within a steady state, aggregate capital stock, aggregate effective labour, wage, interest rate, and taxes are constant and the optimal capital allocation and consumption decisions do not change over time:

$$\{k_t^s\}_{s=1}^{60} = \{k_{t+1}^s\}_{s=1}^{60} = \{\overline{k}^s\}_{s=1}^{60} \tag{4.22}$$

Things get interesting when some parameter suddenly changes; while newborn generations see the variables as default ones, generations alive at the time of the change must change behaviour accordingly. As first explored by Auerbach and Kotlikoff (1987), a permanent change in parameters sets in motion a sequence of transformations within the economy, slowly aiming towards a new steady state. They suggest that the number of transition periods required for an economy to reach a new steady state is at least three times the number of generations, i.e. 180 years in the case of my model. To be sure that the economy reaches a new steady state, I set the number of transition periods to 300.

It should be noted that even though the model implicitly assumes each period to be a year in real terms, it is not quite so. Time, in this case, should be understood as just an auxiliary variable that facilitates computation and allows better orientation in the model rather than an actual output variable. In particular, the requirement of at least 180 periods to reach a new steady state prevents any real-world implications to be made out of the outcomes - the is only usable for a theoretical analysis.

4.5 Computation Approach

I follow Nishiyama and Smetters (2007) and use value function iteration to compute agents' policy functions for respective periods and shocks.[14] Let $v(K)$ be the value function and let it be the discounted sum of all instantaneous utility functions $u(c_1), u(c_2), ..., u(c_{T+TR})$, where $c_1, c_2, ...$ denote household's con-

[14]Generally, the easiest way to approach the computation is to simply take households' lifetime utility function and find its absolute extrema on a bounded interval, with household's capital, income, and labour limitations being the bounds. Such approach is virtually impossible to utilize in the case of stochastic productivity shocks, though, as there are Z^T possible realizations of the shocks over T years of household's employment.

sumption at age $1, 2, \ldots, T+TR$, and K denotes the optimal capital decisions that maximize household's lifetime utility. In other words, given a sequence of capital stocks, $v(K)$ is the maximum discounted level of household's utility.

Assume an optimal sequence of capital stocks from $t = 0$ to time $t = q$, that is, $K = k_0, k_1, ..., k_q$, then the best level of capital K^* in time $t = q + 1$ is given by

$$v(K^*) = \max_{0 \leq K' \leq f(K)} u(f(K) - K') + \beta v(K'), \tag{4.23}$$

where $f(K)$ denotes the production function and $f(K) - K'$ thus denotes consumption in a given period. In case the value function is known, we may compute the solution K^* using a policy function g, i.e.

$$K^* = g(K); \tag{4.24}$$

policy function thus represents the optimal decision regarding the next-period level of capital as a function of the current capital stock.

In a perfect world with no uncertainty, policy functions can be easily computed from time $t = 0$ onwards because households know their future income and may plan consumption and savings accordingly. However, idiosyncratic shocks introduce individual uncertainty into the model so that agents no longer have a perfect foresight. To overcome the obstacle, policy functions must be computed for all prospective capital and income levels. The approach resembles reality, where people predict possible future states of the world - weather, for example - and choose the best response for each possibility.

Let k_{min} and k_{max} denote the minimum and the maximum capital stock a household could posses at a single point in its life. Choosing a number of grid points n, we may discretize the interval; we can assign a level of capital to each of the equidistant points $k_1, k_2, ..., k_n$ in the interval, with $k_1 = k_{min}$ and $k_n = k_{max}$. Obviously, k_{min}, k_{max} and n must be chosen carefully, as a too tight interval may create artificial bounds to households' decisions and thus produce errors in computation. Moreover, higher number of grid points implies growing computation requirements and a low number of grid points (wide intervals between individual points) result in higher computation error due to too rough discretization.

The assumption of finite-horizon model with mortal agents allows search for the optimal consumption and savings decision in the last period of life - that is, in $t = T + TR = 60$. As discussed above, households consume all its remaining wealth to maximize instantaneous utility. Formally, following Stokey (1989), their value function is then

$$v^{T+TR}(k_t^{T+TR}, K_t, N_t) = u(c_t^{T+TR}). \tag{4.25}$$

The level of consumption depends on income from retirement benefits and capital gains from assets owned; therefore, the value varies for different points on the capital grid and we must compute it for all of them. Then, the point with the highest utility value is chosen as the optimum policy function. Similarly, in all prior periods $t = 1, ..., T + TR - 1$, instantaneous utility depends on actual consumption, which, in turn, depends on savings and other income. However, here it also depends on the intended level of capital accumulation at time $t+1$, determined by savings at time t. And as the savings in period $t + 1$ further depend on the projected capital stock in period $t + 2$ in a similar manner, the easiest way to define the value function is using recursive notion as following:

$$v^s(k_t^s, K_t, N_t) = \max_{k_{t+1}^{s+1}, c_t^s} [u(c_t^s) + \beta v^{s+1}(k_{t+1}^{s+1}, K_{t+1}, N_{t+1})]. \tag{4.26}$$

Clearly, at time t, one can always consider such level of capital k_{t+1} for $t+1$ that $k_{t+1} \leq k_t$ as all it takes is to save less than to consume. On the other hand, only some points with higher level of capital are accessible, as it is impossible borrow. The algorithm assesses utility for each point on the asset grid at time t through computation of value functions for all points at $t+1$. Only accessible levels of capital are then considered and compared.

This way, one can start in the period $t = T + TR$ and compute the ideal policy functions $k^{s+1}(k^s, K, N)$ and respective value functions in earlier periods using backward induction to obtain the age-wealth profile of the steady state $\overline{k}^s$. Then, using the set of optimal policy functions, another algorithm simply starts in $t = 0$ (where $k_0 = 0$) and, through forward induction, computes the optimal capital allocation in all future periods according to the actual shock realizations. That is, the former algorithm finds optimal decisions for all possible states of the world in each period, whereas the latter algorithm chooses optimal consumption and savings with respect to the actual earnings.

It also implies that each point on the asset grid must be evaluated separately for every realization of the idiosyncratic shocks. Albeit it is just a minor obstacle regarding implementation, it literally multiplies the computation time needed. Fortunately, model computation may be greatly quickened by narrowing the interval $[k^s_{min}, k^s_{max}]$ after the initial iterations. Generally, the interval must be wide enough for capital stock in each period to be within its bounds. As soon as the approximate age-wealth profile is known, wide interval may be replaced by a higher number of smaller intervals for different periods that would match the suggested capital stocks more closely.

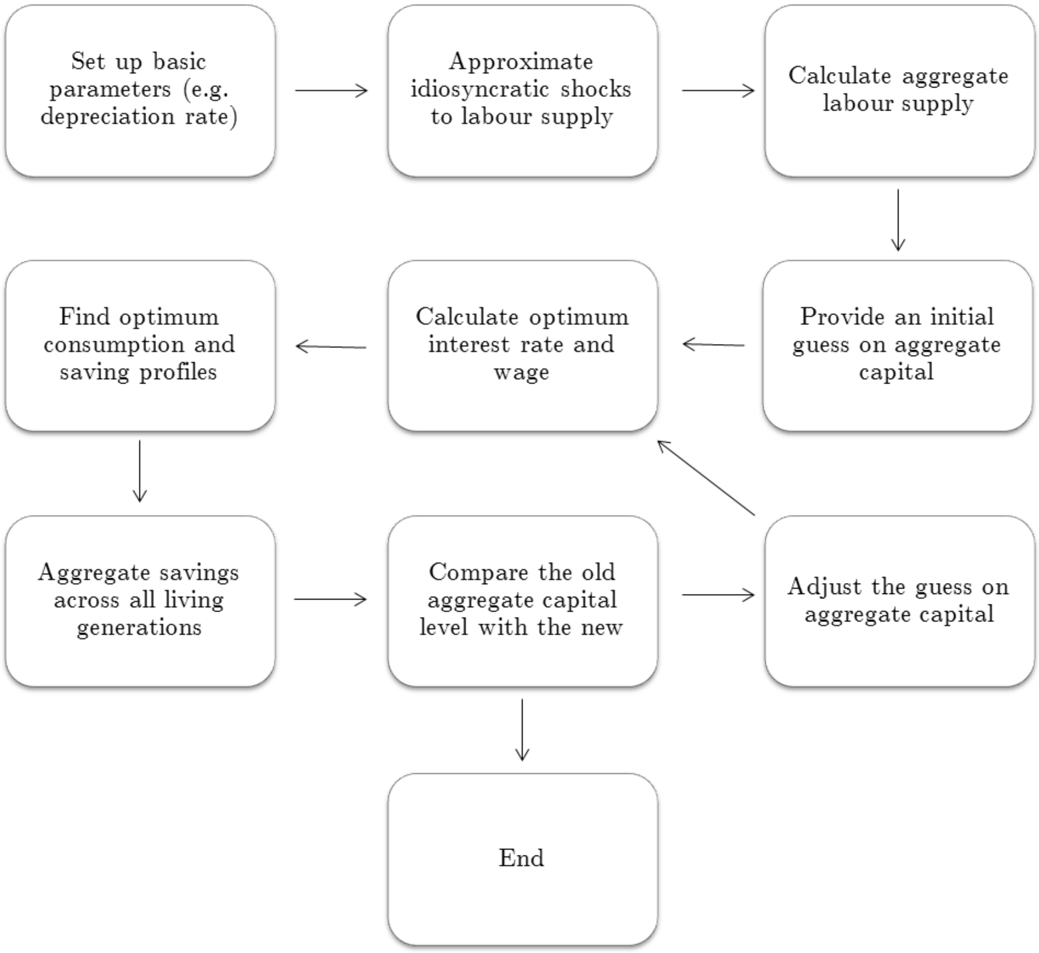
Note, that to assess the value function, households need to know other parameters - particularly wage and interest rate - determined implicitly according to individual behaviour of households. In a sense, it is thus a vicious circle. Hence, besides the already formulated relationships, the user must also provide an initial guess on the aggregate variables (in this case, capital), which are subsequently adjusted through the process of value function iteration. That is, the program computes the optimal individual behaviour according to a guess on aggregate variables, and provides a new, more accurate guess on the aggregate variables. And the process of iterations is repeated until the behaviour corresponds to economy and vice versa (formally, until specific convergence criteria are met). The algorithm is depicted in Figure 4.2.

With respect to computation approach, there is no difference between the initial and the final steady states. In fact, unless there is a substantial change in the economy, the same guess on the aggregate capital accumulation works for both. Moreover, it can be used for computation of transition as well. The basic transition algorithm proposed by Auerbach and Kotlikoff (1987) assumes a prior computation (if possible) of the initial and final steady state variables and a subsequent analysis of all steps between them. Recall, that the model assumes individual uncertainty in form of idiosyncratic shocks but there is no aggregate uncertainty, such as stochastic technology progress or business cycles. Agents thus face perfect foresight on the aggregate level and, unlike in the assessment of policy functions, are able to predict the exact transition path from the point of change until reaching a new steady state.

This fact must be considered in the computation; as shown by Auerbach and Kotlikoff (1987), all transition periods' variables must be computed at the same time. In other words, the user must first provide an initial guess on the aggregate level of capital in all the transition periods $K_2, K_3, \ldots, K_k$, and these must then be updated all at once. That means, for $k = 300$ being the number of transition periods and $c = 601$ being the number of points on the asset grid, computing $(T + TR) \cdot k \cdot c = 10,810,000$ policy functions in each iteration.

To prevent that, Evans and Phillips (2010) propose an alternative algorithm

Figure 4.2: Computation Algorithm.



that utilizes a slightly different approach to households' rationality. Instead of the unrealistic assumption that all agents know all other agents' policy functions and therefore can predict the whole transition period, the alternative approach uses a weaker assumption that agents use some general model to forecast the aggregate capital stock for the remaining periods of their lives. Specifically, Evans and Phillips (2010) assume the function to be in a form of

$$K_{t+1} = K_t + \frac{\bar{K} - K_t}{T - t}, \tag{4.27}$$

where $\bar{K}$ is the final steady state capital stock, t is the current period, and T is the period in which is the economy expected to reach the new steady state. Unlike the approach proposed by Auerbach and Kotlikoff (1987), this algorithm does not require each transition period to be calculated iteratively and so it saves approximately 85% of computation time and causes just a minor error in results due to slight difference between the optimal aggregate capital stock K_t^* and its approximation.

Further, while Auerbach and Kotlikoff (1987) assume that the transition path merely serves as an intermediary stage between the initial and the final steady state, the alternative algorithm proposed by Evans and Phillips (2010) allows for a wider range of usability. In particular, although Evans and Phillips considered the economy to be in a steady state in the last period as well, the assumptions require households to plan their consumption and capital holdings according to an anticipated state of economy, yet the economy itself is not required to reach it. Besides the standard scenarios, it is thus also possible to compute transition using variable interest rates, random stock market volatility, and other *ex ante* unspecified parameters, which is necessary for my analysis.

4.5.1 Investigation of Pension Myths

Recall, that my model does not explicitly assume the effect of disproportion in size of different generations on asset prices as it lacks a mechanism simulating market for bonds, yet it allows a comparison of pension systems and thus a direct assessment of the myth. In other words, even though I cannot comment on the theoretical disproval of Barr (1979), we may simply investigate how PAYG and funded pension schemes fare facing the adverse population changes.

Because individual behaviour has an intratemporal impact on macroeconomic variables and vice versa, retirement benefits clearly depend on the overall state of the economy. Specifically, the level of pensions directly affects consumption savings decisions, these affect the aggregate capital stock and output, which, in turn, determine real wage and interest rate and thus affect calculation of retirement benefits. Therefore, if funded pension schemes' performance substantially decreases as a result of population changes, pensions change as well, no matter that they are actuarially fair.

To test the second hypothesis that funded schemes facilitate savings, I assume the economy to run a balanced PAYG system in the initial steady state, and to approve a fundamental pension reform introducing a funded scheme in period 2, with slow adjustment to a new steady state over a longer time span. Pension transfers in periods after the transformation must be covered from explicit debt; if increase in private savings exceed explicit indebtedness created by the government, we may assume that the myth is indeed true, under given conditions. If, however, debt repayments result in worsening of welfare for future generations, it would be a sound argument against a prospective reform.

4.6 Calibration

To provide the initial data for the idiosyncratic productivity shocks, I follow Huggett (1996) and choose a log-normal distribution of earning for newly born generation with $\sigma_{y1} = 0.38$. The other function parameters are taken from Heer and Maussner (2009) and presented in Table 4.6.

Table 4.1: Function parameters.

Preferences	Production
$\beta = 1.011$	$\alpha = 0.36$
$\eta = 1.5$	$\delta = 0.06$
	$n = 0.3$

To model population changes, I use demographic predictions published by the Czech Statistical Office as presented in Chapter 2. Recall that there are two complementary effects that adversely affect pension systems: population ageing and shrinking.[15] Simulation of the continuous population decrease is straightforward - it only requires to lower the initial amount of people born in the subsequent time periods - that is, $mass_1 < 1$. Because population size is equal to one initially, size of the smaller subsequent newborn generations can easily be interpreted in percentage of the original one.

Using linear interpolation on the demographic data, I distribute the expected decline in fertility among the 300 transition assumed in my periods as $mass_{1,e} = mass_{1,e-1} \cdot \rho$, where e denotes period,

$$\rho = \frac{Population_{2100} - Population_{2013}}{Population_{2013} \cdot c}, \tag{4.28}$$

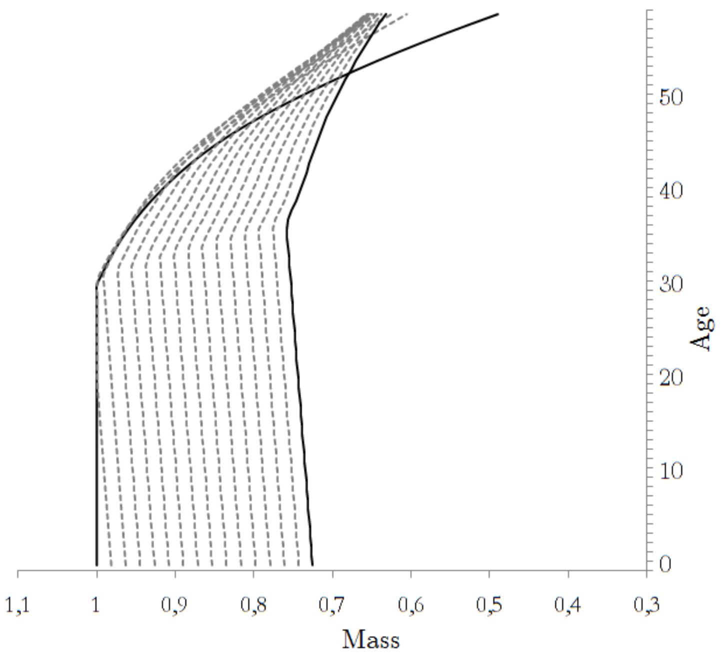
and c denotes the total number of transition periods. This way, the newborn generation in the last period is reduced to 72.54 percent of the initial newborn generation.

To simulate population ageing, I projected a function that changes the age structure of households in a way that it closely resembles Figure 2.1 in the

[15]As opposite to the earlier works (e.g. Muto et al. 2012 or Oksanen 2009), which assumed an increase in life expectancy only, my model simulates both of the effects as once.

respective transition periods. Beginning in $t = 1$ the model's age structure is defined by survival probabilities taken from Krueger and Ludwig (2007). Since $p(death|t) = 0$ for $t \leq 35$, there are as many households of age 1 as of age 35 and only then their numbers decrease; it is an unavoidable result of a long-term steady state. From $t = 2$, though, the number of new-born agents continuously decreases and the vector of survival probabilities changes towards lower probability of death in all periods (with greater change for older generations), so the resulting demographic tree is flatter with more pensioners per active worker than in the initial steady state, as shown in Figure 4.3.

Figure 4.3: Simulation of Population Changes.



Note: The initial population structure is represented by the line on the very left and the final population structure by the line on the very right. Other lines represent selected intermediary steps in the population change.

4.7 Scenarios

In order to illustrate the effects of possible reforms to pension systems, I propose several different policy options. Because computation of the whole transition is very time demanding, it is not included in all variants; I rather only summarize the conclusive impacts of various retirement schemes on the economy in the final steady state (i.e. after the demographic changes) and compare them with values from the initial steady state and among each other. Note, that

I choose a relatively high number of transition periods to guarantee a complete transition to a new steady state, it is therefore satisfactory to observe the final steady state only. Transition is computed for scenarios where it is essential to investigate the gradual transformation, such as transition from the Czech PAYG to Chilean fully-funded system, where we are primarily interested in the level of external indebtedness. The details are as following.

- **Taxation:** Flat or progressive income tax is assumed in all three pension systems, creating six baseline scenarios for the initial steady state and six scenarios for the final steady state in total.

- **Indebtedness:** Non-zero explicit government debt is assumed only in the Swedish and the Czech pension system and only in the final steady state. In other words, I assume the government to set the initial policy rule such that the pension budget is always balanced and that debt can be created only as a result of adverse demographic changes. In the fully-funded pension scheme, government budget remains balanced at all times because I do not account for any form of zero pillar and all pension transfers are therefore financed from savings only.

  All three pension systems may be deficit during the transition, though, as a result of population changes, stock market volatility, or transition costs. In particular, transition from an unfunded pension scheme implies financing of retirement benefits from taxes of contemporary workers (an implicit form of debt) or through government explicit indebtedness. Further, pension system can be loss-making due to inability to adjust pensions or taxes to meet changes in the economy.

- **Budget balancing:** If the government approves a fundamental pension reform, it can either decide to keep taxes and pensions at their initial level and run deficit pension system, or to decrease pensions or increase taxes (or both) to keep pension budget balanced. All such scenarios are assumed by the model, creating three possibilities for each steady state analysis of the Czech pension system.

- **Technology growth:** Continuous technological progress is one of a few factors that may alleviate the adverse effects of population changes in the long-run without having negative impacts on other parts of the economy. I assume three technological scenarios: no growth at all, sluggish growth that does not fully negate the population effects, and intense technological progress that results in higher economic output than in the initial steady state despite declining population.

I specifically do not set the productivity growth at some real-world level for two reasons. First, if we follow the mainstream literature and choose productivity growth at around 1.5 percent p.a. for developed countries (Marek 2007) and assume that each transition period is equal to a year, productivity would increase $1.015^{300} = 87.1$ times, which would totally ruin any comparison between the steady states. Second, the actual level of productivity growth does not matter anyway as the question is qualitative - whether technology growth may serve as a remedy to demographic changes - not quantitative - by what margin can we expect pensions to grow?

- **Assumptions:** Unlike funded schemes, PAYG scheme is virtually immune to volatility of asset returns. Hence, the extended assumptions of market imperfections are only included in analysis of the Swedish and the Chilean plans, which are simulated twice - once using the standard assumptions and once using the extended assumptions.

Another aim of this work is to depict advantages or downsides of transition from an unfunded pension scheme as it is functioning in most of the European countries to a (fully-) funded scheme. Therefore, the selective scenarios always start with the PAYG pension system in the initial steady state and transition into the Swedish or the Chilean pension system beginning in period 2. Further, as I stressed out in the beginning, the prior literature on transition from PAYG to funded pension systems neglects the market imperfections. The bottom line is that I selected four scenarios for transition. These are:

- **Shift from PAYG to the Swedish pension system:** Swedish pension system is introduced in the second period, with PAYG retirement benefits being paid from the general government budget to current pensioners and disadvantaged prospective pensioners. That is, workers will slowly build up savings for retirement for forty periods after the pension reform, and the government will top up pension transfers for those not able to contribute for their whole working life. Because my simulations neglect government spending besides social security payments, I choose to not take implicit debt into account and report explicit indebtedness only. The scenario is simulated twice, with and without financial markets volatility and administrative costs.

- **Shift from PAYG to the Chilean pension system:** The fundamental principle is the same as in the previous scenario: pension reform is introduced in period 2 and the government must suffer the consequences of having no social tax revenue while being responsible for payment of retirement benefits for 59 years in total. The main difference here is the substantial reliance on the *actual* individual account balance that is subsequently transformed into annuity, whereas pension transfers in the multipillar system are financed from tax revenues. The scenario is again simulated with and without the assumption of imperfect markets to reflect the difference.

All simulations were done in Matlab, the code is available upon request.

Chapter 5

Simulation Results

Recall, that my work is mainly concerned with problems of the current unfunded pension schemes and investigates the possible solutions. That is, I aim to compare the three basic pension systems and their effects on economy and households, and look for advantages of a prospective fundamental pension reform introducing some form of a funded scheme instead of a PAYG scheme. The assumption of an initial and a final steady state fits the objective well indeed; not only we can easily see the impact of demographic changes and technological progress on the economy, the respective steady states also represent the pre-reform situation and the long-term post-reform equilibrium state of the economy.

Literally all of the outcome variables stemming from the model simulations are of importance. Starting with microeconomics, we may observe individual consumption and savings decisions made by each generation alive resulting from the lifetime utility maximization principle. Although capital accumulation per generation is merely a secondary variable - we are mainly interested in the aggregate savings - it is interesting to see the progressive hoarding of assets and their subsequent gradual sale. As I discuss below, lower retirement benefits imply necessity to save higher proportion of income in order to maintain satisfactory living standards in retirement. Moreover, the actual savings profile varies across the assumed pension schemes.

From the macroeconomic point of view, the model gives estimates of the aggregate interest rate (representing both, corporate asset returns and interest on government debt), real wage, capital accumulation, output, explicit indebtedness, and pensions paid to the respective income groups. While the variables are individually relevant, it is crucial to investigate also their relations. In particular, retirement benefits greatly depend on wage, taxation, and other economic variables; their simple comparison among pension systems is therefore inappropriate without taking the other factors into account.

All results presented in this chapter (except for taxation and pension/wage ratio) are quoted in specific units and only useful for comparison within the model. That is, they have no connection to the real-world variables and cannot be converted.

5.1 Initial Steady State

The economy is always stable initially, with balanced pension budget, no prior external debt, and no market imperfections or uncertainty. Hence, the outcomes are equal for any conditions set further. This is of particular interest in the case of the Czech pension system, which can be balanced or unbalanced, depending on user preferences. Similarly, there are no differences in the aggregate variables no matter what technology growth is assumed as it only impacts the economy during the latter periods. For simplicity and comprehensiveness of results, I also do not include the extended assumptions here. Table 5.1 summarizes simulation outcomes for the initial steady state.

Table 5.1: Initial Steady State Variables.

	Flat			**Progressive**		
	Czech	Swedish	Chilean	Czech	Swedish	Chilean
Interest Rate	2,28%	2,89%	1,47%	3,11%	3,63%	2,07%
Wage	1,463	1,405	1,551	1,386	1,344	1,484
Output	0,899	0,865	0,954	0,853	0,826	0,912
Capital	220,5	199,4	260,9	191,7	174,6	230,9

Looking at interest rate values, there is a large gap between outcomes given by the respective pension systems. To understand the difference, it is necessary to take the other variables, namely capital accumulation and output, into consideration. Because the interest rate is set according to the marginal product of capital, given by

$$r = K_t^{\alpha-1}\alpha N_t^{1-\alpha} - \delta, \tag{5.1}$$

it is clear that the value is proportional to output/capital ratio, which is much lower for the Chilean pension system. Analogously, wage is set according to labour supply, which is constant for all simulated schemes, and capital accumulation. Finally, output depends on labour supply (constant in all variants), productivity growth (not present initially) and capital accumulation.

The main focus is therefore on capital - the sum of savings of each generation alive. As suggested by the prior literature, the fully-funded scheme indeed results in significantly higher savings than its alternatives. Moreover, the variable does not include savings on individual accounts in pension funds, only private savings stemming from consumption-saving decisions, the total amount of funds that can be invested is thus even higher.

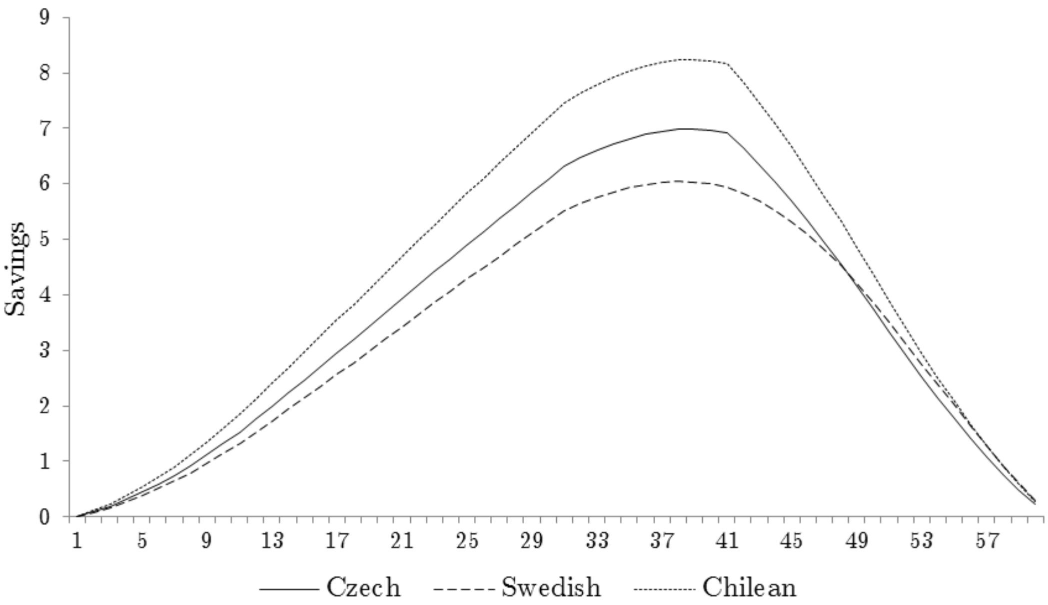
The crucial question is what leads to the differences in savings. The answer lies within the life-cycle hypothesis: rational agents try to smooth their consumption profiles across their lives - i.e. even in retirement. Because the fully-funded pension system does not provide even a basic protection against poverty in my model, all consumption above the minimal level depends on private savings - mandatory and voluntary.[16] Social security tax rates are often set such that savings on individual accounts implicitly support invariable standard of living, yet people must still put additional funds aside to match their own consumption preferences. In the Swedish pension scheme, on the other hand, pensioners are provided with rather generous state funding independent of prior earnings (the guarantee pension), low-income classes thus need not to accumulate excessive savings to maintain satisfactory living standards.

Real wage, of course, plays its role as well. The Chilean pension system leads to 10.4 percent higher wage than the Swedish one, people thus have more disposable income and, given the same propensity to saving, save more. Still, the difference in savings is 30.8 percent and cannot be explained by higher wage only.

Note, that the average effective taxation is very similar in the flat and progressive tax regime, yet the simulations yield considerably different results as the aggregate output is higher in flat taxation settings. The results are slightly skewed because the assumed income distribution does not fully correspond to the real one, though the message is clear: lower taxation on the poorest may alleviate poverty, but only at the cost of higher taxation on wealthier people, reducing the aggregate output and welfare. In my model, transition

[16] The actual Chilean implementation is not much different; low-income classes are offered additional funding from the zero pillar, yet only in limited amount as it is financed from the general government budget.

Figure 5.1: Savings Profiles, Initial Steady State.



to progressive taxation reduces aggregate output by 12 percent and wages by 5 percent on average.

The effect of pension system on savings can be further observed from Figure 5.1, which depicts capital accumulation per generation. Clearly, households put money aside during work years (i.e. up to age 40 in the model) and then continuously consume saved funds during retirement. That is indeed in line with our expectations and the reality. The capital profiles seem similar at the first glance, yet we can see that the Swedish profile is different in the last years. Specifically, it is flatter than the others, suggesting that the pensioners may enjoy higher living standards in the last years.

The last variable to be evaluated is the level of pensions. As noted in Chapter 4, the program calculates pension transfers for all income classes represented among workmen and we are thus able to observe not only the total amount of retirement benefits received, but also their distribution. Results of the initial steady state simulations are shown in Table 5.2. Regarding both, the aggregate level of pensions and pensions per income class, Swedish pension system is most generous as it provides nearly double pensions compared to the Czech PAYG system.

There are three factors behind the difference: first, the Swedish pension system is simulated using real social security tax of 18.5 percent, which is higher than in the other two schemes. Therefore, albeit the basic mechanism is similar

Table 5.2: Initial Steady State Retirement Benefits

	Flat			Progressive		
	Czech	Swedish	Chilean	Czech	Swedish	Chilean
Class						
1	0,081	0,150	0,037	0,091	0,146	0,043
2	0,119	0,159	0,054	0,127	0,163	0,063
3	0,134	0,187	0,080	0,139	0,196	0,091
4	0,155	0,229	0,116	0,157	0,243	0,133
5	0,177	0,290	0,170	0,174	0,313	0,195
6	0,210	0,379	0,248	0,197	0,415	0,284
7	0,257	0,510	0,362	0,229	0,563	0,415
8	0,277	0,700	0,529	0,243	0,781	0,607
9	0,306	0,978	0,772	0,263	1,098	0,886
Total	1,716	3,583	2,369	1,619	3,918	2,717

Note: Class denotes income class, where 1 = poorest and 9 = wealthiest.

to the Chilean scheme, pensions are higher, leading to lower aggregate output. Second, the Swedish system alleviates poverty through the guarantee pension, which reallocates funds among income classes. In reality, funds from the general government budget must be used to cover the expenses, but due to the skewed income distribution in my model, the guarantee pension does in fact bring additional funds to the budget as it decreases pensions for the wealthiest while providing lower additional funds to low-income classes. Finally, the Swedish pension system is a publicly-organized PAYG analogue of the Chilean scheme - person's pension entitlement is in a actuarial relationship with prior contributions, yet the actual transfers are paid from contemporary workers' contributions.

Obviously, the sum of pensions to be paid in the Swedish pension scheme is virtually never the same as the sum of contributions, resulting in a surplus/deficit in pension budget. That is a factual difference from the PAYG system: in the Czech system, retirement benefits essentially depend on government decisions and may be effortlessly adjusted in order to keep pension budget balanced. In the Swedish system, on the other hand, pensions depend only on prior contributions. In my simulations, pension budget runs into a slight deficit in the initial steady state because technology growth and demographics remain unchanged so that contributions would indeed be equal to the needs, but the pension base used for calculation of retirement benefits increases annually by interest rate on savings - expenses are thus greater than contributions.

The bottom line is that under the assumption of no market imperfections or adverse demographics, the Chilean and the Swedish pension systems yield similar results as the differences in retirement benefits and aggregate variables are mostly in line with existence of the guarantee pension and the difference in social security tax rate. Indeed, that is an expected outcome since the respective schemes are essentially equal in their dynamics.

Going back to the results, let us now take a closer look at the distribution of pensions among income classes. The Czech pension system is an example of what would be called "socially just" for it creates but a little inequality between the poorest and the wealthiest, who obtain only 3.8 times higher pension than the poorest, whereas it is 6.3 times higher in the Swedish plan and 20.9 times higher in the fully funded scheme. In this respect, only the Chilean pension plan is truly actuarially fair and resembles the income distribution among workmen, whereas the other two schemes substantially reduce income inequality.

Interesting is also a comparison of flat-taxation vs progressive-taxation pensions. Unlike in aggregate variables, there is no trend across the observed pension systems. All households in economy with the Chilean scheme would greatly benefit from introduction of progressive taxation as the increase of interest rate would result in higher individual account balance in spite of lower real wage and contributions. The Swedish plan also provides higher aggregate pensions in progressive-tax regime, but, as opposite to the Chilean scheme, not all income classes would profit from the change as the poorest households would have their income reduced. Lastly, approximately a half of all households would benefit from transition to progressive tax system in the Czech scheme, while those with above-median salary would suffer, resulting in lower aggregate retirement benefits overall.

One thing to keep in mind is the interconnection of retirement benefits and wage - the higher real wages are in the economy, the higher shall consequently be pensions as they are calculated from lifetime salary. Because there are only minor differences in wages across the three systems, there is virtually no bias in the results and the conclusions made above are accurate. In the final steady state, on the other hand, wages vary considerably not only across pension schemes but also within them as a result of technological progress. Retirement benefits in absolute values therefore no longer have reliable explanatory power and must be replaced with comparison of pension/wage ratio instead.

5.2 Final Steady State

The final steady state allows for a deeper investigation of the actual processes as it assumes all the aforementioned concepts: demographic changes, explicit government debt, various balancing procedures, extended assumptions, and technology level growth. The assumption of balanced pension budget is released in some scenarios of the Czech system, allowing retirement benefits to remain at their original level despite the demographic changes at the cost of external indebtedness.

That being said, the outcomes of the final steady state simulations are extensive and often presented in the Appendix so as not to distract the reader. Specifically, the Appendix includes detailed overview of taxation, pensions, pension/wage ratios, and individual savings profiles for all the respective pension schemes and all the assumptions.

5.2.1 The Czech Pension System

First, let us take a look upon changes in the PAYG scheme. Level of taxation, as shown in Table B.2 in the Appendix, depends on flat or progressive regime just as in the initial steady state, and additionally also on the selected scenario. If we assume that taxation remains unchanged and pensions adjust so that the budget is balanced or if we let the budget to be unbalanced with taxes and pensions at their original levels, taxation is not modified. Importantly, the adjustments are made such that pensions do not change with respect to real wages rather than in absolute value. Therefore, if we look at taxation in the scenario with tax adjustment, the resulting taxation is in fact higher in the case of positive technology level growth.[17] It is also slightly higher even if we assume no technology growth as it reflects demographic changes.

Simulation results of retirement benefits are coherent with the prior findings. If we assume no adjustment in taxes or pensions to balance the pension budget, pensions slightly decrease in absolute values, but remain constant with respect to wages, which decrease over the observed time period (see Tables B.1 and B.3 in the Appendix for detailed results). No adjustments to retirement benefits or taxes must be compensated otherwise, budget deficits thus go hand in hand

[17]In other words, although the growth in productivity implies better overall economic conditions, it particularly leads to substantially higher real wage, which needs to be reflected in pensions as well. In the end, higher tax revenues are not sufficient to finance the excessive pension expenses and the social tax rate must be increased accordingly.

Table 5.3: Czech Pension System, Final Steady State Variables.

	Initial		Final					
	Flat	Prog.	Flat			Progressive		
			PA	TA	NA	PA	TA	NA
Interest Rate								
None			3,16%	3,16%	3,15%	3,88%	3,69%	3,83%
Mild	2,28%	3,11%	3,06%	3,20%	3,04%	3,86%	3,84%	3,82%
Strong			3,30%	3,48%	3,27%	3,89%	3,99%	3,85%
Wage								
None			1,382	1,382	1,384	1,324	1,339	1,324
Mild	1,463	1,386	2,221	2,202	2,227	2,118	2,121	2,123
Strong			3,496	3,459	3,513	3,377	3,358	3,391
Output								
None			0,850	0,853	0,851	0,814	0,820	0,816
Mild	0,899	0,853	1,013	1,003	1,016	0,967	0,967	0,970
Strong			1,183	1,168	1,189	1,143	1,135	1,151
Capital								
None			203,3	202,8	204,1	178,2	174,3	178,9
Mild	220,5	191,7	326,2	315,3	328,3	289,9	288,6	290,9
Strong			506,5	476,0	510,0	461,1	442,2	465,3
Capital/Wage								
None			147,1	146,7	147,5	134,6	130,2	135,1
Mild	150,7	138,3	146,8	143,2	147,4	136,8	136,1	137,0
Strong			144,9	137,6	145,2	136,5	131,7	137,2

PA = Pensions Adjust, *TA* = Taxes Adjust, *NA* = No Adjustment. Columns *Initial* and *Final* represent initial and final steady state, respectively. Rows represent variables and level of technology growth (*None/Mild/Strong*).

with providing for pensioners in an ageing environment; the basic scenario with flat taxation, no technology growth, and unbalanced pension regime results in budget deficit of 31.3 percent the total pensions paid, and for progressive taxation it is as high as 39.2 percent.[18]

Having a balanced budget at the cost of volatile retirement benefits leads to approximately 24.6 percent decrease in absolute pensions (21.2 percent in pension/wage ratio) provided there is no technology growth. This is of particular interest as decrease of pension transfers is one of the possible parametric changes proposed as a remedy to population ageing in PAYG systems. Interestingly, while pensions decrease for all income classes in the flat-taxation regime, they actually increase (in absolute values) for those with lower than average

[18]Note, that also indebtedness levels are for illustration purposes rather than an estimate of the actual situation. In particular, the debt cannot be directly compared to e.g. aggregate output as it is quoted in different units. The lesson to be learnt is that the PAYG system would indeed require substantial external financing to preserve reasonable retirement benefits.

wage if we assume progressive taxation, although they effectively decrease for everyone when compared to real wages.

Table 5.3 contains simulation results of the aggregate variables, and Figures B.3 through B.5 in the Appendix exhibit comparison of capital accumulation per generation in the initial and the final steady states. There are just small differences in savings when we take different approaches to system balancing into account; capital accumulation is highest in the unbalanced pension scheme, as expected, and lowest if taxes adjust in order to keep pension budget balanced. Further, changes to technology growth have a decisive impact on the absolute level of savings. However, savings denoted in monetary units may be misleading if wages change accordingly - one must observe savings per unit of income (i.e. marginal propensity to savings) to get a reliable picture. When we look at the capital/wage ratio, it actually decreases the more the higher technology growth we assume. Like in the initial steady state, savings are lower in the progressive tax regime.

Analogously to absolute savings, output decreases in comparison to the initial steady state if there is no technology growth, which is in line with the overall decrease in population. Mild technology growth leads to a 11.5 percent rise in aggregate output on average, while strong growth increases output by 29.9 percent. To sum it up, pensions would be, according to my simulations and as compared to real wages, always lower in the future as a result of adverse demographics, no matter what technology level growth we assume. Moreover, as described in the next sections, the PAYG system falls behind in promotion of aggregate output in case of positive technological growth as well. The negative effect on future pensions can be partially mitigated through parametric changes, yet only at the cost of detrimental effects on the economy as a whole and also on inter-generation solidarity.

5.2.2 The Chilean Pension System

The fully-funded scheme is simulated using two sets of assumptions: standard - without market imperfections - and extended. Because the flaws basically decrease individual account balances, it is no surprise that retirement benefits under extended assumptions are lower than in the standard scenario. Pensions decrease by approximately 30 percent (see Table B.7 in the Appendix), which corresponds to the administrative costs parameter set up according to the previous findings of Murthi et al. (2001). The fact that pensions increase

Table 5.4: Chilean Pension System, Final Steady State Variables.

	Initial				Final			
	Flat		Progressive		Flat		Progressive	
	SA	EA	SA	EA	SA	EA	SA	EA
Interest Rate								
None					2,30%	2,53%	2,88%	3,20%
Mild	1,47%	1,60%	2,07%	2,27%	2,34%	2,57%	2,90%	3,21%
Strong					2,70%	2,98%	3,12%	3,46%
Wage								
None					1,461	1,402	1,406	1,348
Mild	1,551	1,490	1,484	1,425	2,328	2,235	2,244	2,152
Strong					3,629	3,483	3,534	3,389
Output								
None					0,898	0,808	0,865	0,777
Mild	0,954	0,859	0,912	0,822	1,061	0,955	1,023	0,920
Strong					1,228	1,105	1,196	1,075
Capital								
None					235,2	211,7	211,7	189,6
Mild	260,9	235,9	230,9	207,6	372,1	334,9	336,5	301,8
Strong					560,4	503,9	522,9	469,2
Capital/Wage								
None					161,0	151,0	150,6	140,7
Mild	168,2	158,3	155,7	145,7	159,8	149,9	149,9	140,3
Strong					154,4	144,7	148,0	138,5

SA = Standard Assumptions, *EA* = Extended Assumptions. Columns *Initial* and *Final* represent initial and final steady state, respectively. Rows represent variables and level of technology growth (*None/Mild/Strong*).

proportionally also points out that the other imperfection - irregularity of savings - does not have any significant effect on the level of savings.

The other outcomes are shown in Table 5.4. Interest rate and wage are again implicitly incorporated in output; the final steady state values are lower compared with the initial steady state, meaning that funded plans are not flawless alternatives resolving adverse demographics, as suggested by Barr (1979). Even though the Chilean pension scheme provides better absolute results compared with the PAYG scheme (wage, output, and savings are all higher), population changes actually have worse effects: output decreases by 5.9 percent in case of no technology progress, which is in fact 1.4 percentage point more than in case of the PAYG scheme even under the standard assumptions.

Taking administrative costs into account, output is lower than in the initial steady state by additional 9.4 percentage points. Positive productivity growth leads to absolute increase in output, but other variables decrease nonetheless;

savings exhibit analogous trend to output, decreasing by 9.9 percent in absolute value without technology growth, and by 4.3 percent relative to wages. As in the PAYG system, marginal propensity to saving drops with higher technology level growth.

We must keep in mind that pension systems are not only about aggregate output or impact on social security budget - public opinion is what truly matters and satisfactory pensions are therefore the prime objective. First, consider changes in absolute value of retirement benefits. As in the PAYG system, adverse demographics accompanied by no technological progress lead to overall decrease in pensions compared with the initial steady state, whereas even mild growth raise pensions above their initial level. Moreover, pensions change proportionally for all income classes, as opposite to the Czech and the Swedish scheme (see below), where the low-income groups are eventually better-off than the high-income groups. Yet, wages change accordingly to pensions again and so we shall rather analyse pension/wage ratio. The ratio decreases by 20 percent on average across all technology growth scenarios, which is, surprisingly, more than in the Czech pension system. The drop is even worse under the extended assumptions; the fully-funded scheme thus provides a better link between salary and the subsequent retirement benefits, as well as a better aggregate economy output, but it underperforms in relative indicators.

An important fact is that the model does not assume the expected effect of disproportion in size of workers to pensioners on asset returns suggested by Barr (2002) and Brooks (2000). That is, people in the Chilean pension scheme are worse-off due to internal mechanics of the economy only. Hence, if we additionally assume lower than optimal financial markets returns, the difference between the PAYG and the fully-funded scheme would be even bigger.

5.2.3 The Swedish Pension System

The Swedish pension system is simulated using the standard and extended assumptions as well, although the effect on aggregate variables is not as significant as in the fully-funded scheme thanks to the guarantee pension and different implementation of market imperfections. In particular, individual accounts in the defined contribution plan serve merely accounting purposes (the actual benefits are paid on unbalanced basis - the account balance is only imaginative) and so I included administrative costs only in the premium pension. Simulation results are in Table 5.5.

Table 5.5: Swedish Pension System, Final Steady State Variables.

	Initial				Final			
	Flat		Progressive		Flat		Progressive	
	SA	EA	SA	EA	SA	EA	SA	EA
Interest Rate								
None					3,65%	3,77%	4,32%	4,51%
Mild	2,89%	2,99%	3,63%	3,75%	3,52%	3,63%	4,25%	4,39%
Strong					3,62%	3,72%	4,25%	4,37%
Wage								
None					1,342	1,327	1,292	1,275
Mild	1,405	1,391	1,344	1,329	2,161	2,139	2,072	2,050
Strong					3,430	3,397	3,310	3,278
Output								
None					0,826	0,807	0,794	0,774
Mild	0,865	0,846	0,826	0,808	0,985	0,963	0,945	0,923
Strong					1,160	1,135	1,120	1,096
Capital								
None					187,0	181,2	166,2	161,1
Mild	199,4	194,0	174,6	169,5	303,0	294,9	269,3	261,7
Strong					476,4	464,6	431,7	420,7
Capital/Wage								
None					139,3	136,5	128,6	126,4
Mild	141,9	139,5	129,9	127,5	140,2	137,9	130,0	127,6
Strong					138,9	136,8	130,4	128,4

SA = Standard Assumptions, *EA* = Extended Assumptions. Columns *Initial* and *Final* represent initial and final steady state, respectively. Rows represent variables and level of technology growth (*None/Mild/Strong*).

The overall trend is similar to the previous simulations - aggregate output decreases compared with the initial steady state if there is no technological progress, and increases if there is. The same applies to savings in absolute values, while marginal propensity to saving slightly decreases. Let us now consider magnitude of the changes. In case of no growth, output decreases by 4.5 percent and absolute savings by 6.2 percent, but the capital/wage ratio drops by only 1.8 percent, whereas it decreases by 4.2 percent in the Chilean pension scheme and by 2.4 in the Czech system.

The alternative productivity growth assumptions yield surprisingly similar results as strong growth decreases capital/wage ratio only by additional 0.3 percentage points and mild growth actually increases the value by 0.6 percentage points. Extending the underlying assumptions to include financial market volatility and administrative costs, output and capital drop by approximately 2.5 percent across all scenarios.

Turning attention to retirement benefits shown in B.4 in the Appendix, we can see a 21 percent average drop in absolute pensions compared with the initial steady state values in flat taxation regime without technology growth, a 26 percent increase with mild growth, and a 100 percent increase with strong growth. The values for progressive tax regime are very similar (-20, 28, and 104 percent, respectively). The extended assumptions decrease pensions by 5.3 percent compared with the standard assumptions on average (both in the initial and the final steady state), the change is thus less significant than in the Chilean system. If we look at the results closely, there is a clear disproportion between income classes; while the poorest are nearly unaffected by the extension of assumptions as their retirement benefits consist largely of guarantee pension, wealthy households lose more in this respect.

The difference is also lower when we compare the pension/wage ratio; there is a 4.2 percent drop as a result of assumption change and a 18 percent drop between the initial and final steady state. Lastly, as opposed to the other variables, taxation does not vary across steady states or scenarios for neither the Chilean nor the Swedish pension system. Analogously to the initial steady state, pension budget is deficit as a result of accrued interest on contributions, although he level of deficit is slightly higher here due to the population changes.

5.2.4 Overall Results

Before I continue with the simulations of transition between the two steady states, let us sum up the findings thus far. In line with the previous findings, the Chilean fully-funded system indeed seems to perform best - at least from the aggregate perspective. It exhibits the highest output, savings, and wages as long as we assume no market imperfections. However, it causes immense social disparity in retirement as the wealthiest social class has almost 21 times higher pension benefits than the poorest. It is to be noted that the result is affected by non-existence of a safety net in my model, which would certainly need to be implemented in reality. The Swedish pension system offers the highest retirement benefits and alleviates poverty in retirement, but it can easily run into deficits for it is financed mostly on a PAYG basis and it underperforms in aggregate indicators. The Czech system substantially decreases social disparities and actually provides the highest aggregate output under the extended assumptions, but it underperforms overall, as discussed below.

Following up on the evaluation criteria proposed earlier, let us quantify the systems using a form of composite index defined as

$$CI = P_F \,/\, (1 + \frac{|P_F - P_I|}{P_I}) - D_F + 2.82\, O_F \,/\, (1 + \frac{|O_F - O_I|}{O_I}), \tag{5.2}$$

with F denoting the final steady state, I the initial steady state, P is sum of all retirement benefits paid, O is the aggregate output of the economy, and D is the external debt resulting from no adjustments made to balance pension budget.

Note, that because interest rate, wage, and capital accumulation are all implicitly included in the aggregate output (see section 5.1 for discussion), the index effectively takes all the macroeconomic outcomes of the model into consideration, as well as retirement benefits. Each factor is divided by its relative change between the initial and the final steady state, so that the index also reflects vulnerability of pension schemes to population changes. Finally, the constant is calculated as a ratio between the total retirement benefits across all the pension systems and the total aggregate output; it essentially puts equal weight on both factors. For clarity of results, I calculate it for the outcomes assuming flat taxation and no technology growth only; values for progressive taxation would be nearly identical and different levels of productivity growth lead to comparative increase in output and pensions, so the value would be similar too. The results are shown in Table 5.6.

Table 5.6: The Composite Index Comparison.

	Czech	**Swedish**	**Chilean**
PA	3.31	--	--
TA	3.82	--	--
NA	3.31	--	--
SA	--	4.54	3.85
EA	--	4.37	3.16

PA = Pensions Adjust, *TA* = Taxes Adjust,
NA = No Adjustment, *SA* = Standard Assumptions,
EA = Extended Assumptions.

Indeed, the Swedish pension system seems to be best performing overall, as it provides 14 percent higher value of the composite index than the Czech scheme and 38 percent higher value than the Chilean scheme even under the extended assumptions. Arguably, the results may be slightly biased due to higher social security tax rate assumed in the Swedish system, yet the effect should be minimal as it is mostly negated through lower output at the same time.

5.2.5 Pension Myths

As discussed above, the fully-funded scheme is far from being perfectly immune to population changes. In fact, retirement benefits in the Chilean scheme decrease in a similar manner to the Czech scheme because pensions are directly linked to other variables and the aggregate economic conditions worsen overall. The outcomes are the same no matter what grow of productivity we assume. Further, the situation is even worse if we take the extended assumptions into account. We may therefore disprove the myth even without taking Barr's theoretical explanation into account.

Regarding the second pension myth, savings truly increase following a fundamental pension reform - not only the Chilean pension system results in higher capital accumulation, we must also take retirement savings into account - but the increase is at the cost of extreme external indebtedness even under the assumption of no side costs of transition (see below). The amount of debt accumulated clearly outweighs long-term benefits of system change, implying its disadvantageousness.

5.3 Transition

Unlike in simulations of steady states, aggregate variables are no longer of particular importance here as we already know the overall differences between the respective pension schemes and, to answer whether it would be favourable to transition into another pension plan, we must particularly investigate the impacts on pension budget and generations living during the transition. In other words, we are interested in growth of government debt and changes in pensions.

The transitions are again considered under the two sets of assumptions: standard and extended, which include incompleteness of annuity markets and random financial market imperfections, including improbable market crashes and interest rate volatility. Recall, that interest rate is implicitly set within the model at the level of marginal product of labour and enters the model only as means of capital (i.e. savings) valorization. In reality, interest rates are primarily set by central banks according to their monetary policy, yet large proportion of savings is put directly into capital markets and its return thus depends only on asset returns, not on interest rates.

The two effects are simulated separately using random number generator; market crashes may occur in any period with 2 percent probability, i.e. if

$$X > N^{-1}(98), \tag{5.3}$$

where X is normally distributed pseudorandom number drawn from the standard normal distribution and $N^{-1}(98)$ is inverse cumulative distribution function corresponding to probability $p = 0.98$ and mean and standard deviation of standard normal distribution (i.e. $\mu = 0$ and $\sigma = 1$).

Interest rate fluctuates around its optimal level according to

$$r = 0.006\,X + r^{opt}, \tag{5.4}$$

where r^{opt} denotes the ideal interest rate set within the model and X is normally distributed pseudorandom number same as above.

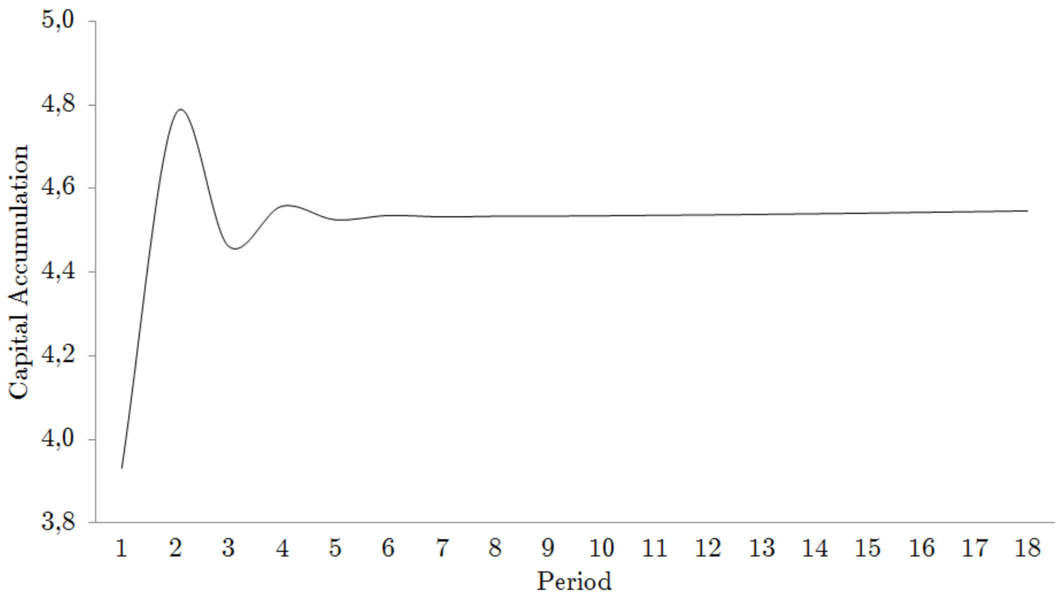
One obvious drawback of models with aggregate uncertainty is dependence of results on the particular outcome of random variables. In other words, it is virtually impossible to get the exactly same outcome twice and it is instead advisable to use additional methods, such as Monte Carlo simulation, in order to obtain precise results. Unfortunately, computation time does not permit it, so we must make conclusions based on one simulation only.

5.3.1 PAYG to Fully-Funded Scheme

The most radical form of a fundamental pension system reform is transition from a PAYG scheme to a fully-funded scheme. The major obstacle here is nonexistence of individual accounts at first; hence, people of work age at the time of transition are disadvantaged as they cannot save up money for the whole lives. Moreover, old pensioners are wholly dependent on government to look after them, without any income coming from contemporaneous taxpayers as all of the contributions go directly to individual accounts. Assuming that people work for 40 years and are in retirement for another 20 years, the government must finance pensions from internal resources for 59 years after the transition so that people are not worse-off. In order to prevent public outrages, I assume that the government would top up retirement benefits at least to the their original level.

Let us now consider no uncertainty or market imperfections and focus on readjustment of the observed variables only. Because the number of observed periods is large enough to prevent precipitous shifts in values and to allow a slow adaptation, the last period of transition always corresponds to the final steady state. Further, the transition algorithm assumes that all households are rational and know precisely what steady state the economy approaches and thus behave accordingly, which results in small gradual changes each period as soon as the newly introduced pension system fully takes its place. Hence, it is uninteresting to observe the whole transition period because from some point on the variables just linearly adjust to the steady state values. That can also be seen from Figure 5.2; savings shoot up in the first period after transition, which corresponds to findings in the previous section. However, the adjustment is not optimal initially and it takes six periods in total for the economy to adapt to the new pension system. Subsequently, the economy stabilizes and slowly approaches the new optimum.[19]

Figure 5.2: Transition from the PAYG to the Fully-Funded Scheme, Standard Assumptions



[19]Needless to say, the mutual relationship between output variables implies a similar development in the other characteristics - output, interest rate, wage, and pensions - as well.

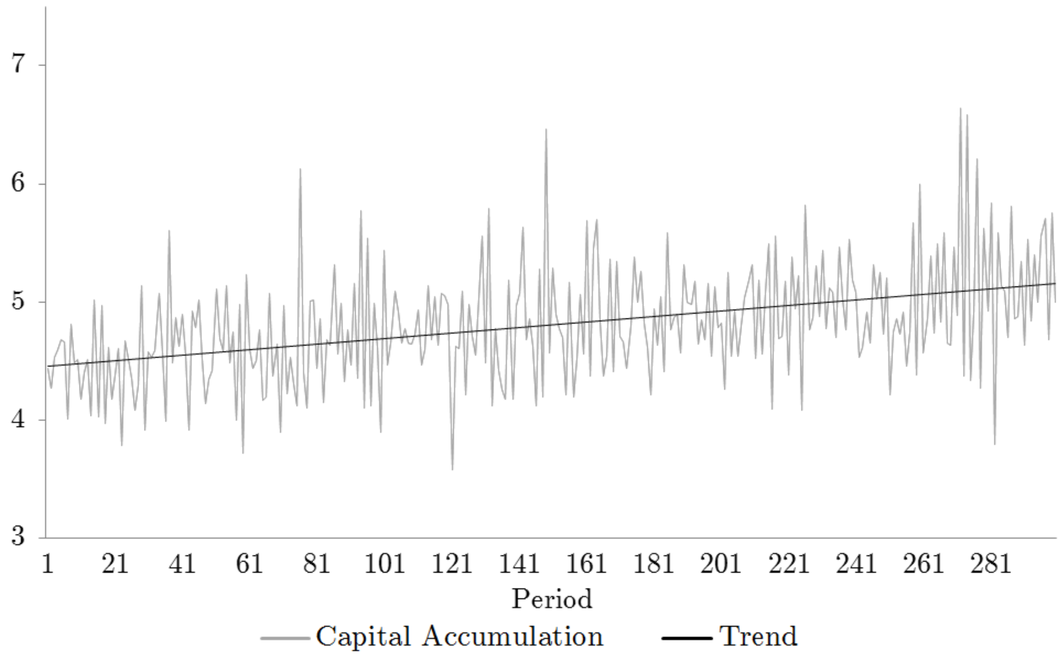
The main focus is then primarily on debt and its growth throughout the transition. Output and debt are not directly comparable in an analogy to debt-to-GDP ratio as they are denoted in different units, although we are able to compare indebtedness with pensions at least. The PAYG system is always assumed to be balanced initially, which implies no external debt in the initial steady state. Total debt accumulated within the first 60 periods after introduction of the fully-funded scheme (that is, during the time that government must finance retirement benefits from the general budget) is approximately equal to 32 times the total amount of pensions paid within a single period in the initial steady state with the Chilean plan. It is improbable that the budget deficit would reach such heights in reality since the government would likely finance the transition from general taxes and cut expenses elsewhere, yet the amount is enormous; for instance, the annual social security expenses in the Czech Republic are currently, as of 2014, equal to approximately 10 percent of GDP. That is, transition towards a fully funded system would cost ca 3 times the GDP of the whole country.

The second transition scenario assumes market imperfections and flat taxation. The main purpose of this simulation is a form of sensitivity analysis of the transition to economic conditions. Essentially, fluctuation of interest rate around its optimal level prevents the economy from setting on the optimal transition path; instead, in each period, households choose consumption and savings according to the current economic conditions, yet with the final steady state optimum as the ultimate goal. The resulting level of aggregate savings is depicted in Figure 5.3.

The actual data are a supplemented by a linear trend for easier examination of the actual adjustment. We can see that even though interest rate does not in fact change by more than 1 percent in most of the periods, it has a distinctive impact on capital accumulation and thus also on the output. Similar volatility in other variables - wages, pensions, labour input - would have analogous results.

The model assumes that interest rate on government debt is equal to interest rate on capital. External indebtedness is slightly higher than in the previous case, particularly because the government needs to finance constant retirement benefits despite fluctuations in the economy. In both scenarios, wealthier people soon (from period 18 on) do not need external financing of retirement benefits as they are able to save up sufficient funds to obtain higher pension than in the

Figure 5.3: Transition from the PAYG to the Fully-Funded Scheme, Extended Assumptions



PAYG system. Low-income classes, on the other hand, want to be subsidized for as long as possible; the pure funded pension would be much lower due to non-existence of a safety net.

5.3.2 PAYG to Multipillar Scheme

Let us recapitulate the two distinctive differences from the previous case. First, the Czech and the Swedish pension schemes are essentially unfunded - the guarantee pension is independent of social security budget and the premium pension serves rather as an addition to the dominant first tier - and vary only in the method of pension entitlement calculation. Consequently, costs of transition are notably lower because the government need not to cover pension expenses during the first years after introduction of alternative pension plan. Second, individual account balances in the model grow at rate equal to the optimum interest rate, not according to the volatile interest rate, to reflect the real-world situation.[20] In the Chilean scheme, on the contrary, balances grow at

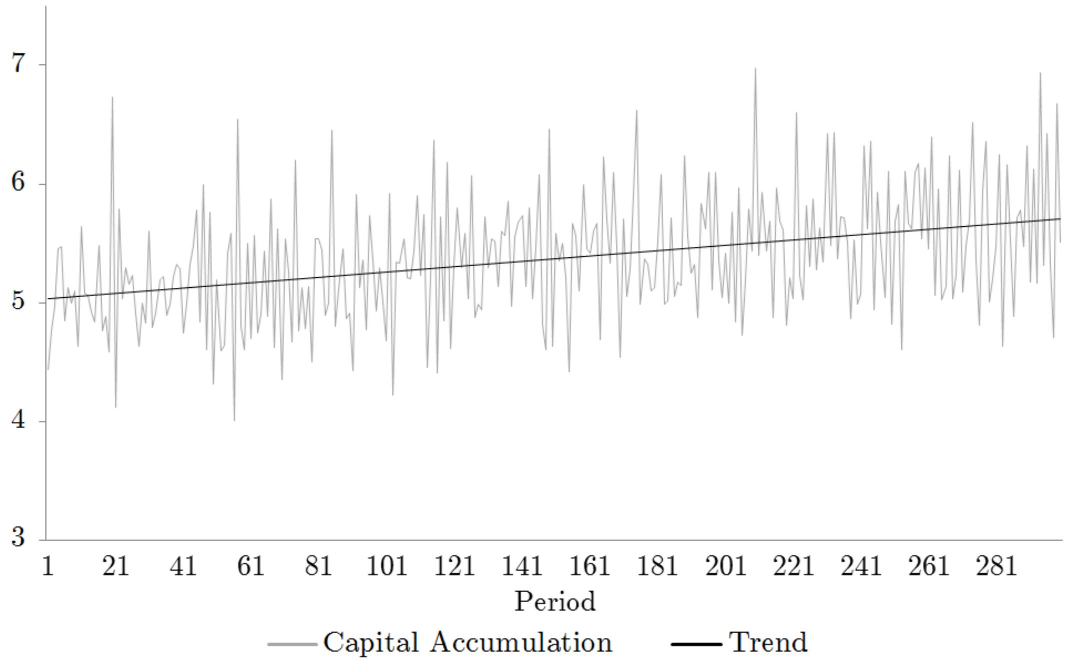
[20]Recall, that individual account balances in Sweden grow by a non-negative rate dependent on the overall state of the pension system. In case of unfavourable conditions, the balance ratio falls below one and indexation decelerates below the growth in real wages. Analogously, in good times, indexation may be higher. Because my model does not assume

the market interest rate.

Just as in the prior example, the transition is simulated twice - with and without market volatility included in the model. Unsurprisingly, the former transition is resembles the first simulation of transition between PAYG and fully-funded scheme; again, the model assumes enough periods for the economy to gradually and linearly adjust to the new optimum thanks to rationality of households and no aggregate uncertainty.

Likewise, if we include aggregate uncertainty in the form of volatile interest rate, the actual transition resembles the second simulation above (see Figure 5.4).

Figure 5.4: Transition from the PAYG to the Multipillar Scheme, Extended Assumptions

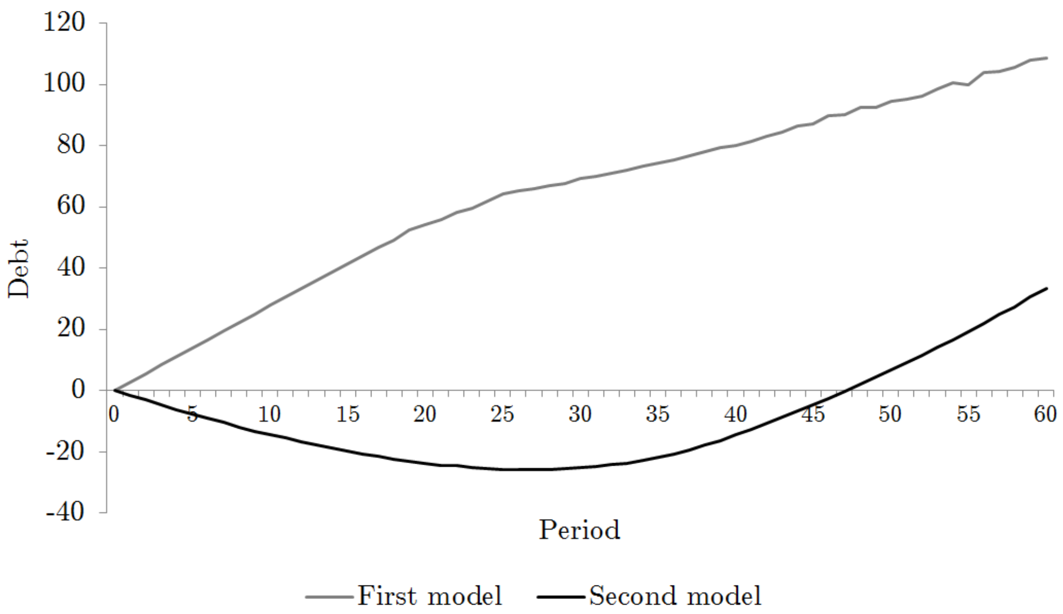


Generally, in periods 2 to 21 pensioners only obtain retirement benefits at the level of the pre-existing PAYG system because they cannot save at all. For several additional periods, most of the people still obtain the old pensions as they are unable to save up sufficient amount of funds. For all this time, pension system is actually in a minor *surplus* as there is higher social security tax imposed on the population since period 2, yet retirement benefits remain at their original level. However, since period 30 the expenses exceed revenues

such a mechanism, I let individual account balances to grow by the optimum aggregate interest rate.

and pension budget slips into deficits. Overall, the explicit government debt accumulated within the first 60 periods is about 70 percent lower than in the case of transition to a fully-funded scheme, although the number may be slightly imprecise due to the disturbance effect related to the change in social security tax rate. For an overview of the government debt, see Figure 5.5.

Figure 5.5: Government Indebtedness as a Result of Pension Reform.



Chapter 6

Concluding Remarks

One aspect of any economic simulations is certain - they are never perfect. OLG models serve as a priceless tool of modelling the economic reality, yet they can always be improved further to provide more precise results. My model is a step forward in this regard as it puts together models used in the prior literature and further enhances them in a functional unit. And it is the versatility what I consider the most valuable; the model can be used for sensitivity analysis, predictions, assessment of policy implications, and much more. In this section, I first discuss the main policy implications of my analysis and then summarize the work with additional ideas and possible improvements to be investigated further.

6.1 Main Policy Implications

In a sense, there is no other known way how to organize a social security system than using some form of the three basic models presented earlier, except for letting children to take care of their elders, which would be nearly impossible to set up. Hence, we must make do with what is available.

According to my analysis, introduction of a fully-funded scheme would be the most advantageous in long-run as it provides ca 6.1 percent higher steady state output than the PAYG system and a 10.2 percent higher output than the multipillar system (see Table 5.1). The results are slightly skewed due to higher social security tax assumed in the multipillar scheme (i.e. it shows lower output and higher pensions), yet the main implication remains unchanged as the difference in output is disproportionally larger than the difference in taxes.

The rankings remain unchanged even when we take adverse demographics

- population ageing and shrinking - into account, although the differences are smaller (5.6 percent and 8.7 percent, respectively). But, as soon as we assume market imperfections (administrative costs, unfair annuity markets, and volatile financial markets), the fully-funded scheme no longer provides the best results and is replaced by the unfunded, which yields 5.2 percent and 5.3 percent higher output than the funded and the multipillar schemes, respectively. The results also imply that there is essentially no difference in output provided by the fully-funded and the multipillar scheme. For a summary of the final steady state aggregate output results, see Table 6.1.

Table 6.1: Final Steady State Aggregate Output.

	Czech	**Swedish**	**Chilean**
PA	0.850	--	--
TA	0.853	--	--
NA	0.851	--	--
SA	--	0.826	0.898
EA	--	0.807	0.808

PA = Pensions Adjust, *TA* = Taxes Adjust,
NA = No Adjustment, *SA* = Standard Assumptions,
EA = Extended Assumptions.

The situation is quite opposite regarding retirement benefits. While the fully-funded system provides substantially higher pensions for wealthy classes thanks to the actuarially fair system of annuity calculation, low-income classes must be subsidized by the government. In fact, while wealthy households obtain equal pensions in the multipillar and the fully-funded systems, the poor obtain three times higher pensions in the multipillar system. Advantageousness of the pension systems thus primarily depends on preferences - the Czech PAYG system yields the lowest retirement benefits in total, but it greatly reduces income inequality among households, while the other two systems yield higher pensions at the cost of lower relative pensions for the poorest. Further, the Chilean pension system essentially overcomes the Swedish plan in a sense that it can yield the same retirement benefits if it had a Swedish-like safety net introduced (since pensions are calculated in the same way) and it shows better macroeconomic results. An overview of the total retirement benefits offered by the respective pension systems is shown in Table 6.2.

Note, that the high pension benefits in the Czech scheme under the assumption of no adjustment or adjustment in taxes are at the cost of explicit

Table 6.2: Final Steady State Total Retirement Benefits.

	Czech	Swedish	Chilean
PA	1.293	--	--
TA	1.621	--	--
NA	1.621	--	--
SA	--	2.813	1.808
EA	--	2.664	1.272

PA = Pensions Adjust, *TA* = Taxes Adjust,
NA = No Adjustment, *SA* = Standard Assumptions,
EA = Extended Assumptions.

government debt and higher social security tax, respectively. Moreover the costs of a transition from any primarily unfunded plan to the Chilean pension system are so high that it would take many decades for the investment to actually return (see Figure 5.5 for a summary for explicit debt resulting from fundamental pension reforms). For additional detailed information on pension distribution and taxation refer to tables in the Appendix B.

The overall results are in line with findings of e.g. Oksanen (2009) and also confirm the theoretical suggestions of Barr (2002). Essentially, emphasis on private savings indeed promotes aggregate growth, but funded schemes cannot implicitly serve as a remedy for population ageing.

Let us now briefly discuss one previously partially neglected aspect of funded systems - dependency on financial markets performance. It is clear from the model's specifications that just as individual account balances grow with positive return on financial markets, characterized by the unified interest rate in the model for the sake of simplicity, any drop would result in an immediate decrease in the balance and thus a proportional change in prospective retirement benefits. Although it may sound as an unrealistically unsophisticated and implausible description of the reality, it is, in fact, quite accurate. In particular, data from the Chilean Pension Supervisor (Superintendencia de Pensiones, www.safp.cl) show about a 65 percent correlation between capital market returns characterized by broad equity indices and individual account balance used for pension calculation.

The relationship explicitly depends on selected investment portfolio, its allocation into equities and bonds, diversification etc., but the message remains the same: funded pension systems do not provide significantly better protection against demographic changes and are substantially riskier than unfunded schemes. For instance, the average account balance of a retiring worker in Chile dropped by 40 percent between 2008 and 2009 as a result of the fall on financial markets. Now, taking the theoretical model proposed by Brooks (2000) into account, we may expect considerable fall in durable assets' returns as a consequence of disproportions in size of working and retired generations. Hence, not only funded systems provide but a little social protections for prospective pensioners, but the simulation results may also overestimate their actual outcomes in case the theoretical model is right.

The main conclusion to be made of my analysis is that while some pension systems provide better partial results than others, there is neither a dominant nor a dominated system overall. Therefore, there cannot be a single best policy implication to be made; instead, each country should adopt a system that would best fit its demographic, financial and structural characteristics. And since transition to a different system would be extremely costly, it may as well be appropriate to introduce parametric changes only. At the same time, any prospective introduction of a (partially) funded scheme must be considered from the perspective of its degree of risk. That is for the politics, though. The conclusion for researchers is rather that none of the systems fares well facing the population changes to come. The question is then if the systems or other institutions can be improved in some way, or if there is simply nothing to be done at all.

6.2 Conclusion

Many authors have recently dealt with rising pension spending in developed countries, threat of pensioner poverty, unsustainability of pension systems, and other related issues stemming from population ageing and inability to exploit the demographic dividend. Despite the number of questions to be answered and approaches used, there has been but a little agreement on the actual results; while it is clear that substantial changes must be made to the current systems so as not to run into extensive budget deficits, more evidence must be found in favour of any of the pension alternatives. As empirical research lacks appropriate observations to draw conclusions upon, numerical simulations of theoretical pension frameworks serve as convenient and promising way to study pension systems, especially due to rapid development of computing technologies.

In his work Oksanen (2009) notes that large-scale simulation models have received some negative responses regarding unrealistic assumptions or results due to their complexity and level of microeconomic relationships that cannot be covered in such complicated models. It is without a doubt that simple models are better in this regard and offer better results when used to explain a specific questions. Researchers should not be discouraged from using complex models though, as only these may bring us closer to understanding of the real world. It is just necessary to have a critical look at the results and to check their transparency and accuracy.

My primary aim was to build a computable theoretical model of pension system that would account for as many microeconomic relationships that affect social security systems as possible. To do that, I presented a considerably complex OLG model based on real-world data and projections. Specifically, the model replicates the Czech, Swedish, and the Chilean pension system with respective links between pension transfers and salary, social security taxation, and so on. The model assumes demographic changes - population ageing and decrease in fertility - using long-term projections for the Czech Republic, flat or progressive taxation, and idiosyncratic shocks to productivity together with various income classes.

To address drawbacks of research done in the prior literature, I build upon the findings of Murthi et al. (2001) and include market imperfections, such as incompleteness of annuity markets and administrative costs related to savings in private funds. In the traditional view, funded pension systems should provide vastly better result facing population changes than PAYG alternatives. However, under the extended assumptions of imperfect markets, privatization of social security system does not seem to be a viable option even when not considering transformation costs. My model presents a comparison of a pure PAYG, multipillar, and a fully-funded scheme to shed more light on the issue.

As discussed in the previous section, the simulations show certain strengths and weaknesses of each pension system, without allowing for a single conclusion to be made, which is in line with the prior literature. However, we may deduce that the multipillar system is indeed the best option for the following reasons. First, it is extremely adjustable to country-specific needs, principally in ability to tune up importance of specific pillars. Second, it provides the best results with respect to the composite index presented in Chapter 5 that puts into contrast the aggregate output, total retirement benefits, and vulnerability to population changes in a single comprehensive indicator (see Table 5.6 for the results). Finally, the multipillar scheme faces risks of poor macroeconomic performance resulting in low tax revenues and consequent budget deficits, poor performance of financial markets resulting in volatility of retirement benefits, and also population changes; yet it can end up substantially less risky than any of the two other schemes if the risks are diversified appropriately.

It is without a doubt that the model is far from being truly comprehensive and would require implementation of more details in order to provide precise estimations of the future. One should particularly keep in mind technological limitations. Computation of the full model used in this work with transition path takes more than a week in the current state and each improvement is likely to multiply rather than add to the time demand. One way to circumvent these challenges is to rewrite the program so that it would enable the computer to use its full potential; choosing between having a simpler code that can only be run on one processor core or complex code that can be run on all cores simultaneously, I decided to opt for the former option. A faster code can greatly speed up the computing process so the model can further be enhanced without excessive simulation time demands.

There are particular issues to be investigated further. First, some exogenous variables, such as labour supply, can be effortlessly substituted by endogenous processes as long as computation time does not constitute a problem. Another straightforward improvement would be broader assumption of uncertainty, namely in maximum life span that would allow true simulation of population ageing, economic growth that would depend on economic cycles, or, for instance, unintentional bequests in case of unexpected death. The model can also be enhanced by inclusion of banking sector offering loans and thus negating the budget constraint on households, or opening the economy to international trade. Second, it would be interesting to model investment opportunities and decisions in a funded pension scheme in a greater detail. Specifically, people are often allowed to have individual accounts in a variety of pension funds at once, to choose a one-off payment of savings upon retirement rather than lifetime annuity, or to save additional resources in the third, voluntary pillar. Finally, unfunded pension schemes are vulnerable to economic shocks, their analysis should therefore include business cycles or other details resulting in fluctuations of pension system revenues.

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Appendix A

Algorithms

A.1 Markov Chain Approximation

The genuine numerically approximation of the autoregressive process of order one algorithm was developed by Tauchen (1986). I only present its main features that are included in my script and kindly ask the reader to refer to Tauchen (1986) for additional information. First, the algorithm requires a grid of equidistant points $z_1 < z_2, ..., < z_m$ that are obtained by from a continuous interval ranging between a chosen multiple of the standard deviation of the process. Each of these points can be assigned a probability of occurrence by integrating over the area below the normal distribution. Formally, the probability of an event z being in the interval $[z_j - x, z_j + x]$ is

$$P(z_j - x \leq z \leq z_j + x) = \pi(z_j + x) - \pi(z_j - x), \tag{A.1}$$

where π denotes the cumulative distribution function of the normal distribution and x is the midpoint between two adjacent grid points. Note, that in the case of z_1, the second term is 0, and because the grid points lie evenly distributed interval with midpoint equal to mean (i.e. the maximum of symmetrical normal distribution), the probability of occurrence of z_1 and z_m is equal, that is $p(z_1) = p(z_m)$. Similarly, the probability of transition from state z_i in period t to state z_j in period $t + 1$ is given by

$$p(z_j|z_i) = \pi(\frac{z_j - \rho z_i + x}{\sigma_\epsilon}) - \pi(\frac{z_j - \rho z_i - x}{\sigma_\epsilon}) \tag{A.2}$$

Having a set of grid points, it is straightforward to compute transition probabilities; the algorithm first computes the midpoint between two adjacent grid points as a half of the distance between them (recall that the points are evenly distributed, so the distance is always the same) and then, using two *for* cycles, computes transition probabilities for every grid point using (A.2). The result is a transition matrix of dimension $n \cdot n$, where n is the number of grid points.

A.2 Golden Section Search Algorithm

An effective way to locate the extremum of a single peaked function on given interval is to repeatedly shrink the range of values around the assumed peak point. The technique derives its name from the search method: the algorithm always assumes triples of points whose distances form a golden ratio.

Consider a continuous and unimodal function f over interval $I = [A, D]$ (i.e. it acquires a single extrema on the interval). Choose points B, C such that $B = pA + (1-p)D$ and $C = (1-p)A + pD$, while $p = (\sqrt{5}-1)/2$ is the golden section parameter that ensures an efficient selection of A and B. Consequently, I may be divided into two sub-intervals $[A, C]$ and $[B, D]$, where $B \in [A, C]$ and $C \in [B, D]$. Evaluating the function in all four points, we may narrow I to either of the two sub-intervals, depending on whether $f(B) > f(C)$ or $f(B) < f(C)$. This way, the interval can be shrunk in successive iterations, always by a constant fraction $p = (\sqrt{5}-1)/2$. The algorithm stops when the interval is sufficiently narrow.

Appendix B

Tables and Figures

Figure B.1: Survival Probabilities.

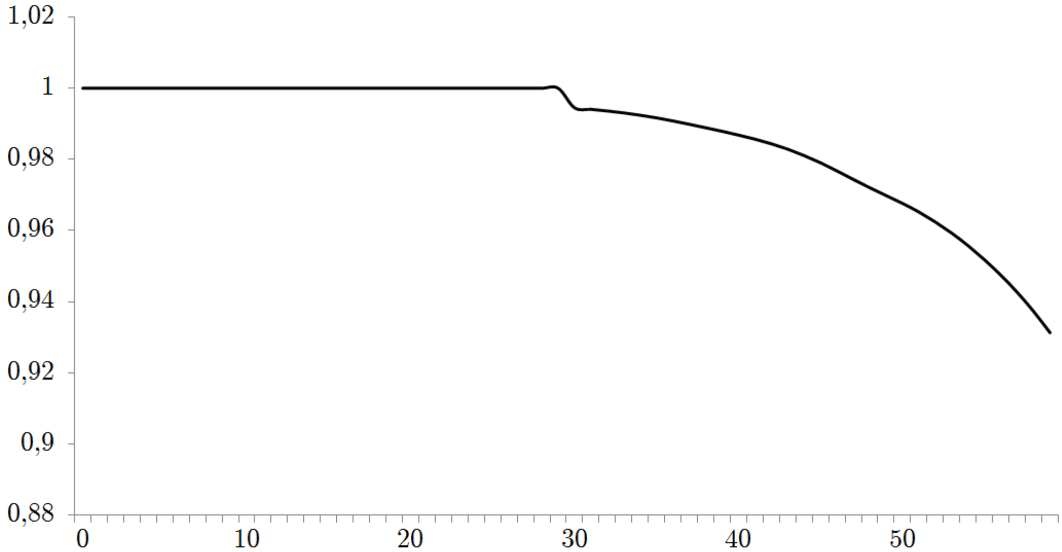


Table B.1: Pensions, Czech Pension System.

	Initial		Final					
	Flat	Prog.	Flat			Progressive		
			PA	TA	NA	PA	TA	NA
None								
1	0,081	0,091	0,061	0,077	0,077	0,069	0,088	0,087
2	0,119	0,127	0,090	0,112	0,112	0,096	0,122	0,121
3	0,134	0,139	0,101	0,126	0,126	0,106	0,134	0,133
4	0,155	0,157	0,117	0,147	0,147	0,119	0,151	0,150
5	0,177	0,174	0,134	0,167	0,167	0,132	0,168	0,166
6	0,210	0,197	0,158	0,198	0,198	0,150	0,190	0,188
7	0,257	0,229	0,194	0,243	0,243	0,175	0,221	0,219
8	0,277	0,243	0,209	0,262	0,262	0,186	0,235	0,233
9	0,306	0,263	0,231	0,289	0,289	0,200	0,254	0,251
Mild								
1	0,081	0,091	0,099	0,122	0,123	0,111	0,139	0,139
2	0,119	0,127	0,144	0,179	0,180	0,154	0,194	0,193
3	0,134	0,139	0,162	0,201	0,203	0,170	0,213	0,212
4	0,155	0,157	0,188	0,233	0,235	0,191	0,240	0,239
5	0,177	0,174	0,215	0,267	0,269	0,212	0,266	0,265
6	0,210	0,197	0,254	0,316	0,318	0,240	0,302	0,301
7	0,257	0,229	0,311	0,387	0,390	0,280	0,351	0,350
8	0,277	0,243	0,336	0,417	0,420	0,297	0,372	0,372
9	0,306	0,263	0,371	0,461	0,465	0,321	0,402	0,402
Strong								
1	0,081	0,091	0,155	0,192	0,194	0,176	0,220	0,221
2	0,119	0,127	0,226	0,281	0,284	0,246	0,307	0,308
3	0,134	0,139	0,255	0,316	0,319	0,270	0,337	0,339
4	0,155	0,157	0,296	0,367	0,371	0,304	0,379	0,382
5	0,177	0,174	0,338	0,419	0,423	0,338	0,421	0,423
6	0,210	0,197	0,400	0,496	0,501	0,383	0,477	0,480
7	0,257	0,229	0,490	0,607	0,614	0,446	0,555	0,559
8	0,277	0,243	0,528	0,655	0,662	0,473	0,590	0,593
9	0,306	0,263	0,584	0,724	0,731	0,511	0,637	0,640

PA = Pensions Adjust, *TA* = Taxes Adjust, *NA* = No Adjustment. Columns *Initial* and *Final* represent initial and final steady state, respectively. Rows represent variables and level of technology growth (*None/Mild/Strong*)

Table B.2: Taxation, Czech Pension System.

	Initial		Final					
	Flat	Prog.		Flat			Progressive	
			PA	TA	NA	PA	TA	NA
None								
1	27,3%	0,124	27,3%	27,4%	27,3%	12,4%	12,5%	12,4%
2	27,3%	0,168	27,3%	27,4%	27,3%	16,8%	17,0%	16,8%
3	27,3%	0,230	27,3%	27,4%	27,3%	23,0%	23,1%	23,0%
4	27,3%	0,282	27,3%	27,4%	27,3%	28,2%	28,3%	28,2%
5	27,3%	0,330	27,3%	27,4%	27,3%	33,0%	33,1%	33,0%
6	27,3%	0,375	27,3%	27,4%	27,3%	37,5%	37,7%	37,5%
7	27,3%	0,419	27,3%	27,4%	27,3%	41,9%	42,0%	41,9%
8	29,5%	0,458	29,5%	29,6%	29,5%	45,8%	46,0%	45,8%
9	31,0%	0,495	31,0%	31,2%	31,0%	49,5%	49,6%	49,5%
Mild								
1	27,3%	0,124	27,3%	30,5%	27,3%	12,4%	12,5%	12,4%
2	27,3%	0,168	27,3%	30,5%	27,3%	16,8%	17,0%	16,8%
3	27,3%	0,230	27,3%	30,5%	27,3%	23,0%	23,1%	23,0%
4	27,3%	0,282	27,3%	30,5%	27,3%	28,2%	28,3%	28,2%
5	27,3%	0,330	27,3%	30,5%	27,3%	33,0%	33,1%	33,0%
6	27,3%	0,375	27,3%	30,5%	27,3%	37,5%	37,7%	37,5%
7	27,3%	0,419	27,3%	30,5%	27,3%	41,9%	42,0%	41,9%
8	29,5%	0,458	29,5%	32,8%	29,5%	45,8%	46,0%	45,8%
9	31,0%	0,495	31,0%	34,3%	31,0%	49,5%	49,6%	49,5%
Strong								
1	27,3%	0,124	27,3%	33,7%	27,3%	12,4%	15,7%	12,4%
2	27,3%	0,168	27,3%	33,7%	27,3%	16,8%	20,1%	16,8%
3	27,3%	0,230	27,3%	33,7%	27,3%	23,0%	26,3%	23,0%
4	27,3%	0,282	27,3%	33,7%	27,3%	28,2%	31,4%	28,2%
5	27,3%	0,330	27,3%	33,7%	27,3%	33,0%	36,3%	33,0%
6	27,3%	0,375	27,3%	33,7%	27,3%	37,5%	40,8%	37,5%
7	27,3%	0,419	27,3%	33,7%	27,3%	41,9%	45,2%	41,9%
8	29,5%	0,458	29,5%	35,9%	29,5%	45,8%	49,1%	45,8%
9	31,0%	0,495	31,0%	37,4%	31,0%	49,5%	52,8%	49,5%

PA = Pensions Adjust, *TA* = Taxes Adjust, *NA* = No Adjustment. Columns *Initial* and *Final* represent initial and final steady state, respectively. Rows represent variables and level of technology growth (*None/Mild/Strong*)

Table B.3: Pensions/Wage, Czech Pension System.

	Initial		**Final**					
	Flat	**Prog.**	**Flat**			**Progressive**		
			PA	TA	NA	PA	TA	NA
None								
1	0,056	0,065	0,044	0,056	0,056	0,052	0,065	0,065
2	0,081	0,091	0,065	0,081	0,081	0,073	0,091	0,091
3	0,091	0,100	0,073	0,091	0,091	0,080	0,100	0,100
4	0,106	0,113	0,085	0,106	0,106	0,090	0,113	0,113
5	0,121	0,125	0,097	0,121	0,121	0,100	0,125	0,125
6	0,143	0,142	0,114	0,143	0,143	0,113	0,142	0,142
7	0,176	0,165	0,140	0,176	0,176	0,132	0,165	0,165
8	0,189	0,176	0,151	0,189	0,189	0,140	0,176	0,176
9	0,209	0,190	0,167	0,209	0,209	0,151	0,190	0,190
Mild								
1	0,056	0,065	0,044	0,056	0,056	0,052	0,065	0,065
2	0,081	0,091	0,065	0,081	0,081	0,073	0,091	0,091
3	0,091	0,100	0,073	0,091	0,091	0,080	0,100	0,100
4	0,106	0,113	0,085	0,106	0,106	0,090	0,113	0,113
5	0,121	0,125	0,097	0,121	0,121	0,100	0,125	0,125
6	0,143	0,142	0,114	0,143	0,143	0,113	0,142	0,142
7	0,176	0,165	0,140	0,176	0,176	0,132	0,165	0,165
8	0,189	0,176	0,151	0,189	0,189	0,140	0,176	0,176
9	0,209	0,190	0,167	0,209	0,209	0,151	0,190	0,190
Strong								
1	0,056	0,065	0,044	0,056	0,056	0,052	0,065	0,065
2	0,081	0,091	0,065	0,081	0,081	0,073	0,091	0,091
3	0,091	0,100	0,073	0,091	0,091	0,080	0,100	0,100
4	0,106	0,113	0,085	0,106	0,106	0,090	0,113	0,113
5	0,121	0,125	0,097	0,121	0,121	0,100	0,125	0,125
6	0,143	0,142	0,114	0,143	0,143	0,113	0,142	0,142
7	0,176	0,165	0,140	0,176	0,176	0,132	0,165	0,165
8	0,189	0,176	0,151	0,189	0,189	0,140	0,176	0,176
9	0,209	0,190	0,167	0,209	0,209	0,151	0,190	0,190

PA = Pensions Adjust, *TA* = Taxes Adjust, *NA* = No Adjustment. Columns *Initial* and *Final* represent initial and final steady state, respectively. Rows represent variables and level of technology growth (*None/Mild/Strong*)

Table B.4: Pensions, Swedish pension system.

	Initial				**Final**			
	Flat		**Progressive**		**Flat**		**Progressive**	
	SA	EA	SA	EA	SA	EA	SA	EA
None								
1	0,150	0,147	0,146	0,142	0,141	0,138	0,138	0,134
2	0,159	0,154	0,163	0,158	0,145	0,141	0,142	0,138
3	0,187	0,181	0,196	0,188	0,158	0,153	0,164	0,158
4	0,229	0,220	0,243	0,232	0,188	0,181	0,199	0,190
5	0,290	0,276	0,313	0,297	0,232	0,222	0,249	0,236
6	0,379	0,359	0,415	0,391	0,297	0,281	0,323	0,305
7	0,510	0,480	0,563	0,529	0,391	0,369	0,431	0,404
8	0,700	0,657	0,781	0,730	0,529	0,496	0,589	0,550
9	0,978	0,916	1,098	1,025	0,731	0,683	0,820	0,763
Mild								
1	0,150	0,147	0,146	0,142	0,227	0,223	0,221	0,216
2	0,159	0,154	0,163	0,158	0,233	0,227	0,228	0,221
3	0,188	0,181	0,196	0,188	0,254	0,246	0,263	0,253
4	0,229	0,220	0,244	0,233	0,302	0,291	0,318	0,305
5	0,291	0,277	0,314	0,297	0,373	0,357	0,399	0,380
6	0,380	0,360	0,416	0,392	0,477	0,453	0,518	0,489
7	0,511	0,481	0,564	0,530	0,629	0,593	0,691	0,649
8	0,701	0,658	0,782	0,732	0,850	0,798	0,944	0,883
9	0,980	0,917	1,099	1,026	1,173	1,097	1,313	1,224
Strong								
1	0,151	0,147	0,147	0,142	0,361	0,354	0,353	0,345
2	0,159	0,154	0,164	0,158	0,370	0,361	0,365	0,353
3	0,188	0,181	0,196	0,189	0,403	0,391	0,420	0,405
4	0,230	0,220	0,244	0,233	0,481	0,462	0,508	0,487
5	0,291	0,277	0,314	0,298	0,594	0,567	0,638	0,607
6	0,381	0,360	0,416	0,393	0,759	0,720	0,827	0,782
7	0,511	0,482	0,565	0,531	1,000	0,943	1,104	1,037
8	0,702	0,659	0,783	0,733	1,352	1,269	1,507	1,411
9	0,981	0,919	1,101	1,028	1,866	1,745	2,097	1,956

SA = Standard Assumptions, *EA* = Extended Assumptions. Columns *Initial* and *Final* represent initial and final steady state, respectively. Rows represent variables and level of technology growth (*None/Mild/Strong*)

Table B.5: Taxation, Swedish Pension System.

	Initial				**Final**			
	Flat		**Progressive**		**Flat**		**Progressive**	
	SA	EA	SA	EA	SA	EA	SA	EA
None								
1	33,5%	33,5%	18,5%	18,5%	33,5%	33,5%	18,5%	18,5%
2	33,5%	33,5%	22,9%	22,9%	33,5%	33,5%	22,9%	22,9%
3	33,5%	33,5%	29,1%	29,1%	33,5%	33,5%	29,1%	29,1%
4	33,5%	33,5%	34,3%	34,3%	33,5%	33,5%	34,3%	34,3%
5	33,5%	33,5%	39,1%	39,1%	33,5%	33,5%	39,1%	39,1%
6	33,5%	33,5%	43,6%	43,6%	33,5%	33,5%	43,6%	43,6%
7	33,5%	33,5%	48,0%	48,0%	33,5%	33,5%	48,0%	48,0%
8	35,7%	35,7%	51,9%	51,9%	35,7%	35,7%	51,9%	51,9%
9	37,2%	37,2%	55,6%	55,6%	37,2%	37,2%	55,6%	55,6%
Mild								
1	33,5%	33,5%	18,5%	18,5%	33,5%	33,5%	18,5%	18,5%
2	33,5%	33,5%	22,9%	22,9%	33,5%	33,5%	22,9%	22,9%
3	33,5%	33,5%	29,1%	29,1%	33,5%	33,5%	29,1%	29,1%
4	33,5%	33,5%	34,3%	34,3%	33,5%	33,5%	34,3%	34,3%
5	33,5%	33,5%	39,1%	39,1%	33,5%	33,5%	39,1%	39,1%
6	33,5%	33,5%	43,6%	43,6%	33,5%	33,5%	43,6%	43,6%
7	33,5%	33,5%	48,0%	48,0%	33,5%	33,5%	48,0%	48,0%
8	35,7%	35,7%	51,9%	51,9%	35,7%	35,7%	51,9%	51,9%
9	37,2%	37,2%	55,6%	55,6%	37,2%	37,2%	55,6%	55,6%
Strong								
1	33,5%	33,5%	18,5%	18,5%	33,5%	33,5%	18,5%	18,5%
2	33,5%	33,5%	22,9%	22,9%	33,5%	33,5%	22,9%	22,9%
3	33,5%	33,5%	29,1%	29,1%	33,5%	33,5%	29,1%	29,1%
4	33,5%	33,5%	34,3%	34,3%	33,5%	33,5%	34,3%	34,3%
5	33,5%	33,5%	39,1%	39,1%	33,5%	33,5%	39,1%	39,1%
6	33,5%	33,5%	43,6%	43,6%	33,5%	33,5%	43,6%	43,6%
7	33,5%	33,5%	48,0%	48,0%	33,5%	33,5%	48,0%	48,0%
8	35,7%	35,7%	51,9%	51,9%	35,7%	35,7%	51,9%	51,9%
9	37,2%	37,2%	55,6%	55,6%	37,2%	37,2%	55,6%	55,6%

SA = Standard Assumptions, *EA* = Extended Assumptions. Columns *Initial* and *Final* represent initial and final steady state, respectively. Rows represent variables and level of technology growth (*None/Mild/Strong*)

Table B.6: Pensions/Wage, Swedish Pension System.

	Initial				**Final**			
	Flat		**Progressive**		**Flat**		**Progressive**	
	SA	EA	SA	EA	SA	EA	SA	EA
None								
1	0,107	0,105	0,109	0,107	0,105	0,104	0,107	0,105
2	0,113	0,111	0,121	0,119	0,108	0,106	0,110	0,108
3	0,133	0,130	0,146	0,141	0,118	0,115	0,127	0,124
4	0,163	0,158	0,181	0,175	0,140	0,136	0,154	0,149
5	0,206	0,199	0,233	0,223	0,173	0,167	0,193	0,185
6	0,270	0,258	0,309	0,294	0,221	0,212	0,250	0,239
7	0,363	0,345	0,419	0,398	0,292	0,278	0,334	0,317
8	0,498	0,473	0,581	0,550	0,394	0,374	0,456	0,431
9	0,696	0,659	0,817	0,771	0,544	0,514	0,634	0,598
Mild								
1	0,107	0,106	0,109	0,107	0,105	0,104	0,107	0,105
2	0,113	0,111	0,122	0,119	0,108	0,106	0,110	0,108
3	0,133	0,130	0,146	0,142	0,117	0,115	0,127	0,124
4	0,163	0,158	0,181	0,175	0,140	0,136	0,154	0,149
5	0,207	0,199	0,233	0,224	0,173	0,167	0,193	0,185
6	0,270	0,259	0,309	0,295	0,221	0,212	0,250	0,239
7	0,363	0,346	0,420	0,399	0,291	0,277	0,333	0,317
8	0,499	0,473	0,582	0,550	0,393	0,373	0,455	0,430
9	0,697	0,660	0,818	0,772	0,543	0,513	0,633	0,597
Strong								
1	0,107	0,106	0,109	0,107	0,105	0,104	0,107	0,105
2	0,113	0,111	0,122	0,119	0,108	0,106	0,110	0,108
3	0,134	0,130	0,146	0,142	0,118	0,115	0,127	0,124
4	0,164	0,158	0,182	0,175	0,140	0,136	0,154	0,149
5	0,207	0,199	0,234	0,224	0,173	0,167	0,193	0,185
6	0,271	0,259	0,310	0,295	0,221	0,212	0,250	0,239
7	0,364	0,347	0,421	0,399	0,292	0,278	0,333	0,317
8	0,500	0,474	0,583	0,551	0,394	0,374	0,455	0,430
9	0,698	0,661	0,820	0,773	0,544	0,514	0,633	0,597

SA = Standard Assumptions, *EA* = Extended Assumptions. Columns *Initial* and *Final* represent initial and final steady state, respectively. Rows represent variables and level of technology growth (*None/Mild/Strong*)

Table B.7: Pensions, Chilean Pension System.

	Initial				**Final**			
	Flat		**Progressive**		**Flat**		**Progressive**	
	SA	EA	SA	EA	SA	EA	SA	EA
None								
1	0,037	0,026	0,043	0,032	0,028	0,019	0,033	0,023
2	0,054	0,039	0,063	0,042	0,042	0,029	0,048	0,033
3	0,080	0,054	0,091	0,070	0,061	0,043	0,070	0,049
4	0,116	0,080	0,133	0,093	0,089	0,054	0,102	0,067
5	0,170	0,113	0,195	0,131	0,130	0,092	0,149	0,103
6	0,248	0,173	0,284	0,202	0,189	0,131	0,217	0,149
7	0,362	0,267	0,415	0,317	0,276	0,199	0,317	0,218
8	0,529	0,348	0,607	0,434	0,404	0,290	0,463	0,318
9	0,772	0,609	0,886	0,648	0,590	0,415	0,676	0,435
Mild								
1	0,037	0,026	0,043	0,033	0,046	0,032	0,052	0,037
2	0,055	0,039	0,063	0,042	0,067	0,046	0,076	0,051
3	0,080	0,054	0,091	0,070	0,097	0,071	0,111	0,077
4	0,116	0,080	0,134	0,093	0,142	0,100	0,163	0,113
5	0,170	0,114	0,195	0,131	0,207	0,136	0,238	0,171
6	0,248	0,173	0,285	0,203	0,303	0,188	0,347	0,239
7	0,363	0,267	0,416	0,318	0,442	0,310	0,507	0,332
8	0,530	0,349	0,608	0,434	0,646	0,481	0,740	0,525
9	0,774	0,610	0,888	0,649	0,943	0,650	1,081	0,710
Strong								
1	0,037	0,026	0,043	0,033	0,073	0,052	0,084	0,060
2	0,055	0,039	0,063	0,042	0,107	0,074	0,123	0,082
3	0,080	0,054	0,092	0,070	0,157	0,115	0,179	0,124
4	0,117	0,080	0,134	0,093	0,229	0,162	0,262	0,182
5	0,170	0,114	0,195	0,132	0,334	0,219	0,382	0,275
6	0,249	0,173	0,285	0,203	0,488	0,304	0,558	0,384
7	0,363	0,268	0,417	0,318	0,713	0,499	0,815	0,534
8	0,531	0,350	0,609	0,435	1,042	0,776	1,190	0,845
9	0,775	0,611	0,889	0,650	1,522	1,049	1,738	1,141

SA = Standard Assumptions, *EA* = Extended Assumptions. Columns *Initial* and *Final* represent initial and final steady state, respectively. Rows represent variables and level of technology growth (*None/Mild/Strong*)

Table B.8: Taxation, Chilean Pension System.

	Initial				**Final**			
	Flat		**Progressive**		**Flat**		**Progressive**	
	SA	EA	SA	EA	SA	EA	SA	EA
None								
1	15,0%	15,0%	0,0%	0,0%	15,0%	15,0%	0,0%	0,0%
2	15,0%	15,0%	4,4%	3,2%	15,0%	15,0%	4,4%	3,2%
3	15,0%	15,0%	10,6%	11,6%	15,0%	15,0%	10,6%	11,6%
4	15,0%	15,0%	15,8%	15,3%	15,0%	15,0%	15,8%	15,3%
5	15,0%	15,0%	20,6%	20,2%	15,0%	15,0%	20,6%	20,2%
6	15,0%	15,0%	25,1%	25,3%	15,0%	15,0%	25,1%	25,3%
7	15,0%	15,0%	29,5%	30,3%	15,0%	15,0%	29,5%	30,3%
8	17,2%	16,5%	33,4%	33,0%	17,2%	16,5%	33,4%	33,0%
9	18,7%	18,9%	37,1%	37,2%	18,7%	18,9%	37,1%	37,2%
Mild								
1	15,0%	15,0%	0,0%	0,0%	15,0%	15,0%	0,0%	0,0%
2	15,0%	15,0%	4,4%	3,2%	15,0%	15,0%	4,4%	3,2%
3	15,0%	15,0%	10,6%	11,6%	15,0%	15,0%	10,6%	11,6%
4	15,0%	15,0%	15,8%	15,3%	15,0%	15,0%	15,8%	15,3%
5	15,0%	15,0%	20,6%	20,2%	15,0%	15,0%	20,6%	20,2%
6	15,0%	15,0%	25,1%	25,3%	15,0%	15,0%	25,1%	25,3%
7	15,0%	15,0%	29,5%	30,3%	15,0%	15,0%	29,5%	30,3%
8	17,2%	16,5%	33,4%	33,0%	17,2%	16,5%	33,4%	33,0%
9	18,7%	18,9%	37,1%	37,2%	18,7%	18,9%	37,1%	37,2%
Strong								
1	15,0%	15,0%	0,0%	0,0%	15,0%	15,0%	0,0%	0,0%
2	15,0%	15,0%	4,4%	3,2%	15,0%	15,0%	4,4%	3,2%
3	15,0%	15,0%	10,6%	11,6%	15,0%	15,0%	10,6%	11,6%
4	15,0%	15,0%	15,8%	15,3%	15,0%	15,0%	15,8%	15,3%
5	15,0%	15,0%	20,6%	20,2%	15,0%	15,0%	20,6%	20,2%
6	15,0%	15,0%	25,1%	25,3%	15,0%	15,0%	25,1%	25,3%
7	15,0%	15,0%	29,5%	30,3%	15,0%	15,0%	29,5%	30,3%
8	17,2%	16,5%	33,4%	33,0%	17,2%	16,5%	33,4%	33,0%
9	18,7%	18,9%	37,1%	37,2%	18,7%	18,9%	37,1%	37,2%

SA = Standard Assumptions, *EA* = Extended Assumptions. Columns *Initial* and *Final* represent initial and final steady state, respectively. Rows represent variables and level of technology growth (*None/Mild/Strong*)

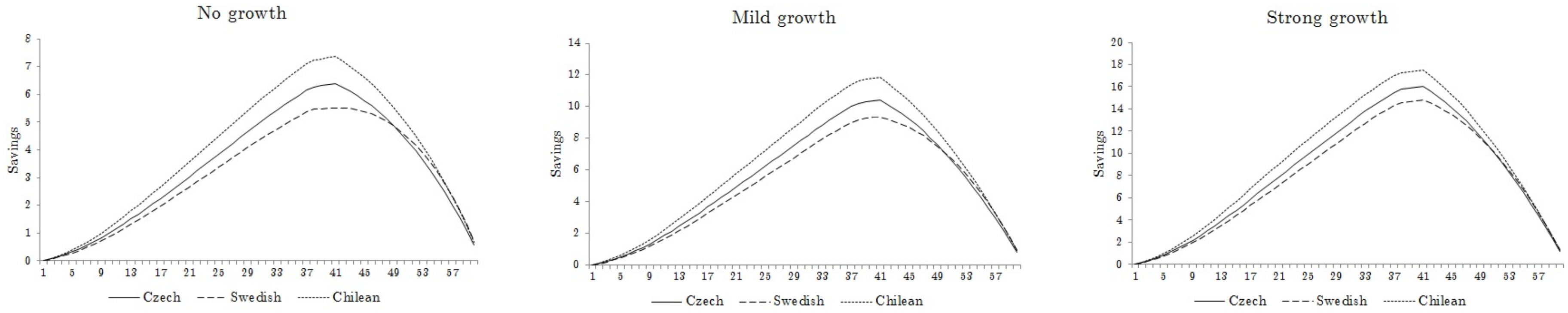
Table B.9: Pensions/Wage, Chilean Pension System.

	Initial				**Final**			
	Flat		**Progressive**		**Flat**		**Progressive**	
	SA	EA	SA	EA	SA	EA	SA	EA
None								
1	0,024	0,017	0,029	0,023	0,019	0,014	0,023	0,017
2	0,035	0,026	0,042	0,030	0,028	0,020	0,034	0,025
3	0,051	0,036	0,062	0,049	0,042	0,030	0,050	0,036
4	0,075	0,054	0,090	0,065	0,061	0,039	0,072	0,050
5	0,109	0,076	0,131	0,092	0,089	0,065	0,106	0,077
6	0,160	0,116	0,192	0,142	0,130	0,094	0,154	0,111
7	0,234	0,179	0,280	0,223	0,189	0,142	0,225	0,162
8	0,341	0,234	0,409	0,304	0,276	0,207	0,329	0,236
9	0,498	0,409	0,597	0,455	0,404	0,296	0,481	0,323
Mild								
1	0,024	0,017	0,029	0,023	0,020	0,014	0,023	0,017
2	0,035	0,026	0,042	0,030	0,029	0,021	0,034	0,024
3	0,051	0,036	0,062	0,049	0,042	0,032	0,050	0,036
4	0,075	0,054	0,090	0,065	0,061	0,045	0,072	0,053
5	0,110	0,076	0,131	0,092	0,089	0,061	0,106	0,079
6	0,160	0,116	0,192	0,142	0,130	0,084	0,155	0,111
7	0,234	0,179	0,280	0,223	0,190	0,139	0,226	0,154
8	0,342	0,234	0,410	0,305	0,277	0,215	0,330	0,244
9	0,499	0,410	0,598	0,455	0,405	0,291	0,482	0,330
Strong								
1	0,024	0,017	0,029	0,023	0,020	0,015	0,024	0,018
2	0,035	0,026	0,042	0,030	0,030	0,021	0,035	0,024
3	0,051	0,036	0,062	0,049	0,043	0,033	0,051	0,036
4	0,075	0,054	0,090	0,065	0,063	0,046	0,074	0,054
5	0,110	0,076	0,132	0,092	0,092	0,063	0,108	0,081
6	0,160	0,116	0,192	0,142	0,135	0,087	0,158	0,113
7	0,234	0,180	0,281	0,223	0,197	0,143	0,231	0,157
8	0,342	0,235	0,410	0,305	0,287	0,223	0,337	0,249
9	0,500	0,410	0,599	0,456	0,419	0,301	0,492	0,337

SA = Standard Assumptions, *EA* = Extended Assumptions. Columns *Initial* and *Final* represent initial and final steady state, respectively. Rows represent variables and level of technology growth (*None/Mild/Strong*)

Figure B.2: Final Steady State Savings.

Final steady state, flat taxation, various technology level growth



Final steady state, progressive taxation, various technology level growth

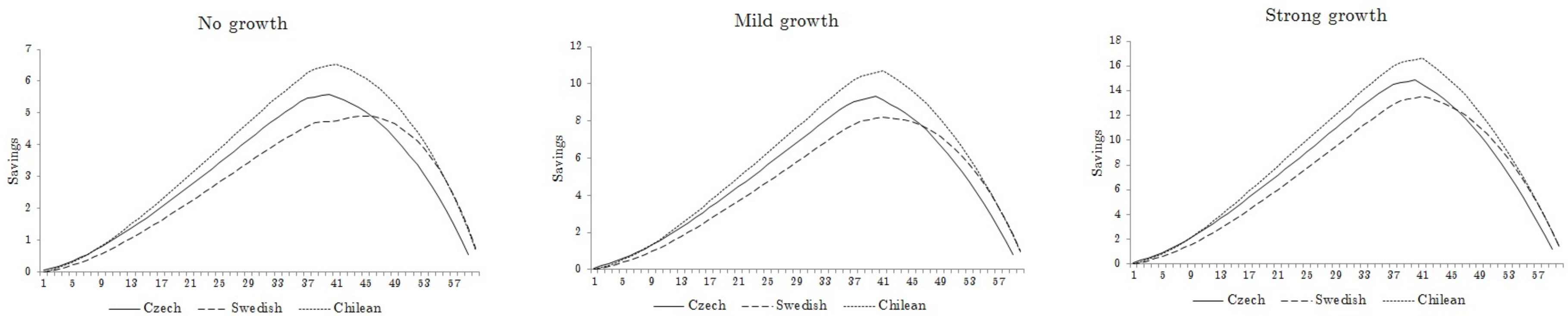
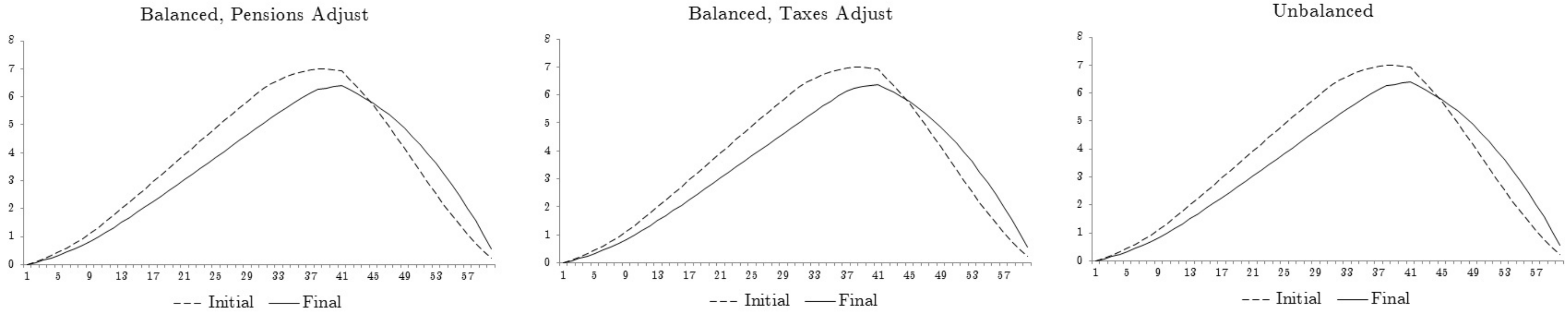


Figure B.3: Savings in the Czech Pension System.

Czech Pension system; flat taxation, no technology level growth



Czech Pension system; flat taxation, mild technology level growth

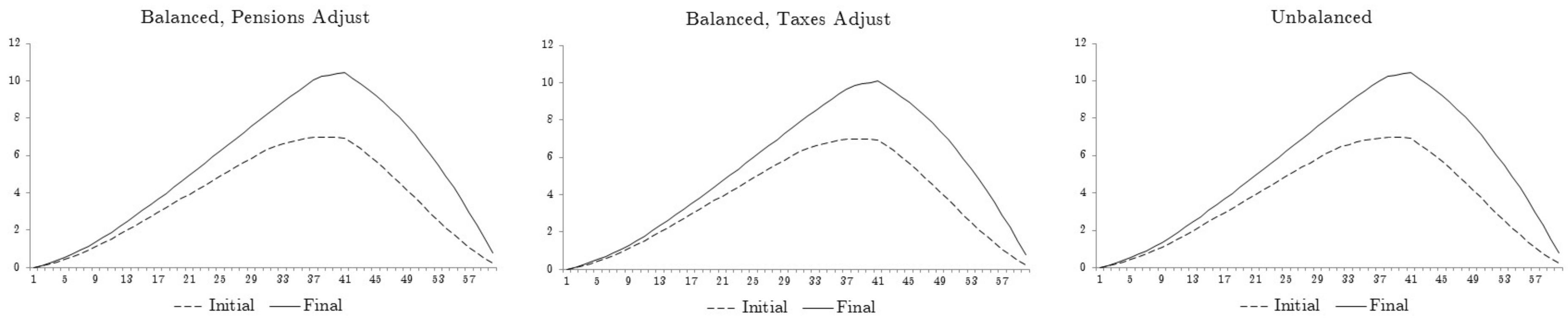
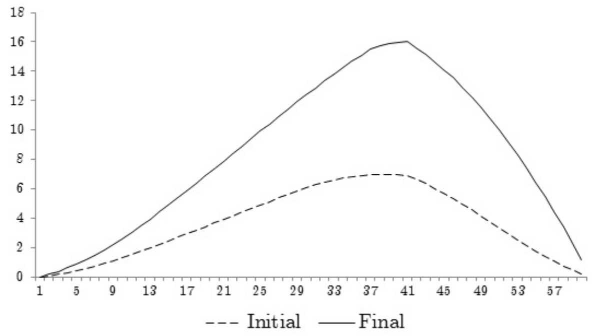


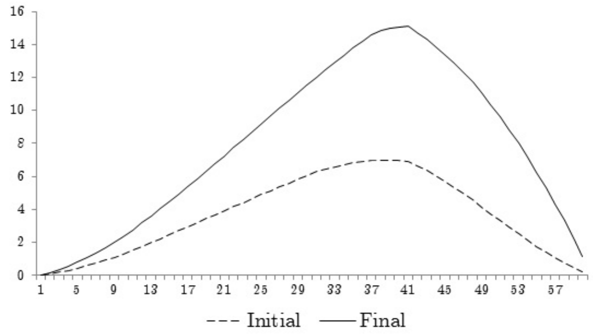
Figure B.4: Savings in the Czech Pension System (Continued).

Czech Pension system; flat taxation, strong technology level growth

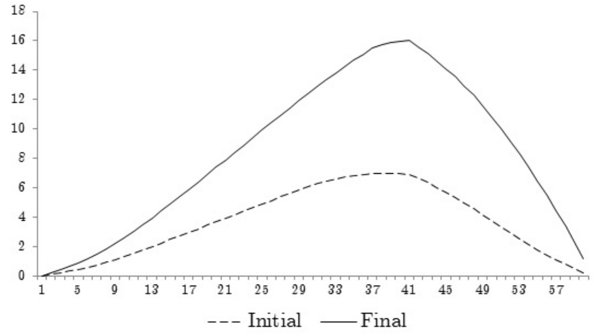
Balanced, Pensions Adjust



Balanced, Taxes Adjust

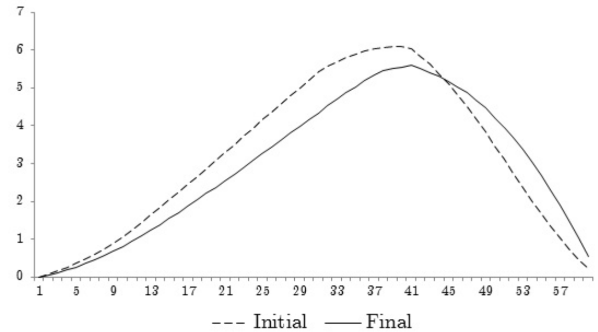


Unbalanced

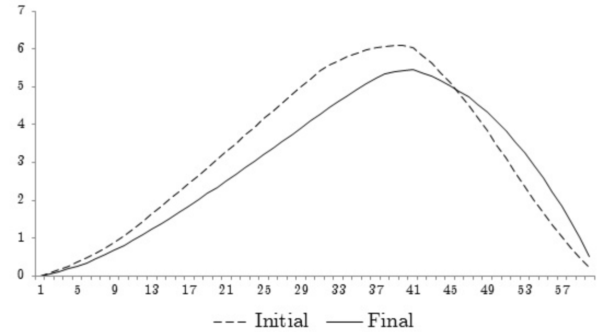


Czech Pension system; progressive taxation, no technology level growth

Balanced, Pensions Adjust



Balanced, Taxes Adjust



Unbalanced

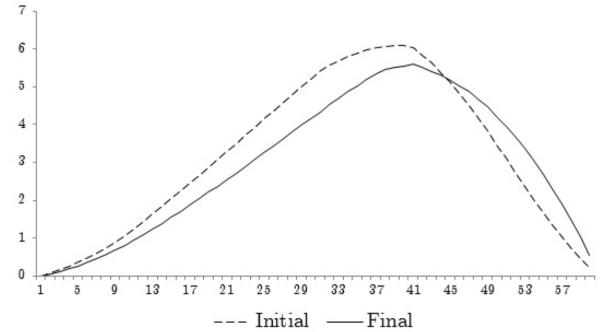
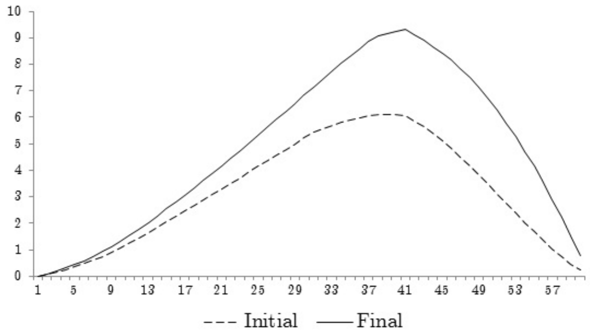


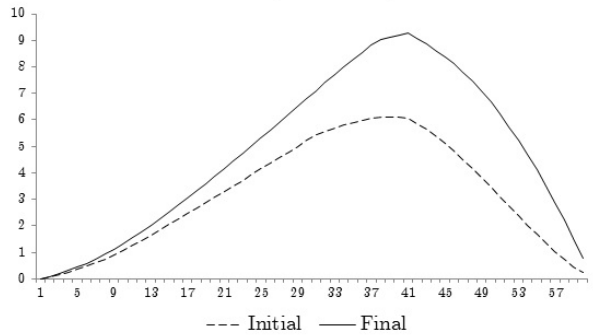
Figure B.5: Savings in the Czech Pension System (Continued).

Czech Pension system; progressive taxation, mild technology level growth

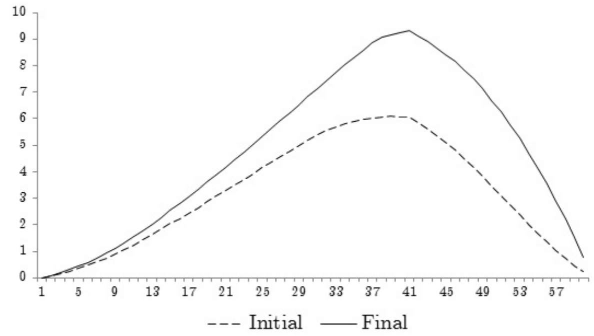
Balanced, Pensions Adjust



Balanced, Taxes Adjust

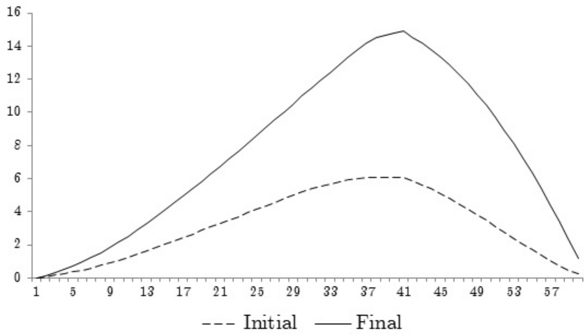


Unbalanced

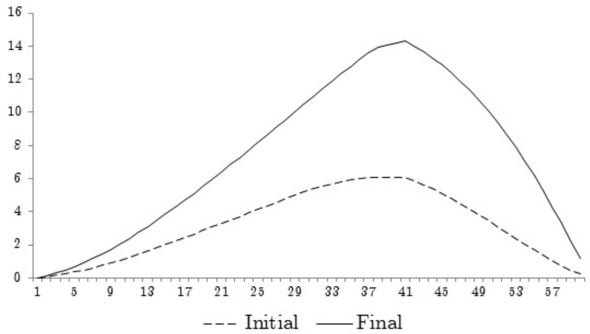


Czech Pension system; progressive taxation, strong technology level growth

Balanced, Pensions Adjust



Balanced, Taxes Adjust



Unbalanced

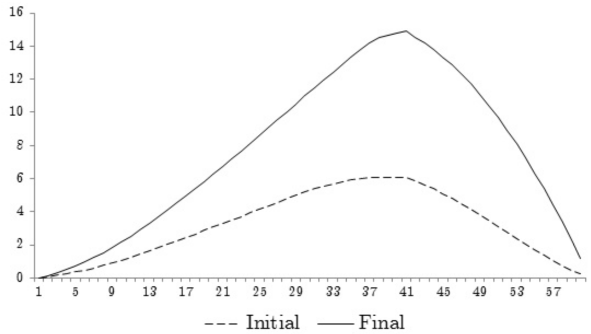
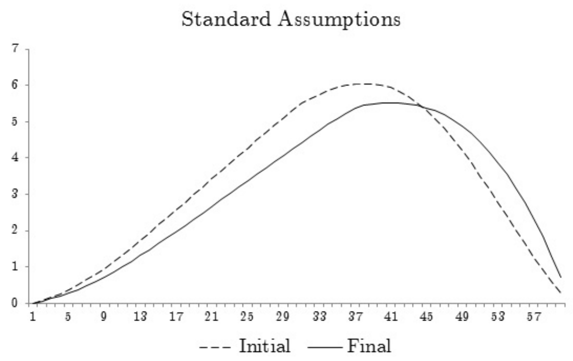
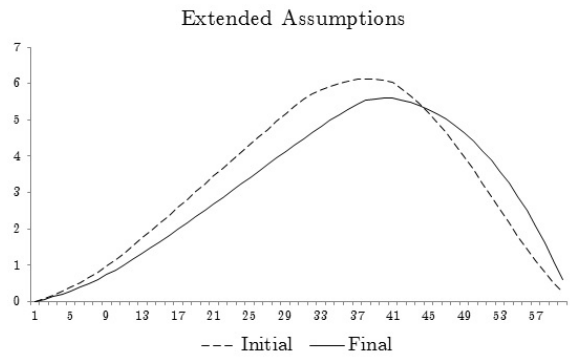


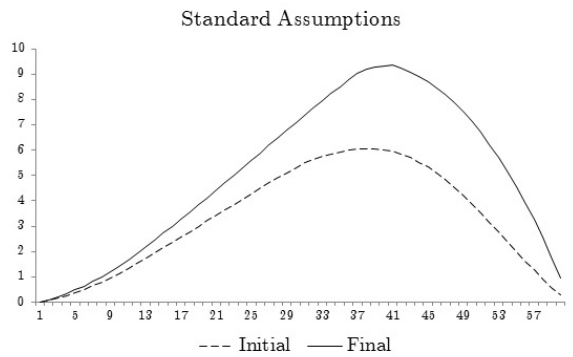
Figure B.6: Savings in the Swedish Pension System.

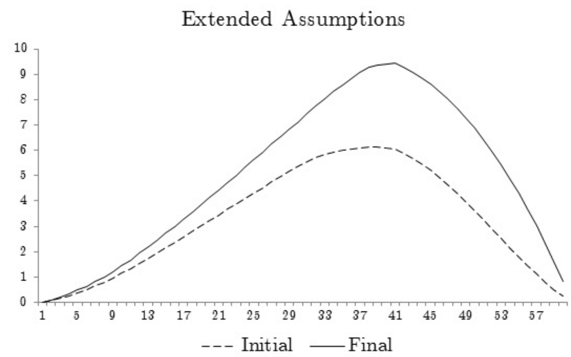
Swedish Pension system; flat taxation, no technology level growth



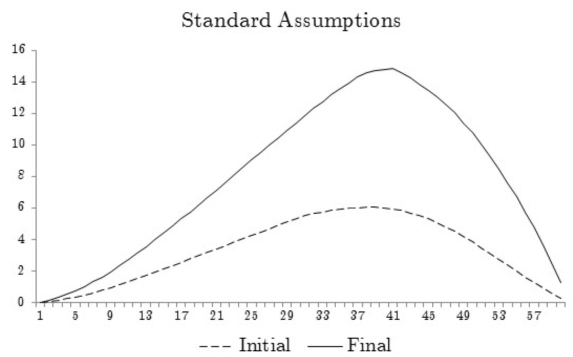


Swedish Pension system; flat taxation, mild technology level growth





Swedish Pension system; flat taxation, strong technology level growth



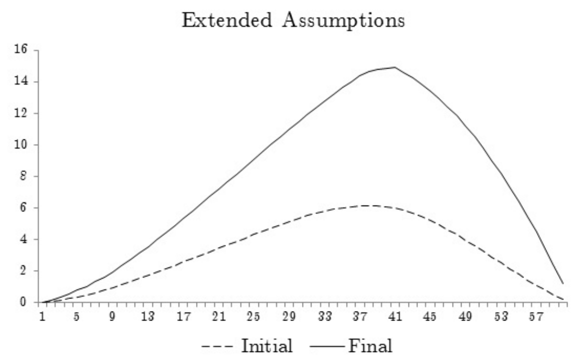
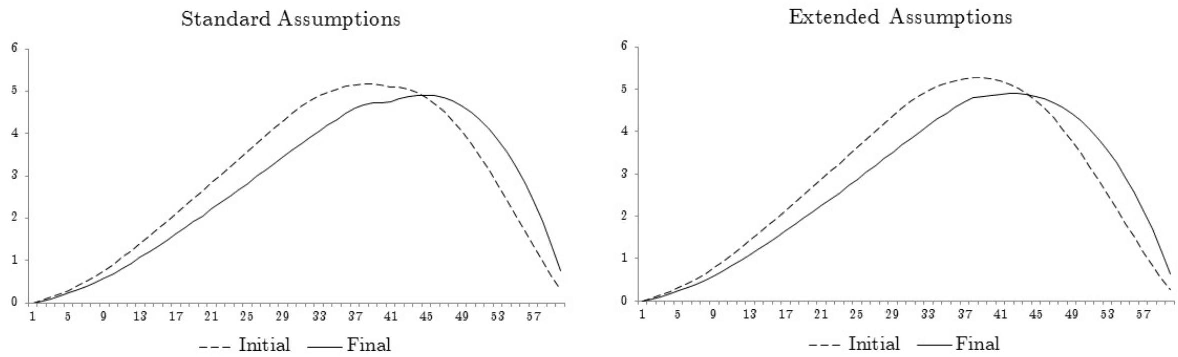
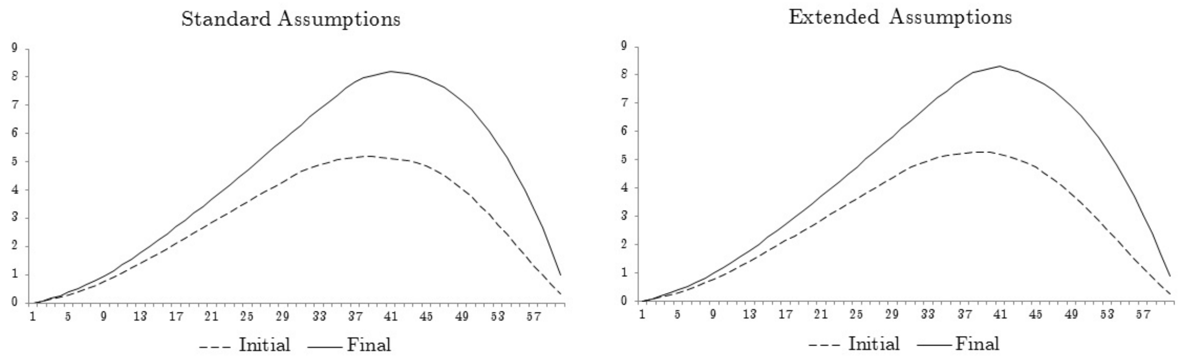


Figure B.7: Savings in the Swedish Pension System (Continued).

Swedish Pension system; progressive taxation, no technology level growth



Swedish Pension system; progressive taxation, mild technology level growth



Swedish Pension system; progressive taxation, strong technology level growth

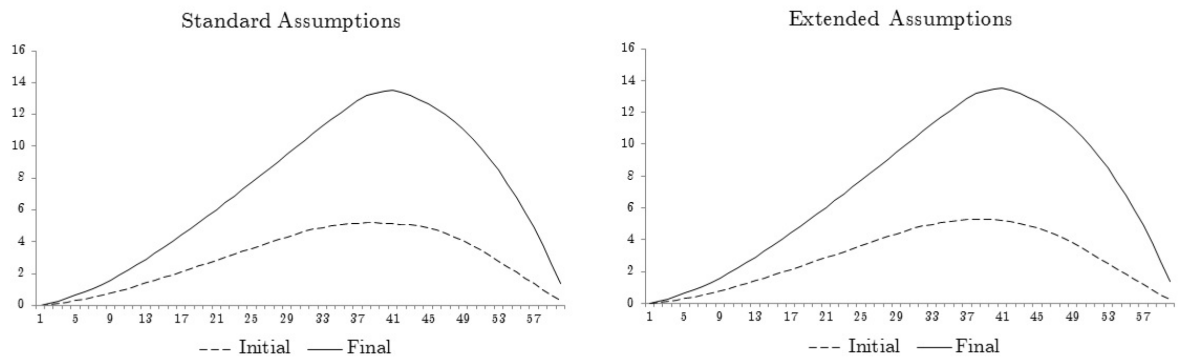
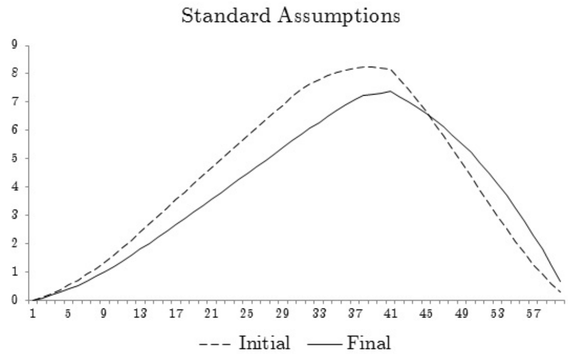
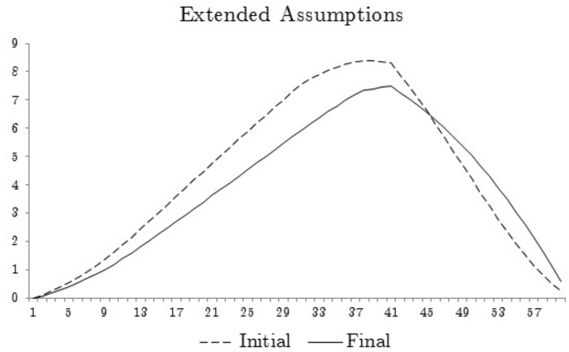


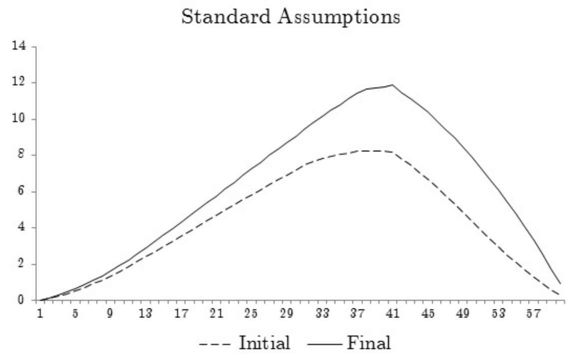
Figure B.8: Savings in the Chilean Pension System.

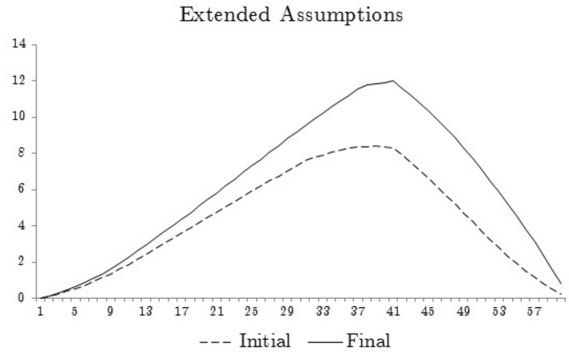
Chilean Pension system; flat taxation, no technology level growth



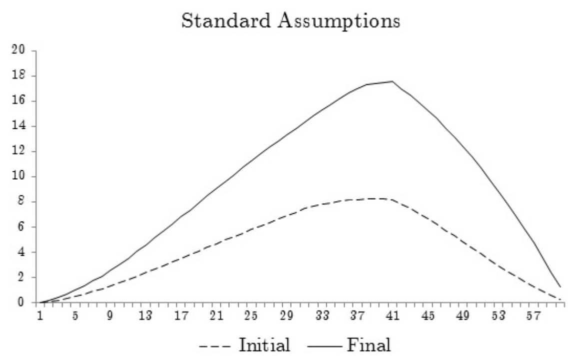


Chilean Pension system; flat taxation, mild technology level growth





Chilean Pension system; flat taxation, strong technology level growth



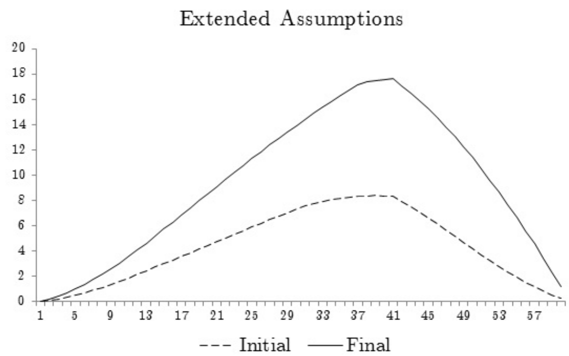
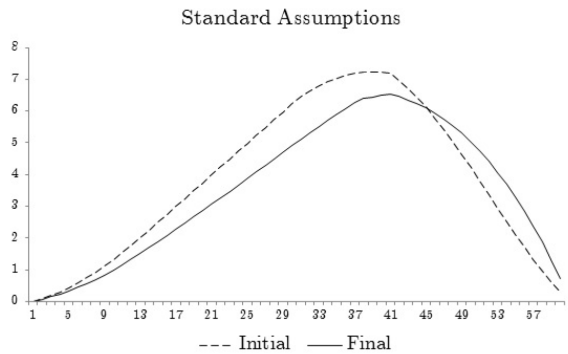
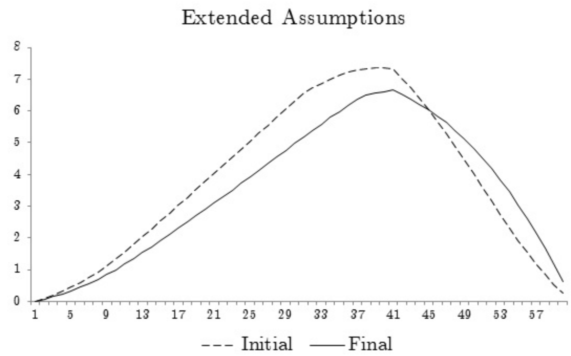


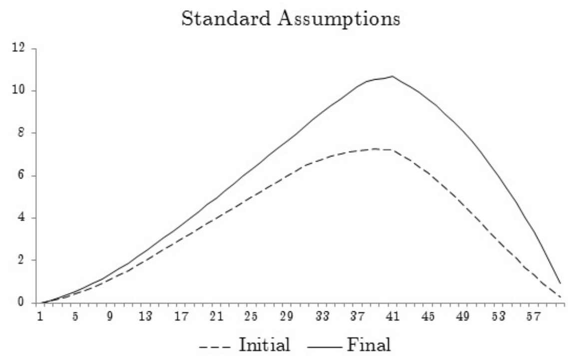
Figure B.9: Savings in the Chilean Pension System (Continued).

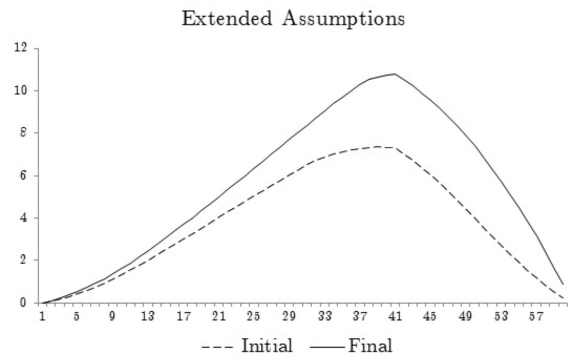
Chilean Pension system; progressive taxation, no technology level growth





Chilean Pension system; progressive taxation, mild technology level growth





Chilean Pension system; progressive taxation, strong technology level growth