Charles University in Prague

Faculty of Social Sciences

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BACHELOR THESIS

Economic Rationale for Damage Functions Entering the Social Cost of Carbon

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Academic Year: **2014/2015**

Declaration of Authorship

1. Hereby I declare that I have compiled this bachelor thesis independently, using only the listed literature and sources.
2. I declare that the thesis has not been used for obtaining another title.
3. I agree on making this thesis accessible for study and research purposes.

Prague, January 2, 2015

Signature

Acknowledgments

Finishing this thesis brings a very rewarding feeling to me and this feeling would not be possible without the extensive support of people around me, whom I would like to thank here for their aid.

I would especially like to thank my supervisor, PhDr. Tomáš Havránek, Ph.D. for introducing me to such an interesting topic, for inspiring debates and for constructive and valuable feedback on my work.

My thanks also belong to Ondřej Kolínský and my dad for proofreading the thesis and especially to my family for their constant support.

Abstract

Climate change studies repeatedly report the present value damage from global warming in the realms of trillion USD. To adopt an efficient climate policy, precise estimates of the costs and damages are essential. This thesis aims to review the most influential social cost of carbon models and to propose for the first time a best practice approach to constructing the damage function. Based on the reliability of the key estimates, two alternative approaches are proposed. The first consists of deriving a highly universal damage function and consequent calibration by multiple point estimates. The latter is based on damage disaggregation to different sectors and subsequent single-point calibration of each contribution separately. Both approaches address the current challenges for the damage function – a flexible functional form and treatment of intangible damages.

JEL Classification	D62, D90, Q51, Q54
Keywords	Social cost of carbon, SCC, damage function

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Abstrakt

Studie klimatických změn opakovaně odhadují diskontované škody z globálního oteplování v řádu bilionů dolarů. Pro přijetí účinných protiopatření je třeba mít přesné odhady škod i nákladů. Tato práce si proto klade za cíl porovnat nejvlivnější modely společenských nákladů uhlíku a navrhnout poprvé co nejvhodnější tvar tzv. ztrátové funkce. V souladu s přesností odhadu klíčových veličin jsou navrženy dva možné postupy. První se skládá z odvození univerzální ztrátové funkce a její kalibrace několika bodovými odhady. Druhý sestává z rozložení škod na jednotlivé příspěvky a následné kalibrace každého příspěvku zvlášť jedním bodem. Oba přístupy reagují na současné požadavky – flexibilní ztrátovou funkci a zahrnutí nehmotných škod.

Klasifikace JEL	D62, D90, Q51, Q54
Klíčová slova	Společenské náklady uhlíku, ztrátová funkce

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Acronyms

CO_2	Carbon dioxide
IPCC	Intergovernmental Panel on Climate Change
SCC	Social cost of carbon
GHG	Greenhouse gas
DF	Damage function
IAM	Integrated assessment model
DICE	Dynamic Integrated model of Climate and Economy
FUND	Climate Framework for Uncertainty, Negotiation and Distribution
PAGE	Policy analysis of the greenhouse effect
BAU	Business-as-usual
CBA	Cost-benefit approach
MCA	Marginal cost approach
GDP	Gross domestic product
WTP	Willingness to pay
WTAC	Willingness to accept compensation
NOAA	National Oceanic and Atmospheric Administration
CRRA	Constant relative risk aversion
PRTP	Pure rate of time preference
SRTP	Social rate of time preference

Bachelor Thesis Proposal

Author	Lukáš Hochmann
Supervisor	PhDr. Tomaš Havránek, Ph.D.
Proposed topic	Economic Rationale for Damage Functions Entering the Social Cost of Carbon

My thesis proposal Estimating the social cost of carbon (SCC) involves a multi-step analysis which projects the changes in greenhouse gas emissions into the change in greenhouse gas concentrations in the atmosphere, and further into the rise of temperature. The rise of temperature is then converted into economic harm by an appropriate damage function. The aim of this thesis is to perform a literature review of various damage functions utilized in influential SCC models and their comparison. Focus will be dedicated to the following models: DICE (Nordhaus, 1992), PAGE (Hope, 1993), FUND (Tol, 1995) and their further modifications. The functional forms of the damage functions used in these models will be assessed by economic theory and risk aversion axioms (Weitzman, 2010). Next, diverse concepts of how to encapsulate reality will be presented and assessed. The differences in the damage function forms in different SCC models will be noted and their impact on the SCC estimate discussed. Finally, a best practice approach to the construction of the damage function for a wide interval of temperature increases will be sought for.

Core bibliography

1. Nordhaus, W. D. (1992). The'DICE'Model: Background and Structure of a Dynamic Integrated Climate-Economy Model of the Economics of Global Warming (No. 1009). Cowles Foundation for Research in Economics, Yale University.
2. Tol, R. S. (1994). The damage costs of climate change: a note on tangibles and intangibles, applied to DICE. Energy Policy, 22(5), 436-438.
3. Tol, R. S. (1995). The damage costs of climate change toward more comprehensive calculations. Environmental and Resource Economics, 5(4), 353-374.

4. Hope, C., Anderson, J. I& Wenman, P. (1993). Policy analysis of the greenhouse effect: an application of the PAGE model. Energy Policy, 21(3), 327-338.
5. Weitzman, M. L. (2010). What Is The" Damages Function" For Global Warming - And What Difference Might It Make?. Climate Change Economics, 1(01), 57-69.
6. Pindyck, R. S. (2013). Climate Change Policy: What do the models tell us? (No. w19244). National Bureau of Economic Research.

Author	Supervisor

Chapter 1
Introduction

Burning of fossil fuels leads to the emission of various gases, most notably of carbon dioxide (CO_2). Carbon dioxide, along with gases like methane, chlorofluorocarbons or water vapor, falls into the category of greenhouse gases. This term relates to the ability of these gases to reflect radiation emitted from Earth back to its surface, which prevents the Earth from cooling down. Such a phenomenon can consequently lead to the heating of our planet which is generally called global warming. Although several scientists continue to deny the heating of our planet (or the anthropogenic cause of this effect), the latest assessment of the Intergovernmental Panel on Climate Change (IPCC) states that human activity is the dominant cause of observed warming during the second half of 20th century with a 95 % certainty (Stocker et al. 2013). Since changes in temperatures and climate in general are likely to have an adverse impact on our quality of life, the burning of fossil fuels is associated with a negative externality. It stems from the theory of market failure that in order to make the markets efficient, a Pigouvian tax should be levied on activities creating negative externalities. To set an appropriate tax onto the burning of fossil fuels, it is essential to quantify the effect of carbon emissions on our climate, as well as to monetize the impact of climate change. This is exactly what is the Social Cost of Carbon (SCC) framework attempts. The SCC is thus an estimate of the damages caused to our society by emitting an additional unit of CO_2.

The SCC framework is a rather complex interdisciplinary field ranging from climatology through anthropology to economics. The analysis of the impact of greenhouse gas (GHG) emissions on welfare consists of the following steps: At first, the level of current CO_2 emissions must be related to the present and future CO_2 levels in the atmosphere. Next, the atmospheric concentrations of GHGs are recalculated into the future changes in average temperatures or other relevant climatic factors. This step is followed by the calculation of damages over time by an appropriate damage function (DF). Finally, the total present value of damages is calculated by depreciating future damages by a suitable discount rate.

In addition to the multitude of steps involved, most of the calculations include relatively high levels of uncertainty. This is caused by the fact that the predictions of future values of diverse variables enter exogenously into the model. Since the SCC models are very long-term (the life cycle of CO_2 in the atmosphere is over 100 years (Falkowski et al. 2000), predictions of the values of necessary variables might differ significantly from the future reality. Furthermore, other values, such as the discount rate, are set arbitrarily by the modeler. Altogether, this leads to a very high spread of estimated values of the SCC. The interval of best guesses of relevant studies for the SCC ranges from 9 to 197 USD per ton of carbon[1] for the last decade (Clarkson and Deyes 2002) and from 0 to 130 USD for the current one (Havranek et al. 2014).[2]

It is rather complicated to enforce any governmental policy based on such a wide interval of estimates. Since each author can defend and reason for his own set of assumptions, it is very difficult to narrow down this interval of SCC estimates. On the other hand, it should be possible to review the models step-wise and identify and comment on the main sources of variance among each SCC estimate. One of the important factor influencing the value of the SCC estimate is the applied DF. For this reason, a review assessing the impact of various DF forms on the SCC value, along with a debate as to which DF form is the most appropriate, would be beneficial.

In this thesis, focus is dedicated to the economic side of the SCC models, most notably to the applied DF. The most influential SCC models (DICE, FUND & PAGE) are presented and assessed in chapter 2. Chapter 3 presents the DF, its properties and derivation. The concept of risk-aversion axioms is introduced and the DF forms are subject to microeconomic theory. In addition, several DF modifications are presented. Chapter 4 builds on the previous sections and analyzes the concepts introduced in the SCC models. Section 4.4 treats the difference among tangible and intangible damage. The advantage of a disaggregated DF is examined in section 4.2 and a highly universal DF is derived in section 4.5. Finally, a best practice approach to the construction and calibration of the DF is presented in section 4.6.

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[1] The SCC estimates are reported in the literature with two different units: price per *ton of carbon* and price per *ton of* CO_2. The ratio among these prices is the following: $1USD/tC \doteq 3.7USD/tCO_2$ In this thesis, I will hereafter report all estimates in USD/tC.

[2] The SCC estimate reduction in Havranek et al. (2014) is caused by the inclusion of publication bias; the SCC estimates themselves do not follow a decreasing trend.

Chapter 2

The framework for estimating the social cost of carbon

The Social Cost of Carbon (SCC) model is a complex, multi-step analysis determining the impact of the emission of greenhouse gases (GHGs) on our planet over time. The whole analysis can be divided into two main parts: the climatic part and the economic part. The climatic part deals with the influence of the level of GHG emissions on the future GHG concentrations in the atmosphere and consequently with the impact on our climate. This impact may consist of many different effects (temperature increases, sea level rise, changes in rainfall, etc.) but temperature rise is generally taken as an aggregate proxy for the climatic impact. The economic side then calculates the economic damages stemming from the climatic changes by an appropriate damage function (DF) and also discounts the future damages to current prices by a suitable discount factor. The DF, which is usually obtained by estimating the damage associated with a benchmark temperature increase and fitting this point estimate by a suitable curve, is a notorious weak link of the SCC framework (Pindyck 2013a).

Even though the above-mentioned process appears to be a linear sequence of steps, the individual variables are highly intertwined and affect each other. For this reason, the calculations are not done step-wise, but by introducing an integrated assessment model (IAM). This model incorporates a large set of information into a single body. The input includes, among others, the values of current emissions and economic output as well as their long-term forecasts, the estimate of future GHG emission abatement costs and climate sensitivity.[1] Next, the modeler must choose a specific DF and discount rate to calculate and discount future damages. After putting in all the input data, the model is able to predict the future climatic and economic development for

[1]Climate sensitivity is a parameter that relates the expected long-term rise in average temperature associated with a 100% increase in CO_2 concentration in the atmosphere (Roe and Baker 2007).

a set of scenarios. These usually include the business-as-usual (BAU) scenario and some alternative GHG abatement scenarios. The SCC estimate is then usually calculated by either the cost-benefit approach (CBA) or marginal cost approach (MCA).

The CBA attempts to calculate the optimal emission trajectory over time. The optimal level of emissions is given by the intersection of the marginal damages to society and marginal costs of abatement. Supposing that private damages from carbon[2] production are equal to zero and no market failures are present, the SCC is obtained as the marginal social damage from carbon emission at the optimal point (Clarkson and Deyes 2002). The mechanism of the calculations is rather obscure though – the IAM has to initially give us the optimum emissions trajectory. For this reason, I will try to demonstrate how to elucidate the SCC estimate from the IAM framework on the alternative approach: MCA.

The MCA opts for a different pathway to obtain the SCC estimate. At first, an expected BAU emissions trajectory is estimated and the carbon concentration development over time is plotted. Next, the carbon concentration development is plotted over time with a marginal increase[3] of carbon emissions at time t_0. The carbon level gradually returns to the BAU concentration plot but, before it does, increased carbon concentration causes cumulative damage over time through increased temperatures. This situation is schematically depicted in figure 2.1.

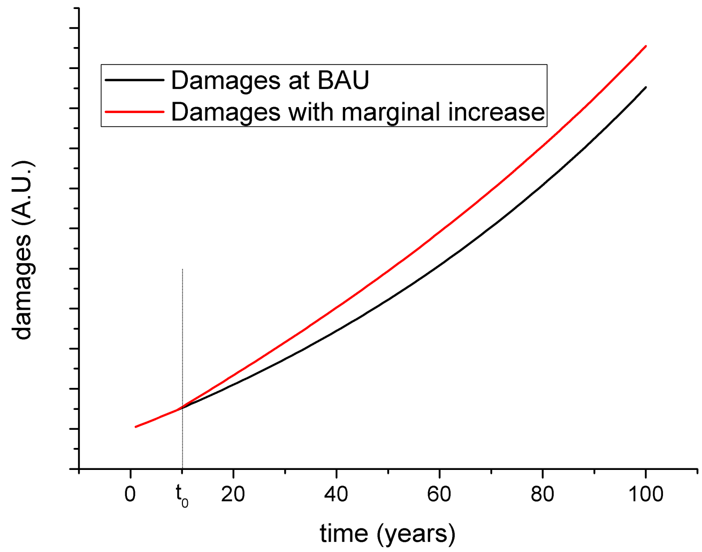
As seen from this figure, a marginal increase in carbon *emissions* in t_0 leads to a long-term increase in the carbon *level*. The difference compared to BAU gradually vanishes, but projects itself into an increasing difference in damages due to elevated temperature throughout this period. These have to be discounted to present values by a formula such as (2.1) in order to obtain a SCC estimate.

$$Damage = \int_{t=0}^{\infty} \left[MarginalIncreaseDamage(t) - BAUDamage(t) \right] e^{-\delta t} dt \qquad (2.1)$$

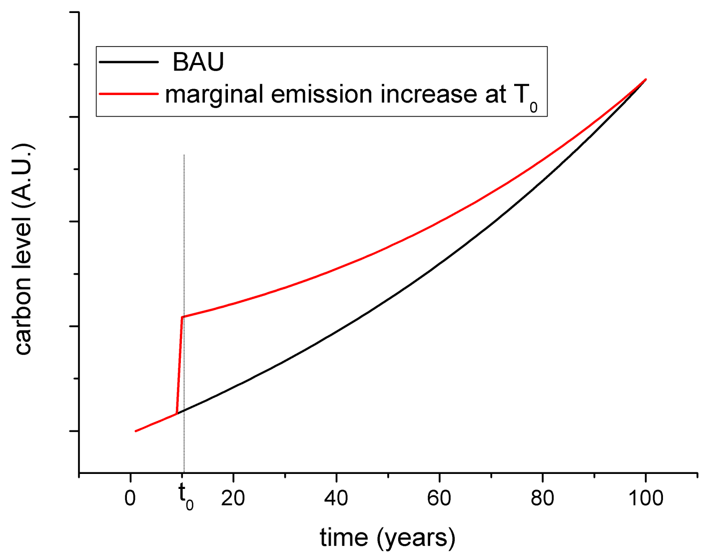
Here I would like to note that the curves in figure 2.1 are not mere cartoons. They are the result of an actual simulation of the impact of a marginal increase in emissions at t_0 for an arbitrarily chosen set of parameters. These include an exponential BAU carbon level profile (1% increase per year), a DF linear in carbon level and linear carbon sequestration from the atmosphere within 90 years. While the MCA is more

[2] Throughout this work, for the sake of brevity, the word carbon will stand for carbon dioxide.

[3] Alternatively, the same can be done for a marginal emission abatement. But a marginal increase leads to equal results and the derivation is more intuitive.



(a) Carbon level at BAU and with marginal emission increase at t_0. The marginal increase is amplified for better visibility.



(b) Damages at BAU and with marginal emission increase at t_0.

Figure 2.1: Visualization of the damage calculation via MCA

suitable for demonstrative purposes due to its transparency, it tends to overestimate the actual SCC, so the CBA approach is often preferred.

In the next section, three important SCC models, along with their important modifications and updates, will be introduced. The Dynamic Integrated model for Climate and Economy will be presented more in detail to demonstrate how such a model works; the remaining models will be presented with regard to their differences from the first one and their DF structure, which is central to this thesis.

2.1 DICE SCC model

The Dynamic Integrated model of Climate and Economy (DICE model, Nordhaus (1992) was introduced by William Nordhaus in February 1992. The aim of this model was to become an economic tool for addressing the threat of global warming. Its fundamental premise stated that the society should adopt some environmental policy only when the (future) benefits exceed the costs of said policy.[4] This premise remains unchallenged until now; accurate assessment of the costs and benefits of abatement policies still remains an issue, though. With respect to the evaluation of climate policies, Nordhaus' DICE model contributed by introducing dynamics into both the climatic and economic part of the model. Since all climatic changes span over a long time horizon, neglecting the dynamics can lead to misleading results (Nordhaus 1992) and the introduction of an albeit simplistic dynamic model is an important step forward. This simplistic DICE model was updated and enlarged various times (Nordhaus (1993a;b), Nordhaus and Boyer (1999), Nordhaus (2011; 2014) and the dynamic approach was also adopted by the competing models (PAGE, FUND and others). I will now briefly present the concept of the original DICE model, followed by a summary of the most important updates.

The DICE model aggregates a large number of segments into one big model. Each segment is a simplified description of the underlying processes. While these simplifications may bring inaccuracies, the advantage of this approach is that the overall model is fairly transparent and any improvement is easy to implement. The core of the DICE model is represented by the Ramsey optimal growth theory (Ramsey 1928). Nordhaus argues that the environmental policy is an exact analogue to the original Ramsey concept – the society opts to reduce its current consumption by investing into

[4]While Nordhaus did stress the negative externalities of GHG emissions, he also tried to address the calls for severe GHG emission cuts. In this study, as well as its updates, he finds these calls unwarranted by economic theory.

GHG reduction to increase future consumption. The election of the optimal trajectory of emission cuts is therefore based on the maximization of discounted societal utility. In order to evaluate the utility over time, it is necessary to have estimates of the climate changes over time under different GHG abatement scenarios as well as the estimates of costs and impacts associated with enforcing a given scenario. The first part is answered by the climatic part of the model, the latter by the economic side.

The climatic part of the model includes a set of sub-models which encompass the temporal evolution scenarios of GHG[5] emission, GHG level and temperature changes. The GHG emission pathway is affected by the economic output and carbon intensity of production under the BAU scenario. In case of an adoption of climate policy, an additional "emissions control" parameter, which is them optimized by running the aggregate model, is introduced. The next step is to translate carbon emissions to carbon levels in the atmosphere. To achieve this, a simplistic form of a multicompartment model is employed. The compartments include land, atmosphere and oceans. Carbon is introduced into the atmosphere from land mainly by the burning of fossil fuels (or biomass) and removed from the atmosphere by various means of carbon sequestration (eg. plant growth). In addition, the atmosphere is in contact with oceans and carbon exchange may occur among them. Similarly, a multicompartment model is used to describe the rises in temperature resulting from increased GHG concentration in the atmosphere. This GHG excess leads to an increase in the radiative forcing and results in an increment of the total heat of the Earth. This increase is then redistributed via the multicompartment model among the land, atmosphere and shallow & deep oceans. Basically, the climatic side of the model predicts the temperature increase evolution for various GHG abatement policies or the BAU scenario.

The economic part focuses of optimization of the societal utility under budget constraints. Societal utility is set as a sum of individual utilities, which in turn are expressed as a logarithmic function of *per capita* consumption. The whole model is optimized to maximize the discounted societal utility U from equation (2.2), where L is simultaneously labor force and population, C is consumption, δ is the discount rate and t denotes time.

$$U = \sum_t \frac{L_t ln(C_t/L_t)}{(1+\delta)^t} \tag{2.2}$$

[5]Here it is suitable to clarify that CO_2 is not the only greenhouse gas. However, since it is the most important contributor, the majority of SCC models focus on CO_2. Other gases are either recalculated to their "carbon equivalent" by multiplying their emissions by an experimentally-set factor taking into account their "warming potential" (eg. chlorofluorocarbons) or their emission trajectories are regarded as exogenous (eg. methane).

Potential output Y is calculated by a classical Cobb-Douglas production function in technology, population and capital. Real output[6] is then distributed by an optimization process between consumption and investment. Consumption is thus given by the following equation:

$$C_t = \frac{\varepsilon_t Y_t}{(1+\alpha T_t^2)} \tag{2.3}$$

where ε is the ratio of consumed potential output, T stands for temperature increase and the term in the denominator represents the potential output reduction due to climate change. The most important part of the model for this thesis are the abatement costs and especially the damage structure. Nordhaus assumed the following relationship between temperature rise T and suffered economic damage:

$$d(t) = Y(t)aT(t)^b \tag{2.4}$$

where $d(t)$ is the total loss of output, $Y(t)$ is the potential output and a and b are parameters representing the proportionality. Likewise, Nordhaus defined the costs of emission abatement as follows:

$$TC(t) = Y(t)c\mu(t)^d \tag{2.5}$$

where $TC(t)$ are the total costs of emission reduction by a fraction $\mu(t)$. As in the case of damages, coefficients c and d describe the proportionality of the given relationship. Alternatively, equation (2.4) and equation (2.5) can be divided by $Y(t)$ to obtain the percentage loss of output and percentage costs of abatement, respectively. This approach is more common nowadays.

Each of the sub-models listed above is described by a set of equations which together form a system subject to optimization. Obviously, the values of all parameters from both the climatic and economic side have to be set exogenously. Their values are assigned either by fitting historical data, taken from previously published articles or estimated from the author's own models. The same applies for several exogenous variables: level of technology over time and population over time. The remaining 15 endogenous variables reflecting the policy maker's choices are subject to optimization. Although many of the sub-models are simplistic and several other aspects are neglected, Nordhaus' pioneer into dynamic integrated climatic models is a remarkable feat in my eyes. It opened the field for further modifications and improvements

[6]Potential output is reduced to real output by the environmental damage ratio coming from the climatic side of the model, as discussed in the next paragraph.

from other scientists. Nordhaus himself remained very active in adjusting and upgrading his original model. In the following section, the changes in the DICE models regarding the DF will be mentioned. It remains to be said that the original DICE model concluded with a DF in the following form:

$$d(t) = \frac{1.3}{9} T(t)^2 Y(t) \tag{2.6}$$

which postulates a quadratic DF relating a loss of 1.3 % of GDP with a temperature rise of 3°C, which in turn corresponds to the doubling of atmospheric CO_2 concentration.

2.1.1 DICE modifications

The DICE model was introduced more than 20 years ago and still remains influential nowadays, which can be attributed to the plethora of modifications and updates it has undergone. I will now briefly mention the relevant changes with respect to the DF. In 1999, Nordhaus and Boyer "Roll the DICE again" and introduce a significantly revisited version of the DICE model (Nordhaus and Boyer 1999). This version introduces a slight change into the DF: the presence of a linear term (equation (2.7).

$$\frac{d(t)}{Y(t)} = aT(t) + bT(t)^2 \tag{2.7}$$

The core of the relationship remains quadratic, but at the expense of an additional degree of freedom, this functional form enables more flexibility at low temperature increases. It is important to realize that a strictly quadratic DF form, such as in equation (2.4), has a zero first derivative for $T = 0$. Thus, the marginal damage from an infinitesimal increase of temperature is virtually null – a constraint which has no justification in the underlying economical or environmental theory.[7] Other important refinements of the DICE model are related to Nordhaus (2011) and Nordhaus (2014).

The DF included in these versions comprises several sources of economic damage: direct adverse effects of rising temperature, damages associated with sea level rise and the impact of augmented CO_2 concentration. Nonetheless, in the following step, these aspects are reduced to the same quadratic form in temperature including the

[7] This is rather straightforward from the following thought experiment: A zero first derivative means the presence of either an inflection point or a local extreme of the DF at the given (ie. current) temperature. It should be possible to rule out these options by underlying assumptions, but this is not necessary. It suffices to realize that these point will be scarce (if any) on the DF profile and that the occurrence of such a point at precisely the current temperature would be only incidental.

linear term. Thus, the DICE models grew significantly over time with respect to their overall content, but the changes in the analytical form of the DF include solely the introduction of a linear term alongside the quadratic one.

2.2 FUND SCC model

The Climate Framework for Uncertainty, Negotiation and Distribution (FUND), introduced by Tol and coworkers, is another influential IAM of the SCC. The general approach towards the evaluation of the SCC is similar to the DICE model but, as the name suggests, this model attempts to address additional issues[8] such as the treatment of uncertainty, regional analysis and the policy enforcement.[9] The discussion of said aspects surpasses the scope of this text. In case of interest, Tol (1995; 1996; 2009) give a good overview of the FUND framework.

In Tol (1995), significant attention is dedicated to the formation of the DF, which is extended in a revolutionary way. At first, he discusses the obvious as well as more obscure weaknesses of the currently-used DFs. The most visible drawback of DFs used in most of the earlier models is that the whole function is formed by one rather *ad hoc* functional form which is calibrated by a single point estimate. Furthermore, Tol argues that one functional form can hardly encompass all types of damages that stem from global warming. For this reason, Tol employs a much less aggregated approach towards the calculation of economic damages from climate change. Most importantly, he does not use temperature as an aggregate proxy for all climate-related damages. In FUND, each category of damage is related to the relevant parameter, such as temperature rise, sea-level rise or increase in hurricane probability. Other authors, such as Nordhaus, do the same when estimating the benchmark damage associated with the doubling of CO_2 in the atmosphere, but then operate with temperature alone when estimating damages associated with different temperature increases via the DF. On the contrary, Tol keeps the different contributions separate even when inserted into the DF. The major advantage of this approach is that it permits to employ DFs of different profiles associated with different damage contributions. Based

[8]It is should be noted that even though the FUND addresses more areas than the original DICE model, major updates of the DICE also address uncertainty and regional issues.

[9]FUND does not conclude with finding an optimal policy and the SCC calculation. It takes into account the fact that the world does not have a single sovereign and employs game theory to determine the optimal policies for either cooperative or non-cooperative games. The step from determining an optimum strategy to enforcing it is generally neglected in the literature and the treatment of this issue by FUND seems to me like one of its most valuable contributions.

Damage contribution	V	V^2	ΔV	ΔV^2	Variable
Agriculture	1		0.75	0.25	Temperature
Coastal defence	0.5		0.25	0.25	Sea level
Dry land loss	1				Sea level
Wetland loss	0.5		0.5		Sea level
Species loss		0.5		0.5	Temperature
Amenity		0.17		0.83	Temperature
Morbidity		0.17		0.83	Temperature
Emigration	1				Sea level
Immigration	1				Sea level
Natural hazards	0.75	0.25			Hurricanes

Figure 2.2: Illustration of Tol's disintegrated approach to postulating the DFs. The table depicts the weight of each component of the corresponding variable V. Adapted from Tol (1995).

on his economic beliefs, Tol attempts to assign the most suitable from the following shapes of the DF to each contribution: a linear DF, purely quadratic DF or a combination of the two. In addition, he differentiates damages provoked by the *level* of climate change and by the *rate* of climate change. This does seem logical: the rate of climate change determines the time domain that the society has to adapt to new climate levels. For instance, an immediate 10°C temperature increase would definitely have more adverse effects than the same temperature increase spread out within a century. Every damage contribution is thus linked to either the *level* of climate change, *rate* of climate change or a combination of the two by either a linear relationship, quadratic one or a mixture of the two. A visualization of this approach is presented in figure (2.2).

Although this approach is extremely laborious, it is noteworthy that it does not require more information or assumptions than the simple aggregate DF of Nordhaus. The *level* and *rate* of climate change are two sides of a single coin. Likewise, the DF profile assignment is just a more complicated analogy of the general quadratic function. Assigning a quadratic profile to an aggregated DF can be viewed as assigning the same quadratic profile to each of its components. Tol does essentially the same – he just assigns a different DF profile to each contribution to reflect the reality more precisely. Each component's DF remains a single-parameter function which can be readily calibrated by the benchmark point estimate.

Other important articles on the FUND model include Tol (2009), where the FUND 1.6 model is introduced and Tol (2011), which serves as an extensive literature review of the SCC estimates until that point. Both articles include interesting observations and conclusions, but no dramatic change in the DF is presented.

2.3 PAGE SCC model

The Policy analysis of the greenhouse effect (PAGE) model, introduced by Hope et al. (1993), is another important SCC model. The PAGE model was elaborated for the European Community policy makers and it differs in many ways from the previous models. Most importantly, PAGE stresses the importance of handling uncertainty – over 80 input parameters are expressed as probability distributions. Uncertainty is then carried through all the calculations. An important side effect of this approach is that optimization becomes extremely difficult. As a consequence, PAGE does not offer any optimization process, but it estimates the implications of any policy the user defines. The inability of estimating the optimal pathway is a setback of the PAGE in comparison with DICE or FUND, but it is a price for a more thorough treatment of uncertainty. In addition, Hope and coworkers do not strive to find the optimal policy scenario (often with absent or large confidence intervals); they rather aim to compare the outcomes of several competing policies.

The policies in question include the classical BAU scenario, an aggressive GHG abatement scenario and an aggressive adaptive policy. Taking into account solely tangible damage, Hope *et al.* conclude that while an aggressive GHG abatement policy might just about pay off, the adaptive policies mitigate significant amounts of damage with a rather low investment.[10] Intangible damage[11] is not estimated by the current model. However, PAGE calculates the amount of intangible damage that would justify an aggressive GHG abatement policy in addition to the adaptive measures. PAGE concludes that an aggressive abatement policy is justifiable only if the intangible damage exceeds tangible damage twice worldwide.

The adaptive policies are centered around one term that the PAGE introduced: tolerable temperature change. This concept asserts that there is a certain level and rate of temperature increase that does not lead to any economic damage if not exceeded. Adaptive policies are then based on expanding this “harm-free” area either by increasing the level or slope of tolerable temperature increase or by reducing the incurred damage when exceeding the tolerable range. This peculiar concept is demonstrated

[10] The adaptive policy consist mainly of sea-level rise protection measures and the ratio of investment to prevented damage ascends to an astonishing 1:35. (Hope et al. 1993)

[11] Intangible damage is discussed in detail in sections 3.1 and 4.4.

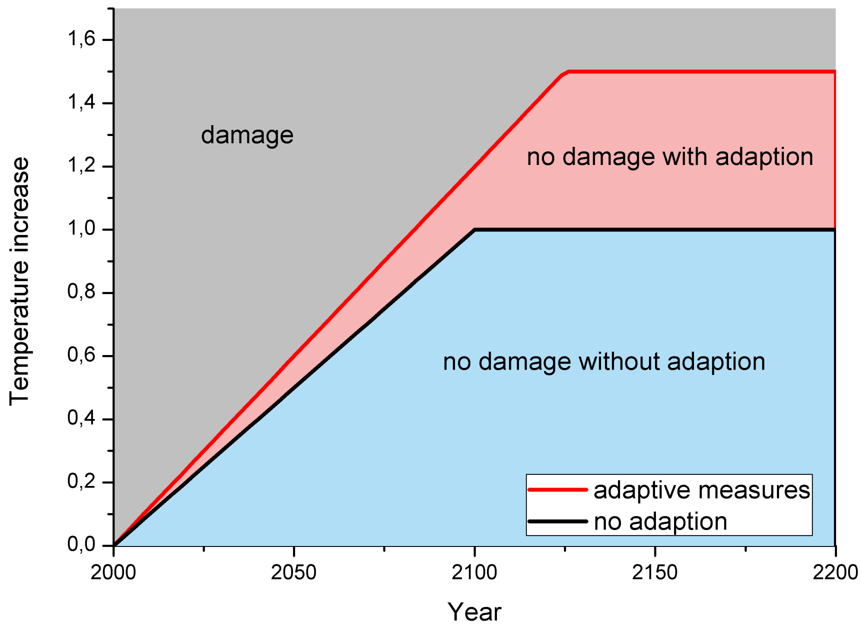


Figure 2.3: The effect of adaptive policy on the increase in tolerable temperature level and rate. Adapted from Hope et al. (1993).

in figure 2.3. This concept is quite unique and will be further discussed in section 4.1. The technical details of this model can be found in Hope (1992).

2.3.1 PAGE modifications

In 1995, the original PAGE model was superceded by PAGE95 (Hope and Plambeck 1996). In this revision, the linear DF was replaced by a polynomial convex curve. Following the course set in their original work – the concept of tolerable temperature and complex uncertainty treatment – Hope *et al.* set the DF as follows:[12]

$$D \approx (T - T_{tol})^n \tag{2.8}$$

where T is the temperature rise, T_{tol} is the tolerated temperature increase and n is an uncertain input parameter with a minimum value of 1, maximum value of 3 and most likely value of 1.3. The value of 1.3 corresponds to the most likely DF exponent advocated by Fankhauser (1994) and the DF was calibrated to yield the

[12] Even though it is not specified in the paper, this relation should only hold for $T \geq T_{tol}$.

same damage estimate as the previously-used linear relationship for the benchmark temperature increase.

Another major PAGE revision is presented in Hope (2006). The presented PAGE-2002 model responds to the concerns formulated by the third assessment report of the Intergovernmental Panel on Climate Change (IPCC), namely the risks from future large-scale discontinuities (McCarthy 2001). The DF remains the same as in Hope and Plambeck (1996). The final major revision of the PAGE model (PAGE2009) is presented in Hope (2013). This paper also focuses on the reasons that cause different SCC estimates resulting from PAGE2002, PAGE2009 and the Stern review,[13] which uses the framework of PAGE2002. The article concludes with a list of factors that have an important impact on the final SCC estimate. The damage function, however, remains the same.

[13] Stern (2007) presents his review on the economy of climate change where his SCC estimates are several-fold higher than usual in the contemporary literature. This was caused mainly by choosing a very low coefficient of SRTP. This was criticized by many authors and led to a complex discussion about the "correct" value of the SRTP coefficient.

Chapter 3

The building blocks of the damage function

3.1 Damages related to global warming

The damages caused by global warming can be essentially separated into two main categories: tangible and intangible damage. Tangible or “real” damage is the actual economic loss directly related to global warming. For example, in agriculture, temperature rise may lead to droughts which cause drops in harvests and a loss of output. Likewise, sea level rise causes damage by flooding farmland or other economically-active assets. Other types of tangible damage are a bit less palpable. For instance, elevated temperatures may lead to the destruction of coral reefs and thus to losses from the decline of tourism in the adjacent regions. Likewise, additional medical costs are associated with the rise of temperatures (heart diseases, spread of tropical epidemics). In a similar way, many other areas can relate some loss in output to climate change.

Nevertheless, the occurrence of “real” damage is not the only adverse effect of global warming. Additional costs are associated with intangible or “ethereal” damage. Returning back to the case of health care, let us assume that we can precisely calculate the costs associated with the treatment of a malady as well as the value of the lost productivity of the ill person. These form the tangible damages associated with the occurrence of the disease to the individual or the society in general. Theoretically, the society could remunerate the individual for the tangible damages he suffers[1]. However, it is unlikely that said individual would be equally happy (enjoy

[1] This is actually the case of health insurance or social security, even though here the individual is not fully compensated.

the same utility) healthy or ill, even with his economic damage compensated. The sole effect of being ill decreases one's utility regardless of the economic implications. The same applies to natural heritage. Even if people were compensated for their losses from the decline in tourism, we can hardly expect that they would be indifferent to living in beautiful nature or a deteriorated environment. These ethereal (intangible) damages form the complement to tangible damages caused by climate change. The main difference is that, unlike tangible damages which decrease our utility via reduced consumption, intangible damages reduce our utility directly. Nevertheless, in order to set an appropriate SCC price, it is necessary to monetize these damages. The possibilities how to accomplish this will be outlined in the following section.

3.1.1 Monetization of intangible damage

Even though intangible damage translates directly into our welfare and bypasses the economic side, it is necessary to monetize it to fully account for damage related to climate change. This is usually done by contingent valuation, a technique first reported by Ciriacy-Wantrup (1947). This approach is based on surveys where economic subjects (persons or households) are questioned how much they value a certain intangible good. The subjects are asked either for the maximum price they would agree to pay to conserve the given good (willingness to pay (WTP) approach) or for the minimum sum they would accept as a compensation for the loss of said good (willingness to accept compensation (WTAC) approach).

Although this method has attracted a lot of criticism related to the accuracy of its estimates – for details consult Hausman (1993) & Diamond and Hausman (1994) – the National Oceanic and Atmospheric Administration (NOAA) panel approved its use under a carefully controlled design. The major points of critique highlight the fact that the estimates often demonstrated a significant upward bias. This was partly caused by misunderstandings between the survey operator and respondent. In addition, when the survey design is not prepared carefully, the respondents could be motivated to intentionally overestimate their WTAC, perhaps with a vision of a higher payoff should the research result in some actual future compensation. Likewise, persons could overestimate their WTP since the results of the survey were not binding. To reduce the upward bias and sensitivity of the results on the survey design, a set of recommendations by the NOAA panel was published in Arrow et al. (1993). This report also concludes, along with Horowitz and McConnell (2002) that the WTP is usually substantially lower than the WTAC and thus closer the the real price of the intangible good.

3.2 Damage function and its trivial restrictions

The DF entering into the SCC model is in general a function which translated temperature increase[2] T and potential output C^* into economic damage D as follows: $D = f(T, C^*)$. The DF is usually declared as a percentage loss of potential output. Such a function could in principle have any form, but there are several sensible restrictions stemming from common sense as well as economic theory which can help narrow down the classes of suitable functions. First of all, it is sensible to differentiate between two different terms which are related to the DF in the literature. The first concept of the DF relates a temperature rise to monetized economic damage (consumption percentage loss). The other interpretation relates temperature rises to a loss of utility. At first, lets us consider the case where a damage function translates a temperature rise into economic damage.

(i) It is a reasonable assumption that the DF is continuous for $C^* > 0$ and $T \geq 0$.[3]

(ii) Likewise, it is a reasonable assumption that the DF is differentiable. This can be demonstrated as follows: let us take the first derivative of the DF with respect to T and C^*:

$$\frac{\partial D}{\partial T} = \frac{\partial f(T, C^*)}{\partial T}; \frac{\partial D}{\partial C^*} = \frac{\partial f(T, C^*)}{\partial C^*} \tag{3.1}$$

The expressions in equation (3.1) are the marginal percentage loss changes associated with an infinitesimal change in temperature or consumption. Following the reasoning from (i), these functions should be continuous.[4]

Next, let us consider the case of a DF converting a temperature rise into a utility loss. This is the case of Weitzman (2010), who also advocates two microeconomy-based axioms which should restrict the functional form of the DF. If we apply the

[2]As mentioned in section 2.2, some models do not use temperature as a sole aggregate proxy for climate change. However, the reasoning presented in this chapter can equally apply to the other climatic factors.

[3]The opposite would require that an infinitesimal rise in either potential output or temperature would result in a jump in the consumption percentage loss. I am not aware of any argument in favor of such behavior.

[4]By the same logic it could be assumed that the DF should fall into the class C^∞, but this restriction is not needed at this point.

law of diminishing returns to transform economic losses into a utility loss, it is clear that the points (i) and (ii) introduced on the previous page also apply for this case as the transformation does not tamper with continuousness or differentiability of said functions.

3.3 Risk aversion axioms of the DF

Taking into account the elementary DF requirements, a wide class of functions still remains eligible for the DF in the SCC model. To round these options down, Weitzman (2010) postulated two risk aversion axioms regarding the consumption and temperature utility functions to deduce a fairly simple and analytically solvable DF form. A brief overview of his ideas will be presented now, along with the implications on the DF form. The assumptions will then be subject to economic theory and the implications of the axioms discussed.

Weitzman's model starts off with the assumption that utility from consumption exhibits constant relative risk aversion (CRRA) behavior. For a constant coefficient η larger than unity, Pratt (1964) deduced a utility function as follows:

$$V(C) = -C^{1-\eta} \tag{3.2}$$

where V is utility,[5] C is consumption and $\eta > 1$ is the coefficient of risk aversion. In the standard SCC framework, the increased temperatures cause economical damage, thus reducing consumption. Consequently, we can express consumption C as a function of the potential consumption C^* in the absence of warming and temperature increase T. The consumption-related CRRA is defined as follows:

$$\eta(C^*,T) = -\frac{C^* \frac{\partial^2 U(C^*,T)}{\partial C^{*2}}}{\frac{\partial U(C^*,T)}{\partial C^*}} \tag{3.3}$$

and Weitzman's first axiom states that $\eta > 1$ is assumed to be constant for any $C^* > 0$ and $T \geq 0$. His second axiom relates to the risk aversion regarding temperature increase. Weitzman reasons that the responsible agent manifests similar risk aversion

[5] An attentive reader might notice that the utility is negative for any positive value of consumption. This is not a problem. Such a utility function obeys the imposed restrictions and it is possible to maximize negative values just like positive ones. Alternatively, one can perform an affine transformation of this utility function to obtain strictly positive values – such a transformation is allowed for the cardinal utility function.

regarding uncertainty over temperature increases. The CRRA of temperature rise is analogically defined in equation (3.4)

$$\mu(C^*,T) = \frac{T\frac{\partial^2 U(C^*,T)}{\partial T^2}}{\frac{\partial U(C^*,T)}{\partial T}} \tag{3.4}$$

where $\mu > 0$ is the relative temperature risk aversion parameter and is supposed to be constant for all $T > 0$ and $C^* > 0$ (please note the missing negative sign in equation (3.4). This is due to the character of temperature increase, which is negatively correlated to utility). Next, we can obtain the general functional forms of utility functions that obey both axioms respectively. Equation (3.3) leads to equation (3.5) where $a(T)$ and $b(T)$ are functions of temperature representing a linear transformation of $C^{*(1-\eta)}$ and equation (3.4) leads to equation (3.6), where $\alpha(C^*)$ and $\beta(C^*)$ are functions of potential consumption representing a linear transformation of $T^{1+\mu}$. The validity of these formulas may be verified by plugging them into the definition of the CRRA axioms.

$$U(C^*,T) = a(T) + b(T)C^{*(1-\eta)} \tag{3.5}$$

$$U(C^*,T) = \alpha(C^*) + b(C^*)T^{(1+\mu)} \tag{3.6}$$

The most general function satisfying both equation (3.5) and equation (3.6) is described by equation (3.7), where α_A and α_M are non-negative constants and at least one of them has to be non-zero (Weitzman 2010). It can be easily verified that equation (3.7) possesses the expected properties of a cardinal utility function, namely $\frac{\partial U}{\partial C^*} > 0$, $\frac{\partial^2 U}{\partial C^{*2}} < 0$, $\frac{\partial U}{\partial T} < 0$ and $\frac{\partial^2 U}{\partial T^2} < 0$ for $T > 0$ and $C^* > 0$.

$$U(C^*,T) = -C^{*(1-\eta)} - \alpha_M C^{*(1-\eta)} T^{1+\mu} - \alpha_A T^{1+\mu} \tag{3.7}$$

Looking at equation (3.7) we can notice that there are two different temperature-related contributions to the (dis)utility. The value of the first contribution depends on the level of potential output. By factoring out $-C^{*(1-\eta)}$ from the first and second term of equation (3.7) we obtain $1 + \alpha_M T^{1+\mu}$. Now, if we recall equation (3.2), it is apparent that this term reduces the utility originating from pure consumption. On the other hand, the magnitude of the second contribution does not relate to the value of potential output, but depends solely on the temperature increase. Let us now assess the economical reasoning behind both of the temperature-related terms in equation (3.7). This equation represents a generalization of the two popular utility functions used in SCC models. By setting α_A and α_M alternatively equal to zero we obtain the following equations:

$$U(C^*,T) = -\left[C^{*(1-\eta)} \times (1+\alpha_M T^{1+\mu})\right] \tag{3.8}$$

$$U(C^*,T) = -\left[C^{*(1-\eta)} + \alpha_A T^{1+\mu}\right] \tag{3.9}$$

The first case, often called *multiplicative*, reduces welfare by reducing potential consumption by some temperature-dependent fraction. Looking back to section 3.1, this corresponds for example to the case of agriculture, where increasing temperatures may reduce the harvest by a temperature-dependent ratio. Even thought the temperature is present in the DF, the damage it causes is indirect via consumption reduction. By contrast, in the second (*additive*) case, the temperature increase causes a decrease in utility directly. This could be the case of losses in "environmental amenity", such as extinction of species, which decreases welfare but which does not cause direct economical damage. This case corresponds to the intangible damages discussed in subsection 3.1.1.

The discrimination between both analogues is very difficult based on experimental data. It is just important to keep in mind the different mechanism by which climate changes affects our utility in the real world. In addition, both systems vary in the substituability between both goods. While it is relatively inexpensive to substitute for climate change by consumption in the *multiplicative* form, it becomes increasingly difficult in the *additive* mechanism. For this reason, the results begin to diverge for high temperature increases even when using the same μ coefficient.

Keeping these differences in mind, it is possible to proceed to the derivation of the damage function. By defining the damage function $D(C^*,T)$ as an implicit solution of equation (3.10) and plugging in from equation (3.2) and equation (3.7), a general formula for the damage function can be obtained (equation (3.11).

$$V\{[1-D(C^*,T)]C^*\} = U(C^*,T) \tag{3.10}$$

$$D(C^*,T) = 1 - \frac{(C^{*(1-\eta)} + \alpha_M C^{*(1-\eta)} T^{1+\mu} + \alpha_A T^{1+\mu})^{\frac{1}{1-\eta}}}{C^*} \tag{3.11}$$

To demonstrate the properties and implications of such a DF, it is now useful to set specific values to the CRRA parameters to simplify the equation. By setting $\eta = 2$ and $\mu = 1$ the equation becomes significantly clearer (equation (3.12). Furthermore, the consumption-related parameter falls well into the generally reported range and setting the temperature-related parameter to unity leads to a quadratic function of temperature – a very frequent assumption in the most influential SCC models (Nordhaus 1992).

$$D(C^*, T) = \frac{\alpha_A C^* T^2 + \alpha_M T^2}{\alpha_A C^* T^2 + \alpha_M T^2 + 1} \tag{3.12}$$

Equation 3.12 presents the general solution for the damage function stemming from the two previously-stated axioms for $\eta = 2$ and $\mu = 1$. Now, by setting the α coefficients alternatively to zero, the DFs related to the *multiplicative* and *additive* utility form may be obtained (equation (3.13) and equation (3.14).

$$D_M(C^*, T) = \frac{\alpha_M T^2}{\alpha_M T^2 + 1} \tag{3.13}$$

$$D_A(C^*, T) = \frac{\alpha_A C^* T^2}{\alpha_A C^* T^2 + 1} \tag{3.14}$$

Both of these DFs represent the *fraction* of potential consumption lost at a given temperature and potential consumption level. Due to the use of equation (3.10) in the definition of the DFs, even the *additive* form utility may now be reduced to a single-variable function of consumption. However, this apparent deviation from the concept of "environmental amenity" is not so troublesome. The *additive* DF still reflects the essential characteristics of the *additive* form utility – an increasing ratio of consumption lost with increasing C^*, which is in accord with the increasing difficulty of substitution of consumption for the deteriorating climate. Thus, the *additive* DF still yields higher losses to utility than the *multiplicative* analogue as potential consumption rises. Furthermore, both DFs may be calibrated in a similar manner, as is shown in the next section.

3.4 Construction of the damage function

The DF in the SCC model must by definition relate any rise of temperature (and possibly other variables) to economic damage (usually as percentage of consumption loss). Forming such a function consists of two important steps. At first, a suitable DF form must be elucidated. This can be done by deduction from economic theory, but great liberty is given to the modeler as to which assumptions he postulates and which form he chooses. Unfortunately, as reported by Pindyck (2013a), the DF is often chosen arbitrarily with no or little economic support. Then, the chosen DF must be calibrated. For instance, if the benchmark estimate assumes a 2% consumption loss for a 2°C temperature rise, the DF presented in equation (3.13) must have the value of 0.02 for $T = 2$, which leads to a value of 0.0051 for α. When this coefficient is set, the expected consumption loss can then be calculated for any temperature scenario. Both the DF form and the value or the point estimate are therefore crucial for the valuation of damages associated to different temperature increases.

3.5 Discounting utility & consumption over time

When comparing utility over a longer time period, it is important to take into account the effect of time preference. It is a well-established fact that economic agents tend to prefer the same amount of utility now rather than in the future. This preference is expressed by the pure rate of time preference (PRTP) coefficient δ. The value of this coefficient has been subject to many discussions since its introduction by Ramsey (1928). Although economists like Ramsey advocated a zero PRTR based on ethical grounds (not discriminating against future generations), some authors such as Marini and Scaramozzino (2000) demonstrate that a minor δ value is justifiable. However, the PRTP operates only on the utility level. When we turn our focus to consumption time preference, other factors come into play, most notably output growth as well as uncertainty over it. A coefficient taking into account these factors is called the social rate of time preference (SRTP) and is defined as follows: $r = \delta + \eta g$, where η is the coefficient of consumption risk-aversion and g the growth rate of consumption. In both cases, the discount rates serve as relative weights of utility or consumption over time and are employed when maximizing the social welfare function W. The utility-based approach assigns consumption distribution under budget constrains through equation (3.15); the consumption-based approach via equation (3.16).[6] The value of these parameters is a hot topic among climate researchers. The reason for this is that the values depend on subjective grounds like ethics, so there is not any “correct” an-

[6]Even though most authors do not distinguish precisely among these methods, Creedy (2007) argues that results from both approaches may differ and argues against the use of SRTP.

swer. In addition, the value of the SRTP influences heavily the overall SCC estimate so the choice of the SRTP value is of great importance.

$$W = \sum_{t=1}^{T} U(c_t)(\frac{1}{1+\delta})^{t-1} \tag{3.15}$$

$$W = U(\sum_{t=1}^{T} c_t(\frac{1}{1+r})^{t-1}) \tag{3.16}$$

3.6 Differences in the additive and multiplicative DF

Sections 3.3 and 3.4 explain the derivation of two different special cases of a DF based on an axiomatic risk-aversion approach and their calibration. In addition, section 3.5 introduces the concept of time preference. Equipped with this knowledge, we can proceed to analyze the differences between these special cases.

At first, it is noteworthy that both damage functions yield the same result if the potential output remains constant over time.[7] Should consumption grow over time though, the results of both models start to differ. This can be demonstrated by answering the following question: How much current consumption should the policy maker be willing to sacrifice in order to reduce future temperature increase from T degrees to a lower value? This question is not equivalent to the calculation of the SCC. While the SCC takes into account the sum of discounted damages over a long period of time, this question debates the willing sacrifice of consumption now to reduce the impact of climate change at some arbitrary point in the future. This simplification has a huge impact. It permits us to completely bypass the climatic side of the SCC model which is necessary to determine the temperature increase *pathway*, but it still reflects well the differences in the damage valuation. What follows is a generalization of Weitzman (2010) for a reduction of temperature to any positive value (Weitzman derives the formula only for the reduction to zero).

Let t denote time and let us set $t = 0$ to the present. Utility U is a two-variable function of potential output C^* and temperature increase T. Potential consumption over time is then defined as $C^*(t)$ with $C^*(0) = 1$ and temperature increase as $T(t)$

[7] This is easy to demonstrate by normalizing C^* to unity in equation (3.14) and comparison with equation (3.13).

with $T(0) = 0$. Next, we assume exponential growth of potential consumption with a rate g ($C^*(t) = e^{gt}$). Finally, let us denote T_A the temperature increase after a certain abatement policy. The maximal fraction of current consumption ω which the policy maker is willing to sacrifice to reduce future temperature increase T to T_A is set by the following condition:

$$U(1,0) - U((1-\omega),0) = e^{-\delta t}\left[U(C^*(t), T_A(t)) - U(C^*(t), T(t))\right] \tag{3.17}$$

where δ is the PRTP. This equation can be solved by plugging either DF (equation (3.13) or equation (3.14) into the implicit solution for the DF equation (3.10). Thus the utility function is obtained and can be plugged into equation (3.17) to get the result. The maximal fractions ω_M and ω_A corresponding to the *multiplicative* and *additive* damage function are set as follows:

$$\omega_M = \frac{\alpha\left[T^2(t) - T_A^2(t)\right]}{\alpha\left[T^2(t) - T_A^2(t)\right] + e^{(g+\delta)t}} \tag{3.18}$$

$$\omega_A = \frac{\alpha\left[T^2(t) - T_A^2(t)\right]}{\alpha\left[T^2(t) - T_A^2(t)\right] + e^{\delta t}} \tag{3.19}$$

The difference in both formulas is the presence of the growth factor g in the multiplicative analogue, which leads to a significantly lower estimate than for the additive version. This can be demonstrated numerically for any gives set of parameters. For the following set of parameters which try to reflect a plausible temperature and economic growth scenario ($\delta = 0.5$ % *p.a.*, $g = 2$ % *p.a.*, $T = 5$ °C, $t = 100$ years, $\alpha = 0.0051$ and T_A ranging from 5°C to 0°C) the values of ω_A and ω_M are plotted in figure 3.1. In addition, 3D graphs depicting the WTP as a function of temperature rise and abated temperature for the multiplicative and additive form are visualized in figure 3.2.

These graphs readily demonstrate the significant impact of the choice of the damage function on the overall SCC. The additive DF does not allow easy substitution of the environmental good for increased consumption, which drives the SCC estimate further up if consumption grows over time.

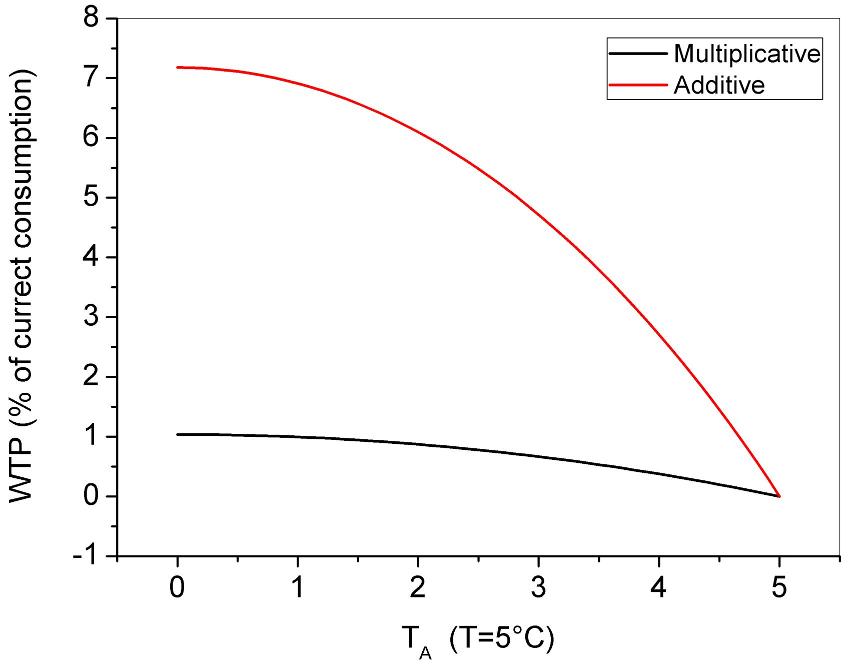
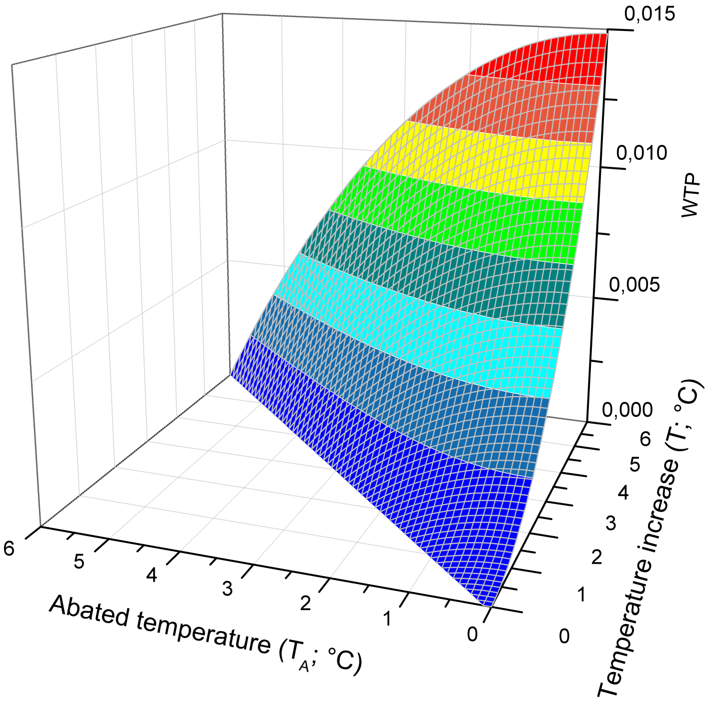


Figure 3.1: WTP for the multiplicative and additive scenario

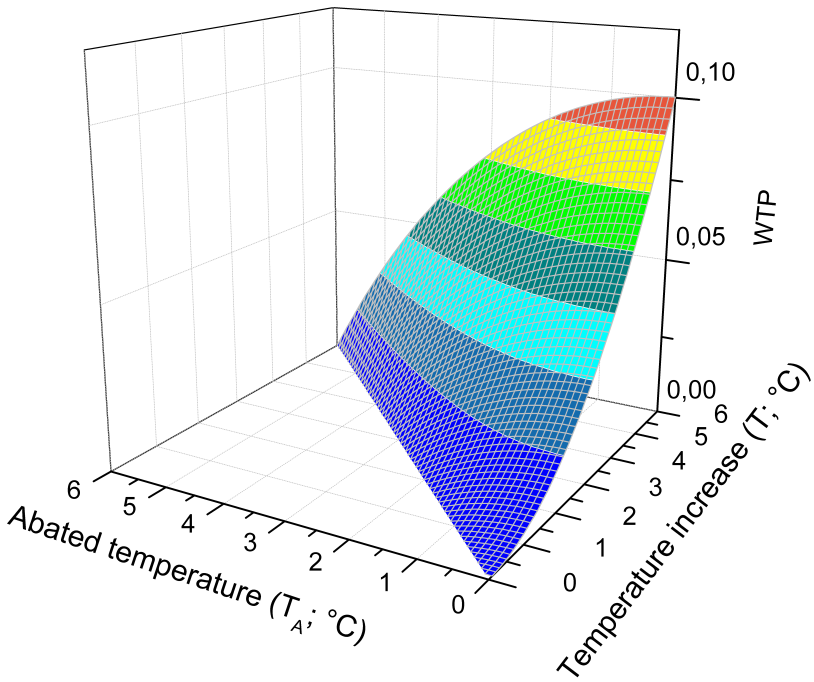
3.7 Quadratic analogues of the DF

Quadratic DFs occupy a dominant position in modeling the impact of rising temperature on economic output. The general form used in Nordhaus (1992) and others is the multiplicative analogue DF (equation (3.13) on page 21) resulting in the following relationship for consumption: $C = C^*/(1+\alpha T^2)$ where α is a positive constant. This form has many advantages. Firstly, it reflects the fact that increasing temperatures cause more than proportional increase in the economic damages. Secondly, such a form follows the general risk aversion axioms. Finally, a single-parameter relation can readily be calibrated by a single point estimate.

Even though these advantages are notable and very useful (any non-harmful simplification is of great use in such complicated models), such a simplistic form also brings some handicaps. For instance, this form imposes the assumption that the damages are always positive and that the DF has a zero first derivative at $T = 0$. This, however, does not have to correspond to reality. It is easy to imagine that a minor increase of ambient temperature in, lets say, Nordic countries might lead to an increase



(a)



(b)

Figure 3.2: 3D graphs depicting the WTP to reduce the temperature increase for the multiplicative case (top) and additive case (bottom)

of economic output, thus yielding negative economic damage[8]. I do not claim here that some rise of temperature definitely leads to an increase in the overall welfare. Keeping this option open, however, does have an economic rationale by itself. The same applies to the restriction of the null first derivative of the DF function at T=0. A null first derivative and positive second derivative would imply a local minimum of the DF. This would mean that our planet is currently at the optimal temperature level.[9] There is no claim I am aware of that would confirm this statement.

Here I will try to demonstrate that it is possible to employ such DFs that allow for both of these possibilities while not creating any major complications for the model framework. Let us postulate a DF in the following form including a linear term in temperature:

$$D_M(C^*,T) = \frac{\alpha T^2 + \beta T + \gamma}{1 + \alpha T^2 + \beta T + \gamma} \tag{3.20}$$

Since the value of this DF must by definition be zero for $T = 0$, γ must be equal to zero. As in the specific case without a linear term, α remains a positive constant. The value of the additional parameter β then governs the behavior of the DF around $T = 0$. A positive value of β leads to an increasing DF with a positive first derivative at $T = 0$, a negative β value gives a DF where a minor temperature increase yields a positive economic effect and a zero β-value reduces the DF to the traditional multiplicative version. In the next section, I will demonstrate the derivation of the DF in equation (3.20), as well as its additive analogue, and assess the impact of the linear term addition on the compliance with Weitzman's risk aversion axioms.

3.7.1 Analysis of a quadratic DF with a linear term

The addition of the linear term into the DF appears like a promising step forward to reflecting the reality more precisely. But does such a modification comply with the risk aversion axioms? And if not, what does it mean? In this section I will try to answer the following issues as well as derive the specific DF forms. At this point I would like to ask the reader for a bit of patience. At first, I will conjure the current utility functions merely by adding a linear term in temperature into the utility functions derived in equation (3.8) and (3.9) on page 20 for $\mu = 1$. Next, I will

[8]Negative damages for low temperature increases are often predicted by regional models for post-Soviet countries.

[9]The benchmark temperature ($T = 0$) is sometimes defined as the present mean temperature, in other cases as the pre-industrial one. In that case the derivative values would imply that the pre-industrial mean temperature was the optimal one.

demonstrate that these utility functions obey the consumption-related risk aversion axiom and explain why they break the temperature-related one. Finally, using the first axiom, I will derive a more general utility and damage function that encompasses the special case postulated in equation (3.20).

By adding a linear term into the reduced-form utility functions in equation (3.8) and (3.9), we obtain the following utility functions:

$$U_M(C^*,T) = -\left[C^{*(1-\eta)} \times (1+\alpha_M T^2 + \beta_M T)\right] \tag{3.21}$$

$$U_A(C^*,T) = -\left[C^{*(1-\eta)} + \alpha_A T^2 + \beta_A T\right] \tag{3.22}$$

By plugging either of them into the definition of the coefficient of relative consumption-risk aversion (η; equation (3.23) it is easy to verify that both utility functions exhibit a constant η coefficient and thus obey the axiom for $\eta > 1$, which is the fundamental condition implying risk aversion.

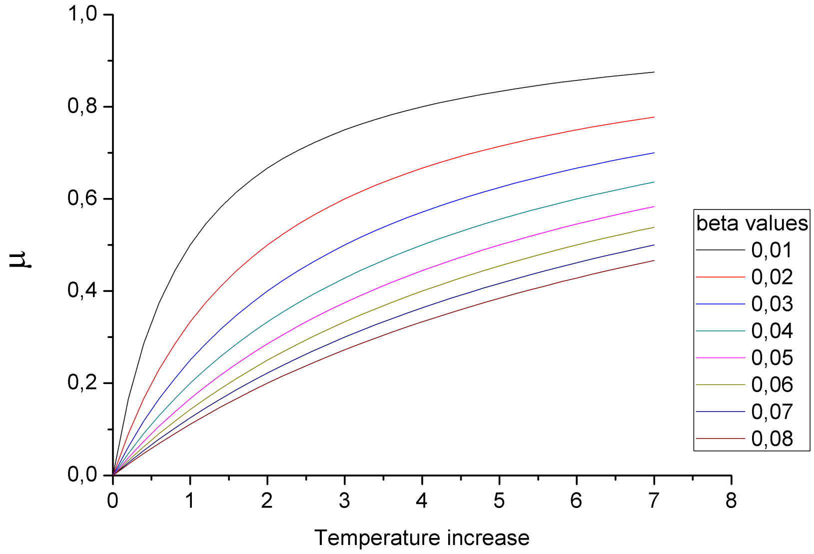
$$\eta(C^*,T) = \eta(C^*,T) = -\frac{C^* \frac{\partial^2 U(C^*,T)}{\partial C^{*2}}}{\frac{\partial U(C^*,T)}{\partial C^*}} \tag{3.23}$$

On the other hand, when plugging either equation into the definition of the temperature-risk aversion coefficient (equation (3.24), we obtain the result formulated in equation (3.25). Since the value of μ is temperature-dependent in this case, it is no longer a constant and the temperature-related axiom does not hold for this utility function apart from $\beta = 0$ (but in this case we obtain the classical quadratic function with no linear term). For a positive β-value the coefficient of temperature-risk aversion is zero for $T = 0$, is increasing with temperature and asymptotically approaches 1 for large temperature increases. For a negative β-value μ diverges when T approaches $-2\beta/\alpha$. This situation is depicted in figure 3.3 for $a = 0.005$.

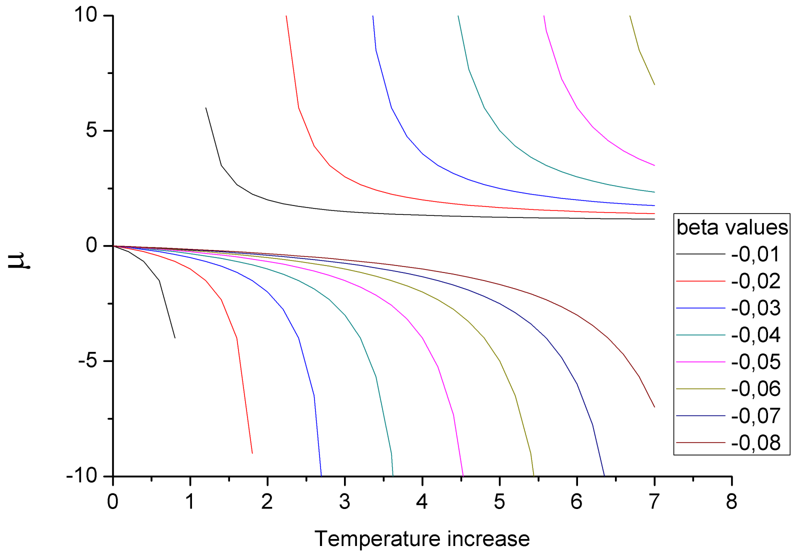
$$\mu(C^*,T) = \frac{T \frac{\partial^2 U(C^*,T)}{\partial T^2}}{\frac{\partial U(C^*,T)}{\partial T}} \tag{3.24}$$

$$\mu = \frac{2T\alpha}{2T\alpha + \beta} \tag{3.25}$$

As seen from figure 3.3a, the coefficient is positive, increasing with temperature and approaches 1 for high temperatures. This is in contrast to Weitzman's second axiom which postulates μ to be a constant (equal to 1 for a quadratic DF). But should



(a) positive β values



(b) negative β values

Figure 3.3: Dependance of the coefficient of temperature-risk aversion μ on temperature for positive β values (top) and negative β values (bottom)

the coefficient really be constant? While there is limited uncertainty about the impact of small temperature changes, the uncertainty rapidly grows for large temperature increases. In addition, the possibility of a catastrophic impact (Weitzman 2012) rises with temperature. Altogether, this should result in the policy-maker being more risk-averse when dealing with large temperature increases than with modest ones. This corresponds to the rising value of μ when temperature increases as is the case of the quadratic DF with a linear term presented here.

A different scenario is associated with negative β-values. As seen from figure 3.3b, the μ values rise above all boundaries for $T \to -\beta/2\alpha$ and whole temperature-risk aversion measurement becomes inapplicable. This stems form the fact that risk aversion only makes sense for monotonous functions while the utility with a negative β-value has a local maximum in temperature. This reflects the fact that with the linear term included, the functional form allows small temperature rises to have a positive economic impact, thus increasing utility. This is one of the reasons why the linear term was introduced, so such an observation is neither surprising, nor disturbing.

Now, when it has been demonstrated that the introduction of the linear term in temperature does not interfere with the validity of the consumption-risk aversion axiom and explained why the temperature-risk aversion axiom can not be employed, it is possible to derive the general utility function. The most general utility function $U(C^*,T)$ obeying the consumption-risk aversion axiom is the following:

$$U(C^*,T) = a(T) + b(T)C^{*1-\eta} \tag{3.26}$$

where $a(T)$ and $b(T)$ are suitable[10] functions of temperature. This is the general formula for any value of $\eta > 1$ and an unspecified function of temperature. By selecting $a(T) = -(\alpha T^2 + \beta T)$ and $b(T) = -1$ we obtain

$$U(C^*,T) = -C^{*1-\eta} - (\alpha T^2 + \beta T)$$

Likewise, by setting $a(T) = 0$ and $b(T) = -(\alpha T^2 + \beta T + 1)$ we obtain

$$U(C^*,T) = -C^{*1-\eta}(\alpha T^2 + \beta T + 1)$$

When setting $\eta = 2$ as in the previous cases we arrive to exactly the *multiplicative* and *additive* utility function form postulated *ad hoc* in equation (3.21) and (3.22) on page

[10] Releasing a rather strict temperature-risk aversion axiom does not mean that any function of temperature is eligible. The functions still should obey elementary requirements stemming from economic theory. For instance, the functions should be continuous, differentiable and concave.

28. Next, it is possible to derive the DF as the implicit solution of equation (3.27) for either utility function (analogy of the procedure described on on page 20).

$$V\{[1-D(C^*,T)]C^*\} = U(C^*,T) \tag{3.27}$$

The solution of equation (3.27) leads to equation (3.28) for the *multiplicative* utility function and to equation (3.29) for the *additive* one. These DFs reflect all the superimposed restrictions: zero damage for $T = 0$, approach 1 (100%) for high temperatures and the additive analogue causes higher damage than the multiplicative DF for increasing potential consumption.

Altogether, this section deals with the addition of a linear term in temperature into the utility function. It has been demonstrated that this addition is consistent with the consumption-risk aversion axiom and explained why the temperature-risk aversion axiom is not applicable for this case. Based on the consumption-risk aversion axiom, the general form of the utility function has been constructed. Next, two special cases, corresponding to Weitzman's *multiplicative* and *additive* analogue, of this utility function were selected and the corresponding DFs derived with the use of equation (3.27). These DFs manifest superior flexibility for low temperature increases and may thus better describe the economic damage in this temperature increase region.

$$D_M = \frac{\beta T + \alpha T^2}{1 + \beta T + \alpha T^2} \tag{3.28}$$

$$D_A = \frac{C^*(\beta T + \alpha T^2)}{1 + C^*(\beta T + \alpha T^2)} \tag{3.29}$$

3.7.2 Higher order polynomials

The general utility function obtained from the consumption-risk aversion axiom (equation (3.26) does not restrict us to the use of a quadratic function. In principle, any other continuous, differentiable and convex function could be eligible. The options include, among others, higher order polynomials. Here, I will try to demonstrate that the use of higher order polynomials is unnecessary.

It is important to keep in mind that the real DF is unknown, possibly even unknowable (Pindyck 2013b). The DFs employed in the models are the attempts to best fit the unknown, "real" DF. An important aspect of a DF is thus its ability to mimic the expected properties and shape of the "real" DF. The reason why I perceive higher order polynomials as excessive is that the quadratic function is equally capable of

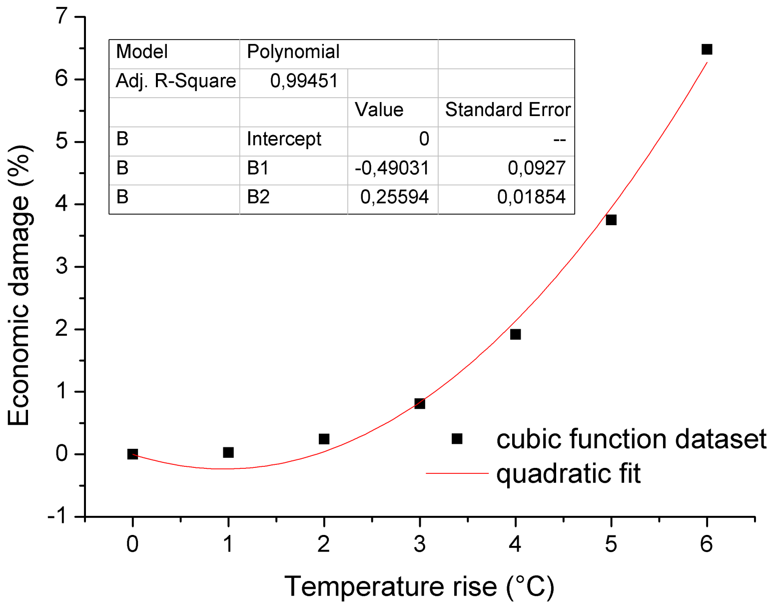


Figure 3.4: Quadratic fit of cubic data – illustration of the ability of a quadratic function to mimic a cubic one within a reasonable region.

mimicking the true DF as its higher order analogues. Let us imagine the unlikely case that the true DF is exactly of third order ($D = \alpha T^3$). This is quite an extreme case – for instance this DF is unbounded and could thus exceed 100%. Even in this case, however, a quadratic function can mimic this behavior remarkably well. This is shown in figure 3.4 by fitting data points corresponding to the third order DF by a quadratic function over a reasonable temperature increase and economic loss region. As seen from the figure, the fit is very good over with an $R^2 > 0.994$. Since no paper I am aware of considers a higher than cubic DF, a discussion of these functions will be omitted.

Chapter 4

Modifications of the damage function

4.1 Adaptation and tolerable temperature increase

The PAGE model (Hope et al. 1993, Hope and Plambeck 1996, Hope 2006; 2013) introduced a new term into the debate over climatic change: tolerable temperature increase. This temperature reflects a boundary within which no economic damage occurs. This boundary is defined by a slope parameter reflecting the maximum allowed temperature increase and by a plateau value indicating the maximum level of temperature increase. This situation is depicted in figure 2.3 on page 13. This boundary may further be expanded by an adaptive policy, which is seen as an important alternative to GHG abatement scenarios in the PAGE models. However, the presence of a damage-free temperature increase domain is contradictory to the assumptions of the remaining climatic models, which present a DF starting at a 0°C temperature rise. In addition, the idea of excluding any possible damages (or benefits) within some temperature range seems counter-intuitive. In this section, I will therefore try to resolve this striking difference.

In my opinion, this disparity is caused by a different perception of adaptation by the PAGE model and the other models (for brevity, I will use DICE as a proxy for the standard view in this section). The DICE is based on investment into abatement policy in order to avoid future damages and increase future consumption. The losses suffered from climate change are either direct economic damage (eg. flooding of farmland) or costs associated with preventing these damages (building dykes). The

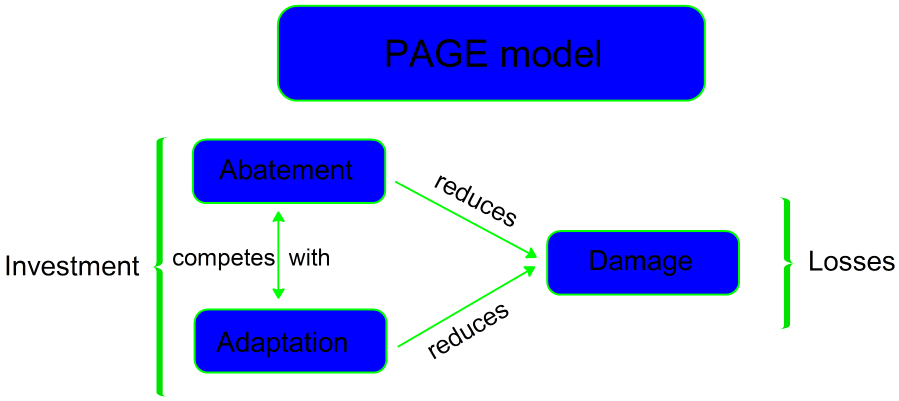
DICE model assumes that only assets economically worth saving are protected; other assets are knowingly sacrificed. Consequently, the losses associated with climate change consist of two parts: protection (adaptation) costs and direct damage.

On the other hand, PAGE views adaptation as an investment alternative to abatement policy and these two policies compete against each other. Both policies are carried out simultaneously to an extent where they are cost-efficient. Since the adaptation policy is more cost efficient than abatement policy for small temperature increases (Hope et al. 1993) and PAGE views adaptation costs as investment and not losses, it is possible that there are no losses associated with a minor temperature increase due to complete mitigation of economic damage by adaptation policy. These alternative approaches are depicted in figure 4.1.

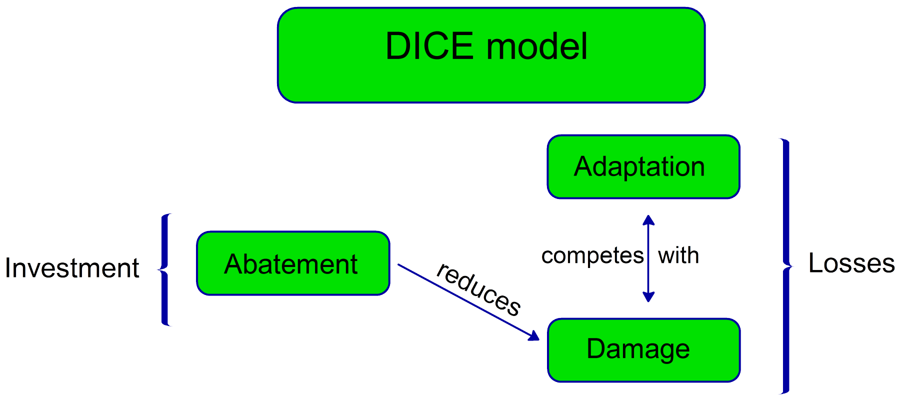
Even though this schematic explains why there can be no costs associated with a minor damage increase from the PAGE perspective, the concept of tolerable temperature increase still remains troublesome. At first, it assumes that the damage provoked by climate change can be fully mitigated by adaptation. However, in the case of intangible damage, this assumption does not seem very viable. As discussed in section 3.1, mitigation of some types of intangible damage is unrealistic. For instance, the destruction of coral reefs cannot be prevented by some investment into preventive (adaptation) measures. Secondly, the tolerable temperature does not allow for a positive impact from climate change for low temperature increases. This seems to be an unnecessary and also problematic limitation. Many climatic models predict a slight net benefit from minor climate change, mostly due to net benefits in former USSR countries. The tolerable temperature increase model thus inhibits the contribution of such benefits, forming an additional drawback to this approach.

4.2 Disaggregation of the damage function

The evaluation of economic damage is based on the calibration of a DF of a certain functional form by a benchmark point estimate of consumption loss related to a specific temperature increase, as described in section 3.4. The benchmark estimate is usually obtained by aggregating different contributions to the overall damage. These contributions are in turn estimated from a literature review or personal calculations. The DF function form then establishes the overall (aggregated) damage profile for any temperature.



(a) PAGE approach to adaptation



(b) DICE approach to adaptation

Figure 4.1: Schematic of alternative approaches towards adaptation. DICE views adaptation as costs; PAGE as investment.

However, this aggregation quietly postulates that all damage contributions follow the same damage profile, such as a quadratic function of temperature with the same coefficients. Nevertheless, as noted in (Tol 1995), this does not have to be the case. Tol argues that different contributions may exhibit different temperature-related behavior and that aggregation of the contributions into one DF inhibits this versatility. In addition, Tol relates the damage from different contributions to the relevant climatic parameter (temperature increase, sea-level rise, hurricane activity) as depicted in figure 3.3 on page 11. Even though such an approach introduces a large number of additional variables and parameters into the economic part of the model, careful reasoning can show that this enlargement of the economic part does not require additional input or estimation of additional parameters. On the other hand, the disaggregation enables the DF to model the reality more precisely, supposing that the modeler is capable of selecting an appropriate DF form for each contribution.[1] Based on the conjunction of climatology, economic theory and the modeler's best guess, a particular DF can be assigned to each contribution. The calibration proceeds as follows: since the individual damages arising from different contributions are calculated as an input into the aggregate benchmark estimate, it suffices to use these values for the calibration of individual contributions.

By the above-mentioned procedure it is thus possible to obtain a more precise DF model which encompasses possibly different temperature-dependent behavior of each contribution without the necessity of additional input data or estimates. On the other hand, two disadvantages should be acknowledged. The first (and minor) one is that the disaggregation of the DF complicates the presentation of the final DF form. Although it has been shown that the IAM can cope with the disaggregation without major problems, the resulting DF becomes difficult to plot into a graph or describe by some equation. This complicates inter-model comparison. However, this drawback alone should not discourage the use of disaggregated DFs.

The second and possibly much more serious issue lies in the question whether the modeler is actually capable of evaluating the benchmark damage contributions precisely. The exact calculation of the benchmark estimates is usually omitted or hidden in supporting materials or technical specifications, so it is hard to answer this question. Alas, since the benchmark estimates differ among each model (even among different versions of the same model), I am concerned that this is not the case. If the benchmark estimate is based on an uncertain best estimate, then said disaggregation

[1] This is no small assumption. However, it can be argued that assigning an appropriate DF form to a single contribution should be no harder (likely easier) than to the aggregated analogue, which is common practice.

makes little sense. Even if the deviation of the estimated damage from the real one were only minor on the aggregate level, the differences on the disaggregated level could be significantly larger as the random errors tend to partially cancel each other out by means of summation. To conclude, the disaggregation appears like a viable option to improve the DF form without major technical or methodological issues. However, the success of the whole procedure is conditional on the modeler's ability to encapsulate precisely the various damage contributions during the benchmark estimate.

4.3 Fitting the damage function

In the usual approach, the DF is calibrated by a single-point benchmark estimate of the damage associated with an arbitrary temperature increase. This method results in the DF being quite sensitive to the value of the benchmark estimate, but especially to the arbitrarily-chosen DF form. Section 4.2 discusses the possibility to improve the DF by differentiating the functional form for different contributions. However, within this approach, the DF coefficients are still based on a single point estimate. For this reason, the DF still remains sensitive to the chosen functional form,[2] even though in this case the functional form should ideally be backed up by economic and climatic rationale. A logical path how to better estimate the DF form would be to obtain multiple benchmark temperature damage estimates and fit them by the DF.

Although this seems to be a viable procedure – if one is able to obtain a reasonable damage estimate for a single temperature increase, he should be able to do so for multiple – this approach is rarely seen in the literature. Fitting of multiple benchmark estimates by a quadratic function appears in several recent papers (Tol (2009), Nordhaus (2011) and others), but these papers are literature reviews and the multiple estimates come from different studies. Obviously, such a fit can give a general idea about the shape of the DF, but since every study works with a different model and set of assumptions, this dataset can hardly be considered consistent. The absence of a systematic study with multiple benchmark estimates stresses the concerns underlined in the previous section – that the benchmark estimate is a rough guess with absent or large confidence intervals. If this is the case, then a logical step to improve the DF prediction, as well as the overall SCC estimates, is to focus more energy into

[2]For instance, in the case where Tol (1995) expected a quadratic function with a linear term, he had to assign the weight of each contribution at the benchmark temperature to allow for a single point estimation.

understanding the direct and indirect impacts of climate change.[3] Unfortunately, the precise values of damage still appear to be uncertain to an extend to discourage researchers from constructing multiple estimates.

4.4 Tangible vs. intangible damage

A further complication of the DF originates from the distinction of the tangible (economic or "real") damage and intangible (non-economic or "ethereal") damage. Since the pilot SCC models (Nordhaus 1993b, Hope et al. 1993), researchers were aware of the presence of intangible damages associated with climate change. However, the full incorporation of intangible damage has proven to be a complicated and controversial process. At first, scientists attempted to treat intangible damage in a similar fashion as tangible damage. For this purpose, non-market good valuation methods, such as the WTP (Arrow et al. 1993), were employed to monetize the ethereal damage. Even though the WTP method was often criticized for unreliable estimates, several researchers began to question this approach for a different reason. They argued that intangible damage decreases our utility directly, unlike tangible damage, which decreases our utility via reduced consumption. In his note on tangible and intangible damage, Tol (1994) demonstrated that plugging the intangible damage directly into the utility function leads to a substantially higher optimal emission reduction pathway in the DICE model. Tol does this by separating the intangible and tangible damage in their assumed ratio and accounting for the different types of damage separately. This approach is very important for the perception of the adverse impacts of climate change. In addition, it can be related to Weitzman's DF prototypes. For this reason, the original DICE utility function as well as Tol's modification will now be presented.

The utility function assumed in DICE is presented in equation (4.1) and the relationship for consumption C is defined in equation (4.2). Both equations are presented along with the variable notation on page 7.

$$U = \sum_t \frac{L_t ln(C_t/L_t)}{(1+\delta)^t} \tag{4.1}$$

$$C_t = \frac{\varepsilon_t Y_t}{(1+\alpha T^2)} \tag{4.2}$$

[3] This does not mean that research in this area is being neglected. Recent advancements in the understanding of damage from climate change originate from Schlenker and Roberts (2009), Deschenes and Greenstone (2007), Dell et al. (2012) and others.

Essentially, the DICE model optimizes investment into climate policy (parameter ε) in order to maximize the joint utility originating from consumption. The same approach is advised in Tol's note for tangible damage. Intangible damage, on the other hand, is plugged directly into the utility function in the following manner:

$$\text{U'}=\sum_t \frac{L_t ln\left[(C'_t/L_t)-\omega\alpha T^2 Y_t/L_t\right]}{(1+\delta)^t} \tag{4.3}$$

where ω is the fraction of intangible damage, α the usual coefficient of proportionality, Y the potential output, C' is the consumption defined by equation (4.4) and other variables follow the notation from the DICE model utility function described by equation (2.2) on page 7.

$$C'_t = \frac{\varepsilon_t Y_t}{[1+\alpha T^2(1-\omega)]} \tag{4.4}$$

Compared to DICE, Tol thus reduces the consumption decrease which he assigns only to tangible damage. On the other hand, he introduces a new term into the utility function which decreases the total welfare directly and associates this term with intangible damage.

Using this approach to recalculate "trustworthingly" the original DICE results requires a rather large leap of faith: Tol assumes that the proportionality coefficient α remains the same when employing this new utility function form to compare the results of the original DICE model and his modification. That this might not be so straightforward is hinted by Tol (1995), where the same author indeed does distinguish between tangible and intangible damage but does not clearly state whether both types of damages are treated differently within the FUND framework. In his note, Tol also does not comment on the specific utility function form he chose. For instance, the occurrence of output rather than consumption is equation (4.3) is not discussed and the decrease in consumption loss is not visible.[4]

Putting aside the potential issues connected to calibration, such a concept has far-reaching implications nevertheless. Most importantly, it renders the term DF obsolete within the current paradigm. Up to now, the DF was understood as a measure of proportionality among temperature increase and consumption loss. This remains the case for tangible damage in this new concept, but is no longer the case for intangible damage, which enters directly into the utility function. Nevertheless, this is not an unresolvable issue – the model can readily optimize the policy-related pa-

[4]Equation (4.4) results from my derivation.

rameters in order to maximize the utility function. As such, it does not necessarily require a DF *per se* to exist. Actually, maximizing directly the discounted utility function is advocated as a superior approach to maximizing discounted consumption by Creedy (2007). Of course, there persists a relationship between climate change and suffered losses (be it consumption or utility) as visible from equation (4.3). The properties of such a relationship remain unchanged and still follow the requirements discussed throughout this work. Solely, in Tol's case of separating intangible damage, the whole climatic impact can not be reduced to a single, easily comparable consumption-reduction relationship: the damage function.

As explained on the last paragraph, the DF in the classical interpretation (absolute or percentage loss of consumption related to climate change) is not necessary for the SCC framework. The optimization can be based on discounted utility with an optimal emission trajectory as the result. The SCC can then be set as the costs associated with enforcing the optimal pathway. However, since this is not the only established method how to obtain the SCC,[5] it would be useful to be capable of transforming the intangible damage utility function to the classical consumption-related DF. Fortunately, such a transformation is possible. By employing the equation for the implicit solution of the DF (equation (4.5), as defined on page 20), the DF can be obtained for a single-period utility function.[6]

$$V\{[1-D(C^*,T)]C^*\} = U(C^*,T) \tag{4.5}$$

Since this procedure becomes rather complicated for Tol's utility function, it will first be shown for the original DICE version. By reducing equation (4.1) and equation (4.2) to a single time period and plugging them into equation (4.5), we obtain the following relationship:

$$Lln\left[(1-D(C^*,T))\frac{\varepsilon Y}{L}\right] = Lln\left[\frac{\varepsilon Y}{L(1+\alpha T^2)}\right] \tag{4.6}$$

Solving this equation leads to a rather simple form of equation (4.7), which exhibits the expected properties, such as zero damage for a null temperature increase and approaching 100% damage for high temperature increases.[7]

$$D(C^*,T) = \frac{\alpha T^2}{1+\alpha T^2} \tag{4.7}$$

[5]Even though the utility-based approach has attracted more attention recently, the consumption-based one still remains more frequently used so far.

[6]Here C^* stands for potential consumption *per capita*.

[7]This is only true for $\alpha > 0$. However, this is implied by the calibration mechanism.

The situation becomes more difficult for Tol's utility function. Plugging into the equation for the implicit solution leads to the following formula:

$$Lln\left[(1-D(C^{*},T)\frac{\varepsilon Y}{L}\right]=Lln[\frac{\varepsilon Y}{L(1+\alpha T^{2}(1-\omega)}-\frac{\omega\alpha YT^{2}}{L}] \tag{4.8}$$

The solution of this formula is presented in equation (4.9).[8] The first term is the contribution of tangible damage. This contribution is identical to the DICE model, which can be demonstrated as follows: by acknowledging only tangible damage (setting $\omega = 0$) we receive the same result as from the DICE model. The second contribution stems from the intangible damage contribution and is a more strongly increasing function of temperature. Altogether, the damage estimate increases. Tol (1994) explains this on the utility level by the fact that the intangible damage is moved from the flexible production function to a more rigid utility function. On a more general level, it becomes harder to substitute between consumption and intangible damages.

$$D(C^{*},T)=\frac{\alpha T^{2}(1-\omega)}{1+\alpha T^{2}(1-\omega)}+\frac{\omega\alpha T^{2}}{\varepsilon} \tag{4.9}$$

In his note, Tol makes one more step forward by realizing that the tangible damages often represent damages to the environment which he assumes to be a luxury good.[9] To account for this is his utility function, he adds an additional term which makes the utility function convex in environmental quality. This is achieved by the following change:

$$U\text{''}=\sum_{t}\frac{L_{t}ln\left[(C_{t}/L_{t})-\omega\alpha T^{2}(Y_{t}/L_{t})\,(C_{t}/L_{t})\,/\,(C_{0}/L_{0})\right]}{(1+\delta)^{t}} \tag{4.10}$$

Now, when consumption *per capita* increases, more value is put on the "environmental" term in the utility function. This function can again be reduced to a single time period and then plugged into equation (4.5). By algebraic rearrangement of said equation and realizing that $C/L = C^{*}$, the following damage function for environment treated as a luxury good is obtained in equation (4.11). This DF is similar to the prior one but it is visible that for growing *per capita* consumption the damages

[8] One can argue that the value of the DF does not make sense for ε approaching zero. However, this is to be expected. Tol notes that his utility function only approximates the real DF for reasonable values of parameters. This can be seen directly from the utility function which includes a logarithm and thus is not defined for extremely high temperatures or low consumption.

[9] There is likely no hard evidence for such a claim. Nevertheless, it can be reasoned that high income countries (EU or USA) dedicate much more effort to mitigating climate change than low income countries where most of the adverse effects will likely occur (Africa, South-east Asia).

become more elevated.

$$D(C^*,T) = \frac{\alpha T^2(1-\omega)}{1+\alpha T^2(1-\omega)} + \frac{\omega\alpha T^2}{\varepsilon}\frac{C^*}{C_0} \tag{4.11}$$

Tol thus concludes that treating the intangible damages separately, and especially as a luxury good, leads to higher optimal abatement scenarios. However, he does not explain why he opted for this specific utility function form. Nevertheless, he opened the interesting debate over different damage contributions and the implications they might have on the SCC results.

4.4.1 Intangible damage and the prototype additive DF

While reading the last section, an attentive reader might have experienced a *déjà vu*. This might be related to the fact that similar reasoning concerning the different contributions or mechanisms of the DF was used by Weitzman (2010) and discussed in the section about risk-aversion axioms (section 3.3). It is remarkable that two considerably different articles are in accord to such an extent. Tol (1994) simply adjusts the utility function of the DICE model of Nordhaus (1992) to highlight the impact of proposed tangible damage of the optimal policy. The utility function is rather *ad hoc* and the paper serves mainly empirical purposes. On the other hand, Weitzman (2010) is a mainly theoretical work which builds the utility function by a microeconomy-based axiomatic approach. Nevertheless, both papers have a lot in common and a conjunction of their results might be an important step towards a better understanding of the DF.

The standard DICE utility function for a single time period and one person is $U = ln(C^*/1+\alpha T^2)$[10] and the corresponding DF is $D(C^*,T) = \frac{\alpha T^2}{1+\alpha T^2}$. The same applies for any of Tol's modifications for $\omega = 0$. This can be readily related to Weitzman's multiplicative utility function $U = -\left[C^*/(1+\alpha T^2)\right]^{-1}$ yielding an identical DF. Both functions relate climate change to reduced consumption in the same manner. In addition, both the logarithmic and negative inverse function are of the CRRA class so both transformations of consumption to utility yield equal results.

The situation in less similar in the case of the second contribution. Tol associates this contribution to intangible damage and introduces a negative term into the utility function whose magnitude rises with temperature and consumption. On the other hand, Weitzman, in his additive analogue, adds a consumption-independent negative

[10] This stems from equation (4.1) and equation (4.2). C^* still denotes potential consumption *per capita*.

contribution to the utility. However, this contribution stands outside the transforming function (for details see equation (3.9) on page 20) so that substitution of consumption and environmental amenity is not possible. Even though both utility functions look very different, the resulting DF has remarkably similar properties. For strictly intangible damages ($\omega = 1$) and normalizing C_0 to unity, Tol's final DF reduces to equation (4.12) while Weitzman's pure additive DF is expressed by equation (4.13). The only differences are the presence of ε in Tol's version and the denominator in Weitzman's. The consumption ratio ε originates from Tol using output instead of consumption in weighing the intangible damage. The denominator difference is caused by a better-designed utility function in Weitzman. Tol's utility function is plausible only for modest values of consumption and temperature increase. For extreme values, the logarithm is not defined and the damages can exceed 100 %. On the contrary, the additive DF only converges to 1 for high temperatures. Nevertheless, for low temperature increases the denominator is close to one and both DFs yield similar results.

$$D(C^*,T) = \frac{\alpha T^2 C^*}{\varepsilon} \tag{4.12}$$

$$D_A(C^*,T) = \frac{\alpha_A T^2 C^*}{\alpha_A T^2 C^* + 1} \tag{4.13}$$

When considering both contributions simultaneously, the DFs begin to look more different. This originates by a different approach in incorporating the intangible damage into the utility function, as discussed above. However, some similarities are still notable. In addition, the comparison of both DFs can be helpful when interpreting the α parameters in Weitzman (2010). By comparing equation (4.11) with Weitzman's general DF (equation (4.14) there is an apparent relationship among the α parameters: $\alpha_M = \alpha(1-\omega)$ and $\alpha_A = \alpha\omega$. While Weitzman operates with these coefficients, he does not discuss their magnitude. The use of these relationships, however, relates their magnitude to the fraction of (in)tangible damage. The justifiability of such an approach can be shown as follows: by substituting these relations into equation (4.14) and for constant potential consumption normalized to unity, the DF reduces to: $D(C^*,T) = \frac{\alpha T^2}{1+\alpha T^2}$ for any value of ω. This is in accord with Weitzman, who claims that both DF analogues yield the same result for constant consumption. The comparison of the DFs thus indicates a way to calibrate the coefficients in Weitzman's more thought-through DF, which can be done by weighing the tangible and intangible damage.

$$D(C^*,T) = \frac{\alpha_A C^* T^2 + \alpha_M T^2}{\alpha_A C^* T^2 + \alpha_M T^2 + 1} \tag{4.14}$$

While the mathematical results are similar in both papers, the reasoning behind is even more so. Even though one mentions tangible and intangible damage while the other operates with the multiplicative and additive DF analogue, both authors contribute the distinction to the same phenomenon: the presence of damage which does not reduce consumption yet reduces our utility. In addition, by using Tol's concept it is possible to weigh both contributions and obtain a calibrated, Weitzman's-axioms-abiding DF:

$$D(C^*,T) = \frac{\alpha\omega C^* T^2 + \alpha(1-\omega)T^2}{1 + \alpha\omega C^* T^2 + \alpha(1-\omega)T^2} \tag{4.15}$$

4.5 Derivation of a versatile damage function

Section 4.4 explains in detail the concept of intangible damage and subsection 4.4.1 compares Tol's intangible damage with Weitzman's additive DF. Furthermore, the ratio of intangible damage ω is introduced into the additive DF to obtain a general and calibrated risk-aversion-axiom-abiding DF. Now everything is set to derive a general consumption-risk-aversion-axiom-abiding DF with an introduced linear term in temperature. This can be done by relaxing the temperature-risk aversion axiom, which is justified in section 3.7.1. The same section deals with the general derivation of the utility function under this axiom, which results in a general function in the following form:

$$U(C^*,T) = a(T) + b(T)C^{*1-\eta} \tag{4.16}$$

Section 3.7.1 then proceeds to derive an analogue to the classical additive and multiplicative DF. However, a more general treatment can result in a highly universal DF which encompasses all the important concepts mentioned so far: the differentiation between tangible and intangible damage, the presence of a linear term in temperature and the consumption-risk aversion. By choosing $a(T) = -(\alpha\omega T^2 + \beta\omega T)$ and $b(T) = -\left[1 + \alpha(1-\omega)T^2 + \beta(1-\omega)T\right]$ we arrive to the following equation for the utility function for $\eta = 2$:

$$U(C^*,T) = -(\alpha\omega T^2 + \beta\omega T) - \frac{\left[1 + \alpha(1-\omega)T^2 + \beta(1-\omega)T\right]}{C^*} \tag{4.17}$$

Next, the DF is elucidated accordingly to the procedure described in 3.7.1. Plugging the utility function into equation (3.27) on page 31 leads to the following DF:

$$D(C^*,T) = \frac{\omega C^*(\alpha T^2 + \beta T) + (1-\omega)(\alpha T^2 + \beta T)}{1 + \omega C^*(\alpha T^2 + \beta T) + (1-\omega)(\alpha T^2 + \beta T)} \tag{4.18}$$

where ω is the ratio of intangible damage. The reader can verify that the damages are equal to zero for $T = 0$, are increasing with potential consumption C^*(due to the intangible damage contribution) and asymptotically approach 1 for high temperature increases T. In addition, the linear term advocated in section 3.7 is introduced in both the intangible and tangible damage contribution and the consumption-risk aversion axiom holds for the original utility function. The former can be verified by alternatively setting ω to 0 and 1, which leads to the special cases derived in subsection 3.7.1, the latter by plugging equation (4.17) into equation (3.23). The DF presented in equation (4.18) thus represents an exceptionally versatile, yet analytically tractable version of a DF.

4.6 Best practice

So far, different modifications to the DF have been presented and various concepts of damage treatment introduced. Now it is time to draw conclusions as to which of them is the most suitable for the estimation of the SCC. Section 3.4 explains the construction of the DF and notes that two different estimates are necessary for its construction. At first, the functional form must be elucidated. Next, this functional form has to be calibrated by one or more point estimates.[11] In this section I suggest that depending on the preciseness of both steps, different best practice approaches may be recommended.

If several precise point estimates are obtainable, then it seems rational to use the universal DF (equation 4.17 on the current page). This DF accounts for all the important effects discussed throughout this work: it obeys the consumption-risk aversion

[11] Point estimate for brevity refers to the damage as percentage of consumption loss associated with a benchmark temperature increase.

axiom, discriminates among tangible and intangible damage and allows for an economic surplus for low temperature increases while simultaneously yielding significant damage for high temperature increases. The calibration by these several[12] point estimates should be sufficient to model the reality with reasonable accuracy and no assertions about the exact DF form are required.

On the other hand, if the functional forms of the different damage contributions can be asserted well from economic theory, the specific functional form can be assigned directly to each contribution. This should lead to more precise estimates for each contribution, especially for large temperature increases where the functional form becomes a dominant driving force for the damage estimate. A precise overall damage estimate could thus be obtained by aggregation of the individual damage contributions. An additional advantage of this approach is that only one point estimate is needed in this case.

Both above-mentioned approaches offer a versatile damage function form, which is especially important for high temperature increases which drive the SCC estimate upwards. Since confidence intervals are missing in many reports, it is hard to judge the accuracy of either the point estimate values or the DF functional forms. Nevertheless, this thesis offers alternative best-practice approaches for both scenarios.

[12] Two estimates are sufficient for calibration of the model. If more estimates are available, then the parameters may be obtained as a fit of the available point estimates.

Chapter 5
Conclusions

The present value damages resulting from climate change as well as the investment into GHG abatement are often reported in the realms of trillion USD. Since decisions about such astronomical investments are based on the social cost of carbon (SCC) estimates, it is of utmost importance to have reliable SCC models. For this reason I have chosen the damage functions (DFs) entering the SCC as the topic for my thesis. My main aims are the following: to conduct a review of the most influential SCC models, assess their strengths and weaknesses and propose for the first time a best practice for the construction of a DF.

Chapter 2 presents the most influential SCC models (DICE, FUND and PAGE) and discusses the differences among them. Special attention is dedicated to the DFs utilized in these models. The most significant deviation in the DF is the concept of tolerable temperature increase in PAGE. This concept is assessed in section 4.1. The use of tolerable temperature increase is not recommended for the following reasons: (i) this concept disregards costs associated with adaptation to climate change from the SCC estimate and (ii) it *a priori* excludes a net positive impact of climate change. On the other hand, the possibility to mitigate climate damage by adaptation expenditures – a feature heavily used by PAGE – should resonate in the other models.

The most frequently-used DF appearing in the SCC models is a quadratic DF in the following form: $D(C^*, T) = \frac{\alpha T^2}{1+\alpha T^2}$. This corresponds to the *multiplicative* DF analogue resulting from Weitzman's risk aversion axioms. This DF, along with its *additive* analogue, is derived in section 3.3. In addition, section 3.6 presents numerical examples of the impact of the DF choice on the total damages and concludes that rising potential consumption leads to a significantly higher damage estimate from the *additive* analogue.

Weitzman's DF analogues exhibit several desirable properties. They are derived from economic theory, account for two different damage pathways and are analytically tractable. However, their shape is highly restricted, most notably for low temperature increases. For this reason, I introduce a linear term in temperature into the DF. Section 3.7 describes this modification within the axiomatic framework. A new DF including a linear term in temperature is introduced and the compliance with the consumption-risk aversion axiom is demonstrated. This DF violates the temperature-risk aversion axiom, but it is shown that this is in accord with economic theory. The special cases of the resulting DF are presented on page 31.

The division of *multiplicative* and *additive* damages roughly corresponds to the distinction of tangible and intangible damage by Tol. Section 4.4 presents my derivation of the DFs stemming from Tol's modifications of the DICE utility function. Subsection 4.4.1 then compares the DFs resulting from Tol's utility function with those originating from Weitzman's. I conclude that the latter exhibits better properties, especially for higher temperature increases. However, by comparison of both DFs, it is visible that the damage ratio ω used in Tol can readily be used to calibrate the α coefficients used by Weitzman to obtain the following DF:

$$D(C^*,T) = \frac{\alpha\omega C^* T^2 + \alpha(1-\omega)T^2}{1+\alpha\omega C^* T^2 + \alpha(1-\omega)T^2} \tag{5.1}$$

The final step in the derivation of a universal function consists of the addition of the linear term into this equation. This issue is treated in section 4.5. Starting off with the consumption-risk aversion axiom and by an appropriate selection of the temperature-dependent functions, it is possible to combine all the above-mentioned notions (consumption-risk aversion, differentiation between tangible and intangible damage and the presence of a linear term) into one vibrant DF. The final universal damage function is defined as follows:

$$D(C^*,T) = \frac{\omega C^*(\alpha T^2 + \beta T) + (1-\omega)(\alpha T^2 + \beta T)}{1+\omega C^*(\alpha T^2 + \beta T) + (1-\omega)(\alpha T^2 + \beta T)} \tag{5.2}$$

While this universal DF encompasses all the above-mentioned issues at the sole expense of two degrees of freedom, it is noteworthy that there is an alternative approach to a possibly very precise approximation of the true DF: the disaggregation method. This approach is presented in section 2.2 and further discussed is section 4.2. Essentially, each damage contribution can be accounted for separately and with a different DF form. If the modeler is able to assert well the DF form for each contribution, this method could yield very precise results.

So far, I have laid out two alternative methods to construct a viable DF: the calibration of a highly universal DF (equation (5.2) by several point estimates and a disaggregated approach where a simple DF is attributed to each damage contribution, each contribution is calibrated by a single point estimate and the total damages are summed up. The question is: which one to opt for?

In section 4.6, I reason that the answer depends on our ability to estimate precisely the benchmark temperature increase impact and our ability to assign a correct functional form to a specific damage contribution. If we are able to assert well the DF form associated with the various damage contributions, then the disaggregated approach can easily lead to a well-structured DF with the use of a single benchmark temperature increase estimate. On the other hand, when the estimation of specific damage contributions is troublesome but the benchmark temperature increase estimates are reliable, multiple estimates can be used to calibrate the universal DF presented in equation (5.2) to obtain a precise approximation of the true DF.

The employed damage function can influence heavily the social cost of carbon estimates, which in turn effects the decision-making in climate policy. Since these decisions involve astronomical figures, it is essential to base these decisions on formidable grounds. The choice of a reasonable DF is therefore very important. For this reason, this thesis reviews the DFs used in various SCC models and concludes with two different best practice approaches towards the construction of a DF which could be used in further climate change studies.

Bibliography

Arrow, K., Solow, R., and others (1993). *Report of the NOAA panel on contingent valuation*. National Oceanic and Atmospheric Administration Washington, DC.

Ciriacy-Wantrup, S. V. (1947). Capital returns from soil-conservation practices. *Journal of farm economics*, 29(4 Part II):1181–1196.

Clarkson, R. and Deyes, K. (2002). Estimating the social cost of carbon emissions.

Creedy, J. (2007). Discounting and the social time preference rate.

Dell, M., Jones, B. F., and Olken, B. A. (2012). Temperature shocks and economic growth: Evidence from the last half century. *American Economic Journal: Macroeconomics*, 4(3):66–95.

Deschenes, O. and Greenstone, M. (2007). The economic impacts of climate change: evidence from agricultural output and random fluctuations in weather. *The American Economic Review*, pages 354–385.

Diamond, P. A. and Hausman, J. A. (1994). Contingent valuation: Is some number better than no number? *The Journal of economic perspectives*, pages 45–64.

Falkowski, P., Scholes, R. J., Boyle, E. e. a., Canadell, J., Canfield, D., Elser, J., Gruber, N., Hibbard, K., HÃ¶gberg, P., Linder, S., and others (2000). The global carbon cycle: a test of our knowledge of earth as a system. *science*, 290(5490):291–296.

Fankhauser, S. (1994). The social costs of greenhouse gas emissions: An expected value approach. *The Energy Journal*, 15(2).

Hausman, J. A. (1993). *Contingent valuation: A critical assessment*. Elsevier.

Havranek, T., Irsova, Z., Janda, K., and Zilberman, D. (2014). Selective reporting and the social cost of carbon. Department of Agricultural & Resource Economics, UC Berkeley, Working Paper Series qt8wk3t1c8, Department of Agricultural & Resource Economics, UC Berkeley.

Hope, C. (1992). *A fast and simple model of global warming from greenhouse gas emissions*. University of Cambridge, Cambridge.

Hope, C. (2006). The marginal impact of CO2 from PAGE2002: An integrated assessment model incorporating the IPCC's five reasons for concern. *Integrated assessment*, 6(1).

Hope, C. (2013). Critical issues for the calculation of the social cost of CO2: why the estimates from PAGE09 are higher than those from PAGE2002. *Climatic Change*, 117(3):531–543.

Hope, C., Anderson, J., and Wenman, P. (1993). Policy analysis of the greenhouse effect: an application of the PAGE model. *Energy Policy*, 21(3):327–338.

Hope, C. and Plambeck, E. L. (1996). PAGE95: An updated valuation of the impacts of global warming. *Energy Policy*, 24(9):783–793.

Horowitz, J. K. and McConnell, K. E. (2002). A review of WTA/WTP studies. *Journal of Environmental Economics and Management*, 44(3):426–447.

Marini, G. and Scaramozzino, P. (2000). Social time preference. *Journal of Population Economics*, 13(4):639–645.

McCarthy, J. J. (2001). *Climate change 2001: impacts, adaptation, and vulnerability: contribution of Working Group II to the third assessment report of the Intergovernmental Panel on Climate Change*. Cambridge University Press.

Nordhaus, W. (2014). Estimates of the social cost of carbon: Concepts and results from the DICE-2013r model and alternative approaches. *Journal of the Association of Environmental and Resource Economists*, 1(1).

Nordhaus, W. D. (1992). The 'DICE' model: Background and structure of a dynamic integrated climate-economy model of the economics of global warming. Cowles Foundation Discussion Paper 1009, Cowles Foundation for Research in Economics, Yale University.

Nordhaus, W. D. (1993a). Optimal greenhouse-gas reductions and tax policy in the "DICE" model. *The American Economic Review*, 83(2):313–317.

Nordhaus, W. D. (1993b). Rolling the DICE: an optimal transition path for controlling greenhouse gases. *Resource and Energy Economics*, 15(1):27–50.

Nordhaus, W. D. (2011). Estimates of the social cost of carbon: background and results from the RICE-2011 model. Technical report, National Bureau of Economic Research.

Nordhaus, W. D. and Boyer, J. (1999). Roll the DICE again: the economics of global warming. *Draft Version*, 28:1999.

Pindyck, R. S. (2013a). Climate change policy: What do the models tell us? Working Paper 19244, National Bureau of Economic Research.

Pindyck, R. S. (2013b). The climate policy dilemma. *Review of Environmental Economics and Policy*, 7(2):219–237.

Pratt, J. W. (1964). Risk aversion in the small and in the large. *Econometrica*, 32(1/2):122–136.

Ramsey, F. P. (1928). A mathematical theory of saving. *The economic journal*, pages 543–559.

Roe, G. H. and Baker, M. B. (2007). Why is climate sensitivity so unpredictable? *Science*, 318(5850):629–632.

Schlenker, W. and Roberts, M. J. (2009). Nonlinear temperature effects indicate severe damages to US crop yields under climate change. *Proceedings of the National Academy of sciences*, 106(37):15594–15598.

Stern, N. (2007). *The economics of climate change: the Stern review*. Cambridge University press.

Stocker, T. F., Qin, D., Plattner, G.-K., Tignor, M., Allen, S. K., Boschung, J., Nauels, A., Xia, Y., Bex, V., and Midgley, P. M. (2013). Climate change 2013: The physical science basis. *Intergovernmental Panel on Climate Change, Working Group I Contribution to the IPCC Fifth Assessment Report (AR5)(Cambridge Univ Press, New York)*.

Tol, R. S. (1994). The damage costs of climate change: a note on tangibles and intangibles, applied to DICE. *Energy Policy*, 22(5):436–438.

Tol, R. S. (1995). The damage costs of climate change toward more comprehensive calculations. *Environmental and Resource Economics*, 5(4):353–374.

Tol, R. S. (1996). The damage costs of climate change towards a dynamic representation. *Ecological Economics*, 19(1):67–90.

Tol, R. S. (2009). The economic effects of climate change. *The Journal of Economic Perspectives*, pages 29–51.

Tol, R. S. (2011). The social cost of carbon. Technical Report 377, ESRI working paper.

Weitzman, M. L. (2010). What is the" damages function" for global warming-and what difference might it make? *Climate Change Economics*, 1(01):57–69.

Weitzman, M. L. (2012). GHG targets as insurance against catastrophic climate damages. *Journal of Public Economic Theory*, 14(2):221–244.