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Financial markets modeling –  
experimental and agent based approach

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## Abstrakt

Tato práce se zabývá problémem modelování finančních trhů. K modelování používáme dva přístupy: simultánní a experimentální. Nejprve představíme agentní modelování a experimentální ekonomii. Poté vysvětlíme silné a slabé stránky těchto přístupů a ukážeme jejich společný přínos v oblasti modelování finančních trhů. Aby čtenář získal komplexnější představu o celé problematice, uvedeme několik modelů používajících kombinovanou metodologii. Následně představíme model dvojité aukce, jehož autory jsou Gode a Sunder (1993). Naši práci zakončíme výsledky experimentu, který jsme sami provedli, a jehož základní myšlenkou je právě práce od Goda a Sundera.

## **Abstract**

This thesis deals with the problem of financial markets modeling. We use two approaches to the modeling: agent-based simulation and experimental economics. First, we introduce the agent-based models and the experimental economics. Then we discuss strengths and weaknesses of both approaches and show their mutual contribution to this area of financial markets modeling. In order to provide the reader with more complex picture, we give examples of some existing models that use this combined methodology. Subsequently, we explain the model of a double-auction market originally presented by Gode and Sunder (2003). We finish our thesis by providing results of our own experiment conducted with the idea based on Gode and Sunder's paper

## **Klíčová slova**

Modelování finančních trhů, experimentální ekonomie, agentní modelování.

## **Keywords**

Financial markets modeling, experimental economics, agent-based simulation.

## **Declaration of Authorship**

I hereby proclaim that I wrote my bachelor thesis on my own under the leadership of my supervisor and that the references include all resources and literature I have used.

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# Contents

<b>Introduction</b>	<b>2</b>
<b>1 Agent-based Models and Experimental Economics</b>	<b>4</b>
1.1 Agent-based Models and Experimental Economics as a Research Method . . . . .	4
1.1.1 Experimental Economics . . . . .	4
1.1.2 Agent-based Models . . . . .	7
1.2 The Advantages of Combining Both Methods . . .	12
1.3 Examples . . . . .	16
1.3.1 Near-Zero-Intelligence Traders—Model by Duffy and Ünver (2006) . . . . .	16
1.3.2 Learning Competitive Equilibrium—Model by Crockett et al. (2004) . . . . .	20
<b>2 The Model</b>	<b>24</b>
<b>3 The Experiment</b>	<b>28</b>
3.1 The Experimental Setup . . . . .	28
3.2 The Results . . . . .	31
<b>Conclusion</b>	<b>38</b>
<b>References</b>	<b>40</b>
<b>List of tables and figures</b>	<b>44</b>
<b>Appendix A</b>	<b>45</b>

# Introduction

Financial markets are one of the most followed up areas of economics. Economic theorists try to understand the way financial markets work and evolve. If we understand the financial markets better, we can regulate them effectively and prevent such market failures as the U.S. subprime mortgage crisis in 2008.

To reach a deeper understanding, we need to find the suitable instruments for financial markets modeling. There are many ways, how to model financial markets. However, the mainstream theories, e. g. the Efficient Market Hypothesis, are in an unsatisfactory condition. This theory assumes that the market is composed of identical individuals, who behave strictly rationally. Therefore, we use two bottom-up methodologies in our thesis: the agent-based simulation (ABM) and the experimental economics (EXP). A bottom-up methodology is a way of modeling, where rather than using unrealistic assumptions and simplifying the complexity of human behavior, the model is built of agents deciding according to the set of given rules. Then the output on the macro-level results from these agents' behavior.

The core of this thesis lies in the combination of these two approaches, which ideally complement each other. Such an approach to modeling is more valid and brings the models closer to reality.

This bachelor thesis is organized as follows. First, we introduce

agent-based models and experimental economics. Then we discuss strengths and weaknesses of both approaches and show their mutual contribution to this area of financial markets modeling. We give examples of some existing models using this combined methodology. In the next part we explain the model of a double-auction market originally developed by Gode and Sunder (2003). We finish by providing results of our own experiment conducted with the idea based on Gode and Sunder's paper.

# 1 Agent-based Models and Experimental Economics

## 1.1 Agent-based Models and Experimental Economics as a Research Method

### 1.1.1 Experimental Economics

In this chapter we introduce the basic principles of experimental economics and agent-based models. Further, we describe its general properties and explain its advantages.

Experimental economics emerged in the 1960's. At that time, economists took interest in testing microeconomic assumptions such as human rationality, which lie under the most contemporary economic theories (Friedman and Cassar, 2009). With the new theories (game theory, industrial organization, general equilibrium, etc.) new ways of understanding and interpreting microeconomic data occurred. The economists started to explore all the possibilities that the usage of experimental economics (EXP) can bring. Since that time laboratory experiments have received increasing attention. Economists use experiments for many reasons: understanding price setting mechanism, occurrence of competitive equilibrium, testing the validity of economic theories or examining a theoretical parameter such as the degree of risk aversion. According to Wilde (1981) conducting a laboratory experiment means creating "a small-scale microeconomic environment in the

laboratory where adequate control can be maintained and accurate measurement of relevant variables guaranteed" (Wilde 1981, p. 138). Or in words of Gaechter (2009) it means "to uncover empirical regularities, to test the behavioral implications of institutions and incentives, to uncover the structure of peoples' attitudes towards risk and uncertainty, their time preferences and their social preferences" (Gaechter 2009, p. 3).

One of the first works, that tried to formulate a theory of laboratory experiments in economics, is a seminal work of Smith (1982). He distinguishes between the microeconomic system itself and the agent behavior, which is observed within this system. A microeconomic system is defined by two basic components—an environment and an institution. The environment consists of agents, traded assets and a set of rules for these agents. The institution includes a certain language, allocation rules, imputation rules and adjustment process rules (starting, transition and stopping rule). Without these rules it would be impossible to run an experiment. Regarding the agents behavior, three types of behavior are differentiated by Smith—an outcome behavior, a response behavior and a system performance. The outcome behavior represents the final result of the message exchange, which through the institution, determines the outcomes. "Agents do not choose direct commodity allocations. Agents choose messages, and institutions determine allocations via the rules that carry messages into allocations" (Smith 1982, p. 962). The response behavior

makes decisions based on the past events. It can be completely random, based on some decision rule, or just unaccountable. The criterion for the system performance is Pareto efficiency. It is, by the definition, a state of allocation of resources in which it is impossible to make any individual better off without making at least one individual worse off. Utility functions and production frontiers are not observable. Therefore, the evaluation of the outcomes according to Pareto efficiency is only meaningful if we consider the assumptions about preferences, technology, and behavior. It means, that Pareto optimality is satisfied if the experimental environment meets certain requirements such as continuity and convexity, and the institutions and the agent behavior are consistent with the competitive mechanism.

Smith also determines following sufficient conditions for a microeconomic laboratory experiment:

**Nonsatiation** is a classic economic assumption which states that if a consumer is given a costless choice between two alternatives, he will always choose the alternative which yields more of a desired good over the second alternative.

**Saliency** means that human subjects participating in the experiment can claim a reward, which depends on their decisions and the realized experimental outcomes. The institutions of the experiment define how the decisions will be translated into outcomes.

**Dominance** is the condition that was firstly proposed by Wilde (1980). He suggested that the reward earned by human subjects always dominates any subjective costs connected to the participation in the experiment. In other words, the subject participating in the experiment finds himself on the higher utility curve than if doing any other activity. This condition is sufficient to maintain the control over subjects preferences.

**Privacy** is another assumption that helps maintaining the control over the preferences. A subject  $i$ 's utility can namely depend on  $i$ 's reward as well as on another subject  $j$ 's reward. Thus each subject in an experiment is given information only about his own payoff. This information is always transmitted in privacy.

The experimental economics has several advantages. We can control the experimental environment and replicate the experiments. Also, we cannot forget the advantage of using human subjects who bring the validity to the experiment. The possible downside of using the experimental approach is the fact that while studying a more advanced research question the arrangement of an experiment can become too complex and expensive.

### 1.1.2 Agent-based Models

Agent-based modeling is a simulation technique, that has a wide range of application. It can be used in many areas of the social

sciences, including economics. In agent-based modeling (ABM), every system or model is a set of agents who, facing a specific situation, make decisions based on a set of given rules. They can represent individuals, groups or even countries and do various things like buying, selling, producing, or consuming. This approach to modeling brings many benefits. It allows us to model situations that are difficult to describe by the traditional equation modeling. Due to the all specifics of the human behavior, capturing social interactions by a set of mathematical equations can be tricky. The requisite computational technique used is called multi-agent simulation (MAS) or agent-based computational modeling (ABC). Agents, who are representing individuals can be given many properties, such as competition and fighting ability, memory and future expectation, possibility of trading and exchange, or learning abilities.

To get a clearer picture of ABM we will review an example by Bonabeau (2002). This simple game examines the influence of initial rules on group behavior. The game is suitable for a medium group of people (about 20). One asks each member of the group to randomly choose two other persons, A and B. One then asks them to keep person A like a shield between him and person B. People will move slowly and randomly and will get bored in a while. A completely different situation occurs when one asks everybody to keep themselves between A and B, so they represent the shield. The room will implode with everyone joining

one big group which will emerge in the middle. This simple game shows how a simple set of rules given to individuals can lead to a coherent group behavior. Also, it shows that a small change in the initial conditions can have a large impact on the group behavior causing a significantly different outcome.

Besides this example, there are other considerable benefits of this approach. First of all ABM is a natural way of describing and simulating a particular system that, due to the complexity of human behavior, cannot be precisely described only by the differential equations. Whether one is trying to describe a traffic jam, a stock market, or a group behavior, the use of such a method can get it closer to the reality. Secondly ABM is flexible—it is simple to add more agents to the model or to change the set of rules that influences the agents' decisions. Also, they allow one to test a hypothesis in detail. As well as in this thesis, the data from agent-based models can be compared with actual data in order to gain a better understanding of reality. The controllable environment and additionally completely controllable agents represent another advantage of ABM. The models have also perfect replicability and thus allow the researches to study all parameters and variables more deeply.

As for disadvantages, ABM is said to be too assumption driven, i.e. to make simplifying assumptions about agents' behavior. Another problem with ABM is validity, i.e. how well the model represents the real world. In a great paper by Heat, Hill, and Ciarallo

(2009) is the validity described as “one of the most important aspects of model building” and the lack of validation in economic simulations is pointed out. They claim that statistical validation is used only in 2.5% of ABM publications included in the survey. This can be explained by the impossibility of using the statistical methods on the output data, due to their form and kind.

According to Helbing and Balmelli (2012), there are three classes of ABM: physical, economic and sociological models. Physical models suppose that individuals’ reactions are based directly on their current and past interactions. Sociological models, contrariwise, assume that individuals respond according to their own and others future expectations. For our purposes we will use mainly the economic models which are characterized by agents responding as a pure homo economicus, i.e. in a selfish way and maximizing their utility. Tesfatsion (2006), in a great introduction to agent based modeling, describes four main objectives of the research in the area of agent based models.

**Empirical understanding:** The first purpose of agent modeling is to describe and explain the existence of global regularities, that are still to be found despite the lack of centralized planning and control. The methodology lies in the effort to create conditions for agents that simulate the conditions in the real world. The question is whether or not agents in these conditions can reach the observed global regularities.

**Normative understanding:** The aim of this approach is to use ABM as a kind of a laboratory for testing whether a proposed economic design could work and could lead to a system, that is socially desirable. Agents in the artificial world are given a private motivation and a learning capability. The goal is to design a good and fair working system despite the fact that everyone is selfish and tries only to maximize his utility.

**Qualitative insight and theory generation:** This area examines the changes in the initial conditions. It is focused here is on the question if slightly changed initial conditions can change the dynamics and the outcomes of the model. The methodology is to create a world with privately motivated traders who have the capability to learn and observe which types of market behavior persist and thus are necessary for the emergence of such an organization.

**Methodological advancement:** The concept of ABM is relatively new in the area of economics; therefore, there is a need to provide researchers with a good methodology. An economist who wants to use ABM as a tool has to be educated in behavioral and computer sciences to be able to create a model. With better tools it would be easier to work with ABM. More frequent use of ABM could lead to more accurate modeling in the area of the financial markets.

## 1.2 The Advantages of Combining Both Methods

In this chapter, we describe the combination of agent-based models and human subject experiments, the methodology that lies at the core of this thesis. To demonstrate all the advantages we review some examples from this research area.

The emergence of both research techniques—agent-based models and human subject experiments—is bounded to the development of computers and computing power. These techniques have been developed for the same reason: a frustration from highly centralized, top-down, deductive approach, which is based on the traditional neoclassical economic theory (Duffy 2006). On the contrary ABM and EXP are both considered decentralized, bottom-up approaches, where agents are heterogeneous and boundedly rational. Bounded rationality means, that the ability of individuals to make decisions is limited by the amount of information available, their intellectual ability and the finite amount of time for a decision. On the output of these simulations, there are the results of the interaction between agents/human subjects. Although controlled human subjects experiments do not require the use of computers and can be done with the help of paper and pencil, only the use of computer programs allows the researches to do more advanced and more complicated experiments. Thus, without the modern technology, some great projects that have been done in the past decades would not be feasible. Considering

the advantages of computing power many researchers have used the combination of these two methodologies to obtain better and more complex results.

To explain all benefits of combining ABM and EXP, we first need to explain the logic that lies under this process. At the beginning, we have a target, some (economic) phenomenon that we want to examine. This target leads directly to our research question. If using only ABM approach we would build a simulation model and then analyze the data gained from this model. By adding experimental economics to this process we can "integrate a second stream into the research process" (Klingert and Meyer 2011, p.68). Experimental setup is used to answer the same research question. It should be as similar as possible to the simulation model. The combination of these two "streams" allows us to compare the results from both—the simulation and the experiment—and to examine in detail the differences and similarities at the micro (agent and human subject behavior) and macro (output) level. To demonstrate this we present the concept from Klingert and Meyer 2011, p.68

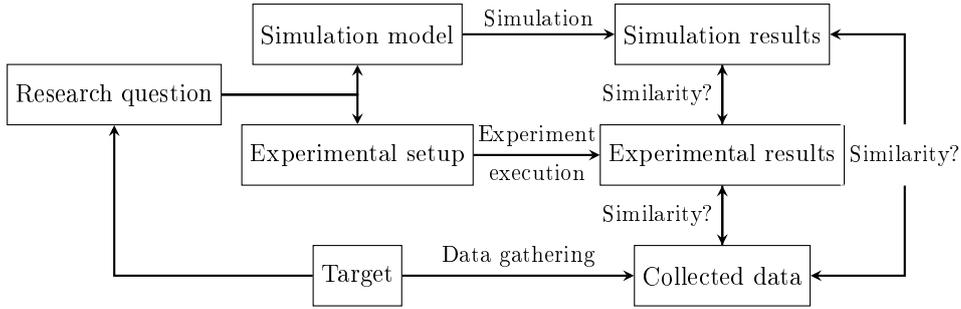


Figure 1: Logic of combining EXP and MAS research (Klingert and Meyer 2011, p.68)

One of the main advantages of this combined approach is that agent-based and experimental model can support each other. ABM is able to support EXP by helping it to find the key parameters or variables that are necessary to the research target and thus sometimes reduce otherwise high costs of the experiments. EXP can support ABM by increasing its validity at the both—micro and macro level. ABM findings are also widely used to explain laboratory findings, because they allow greater control over subjects learning abilities and preferences than the human subjects experiments. On the other hand the data from human subjects experiments can be used to calibrate or test agent-based models and make them closer to the reality.

One of the first methods, how to combine ABM with EXP, was to replace some or all human subjects in a given experiment with programmed agents. For example, Roth and Murnighan (1978) decided to “improve” the famous experiment called Prisoner’s dilemma. They let the human subjects to play against

computer-programmed agents for a different expected length of time, so that they could better explore the impact of the expected duration on human behavior. Coursey, Isaac, Luke and Smith (1984) tested the market efficiency in a contestable market. They conducted 12 experiments, 6 with human subjects and 6 with computer simulated demand. The only difference was the possibility of human error or, on the contrary, some kind of strategic behavior. In violation of their initial theory, they have proved that while using human subjects, there is a “stronger degree of convergence to a competitive price range”. Their work was followed by Brown-Krause (1991). He did a very similar experiment with the difference that humans were informed whether they are playing against computer-simulated or human buyers. Human subjects were found to be playing strategically, same as in Coursey, Isaac, Luke and Smith (1984). Both of these experiments have proved that there is a certain complexity in the decision process, when human subjects are involved. Gode and Sunder (1993) were the first researchers who decided to do two completely separated experiments—one with human subjects and the second one with programmed agents. They named the programmed agents "zero-intelligence" for their no capability of learning, remembering or strategic behavior. Their approach was innovative in many ways. That is the reason why we decided to build our work on it.

## 1.3 Examples

### 1.3.1 Near-Zero-Intelligence Traders—Model by Duffy and Ünver (2006)

The idea of ZI agents (originally from Gode and Sunder (1993) and discussed below) was used by Duffy and Ünver (2006) to examine asset price bubbles and crashes. This experiment is interesting because Duffy and Ünver did not organize an experiment with paid human subjects to compare the results with an agent-based model. Contrariwise, they used agent-based computational approach (ACE) as a bottom-up methodology to understand and explain many laboratory findings about price bubbles and crashes, beginning with the work of Smith, Suchanek and Williams (1988)<sup>1</sup>.

They used a simple environment, which was proposed for the first time by Smith, Suchanek and Williams (1988). In such a market environment there are 9 or 12 inexperienced traders<sup>2</sup> who are trading for limited number of trading periods  $T$  (usually 15 or 30). Subjects can be either a buyers or a seller, this depends upon them and they are given  $x$  units of cash and  $y$  units of the asset at the beginning of each experiment. An only single asset is traded. Subjects are free to sell or buy this asset at the current best bid or ask price. These "best" prices are a public knowledge

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<sup>1</sup>see e.g. King, Smith, Williams and Van Boening (1993) or Lei, Noussair and Plott (2001)

<sup>2</sup>The fact, that traders are inexperienced, is crucial to all experiments in this environment, see Duffy, J., Ünver, M.U. (2006): Asset price bubbles and crashes with near zero-intelligence traders, *Economic Theory* 27, p. 5

because they are shown at everybody's screen. Once the deal is closed the levels of trader's cash and assets are adjusted and the price for which the asset was sold is revealed to other players. For holding an asset at the end of the trading period the subject is rewarded by dividend payment. The amount of dividend paid is a random variable with expected value  $\bar{d}$ . Before the start of the first trading period ( $t = 1$ ) the trader  $i$ 's initial amount of cash  $x_i$  and endowment of the single asset  $y_i$  has to satisfy the condition:

$$x_i + D_1^T y_i = c,$$

where  $c$  is a constant, which is the same for all subjects. Hence giving all subjects the same expected value for their initial endowment and assuming that all of them are risk neutral, they should be indifferent between trading and not trading at the fundamental market price. The fundamental expected market value of the asset at the beginning of each trading period  $t$  is given by the equation:

$$D_t = \bar{d}(T - t + 1) + D_{T+1},$$

where  $D_{T+1}$  is the expected buy-out value of an asset which is the subject holding at the end of period  $T$ . According to this equation the fundamental market price is decreasing over time as  $t \rightarrow T$ . While conducting this type of experiment with inexperienced participant the process is usually very similar, according to Smith, Suchanek and Williams (1988). At the beginning of trading, the

price tends to be below the fundamental price, but gets quickly above this level. Thus in the middle of trading periods so called "bubble" occurs. As the end of trading approaches, the price falls to or below the fundamental value. They called this price path "humped-shaped" because of its typical profile.

Duffy and Ünver used a similar experimental setup as Gode and Sunder (1993) with some alternations. A first alternation was the usage of "loose" budget constraint. Gode and Sunder prohibited their buyers and sellers from unprofitable deals, so they had to trade only below, respectively above their private values. On the contrary, Duffy and Ünver's method was to let the trader submit an ask price whenever he has a unit available for sale and submit a bid price if he has sufficient cash balances. A second alternation was that bid and ask prices were not completely random. They were dependent on the mean transaction price from the previous period, denoted by  $\bar{p}_{t-1}$ . This fact inspired them to qualify their agents as "near-zero-intelligence" traders. As in Gode and Sunder there were bounds imposed on bids and asks. The interval was  $[0; \kappa D_t^T]$ , where  $\kappa > 0$  was a parameter<sup>3</sup>. As we mentioned before bids and asks were not random, but were given by equations. The trader  $i$ 's ask in period  $t$  is given by:

$$a_t^i = (1 - \alpha)u_t^i + \alpha\bar{p}_{t-1},$$

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<sup>3</sup>Considering the equations for bids and asks, price should converge to  $\frac{\kappa D_t}{2}$  so  $\kappa = 2$  might be a good idea. However Duffy and Ünver decided to choose  $\kappa = 2$  due to the calibration of the model which will be discussed later in this chapter.

where  $u_t^i$  is a random draw from  $[0; \kappa D_t^T]$  and  $\alpha \in (0; 1)$  is a constant parameter, which is used to consider the weight of the mean transaction price from the previous period. Seller  $i$  can submit his asks as long as he has amount of asset  $y_i$ , i.e.  $y_i > 0$ . Similar equation was used for buyers. The trader  $i$ 's bid in period  $t$  is given by:

$$b_t^i = \min(1 - \alpha)u_t^i + \alpha\bar{p}_{t-1}, x_t^i,$$

where  $x_i$  has to be greater than 0 for a bid to be submitted. Duffy and Ünver also gave their agents a "weak foresight". This means that the initial probability that a trader chooses to be buyer is 0.5 but decreases over time. This happens due to the fact that as the end of trading approaches the fundamental value of an asset is declining.

### 1.3.2 Learning Competitive Equilibrium—Model by Crockett et al. (2004)

Crockett et al. presented an excellent example of combining agent-based modeling and experimental economics. Firstly, Crockett, Spear, and Sunder (2004) used ZI-agents to model an environment, where agents were learning a competitive equilibrium. Then on the basis of their results Crockett (2004) organized an experiment with paid human subjects to test their hypothesis and to bring an external validity to the simulation.

Crockett, Spear, and Sunder were dealing with the additional intelligence for ZI-agents. The ZI-agents require to reach a competitive equilibrium on the market. Their question was if agents can price a found optimal allocation by learning the common normalized utility gradient at the optimal allocation. The search for this allocation is given by algorithm which combines a direct random search with the Welfare Theorems application that can be used when agents learn their gradients. The First Welfare Theorem claims that every competitive equilibrium is a Pareto optimum. Pareto optimality is the allocation, where it is impossible to make anyone better off without making anyone worse off. Thus, every algorithm starts by searching for Pareto optimum. The Second Welfare Theorem says that all possible Pareto-optimal outcomes can be achieved by enacting lump-sum wealth redistribution, i.e. by redistribution of endowments. With the common

normalized utility gradient used to price a Pareto optimum allocation, these prices can be used for endowment redistribution connected with the allocation. They implemented this idea as an algorithm for repeated trading rounds.

The algorithm is named  $\epsilon$ -intelligence. It is divided into stages which consist of sequences of steps. There is a finite number of agents  $M$ , a finite number of goods and services  $l$ , and no bids or asks are sent. Each agent is characterized by his preferences and endowments. Preferences are represented by utility function. At the beginning of each round, every agent  $i$  receives a certain number of  $l$  goods according to a random draw. Then a proposed allocation of  $l$  goods across  $M$  agents is made. Agent  $i$  then compares his utility from his current endowment and a proposed allocation. If his utility from the proposed endowment is higher, he is willing to accept it. If all  $M$  agents are willing to do so, the proposed allocation becomes a new endowment. This process repeats until no more utility increase is achieved. At this point a stage  $t$  is formed. The economy has reached a near-Pareto optimum (an allocation which lies approximately in the Pareto set) and not necessarily by competitive equilibrium. Crockett, Spear and Sunder also assumed that once this near-Pareto optimum is reached, the ZI-agents are able to compute the common normalized utility gradient. With this knowledge they can state if this gradient passes through their initial endowment point. If not some agents are subsidizing others in the Pareto optimum allo-

cation. The process of reducing the rate of subsidizing is stage  $t + 1$ .

To explain this stage, let's consider a simple example. By the end of each trading period  $t$ , agent  $i$ 's approximate Pareto-optimum allocation is  $\hat{x}_i^t \in R_+^2$ . The  $i^{th}$  agent's gain at this allocation is written as:

$$\lambda = p^t(\hat{x}_i^t - \omega_i),$$

where  $p^t$  is a price of allocation from stage  $t$  and  $\omega^i \in R_+^2$  is  $i^{th}$  agent's initial endowment. If  $\lambda = p^t < 0$  agent  $i$  is said to be subsidizing other agents, i.e. agent  $i$  is not able to purchase his initial endowment at  $p^t \gg 0$ . The main point in the stage  $t + 1$  is that agents are restricted to accept only those allocations  $x^{t+1}$  that increase their utility and also satisfy the condition:

$$0 \geq p^t(x_i^{t+1} - \omega_i) \geq \lambda_i^t + \eta_i,$$

where  $\eta_i$  is a small and positive bound. Under this condition  $\lambda_i^{t+1} > \lambda_i^t$ , so agent  $i$ 's subsidization to other agents is smaller in period  $t + 1$  than it was in the period  $t$ .

By this simulation Crockett, Spear, and Sunder proved that their  $\epsilon$ -intelligent algorithm converges to a competitive equilibrium. Furthermore, they showed that to be able to price Pareto optimal allocations, an agent only needs to be provided with the information about his utility function and ability to count com-

mon normalized utility gradient (he does not need information about first- and second-derivatives of all agents' utility function). With this information provided, it is possible for an agent to recognize his role as a subsidizer and change that.

Crockett (2004) designed an experiment with human subjects to add more validity to this simulation. He used several experimental treatments that varied in the number of subjects and in the parameters of the utility function. Human subjects were provided with the information about their utility function and instructed how to recognize that a proposed allocation will improve their utility. They were given the marginal rate of substitution and a value of allocation valid at the end of period  $t$ . This information was available in the period  $t + 1$ , although without any further instructions. They were able to process the subsidization problem. In a nutshell, human subjects had all the information necessary to behave like the programmed agents in the Crockett, Spear, and Sunder (CSS) simulation. The convergence to competitive equilibrium was slower than expected. Although agents were able to recognize which proposed allocation was utility improving and use this fact, not all of them were able to work with the subsidization constraint. Only some of them behaved like "CSS-ZI" agents. Hence, this experiment brought some support for  $\epsilon$ -intelligent algorithm designed by Crockett, Spear, and Sunder, although the convergence to the competitive equilibrium was slower than predicted.

## 2 The Model

Before describing the original model by Gode and Sunder (1993), we would like to summarize the characteristics of the double auction environment in which the model is realized. The economists use the double auction environment for many kinds of experiments. The first work in this field was written by Smith (1962). Smith, using the findings from previous work of Chamberlain (1948), was able to simulate a market, where equilibrium predicted prices are followed by the real prices from the experiment. Despite the fact that human subjects had a limited amount of information and thus a limited ability to learn during the experiment<sup>4</sup>, they managed to create a stable market. The prices and trading volumes in this market were close to their predictions.

The double auction environment is a simple environment where a single good can be bought and sold for a fixed amount of trading periods. The subjects are either buyers or sellers (sometimes they can be both) and they act according to their given private values. Buyer  $i$  has a private value of buying a unit  $j = 1, 2, \dots$  of a good  $v_{ij}$ , where according to the principle of diminishing marginal utility  $v_{ij} \geq v_{ik}$  for all  $j < k$ . The situation is similar in the case of a seller. Seller  $i$  has a private value of a selling unit  $j = 1, 2, \dots$  of a good  $c_{ij}$ , where according to the principle of diminishing

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<sup>4</sup>There is difference between Smith (1962) and Chamberlin (1948). Chamberlin (1948) use only one trading period, so there was a little opportunity for traders to learn something and used their experience in the next round.

marginal utility  $c_{ij} \leq c_{ik}$  for all  $j < k$ . Using the alignment of prices according to individual private values we obtain demand and supply market curves. The intersection of these curves states the equilibrium price and quantity. Both groups—buyers and sellers—are getting a payoff which is equal to the difference between their private value and the trading price. Subjects are instructed that even when buying/selling a good exactly for their private value they are still better off compared to no trade situation. All information about past transactions is publicly known. It was proved by many experiments, that under these conditions market converges to the equilibrium price and quantity.<sup>5</sup>

Gode and Sunder (1993) were interested in the question of getting to this market equilibrium. They have the hypothesis that the rules of double-auction market might be responsible for the convergence to the equilibrium. Thus they decided to compare the influence of this set of rules to human subject traders and programmed robot traders. Human subjects were graduate students of business, motivated by getting credits at their university. Although the robot traders followed the rules, they decided randomly about their bids and asks. They were not able to “observe, remember or learn anything”<sup>6</sup>. Hence Gode and Sunder decided to name them “zero-intelligence (ZI) traders”. To make the experi-

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<sup>5</sup>see Duffy, J. (2006): Agent-based models and human subject experiments, Handbook of Computational Economics, Volume 2, p. 957

<sup>6</sup>Gode, D.K., Sunder, S. (1993): Allocative efficiency of markets with zero-intelligence traders: Market as a partial substitute for individual rationality, Journal of Political Economy 101, p. 121

ment more conclusive they introduced two types of the ZI-traders: constrained (ZI-C) and unconstrained (ZI-U) traders. For the ZI-U traders the buyer's bids and seller's asks were random draws from a uniform distribution  $U[0; B]$ , where B is the upper bound. The upper bound was determined to be higher than the highest private value of a good among all buyers. These traders were unconstrained due to their lack of interest in profit. On the contrary, the ZI-C traders were prevented from making unprofitable operations on the market. This was assured by randomly drawing their bids and asks from the uniform distribution  $U[0; v_{ij}]$  for buyers and  $U[c_{ij}; 0]$  for seller where  $ij$  mans  $i$ 's bid/ask for unit  $j$ . The difference in the performance of the ZI-C buyers and human subjects was supposed to show the importance of human rationality and strategic behavior, that programmed traders do not posses. Further, the disparity between ZI-C and ZI-U should have proved the influence of the market rules on the market itself.

They worked with 12 ZI-traders, 6 buyers and 6 sellers, under standard double auction rules. As we have described above, any buyer was free to enter a bid and other buyers or the buyer himself was allowed to raise the bid. The situation was the same for sellers. A transaction occured when prices matched or crossed. Buyers and sellers were also instructed to buy/ sell their current unit before buying/selling the next one. The time for one trading period was set up for 30 seconds for the ZI traders and 4 minutes for human subjects, to make it long enough for a thoughtful

decision.

The results from this experiment were very interesting. In the experiment with the ZI-U traders, transaction prices were very volatile and there was no trend, no sign of any convergence. This can be explained by the lack of market discipline. On the contrary, the markets with ZI-C agents and human traders followed a really similar price path. A market with human subjects was stable moving quickly to the equilibrium which was a consequence of human intelligence and a motivation to earn money. The main question of this experiment was, if without these factors market equilibrium could have occurred. In that case the market rules themselves would have been the only responsible factor. A market with ZI-U agents turned out to be capable of reaching the equilibrium. The process was a little bit slower and there was greater volatility to be seen but it still led to the convergence. Nevertheless, ZI-U traders had no capability of learning and thus they could not remember the past transactions. The experiment proved, that market rules and agents with budget constraint are sufficient enough to reach the equilibrium. As Gode and Sunder wrote: “Adam Smith’s invisible hand may be more powerful than some may have thought: when embodied in market mechanisms such as a double auction, it may generate aggregate rationality not only from individual rationality but also from individual irrationality” (Gode and Sunder 1993, p.136).

## 3 The Experiment

### 3.1 The Experimental Setup

The experimental part of the thesis is built on economic experiment. The main idea of this experiment is to examine how much is the market efficiency influenced by intelligence and profit motivation of the human traders and how much by the market discipline. The idea and the design of the experiment are based on the article from Gode and Sunder (1993), described above. The experiment consists of two parts that both simulate a double auction market: human subjects experiment and computer simulation with ZI-traders. Performances of both parts are compared.

Some parameters of the market are similar to the parameters used by Gode and Sunder (1993). We simulate 3 markets; each market consists of 6 periods. In each market there are 12 participants, 6 buyers and 6 sellers. The number of the markets is chosen in order to see how human traders can profit from learning in later periods. While human traders are unique, the ZI-traders participating in each market are identical.

We use different supply and demand functions. This leads to a different equilibrium price and number of trades. Also, the range of trading prices is reduced from 1 to 100 (Gode and Sunder used a range from 1 to 200). The time for trading is shorter, 2 minutes for human traders and 4 seconds for computer traders (Gode and Sunder used 4 minutes and 30 seconds). This time was

sufficient enough for trading. The subjects in our experiment do not receive the same endowment of private values/costs at the beginning of each period in the same market. They are informed that these values/costs will stay the same for the whole market, but will vary across the participants (i.e. the sets of values for each round are fixed, but are randomly distributed among the participants at the beginning of each period).

In the human subjects experiment mostly undergraduate students of different universities are used, thus they can be considered inexperienced traders. The experiment is programmed and conducted with the software z-Tree (Fishbacher 2007). The experiment is conducted in the premises of the Laboratory of Experimental Economics (LEE) in the Czech Economical University. The same institution also provides funding for the experiment. The participants are given a constant show-up fee plus their earning from the experiment based directly on their performance. The amount of money given to them is in average much higher than the average hour salary for student's job. Computers in the LEE are separated from each other by partitions and the participants are given their pay-off in private. Due to these facts experiment fulfills all four sufficient conditions for a microeconomic laboratory experiment (i.e. nonsatiation, saliency, dominance and privacy) laid down by Smith (1982). Full instructions, given to the participants, can be seen in Appendix A.

The computer simulation is programmed and conducted with

the Visual Basic 2013 software. As in the Gode and Sunder (1993) traders are divided into 2 different categories. ZI-U traders have no budget limitation, unlike the ZI-C traders, who have to respect their private values and are constrained from any unprofitable trades. They have no memory of any of their previous actions, nor any strategic thinking. The simulation is programmed to be as close as possible to the market with human subjects.

## 3.2 The Results

### *Prices*

In this chapter we present the results of our experiment and simulation. The results are not far from our expectations. The transaction prices are presented in the figures 2-4. The panel (a) in each figure represents a supply and a demand function for each market as well as the equilibrium price, which is represented by the horizontal dotted line. The panel (b) shows the transaction prices for ZI-U traders, panel (c) for ZI-C traders, and panel (d) for human traders. Two key parameters were used to examine the price paths. The price volatility is described by standard deviation (SD) in Table 1. The convergence to the equilibrium price is represented by root mean square deviation of prices from equilibrium (RMSD) computed for each period in Table 2<sup>7</sup> and graphically displayed in Figure 5.

Let us start with the markets with ZI-U traders. Some kind of a systematic pattern can be seen in these markets, but no sign of convergence to the equilibrium price. As a consequence we observe high SD as well as RMSD, which remain invariant in all markets. The invariance can be seen in Tables 1 and 2. We observe price instability which leads to high price volatility. Such an outcome is the result of trading done by ZI agents without any budget constraint. Also without any budget constraint all units

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<sup>7</sup>Let  $p_{ai}$  and  $p_{ei}$  be the actual and the equilibrium price and  $n$  the number of traders. Then root mean square deviation is computed as  $RMSD = \sqrt{\frac{\sum_{i=1}^n (p_{ei} - p_{ai})^2}{n}}$ .

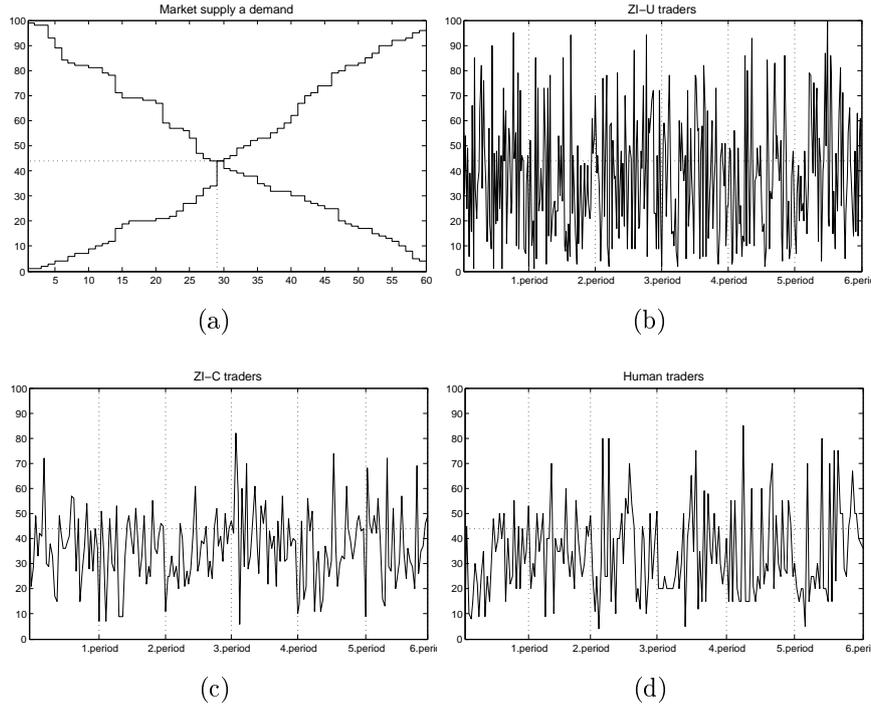


Figure 2: Market 1

are traded.

In contrast the markets with human traders, show a development over time. First market shows high price volatility. SD in this case is even higher than for ZI-C traders. In the second and third market, one can see smaller price volatility. According to the RMSD these markets converge to the equilibrium price. This can be explained by the human trader's ability to learn. There are no practice periods in the experiment, which is reflected in the first market, where traders are getting used to the market mechanism and learning how to make profit. Overall, these markets are provided with high stability of price and volume, due to the profit-motivated, rational and intelligent human traders

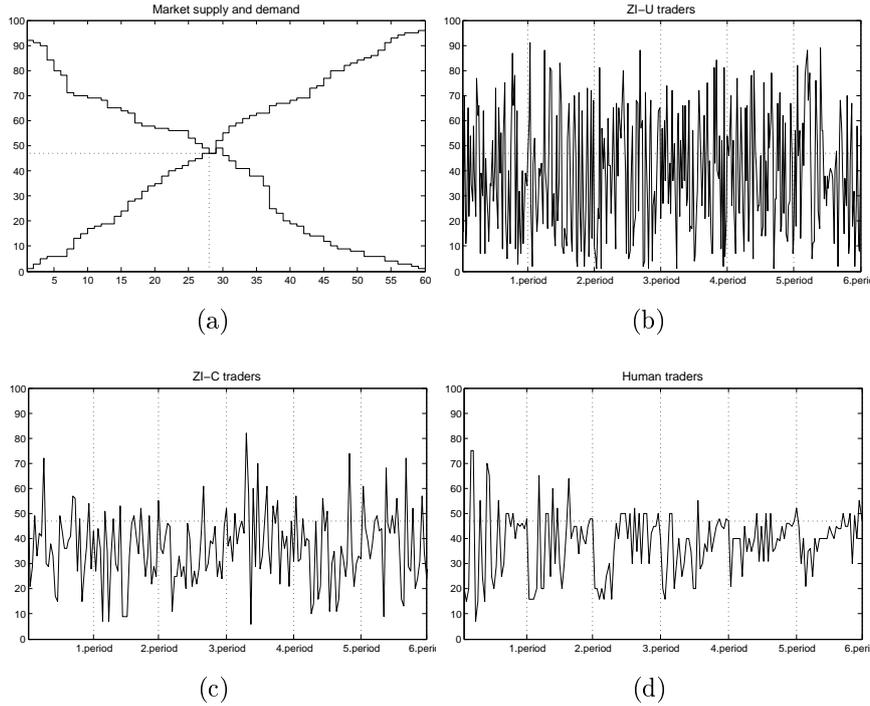


Figure 3: Market 2

facing market rules.

Regarding the markets with ZI-C traders, the price volatility in these markets is lower than the price volatility in the ZI-U markets, but higher than the one in the human traders markets. There is a systematic pattern, but also no convergence. Although RMSD is significantly different for each period, values are neither rising nor declining. This is an expected outcome, because ZI-C agents cannot learn from their decisions. However, the higher stability of price and volume shows, that imposing market discipline on ZI-agents can move the market closer to the market with human traders.

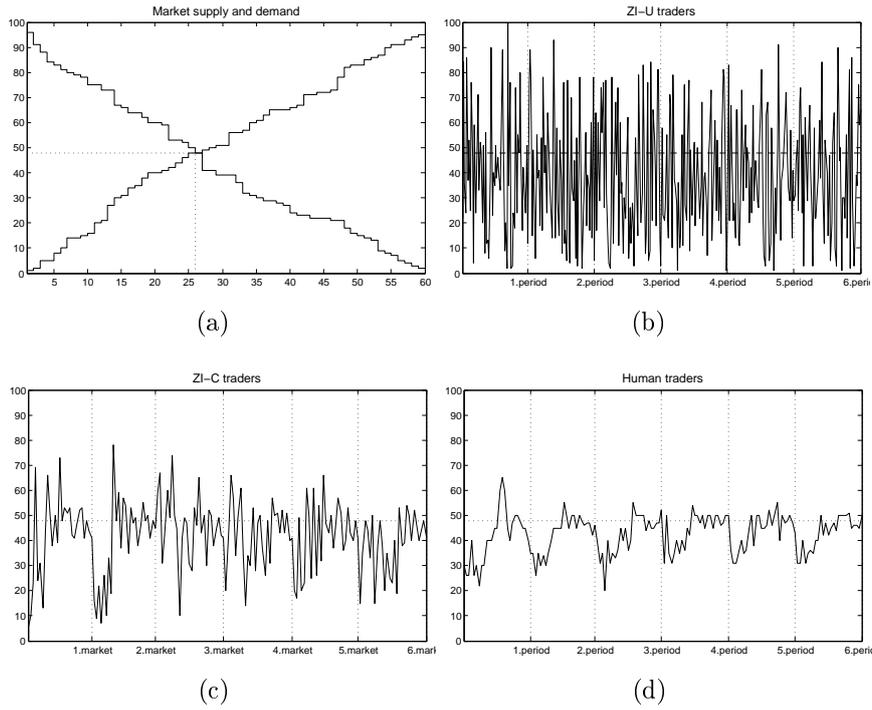


Figure 4: Market 3

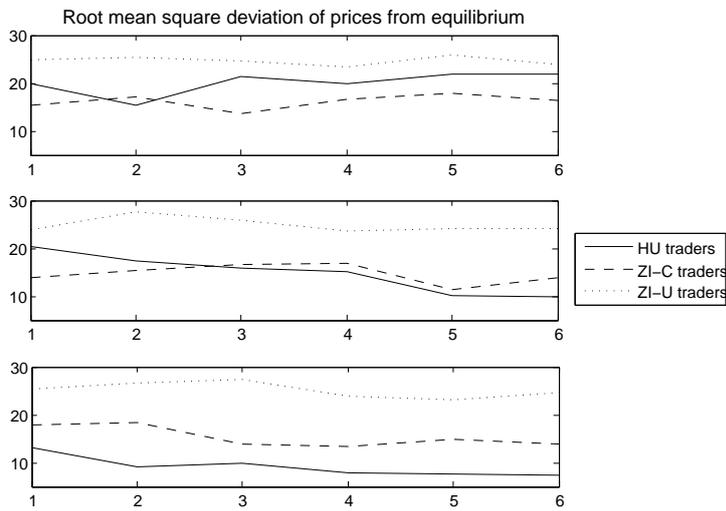


Figure 5: Root mean square deviation of prices from equilibrium

Table 1: Standard deviation

Traders	Market 1	Market2	Market 3
HU	17,7	12,7	7,9
ZI-C	14,5	13,8	14,2
ZI-U	23,9	23,7	23,9

Table 2: Root mean square deviation of prices from equilibrium

	Market 1			Market 2			Market 3		
Period	HU	ZI-C	ZI-U	HU	ZI-C	ZI-U	HU	ZI-C	ZI-U
<b>1</b>	20.0	15.3	25.0	20.3	13.9	24.0	13.1	17.8	25.5
<b>2</b>	15.3	17.2	25.5	17.3	15.5	27.7	9.1	18.3	26.6
<b>3</b>	21.4	13.6	24.6	15.8	16.7	25.8	10.0	13.9	27.4
<b>4</b>	19.8	16.6	23.4	15.2	16.8	23.7	7.9	13.4	23.9
<b>5</b>	21.9	17.8	25.8	10.2	11.5	24.2	7.6	15.0	23.1
<b>6</b>	21.8	16.5	23.8	9.8	13.8	24.1	7.5	13.8	24.6

### *Efficiency*

The key parameter in our experiment is the allocative efficiency of markets. Same as Gode and Sunder (1993), we follow Smith's (1962) definition of allocative efficiency. Smith defines it as "the total profit actually earned by all the traders divided by the maximum total profit that could have been earned by all the traders" (Smith, 1962, p. 120). The efficiency of all markets is shown Table 3.

As expected, the allocative efficiency of ZI-U traders is the lowest, 70.5% in average. This is due to the fact, that without any budget constraint extramarginal units are traded with a negative

Table 3: Mean allocative efficiency of markets

Traders	Market 1	Market 2	Market 3	Average
ZI-U	77.4	69.2	64.9	70.5
ZI-C	98.6	98.7	98.0	98.5
Human	94.8	98.4	99.0	97.4

profit. The efficiency depends on the shape of supply and demand function. It can explain decline in efficiency, because in our experiment the equilibrium price rises and number of trades declines with later markets.

What is interesting is the development of efficiency in the markets with human traders. It starts a lot below the level of ZI-C traders, but as the subjects learn, it ends up even higher in the third market. This proves that human rationality and ability to learn from past decisions and plan the future ones can play an important role in the financial markets. This is not a new idea in economics. However, the fact that human traders managed to increase efficiency from 94.8% to 99.0% during only three rounds in a simple experiment suggests, that on a large scale, with sufficient time human abilities are the key factor for trading in the financial markets.

However, the overall efficiency of human traders' markets is 97.4%, which is less than 98.5% achieved by ZI-C traders. Thus the set of market rules and budget constraint imposed on non-intelligent behavior together are able to create very effective market similar to the human one.

This supports the results of Gode and Sunder's (1993) research. We can only agree with their quote, that Adam Smith's invisible hand can be more powerful than we thought.

## Conclusion

In our thesis we manage to give a broad introduction to the area of financial markets modeling. We describe two methodologies: the agent-based simulation and the experimental economics and show their mutual contribution to the area of financial markets modeling.

The experimental part of our thesis presents the results of our economic experiment. The main idea of this experiment is to examine how much the market efficiency is influenced by intelligence and profit motivation of the human traders and how much by the market discipline. The idea and the design of our experiment are based on the article by Gode and Sunder (1993). The experiment consists of two parts that both simulate a double-auction market: a human subject experiment and a computer simulation with ZI-traders. The ZI-traders are distinguished into two kinds: the ZI-U traders, who do not have any budget constraint, and the ZI-C traders, who have one. Performances of both parts are compared.

The results capture two important phenomena. Firstly, markets with human traders evolve over time. Price volatility decreases and allocative efficiency rises with later markets. That proves that human rationality and ability to learn from past decisions and plan the future ones can play an important role in the financial markets.

Secondly, a set of market rules and budget constraint imposed on non-intelligent behavior together are able to create very effective market similar to the human one. The results of our experiment support the results of Gode and Sunder's (1993) research.

## References

- [1] Brown-Kruse, J.L. (1991): Contestability in the Presence of an Alternate Market: An Experimental Examination, *The RAND Journal of Economics*, vol.22, no.1, pp. 136-147
- [2] Chamberlain, E.H. (1948): An experimental imperfect market, *Journal of Political Economy* vol.56, no.2, pp.95-108
- [3] Coursey D., Isaac, R.M., Luke, M., Smith, V.L. (1984): Market contestability in the presence of sunk (entry) costs, *The RAND Journal of Economics*, vol.15, no.1, pp. 69-84
- [4] Crockett, S., Spear, S., Sunder, S. (2002): A simple decentralized institution for learning competitive equilibrium, *Tepper School of Business*, paper 487
- [5] Gaechter, S. (2009): Improvements and future challenges for the research infrastructure in the field ‘experimental economics’, *RatSWD Working paper no. 56*, available at SSRN: <http://ssrn.com/abstract=1445359>
- [6] Gode, D.K., Sunder, S. (1993): Allocative efficiency of markets with zero-intelligence traders: Market as a partial substitute for individual rationality, *Journal of Political Economy*, vol. 101, no.1, pp. 119-137

- [7] Duffy, J. (2006): Agent-based models and human subject experiments, Handbook of Computational Economics, Volume 2, pages 829-1660, ISBN: 978-0-444-51253-6
- [8] Duffy, J., Ünver, M.U. (2006): Asset price bubbles and crashes with near zero-intelligence traders: Towards an Understanding of Laboratory Findings, Economic Theory, vol.27, no.3, pp.537-536
- [9] Fischbacher, U. (2007): z-tree: Zurich Toolbox for Ready-made Economic Experiments, Experimental Economics 10(2), 171-178
- [10] Friedman D., Cassar, A. (2004): Economics lab: an intensive course in experimental economics, Routledge, London, ISBN: 0-415-32401-7
- [11] Heath, B., Hill, R., Ciarallo, F. (2009): A Survey of Agent-Based Modeling Practices (January 1998 to July 2008), Journal of Artificial Societies and Social Simulation, vol.12, no.4, p. 9
- [12] Hommes, C., Sonnemans, J., Tuinstra, J., van de Velden, H. (2004): Coordination of Expectations in Asset Pricing Experiments, The Review of Financial Studies, vol.18, no. 3, pp. 955-980
- [13] Hommes, C.H. (2005): Heterogeneous Agent Models in Economics and Finance,

- [14] Klingert, F.M.A., Meyer, M. (2011): Effectively combining experimental economics and multi –agent simulation: suggestion for a procedural integration with an example form prediction market research, Computational and Mathematical Organization Theory, March 2012, vol. 18, no. 1, pp. 63-90
- [15] LeBaron, B. (2001): Evolution and time horizons in an agent based stock market, Macroeconomic Dynamics, vol. 5, no. 2, pp. 225–254.
- [16] Roth A.E., Murnighan, J.K. (1978): Equilibrium behavior and repeated play of the prisoner’s dilemma, Journal of Mathematical Psychology vol. 17, no. 2, pp. 189-198
- [17] Smith, V.L. (1962), An experimental study of competitive market behavior, Journal of Political Economy vol. 70, no. 2, pp. 111-137
- [18] Smith, V.L. (1982): Microeconomic systems as an experimental science, American Economic Review vol. 72, no. 5, pp. 923-955
- [19] Tesfatsion, L. (2006): Agent-based computational economics: A constructive approach to economic theory, Handbook of Computational Economics, Volume 2, pages 829-1660, ISBN: 978-0-444-51253-6

- [20] Wilde, L.L. (1981): On the use of laboratory experiments in economics, *Philosophy in economics: Papers Deriving from and Related to a Workshop on Testability and Explanation in Economics* held at Virginia Polytechnic Institute and State University, 1979, ISBN 978-94-009-8396-0

## List of Figures

1	Logic of combining EXP and MAS research . . . .	14
2	Market 1 . . . . .	32
3	Market 2 . . . . .	33
4	Market 3 . . . . .	34
5	Root mean square deviation of prices from equilibrium . . . . .	34

## List of Tables

1	Standard deviation . . . . .	35
2	Root mean square deviation of prices from equilibrium . . . . .	35
3	Mean allocative efficiency of markets . . . . .	35

# Appendix A

## Experiment instructions

Welcome everyone to the economic experiment. It is a simple market, where both sides - buyers and sellers - can influence the price. Only a single type of good will be traded and your goal will be to earn as much money as possible on these trades. The whole system of trading will be described properly later on. As the first round starts, you will see one of these two tables:

Period:	1
You are a:	<b>SELLER</b>
Cost of Good 1:	20
Cost of Good 2:	27
Cost of Good 3:	35
Cost of Good 4:	40
Cost of Good 5:	49
Cost of Good 6:	56
Cost of Good 7:	63
Cost of Good 8:	65
Cost of Good 9:	71
Cost of Good 10:	83
Trading will start soon!	

Period:	1
You are a:	<b>BUYER</b>
Value of Good 1:	83
Value of Good 2:	75
Value of Good 3:	52
Value of Good 4:	50
Value of Good 5:	39
Value of Good 6:	32
Value of Good 7:	29
Value of Good 8:	28
Value of Good 9:	22
Value of Good 10:	18
Trading will start soon!	

The most important thing is whether you are a buyer or a seller. This role is given to you for the whole experiment. Next important information is about the values/costs of goods, which were assigned to you. These values/costs will stay the same for the whole round, but will vary across the participants (i.e. the sets

of values for each round are fixed, but are randomly distributed among the participants at the beginning of each period).

**If you are a seller:** your cost of good means how much you appreciate the good you own. Your goal is to try to *sell above its value* or at least for the same one. It is not possible to sell a good for a lower price than its cost. The amount of money you earn is equal to the difference between the price of the good sold and your private cost. You can sell a good anytime during a trading period by the button "sell at this price". You can make offers by the button "make a lower offer" as well. This offer must be always lower than the current lowest offer. You can lower your own offers too.

**If you are a buyer:** your value of good means, how much you appreciate the good you want to buy. Your goal is to try to *buy it below its value* or at least for the same one. It is not possible to buy a good for a higher price than its value. The amount of money you earn is equal to the difference between the price of the good sold and your private value. You can buy a good anytime during a trading period by the button "buy at this price". You can make bids by the button "make a higher bid" as well. This bid must be always higher than the current highest bid. You can higher your own bids too.

When trading starts you will see the following table:

Market		Period 1	Time Left: 8
<p>You are a: <b>BUYER</b></p> <p>Value of Good 1: SOLD</p> <p>Value of Good 2: SOLD</p> <p>Value of Good 3: 81</p> <p>Value of Good 4: 71</p> <p>Value of Good 5: 69</p> <p>Value of Good 6: 53</p> <p>Value of Good 7: 28</p> <p>Value of Good 8: 13</p> <p>Value of Good 9: 12</p> <p>Value of Good 10: 4</p>		<p><b>Prices of goods sold:</b></p> <p>40</p> <p>42</p>	
<p>Your earnings so far in this round are: 98</p> <p>Number of units purchased: 2</p>			
<p>The highest bid: No offer yet</p>		<p><input type="button" value="Make a higher bid"/></p>	
<p>The lowest offer to sell: No offer yet</p>		<p><input type="button" value="Buy at this price!"/></p>	

**How the trading is done:** You are all trading in one group. There are no pairs. Every round contains 6 periods. Every period lasts 120 seconds. The remaining time is in the upper right corner. You can only trade a good which is currently in the highest row. Until you buy/sell it you cannot trade with the next good. You cannot do any trades after the end of the period. You can buy/sell any number of goods from 0-10, but the higher number of deals you make the more money you can earn. The prices of goods sold in the past trades are on the right side of the screen. After each period you will see a table with all participants profits.

**How your profit will be computed:** At the end of the experiment you will receive the sum of your profits from all periods divided by 10 plus the show-up fee. So there is no random draw of periods at the end, every period counts.

**If you have any questions please raise your hand.**