

Title: Selected views of pupil mistakes in the teaching of mathematics

Author: Derek Pilous

Department: Department of mathematics and mathematics education

Supervisor: Prof. RNDr. Ladislav Kvasz, Ph.D.

Abstract: The main goal of the thesis is to describe in depth the problem of pupil mistakes in mathematics, both from the psychological and mathematics education points of view and from the point of view of mathematics teachers. Four questions were addressed: How does research in mathematics education view mistakes and what is the difference between this view and that of research of general mistakes? How can mistakes made by future mathematics teachers be classified? How do (future) mathematics teachers see the erroneousness of some border phenomena and the seriousness of mistakes? Are there differences between respondents with some teaching experience and without it?

The theoretical part of the thesis consists of (1) characterisation of a general mistake, (2) pupil mistakes in mathematics including their seriousness, (3) psychological view of mistakes including learning without and with mistakes and a more detailed description of a cognitivist viewpoint of mistakes, (4) mistakes from the point of view of mathematics education research. It is shown that pupil mistakes in the teaching of mathematics are not the same as mathematical mistakes as is intuitively understood; the reason being that they are of a quantitative, not qualitative nature. Their quantitative character is depicted by the concept of seriousness.

The distinction between inner and outer mistakes and a qualitative and quantitative character of mistakes as well as the explicit formulation of the validity of the concept of mistakes can be seen as the original contribution of this work to the general theory of mistakes. The step model of the transfer of information explicitly formulates informational levels on which the mistake can occur, including those which are often neglected (that is, the interpretative and syntactic ones). Another theoretical contribution of the work is the division of mistakes according to the state of the knowledge of the one who makes mistakes on the scale conscious ignorance – unconscious ignorance – slip.

The practical part of the thesis consists of two research studies. The first focuses on the classification of mistakes in future teachers' written solutions. Their analysis is made from the point of view of some concepts introduced in the theoretical part. The second study consists of a questionnaire survey with (future) mathematics teachers. Using a qualitative analysis of their answers, factors which influence the evaluation of erroneousness and seriousness are described, perceived seriousness of different types of mistakes is compared and blurred boundaries of erroneousness as perceived by teachers are explored. The sample was not a representative one, thus the quantitative characteristics of the sample cannot be generalised. However, the qualitative characteristics are valid ones, mainly the identification of the mere presence of different attitudes, justifications and other phenomena which can be taken as the basis of wider quantitative research. The conclusions consist of the limitation of research and its possible continuation.

Keywords: pupil mistakes, classification of mistakes, general mistakes, learning with mistakes, seriousness of mistakes, (future) mathematics teachers