BACHELOR THESIS

Pay-to-Play Lobbying

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Declaration of Authorship

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Signature
The author would like to express his sincere gratitude to PhDr. Martin Gregor Ph.D. for his patient supervision and immense support with this thesis, and to Bc. Miroslav Palanský for his insightful comments.

It would not be possible to complete this thesis without the unwavering support of my family.

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Abstract

We consider how the shape of the decision maker’s objective affects the pay-to-play lobbying model with the decision maker as a discriminatory price setter. First, we summarize the important characteristics of two instruments of lobbyists, contributions and information. We then place the theme of pay-to-play lobbying with endogenous access fees within the context of general lobbying literature as well as the narrower access-lobbying literature. Next, we augment the model used in Cotton (2012) and Gregor (2015) and introduce a new policy function to the model, which allows us to analyze the robustness of the main conclusions from the two papers, i.e. the curse of the ex ante advantage and the destruction of the lobbying industry as a result of endogenous private devaluations. We test the robustness of both phenomena with regard to decision maker’s objective and find that (i) the relative sizes of the ex ante utilities do not always completely determine the ex post expected utilities and (ii) the introduction of endogenous private valuations can produce equilibria where the destruction of private values does not occur. In the last section we generalize our findings and introduce a necessary and sufficient condition for the existence of equilibrium devaluation.

JEL Classification D72

Keywords access fee, lobbying, pay-to-play, policy function

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Abstrakt


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**Acronyms**

**DM** decision maker, legislator, politician, policy maker  
**FEC** Federal Election Commission  
**IG** interest group, lobbyist, special interest group  
**PAC** political action committee  
**USA** United States of America


**Thesis proposal** In this primarily theoretical thesis, we consider informational lobbying, namely lobbying as conveying verifiable evidence. Our specific perspective is that lobbying is conditional upon pre-payment of access fees (pay-to-play lobbying). The literature discusses several options how to set an access fee for informational meetings. The fee can be set universally to all competing groups, or can be group-specific. Or, the fee can be endogenous when access is auctioned. These mechanisms differ in how much the interest groups are willing to pay for access, and how much the decision-maker learns.

The aim of the thesis is to review the literature of pay-to-play lobbying and its instruments, and our ambition is to classify the policy functions used in the recent literature and develop their minor variants in a unified model. The idea is to see how changes in the design of the policy function of our decision maker affect the cost of lobbying for the interest groups and the game-theoretic equilibria, with an emphasis on the curse of ex ante advantage as well as destruction of lobbying industry under endogenous private valuations.

**Core bibliography**


Chapter 1

Introduction

The lobbying process is a quite intriguing topic for several reasons. It is a prime example of political economy and an application of game theory. The topic has attracted a lot of interest in the fairly recent literature of economics and political science, though the secrecy of the involved players still veils the delicate mechanisms in a shroud of mystery. More importantly, lobbying interactions between a decision maker and an interest groups are arguably one of the most influential forces that control what legislature is drawn up and passed and, as such, should be of much interest to constituents in any democratic system.

We choose to delve into the delicacies of pay-to-play lobbying partly because it offers a fresh and perchance a little more optimistic perspective on the purpose of contributions, i.e. the money flows between interest groups and politicians’ campaign funds. In the traditional lobbying literature, contributions are often viewed as an instrument used for buying policy decisions from willing decision makers, which blurs the line between lobbying and corruption in the eyes of the general public. Pay-to-play lobbying literature, on the other hand, offers a new paradigm where contributions have a different purpose - they enable the transfer of information between interest groups and politicians, give some level of credibility to the information, and perhaps allow the decision maker to make better informed policy decisions.

In our thesis, we set ourselves two goals. The first is to place the pay-to-play lobbying models within the broader field of lobbying literature. We do so by analyzing the fundamental building blocks of influence - contributions and information, and their nuances as well as applications in the available models.
The second and the main goal of our thesis is to analyze what happens to the results presented in the literature when we change the politician’s preferences over information and money. We capture the preferences using a policy function that forms a part of the equilibrium-determining process. We test the robustness of the findings in Cotton (2012) and Gregor (2015) with regard to policy function choice. Special emphasis is made on Gregor’s “curse of the moderate lobbyist”, which equates the ex ante advantage of an interest group to her ex post disadvantage. Finally, we extend our discussion to the crucial analysis of endogenous valuations where we test the robustness of the competitive devaluation and the destruction of the lobbying industry with regard to decision maker’s preferences.

The remainder of this thesis has the following structure. Chapter 2 introduces interest groups and analyzes the important concepts and properties of the tools of political influence, contributions and information. Chapter 3 introduces the concept of access fees and their structure, and summarizes the available pay-to-play lobbying models. Chapter 4 presents the setting of our model, a motivating example, and the core discussion of the effect of policy maker’s objective on the equilibrium results, the curse of the ex ante advantage and endogenous valuations. Chapter 5 summarizes our findings and concludes.
Chapter 2

Instruments of political influence

2.1 Lobbying and its types

Historically, the literature on politics often downplayed the ability of lobbyists to effect change in policy decisions. However, subsequent studies in the past 50 years found great amount of evidence in favor of the importance and the influence of lobbying efforts. The modern literature on lobbying distinguishes several forms of lobbying (Austen-Smith & Wright 1994; Grossman & Helpman 2001).

Interest groups try to influence political processes and decision making such that it suits their preferences. Their tools can be broken down into three categories - interest groups can provide information to policy makers, contribute to decision makers by making donations to their political campaigns, and finally they can persuade voters and change public perceptions in order to pressure the politician indirectly, be it through issue ads, labour unions etc. (Mueller 2003; Bennedsen & Feldmann 2006). And as Dahm & Porteiro (2008) point out, these tools can be both substitutes and complements when dealing with the decision maker depending on the nature of the particular lobbying process.

Assuming that politicians are mainly motivated by implementing a successful policy and by increasing their chance of (re)election, the lobbying process generally benefits the decision maker, e.g. she can use information to implement a policy that reflects the state of the world or by use contribution money
to fund their (re)election campaign (Mueller 2003; Wright 1996).\(^1\)

### 2.2 Interest groups

Lobbying is an intriguing form of political interaction in the sense that lobbyists face significant barriers such as the mentioned donations that limit the number of players that take part in the lobbying game. In Chapter 4, we start by discussing equilibria under exogenous number of interest groups. We base that on the empirical evidence provided by Kerr et al. (2011), who looked at how firms enter the lobbying process and found that the costs of lobbying (laws, signaling continuous support to a legislator) play a significant role in the decision whether to lobby. Their research suggests that few firms lobby and that they are then likely to be persistent in their efforts. This is a very useful observation because it allows the decision maker to form an expectation of alignment of a particular interest group on a given issue. Only after the discussion of exogenous number of interest groups, we allow for endogenous lobby formation under endogenous valuations.

Some papers, such as De Figueiredo & Kim (2004) and Bertrand et al. (2011)\(^2\), distinguish the terms interest group and lobbyist in the sense that a lobbyist serves as a middle man between an interest group and the legislator. Moreover, De Figueiredo & Kim (2004) argue that firms tend to hire external (middle-man) lobbyists when the probability of internal information leak is minimal but elect to lobby on their own when the probability reaches a critical level. In our thesis, we use the terms interest group and lobbyist interchangeably to denote an agent that is dealing with the decision maker (or legislator, politician) and we do not interest ourselves in the distinction. Instead, for the study of agency problems in lobbying, we refer to (Groll & Ellis 2014).

The traits of interest groups most relevant to our discussion of pay-to-play lobbying, e.g. private values and wealth, ability to find evidence etc., are discussed later in the context of the individual models. Although generally,

\(^1\)The exception is when an interest group has full bargaining power and is able to blackmail the decision maker.

\(^2\)Their paper aims to answer the question whether a lobbyist’s value comes from issue expertise or connections, and they conclude that it is more important for a lobbyist to have a connection to a politician based on the observation that lobbyists frequently change their issue focus as they follow the connected politician’s issue deployment.
interest groups can be divided into two sets - state-independent and state-dependent, where state-dependent interest groups prefer a policy regardless on how well it matches the state of the world (Austen-Smith 1998). As we explain later in our thesis, our model only considers interactions of state-independent interest groups.

Austen-Smith (1998) also highlights the collective action problem in lobbying, first documented by Olson (1965). Larger interest groups face stronger free-riding costs from their members, which results in an offset of welfare gained by the group. On the other hand, size also has a positive effect when the interest groups can use economies of scale when conducting search for information (Grossman & Helpman 2001). Again, given the scope of our thesis, we do not study the collective action problem in our analysis of the pay-to-play lobbying model.

2.3 Contributions

Contributions are arguably the most controversial of the lobbying instruments as they evoke a strong appearance of corruption. However, it is important to distinguish the two for the sake of clarity of the thesis and its discussion of the lobbying models. In the case of the USA, campaign contributions (campaign finance, political contributions) are legal and regulated by the Federal Election Commission (FEC). And though corporations are prohibited by the FEC to make direct donations to candidates, they may do so indirectly through PACs (Federal Election Commission 2015).

2.3.1 Role of contributions

The traditional branch of the literature on influence views considers contributions in two mechanisms - as a tool to help elect a like-minded politician, i.e. “a politician for sale”, or as a tool to buy a policy decision in the form of a quid-pro-quo policy favor, i.e. “policy for sale” (Baye et al. 1993; Gavious et al. 2002; Grossman & Helpman 1994; Hillman & Riley 1989; Tullock 1980).

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3There is some empirical evidence on the distinction between lobbying and contributions as shown by Campos & Giovannoni (2007), who found that lobbying and corruption are substitutes at least in the realm of developing countries.
The pay-and-play lobbying (or access lobbying) literature, i.e. Austen-Smith (1998); Cotton (2009; 2012); Gregor (2015), on the other hand argues that one of the roles of contributions is to secure access to a decision maker and allow an interest group to present information in order to affect the choice of policy in accordance with her preferences, i.e. “access for sale” instead of “policy for sale”.

Some support for the use of contributions as a mean of access are provided by studies by Esterling (2007) and Ansolabehere et al. (2003). The latter look at the amounts and sources of money in the US politics and offer a quite an informative insight into the use of contributions as political investment. The study discovers that there is significantly less money contributed in US politics than what would be expected if contributions were solely a political investment. Rather, on the margin, candidates get more from individuals than from interest groups, which hints at contributions serving as a consumption good for the general public or possibly as an instrument in securing access to a politician.

### 2.4 Information

Interest groups are able to disclose private evidence (or information, experiment) to the politician in order to shift her prior beliefs on a given issue. This information transmission can be conditional on being granted access to a politician in the pay-to-play lobbying game, or unconditionally in other games (Grossman & Helpman 2001). In order to capture the nuances of information that can be used, the literature defines several properties - hard vs. cheap evidence, informativeness and influence, verifiability and credibility.

#### 2.4.1 Hard and cheap evidence

The first property of evidence is that it can be either cheap or hard (Bull & Watson 2004). Cheap evidence is one that exists in all states of the world, though it can still have value for the decision maker under certain conditions. Crawford & Sobel (1982) represent the literature of cheap evidence (cheap talk) and assert that cheap evidence is informative if an interest group has imperfectly correlated preferences with the decision maker and the cheap evidence becomes a reliable signal of the state of the world.
On the other hand, hard evidence exists only in some states of the world and not in others. Hard evidence can convey two distinct messages about possible states of the world - positive and negative. To illustrate that, let us suppose that there are two possible, mutually exclusive states of the world - A and B. First, if a document exists only in the state of the world A, then disclosure of such document is positive evidence of state A being realized. Second, if such document is expected by the politician to exist in state A but is not disclosed to her, the non-disclosure serves as negative evidence of state B being realized. What follows is that positive (disclosed) evidence can have stronger persuasive value than negative (non-disclosed) evidence as the latter might lead to less precise posterior beliefs depending on the equilibrium.

2.4.2 Informativeness and influence

Austen-Smith (1998) describes two additional properties of experiments submitted to a legislator under a binary policy - they can differ in how informative and influential they are. An experiment is informative when the probability that it generates a signal in favor of the preferred position is greater than the probability that it signals the opposite; a fully informative experiment then signals in favor with certainty.\(^4\) On the other hand, an experiment is influential if and only if a signal towards one policy induces the legislator to change her beliefs.\(^5\) A policy that is based on an experiment other than fully informative is informationally inefficient. Austen-Smith also supposes that different groups use different aspects of informativeness and influence - state-dependent lobbyists are expected to use fully informative evidence whereas state-independent lobbyists are expected to provide evidence that maximizes the probability of being influential regardless of the true state of the world. This discussion obviously rests on Austen-Smith’s assumption of endogenous signals, which we do not consider in our model.

2.4.3 Verifiability and credibility

The credibility of the interest group’s messages and the decision maker’s ability to verify information are two closely linked phenomena and they are crucial

\(^4\)A fully uninformative experiment signals both states with the same probability, i.e. \(Pr = 1/2\).

\(^5\)With regard to an experiment, being informative is a necessary condition for being influential.
for the lobbying process (Austen-Smith & Wright 1994; Wright 1996). Bull & Watson (2004) define verifiable evidence as evidence whose aspects and validity can be observed by a third party and then relayed to the politician. Dahm & Porteiro (2008) further divides verifiable evidence into public tests and private tests. Public tests are conducted by an independent third party (external expert) and are easier to verify whereas private tests are conducted by an expert with some ties to the interest group and are more difficult to verify.

The literature considers two contrasting views on verifiability of the evidence and its ability to sway the politician one way or another. Austen-Smith (1995) and Lohmann (1995) assume in their analyses that evidence is unverifiable and argue that it cannot persuade the politician on its own. Rather, they conclude that the only factors that persuade the politician are the knowledge of interest group’s preferences or credible signals, i.e. the signaling effect of the amount of money is associated with the information (the size of the contribution/access fee paid). Another way interest groups can add credibility to their evidence is by costs of evidence, which can be represented by positive costs associated with conducting the search of information about the realized state of the world (Krishna & Morgan 2001; Grossman & Helpman 2001).

On the other hand, Cotton (2009) and Bennedsen & Feldmann (2006) assume that the evidence disclosed upon access is verifiable and can influence politician’s view on its own. What is more, in case of Cotton (2009) where interest groups compete in an all-pay auction, the decision maker is able to learn even the non-invited group’s evidence simply on the basis of the non-winning payment.

Natural extension of the previous discussion is the verification cost and verification strategy. For example, if the hard evidence is unverifiable or if the decision maker does not verify the information often, the interest group might have an incentive to distort the message from its experiments to the politician. Then the assumption is that the decision maker can observe the distortion ex post from a lower-than-expected payoff from a policy decision that does not match the believed state of the world. If the interest group always distorts

\[ \text{6 The role of credibility under the assumption of unverifiable evidence is supported by an empirical analysis by Bertrand et al. (2011), who show that lobbyists are not pure messengers given the large fees they demand and thus they might also give credibility to the evidence they present to a legislator.} \]
the information, then the decision maker disregards the evidence in a repeated game (Grossman & Helpman 2001; Mueller 2003).
Chapter 3

Pay-to-play lobbying

What follows from our discussion in Chapter 2 is that we are interested in both evidence and contributions as means of political influence. The earliest model of influence that fits the framework we use in our thesis is Austen-Smith (1998), where interest groups pay contributions as access fees to the decision maker for the opportunity to disclose evidence, although some support for the notion that contributions and access are related came much earlier and is documented in Langbein (1986), who finds a link between a politician’s time spent on meetings with interest groups and the value of their contributions to her. Additional support is available in Herndon (1982), who interviewed interest groups, and Schram (1995), who interviewed retired politicians. Both books state that the people who have been involved in the lobbying process strongly emphasise the importance of access to a decision maker. The literature of influence that followed, i.e. Baron (1989), Snyder Jr. (1990), Austen-Smith & Wright (1992), Austen-Smith (1995), Lohmann (1995) etc., gave a theoretical foundation for the idea that money buys access to the legislator. However, the pay-to-play lobbying literature is still a fragment of the models of influence.

3.1 Access to politicians

The general assumption of access models is that a decision maker’s time is limited and that if she chooses to listen to evidence of interest groups, it is not possible to give access to everyone. This attention limit helps the decision maker to commit to prices and allows her to set up a game where she sells access for a fee to a number of interest groups.
The access fee is defined as a contribution to a politician’s campaign in exchange for an access to the politician. Access then allows an interest group to disclose evidence in order to influence the ultimate policy decision (Cotton 2008). The literature offers three possible cases of access fees - the fees can be exogenous (Austen-Smith & Wright 1992; 1994), entirely absent, i.e. exogenous at zero (Milgrom & Roberts 1986), or endogenous as is the case in Austen-Smith (1998); Cotton (2009; 2012); Gregor (2015). From this point onwards, we will assume that the access fees in our thesis are endogenous, i.e. set by the decision maker.

Cotton (2008) states that the politician has certain incentives to set the access fee at a positive amount. First, a politician values contributions, and second, a higher access fee might discourage an interest group from seeking access with low quality evidence as it forces the group to evaluate the persuasiveness of its evidence vis-à-vis the cost of the necessary contribution.

Although, an argument could be made that access fee does not necessarily have to be positive. Bertrand et al. (2011) discuss this possibility of a politician offering free access (zero access fee) via family connections in lobbying. But even though the access is costless in the short term, they argue that the strong family connection is likely to be a first string to pull when a politician is in need of funding, and thus in effect the access is not free from a long term perspective.\footnote{Moreover, access fee can also be negative when the decision maker seeks an information from an expert who does not wish to disclose it and the politician thus has to pay the expert.}

### 3.2 Structure of access fees

As to the structure of endogenous access fees, access literature offers a multitude of ways a politician can set the game up. Politician can commit to a single common price for every interest group beforehand (Austen-Smith 1998). The decision maker is, however, able to price-discriminate at least in the sense that he sets multiple prices - one for each of the type of lobbyists, though the individual agents of the same type face the same access fee. Agents then simultaneously decide whether to pay the access fee and the politician selects a single one from the pool and grants her access. A generalized case is when the decision maker can introduce a fully discriminating access fee for each interest.
Another possible setup of access fees, used by Cotton (2009), is an all-pay auction where each interest group contributes the access fee to the politician based on the quality of the evidence and the politician then grants access to the highest bidder. Here Cotton proposes that, given the noiseless information structure, a group with better evidence has an incentive to pay more than the other group, assuming that group with access has to reveal her evidence.

### 3.3 Pay-to-play lobbying models

There are two possible timings of the instruments of influence. The aforementioned Dahm & Porteiro (2008) assume that the payments (contributions) occur after evidence disclosure as an interest group is allowed to respond to an unfavorable outcome of such disclosure by contributing more money. In contrast, the four pay-to-play models with endogenous access fees (Austen-Smith 1998; Cotton 2009; 2012; Gregor 2015) have an interim timing where the payments occur at the start of the game before the evidence disclosure. Furthermore, they are also specific in that the decision maker makes a simple commitment to listen to the participating groups only. We will now briefly introduce the four models.

- The pay-to-play model proposed by Austen-Smith (1998) considers interactions of two sets of interest groups, state-dependent moderates and state-independent extremists, and a legislator with a localized monopoly over an issue. In this particular model, the legislator faces a trade-off between granting access to acquire information or to get financial contribution. Thus, money only influences who is given access and policy is influenced only through the submission of information (information lobbying).

  Here the lobbying process has five stages. The legislator first commits to a list of access prices for groups of lobbyists, where the politician can discriminate each group through the price. Then, individual lobbyists in each group independently choose whether to contribute at the given

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2 An important remark here is that Austen-Smith (1998) considers the amount of players
access price. Thirdly, the legislator chooses one lobbyist from the set of all individuals who contributed above or at their respective price. In the following stage, the selected lobbyist presents evidence (argument, experiment), all the while having control over the legitimacy of the evidence.\(^3\) Finally, the legislator decides on a binary policy.

The crux of the discussion in Austen-Smith (1998) is if and when the decision maker prefers to grant access to a state-independent interest group. The proposition then is that the decision maker grants access to an extremist only if no moderate contributes above the set price, and the probability that the selected information will be informationally inefficient corresponds to the probability that no moderate seeks access to the legislator.

The model also provides a ground for an interesting phenomenon - lobbyists might not seek access only to present information to a legislator. Under an assumption of access being granted to more than one individual, Austen-Smith (1998), building upon Austen-Smith & Wright (1994), suggests that extremists holding mainstream opinions have an incentive to lobby counteractively, that is, to prevent extremists on the opposite side to present influential evidence to the legislator.

- Cotton’s (2009) model presents an intriguing approach to address the apparent dichotomy in understanding the role of contributions. He presents a scenario where the policy maker makes an active choice between selling access to get information or quid pro quo exchange for policy favor.\(^4\) The choice is made ex ante and the politician trades off maximization of policy utility of her and the constituents, and maximization of total contributions. Cotton (2009) then uses this framework to analyze the consequences of tax and limits on contributions to politician’s campaign funds.
The decision to sell policy favor leads to an all-pay auction and the winner gets their optimal policy at the extreme of the single-dimensional policy spectrum. On the other hand, when selling access, the politician observes the contributions made by interest groups and then grants access to the winning group, which then presents her evidence to the politician. To address the loss of information from not granting access to the loser, Cotton (2009) introduces a monotone increasing contribution function that enables the politician to correctly infer the quality of the loser’s evidence simply by observing the level of his or her contribution.

The politician decides on her choice between selling access and favor based on the expected utility of each setup. Cotton (2009) proposes that the choice of the setup is a function of the importance of the issue at hand, while the final policy is a function of the issue’s importance, contributions from each group, and the evidence presented by the winning side (if the politician sells access). Each group’s decision on the level of its contribution is then a function of the issue’s importance, group’s evidence and the setup of the game.

Based on this logic, the model proposes that there exists a unique level of issue importance where politician’s utilities of selling access and policy favor equal and the politician is indifferent. The politician prefers selling access for issues of greater importance and she sells policy favor for less important issues.

The core of the paper is to address the effects of contribution caps and tax\(^5\) within the context of the dual framework. Cotton (2009) assumes that the choice and specific implementation are exogenous to the model and rather discusses the effects of both cap and tax on the equilibrium welfare of the agents. He finds that both cap and tax increase the probability of the politician selling access and concludes that tax is always better for constituents even in comparison with no regulation, but the decision maker has incentives to prefer cap to make herself better off at the expense of the public.

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\(^5\)The discourse over the need of contribution regulation is an underlying theme of the recent literature but it is not within the scope of our thesis.
The other two pay-to-play lobbying models, Cotton (2012) and Gregor (2015) are closely examined in the following chapter because we use them to observe the effects of the decision maker’s objectives (preferences), which are summarized in her policy function.

- Cotton (2012) centers his discussion around the unfairness of the money-oriented politics formulated by Makinson (2003) and aims to answer three questions - (i) whether rich interest groups have better access to decision makers, (ii) whether such access privilege translates into more favorable equilibrium outcome, and finally (iii) whether an introduction of contribution limits levels the playing field.

As we show in greater detail in Section 4.2.2, the setup of the model always leads to the decision maker granting access to the rich group. The decision maker chooses between two interactions with the rich group; she can either give the rich the benefit of the doubt, meaning that she positively discriminates the rich whenever the rich and poor groups perform the same action, or she can issue a burden of proof to the rich group, where the decision maker completely disregards the poor group’s actions.

This access advantage, however, does not always favor the rich group ex post, as we show later in our thesis. The decision maker’s ability to set discriminatory access fees gives her an immense ability to extract rent from the interest groups, which then ex post offsets the apparent access advantage.

In contrast with his previous paper Cotton (2009), here Cotton (2012) views contribution limits from the viewpoint of rich interest groups. He concludes that, rather than helping to breach the wealth gap between the rich and poor interest groups, contribution cap helps the rich as it only limits the decision maker’s ability to extract rent from the group. Similarly to the previous paper, introduction of contribution limits makes the decision maker worse-off as it decreases her payoffs.

- Gregor (2015), on the other hand, predominantly focuses on contribution caps as a game-theoretical foundation for the Tullock’s puzzle, i.e. the
apparent small amounts of money contributed in the US politics given that the stakes are often incredibly high.

Rather than discussing access advantage of rich groups, Gregor looks at how an ex ante advantaged group, i.e. group who prefers status quo policy, fares in the equilibrium. As is the case in Cotton (2012), the price discriminating decision maker has a strong ability to extract rent from the invited interest groups and Gregor shows that ex ante advantage and ex post disadvantage are equivalent, which he defines as the curse of the moderate lobbyist.

The curse has a significant effect on the equilibria with endogenous valuations, where the interest groups can strategically devaluate and Gregor shows that the curse always creates an incentive for each group to completely devalue to zero, which leads to a destruction of the lobbying industry.

In contrast with Cotton (2009), but similarly to Cotton (2012), the regulation of contributions is here in the common interest of the decision maker and the interest groups as it supports the lobby formation. The decision maker is willing to impose a contribution cap on herself to prevent the destruction of the access game, which would explain the small size of the observed contributions in the U.S. politics.
Chapter 4

Policy functions in pay-to-play lobbying

4.1 Pay-to-play lobbying model with discriminatory access fees

The analysis of the effects of the decision maker’s objective in our thesis is based on the pay-to-play lobbying model derived in Cotton (2012) and Gregor (2015), which in turn build upon Austen-Smith & Wright (1992). The model deals with lobbying interaction between two interest groups $IG_i, i = 1, 2$, and a decision maker DM. IGs seek access to inform the decision maker, who commits not to listen to an interest group’s information if it does not have access to the politician, as is also assumed in Austen-Smith (1998) and Cotton (2009).

4.1.1 The setting of the model

The state of the nature exists in two binary dimensions and is described by $\theta_i \in \{0, 1\}, i = 1, 2$; that is, a state either exists or not and the two states are uncorrelated. Each $IG_i$ is able to privately observe her own $\theta_i$ only and the model assumes that such observation is fully informative and considered hard evidence with the properties we described previously in our thesis. Gregor (2015) characterizes an $IG_i$ with the private information $\theta_i = 1$ as a high type of lobbyist and $\theta_i = 0$ as a low type of lobbyist. The ex ante beliefs about the state of the nature are common knowledge among all agents and are described by $\pi_i = Pr(\theta_i = 1)$ and we define $\beta = \pi_2 / \pi_1$ for the sake of further discussions.
Moreover, these priors are independently distributed, i.e.

\[ \pi_i = Pr(\theta_i = 1|\theta_{-i} = 0) = Pr(\theta_i = 1|\theta_{-i} = 1) \]

Our decision maker selects a non-contractible\(^1\) policy \( P = \{P_1, P_2\} \). Each \( IG_i \) has state-independent policy preferences, that is, each prefers her \( P_i \) regardless of the actual state of the world.\(^2\) \( IG_i \) always values her own policy positively (the private value is \( v_i(P_i) > 0 \)) and has no positive value from \( IG_{-i} \)'s policy (\( v_i(P_{-i}) = 0 \)). The size of each IG’s private value of her policy \( v_i \) are common knowledge and we further define \( \alpha = v_1/v_2 \).

The DM’s policy value can be both state-independent and state-dependent based on the DM’s policy function \( V(P_i, \theta) \), which reflects her objectives and preferences and enables us to determine what policy the DM chooses for each state of the world \( \theta = (\theta_1, \theta_2) \). Gregor introduces state dependent policy loss:

\[ l(P_i, \theta) = \max\{V_i(P_1, \theta), V_2(P_2, \theta)\} - V(P_i, \theta) \]

the expected policy loss for event \( \epsilon \):

\[ \lambda(P_i|\epsilon) = \sum_{\theta \in \epsilon} l(P_i, \theta) * Pr(\theta) \]

and the expected policy loss in any \( \sigma \)-subgame \( \Lambda(\sigma) = \lambda(P_i|\sigma) \).

In his paper, Gregor (2015) further defines a moderate (or ex ante advantaged) IG as an interest group who prefers the status-quo policy obtained by minimizing the expected policy loss in the absence of observation, i.e.

\[ P_{\emptyset} = \arg\min_{P_1, P_2} \lambda(P_i|\emptyset) \]

and defines the other IG as extremist (ex ante disadvantaged).\(^3\)

The outcome of the mechanism above is a set of four policies selected for each

\(^1\)Ex post choice cannot be influenced by ex post contributions as we find in Dahm & Porteiro (2008) and Bennedsen & Feldmann (2006).

\(^2\)Using the framework of Austen-Smith (1998), both IGs here are extremists.

\(^3\)Terms “moderate” and “extremist” are obviously used in different a meaning in this sense and we will refrain from using this definition. Rather, we will use the terms “ex ante advantaged” and “ex ante disadvantaged”.

\( \theta = (\theta_1, \theta_2) \) and it is mapped by \( \psi(\theta) \) into a \( 2 \times 2 \) matrix of state-dependent policies, where \( \psi(\theta) \in \{ P_1, P_2 \} \).

\[
O = \begin{pmatrix}
\psi(0,0) & \psi(1,0) \\
\psi(0,1) & \psi(1,1)
\end{pmatrix}
\]

All possible mappings using \( \psi(\theta) \) are given by matrices \( A_i, i = 1, 2, \ldots 16: \)

\[
A_1 = \begin{pmatrix}
P_1 & P_1 \\
P_1 & P_2
\end{pmatrix},
A_2 = \begin{pmatrix}
P_1 & P_1 \\
P_2 & P_1
\end{pmatrix},
A_3 = \begin{pmatrix}
P_2 & P_1 \\
P_1 & P_1
\end{pmatrix},
A_4 = \begin{pmatrix}
P_1 & P_2 \\
P_1 & P_1
\end{pmatrix},
A_5 = \begin{pmatrix}
P_1 & P_1 \\
P_2 & P_2
\end{pmatrix},
A_6 = \begin{pmatrix}
P_2 & P_1 \\
P_2 & P_1
\end{pmatrix},
A_7 = \begin{pmatrix}
P_2 & P_2 \\
P_1 & P_1
\end{pmatrix},
A_8 = \begin{pmatrix}
P_1 & P_2 \\
P_1 & P_1
\end{pmatrix},
A_9 = \begin{pmatrix}
P_1 & P_2 \\
P_1 & P_2
\end{pmatrix},
A_{10} = \begin{pmatrix}
P_2 & P_1 \\
P_2 & P_2
\end{pmatrix},
A_{11} = \begin{pmatrix}
P_2 & P_2 \\
P_2 & P_1
\end{pmatrix},
A_{12} = \begin{pmatrix}
P_2 & P_2 \\
P_1 & P_2
\end{pmatrix},
A_{13} = \begin{pmatrix}
P_1 & P_1 \\
P_1 & P_1
\end{pmatrix},
A_{14} = \begin{pmatrix}
P_2 & P_2 \\
P_2 & P_2
\end{pmatrix},
A_{15} = \begin{pmatrix}
P_1 & P_2 \\
P_2 & P_1
\end{pmatrix},
A_{16} = \begin{pmatrix}
P_2 & P_2 \\
P_1 & P_2
\end{pmatrix}.
\]

Our DM indirectly selects her desired outcome mapping by imposing either acceptable or unacceptable fees, by which she manipulates the amount of information described by \( \sigma \in 2^\mathcal{L} \), where \( \sigma \) is set of participating interest groups and \( \mathcal{L} = \{ IG_1, IG_2 \} \) is set of interest groups in the model. The amount of information thus depends on whether both, one, or no IG participate in the disclose of hard evidence.

Gregor’s model considers the DM as a price setter who assigns each \( IG_i \) a specific access fee (price of access) \( f_i \geq 0, i = 1, 2 \), with the assumption that utility is quasilinear in money. Moreover, we define the DM’s expected utility from set of invited lobbyist as

\[
W(\sigma) = \sum_{IG_i \in \sigma} \pi_i * f_i - \Lambda(\sigma)
\]

and \( IG_i \)’s utility as

\[
u_i = Pr(P = P_i | O) * v_i - \pi_i * f_i
\]

Stages of the game are as follows:
Stage 1 - DM announces $f = (f_1, f_2)$ and commits to not listen to any $IG_i$ who does not pay her $f_i$.
Stage 2 - IGs privately observe their own $\theta_i$.
Stage 3 - Each IG either pays her $f_i$ or does not participate.
Stage 4 - Participating IGs disclose their hard evidence. \(^4\)
Stage 5 - DM selects her ex post optimal policy.

\[\textbf{4.1.2 Motivating example}\]

An opportunistic investor wants to build a hockey arena on a brownfield site in Toronto, ON, Canada with the expectation that the hockey league will relocate a franchise from Glendale, AZ, USA, to Toronto as part of the league’s centennial season in 2017. At the same time, a large manufacturer wants to use the same site to build a factory. Since the decision rests on the city hall in Toronto, both parties want to lobby the mayor to give permission for their project. The investor argues that the new team will bring more tourists to Toronto and create business opportunities for the service sector, while the manufacturer argues that the market is expected to boom and that the factory will create a large amount of jobs for people in the Greater Toronto Area.

The investor can use the arena for other purposes than just hockey and thus her private value is state-independent. Moreover, since there already is one arena in Toronto, the expected profit of the new one can be predicted. The same applies to the factory as they have comparable competition in the same area. Thus, private values are common knowledge. The decision maker however still faces uncertainty in public values. First, the hockey league has not committed to relocate the team to Toronto and there is a positive probability that the arena will not be used and tourism would not increase due to hockey. As for the factory, the development of the manufacturer’s market is similarly uncertain. In contrast with the decision maker, both parties have their specific knowledge and can find and provide private hard evidence to the decision maker with a positive probability in order to alleviate her uncertainty in their particular case.

In our model, the decision maker sets up an access game where, based on the

\[\text{\textsuperscript{4}Alternatively, we could assume voluntary disclosure where we would follow Gregor (2015) in the focus on equilibrium where non-participation is interpreted by the DM as a signal of the information } \theta_i = 0 \text{ in accordance with Milgrom (1981).}\]
common knowledge of the ex ante probabilities, private values, and her policy function, she announces specific access fees for each party and sells access.

4.2 Policy functions

We will now dedicate a part of our thesis to consider the effects of different policy functions on the equilibria of our pay-to-play lobbying game. As is mentioned in the previous discussion, policy function $V(P, \theta)$ shows how our decision maker decides over the available policies in each of the events $(\theta_1, \theta_2)$. The policy functions we analyze differ in the way the DM relates private values of the interest groups and their private information to the public value.

We first consider a trivial policy function $V(P, \theta) = v_i$, where our DM chooses her optimal policy in each event based solely on IG’s state-independent private values and independently on their hard evidence. Next, we analyze $V(P, \theta) = \theta_i$, a policy function used by Cotton (2012), where the DM relates the public value of a policy in each event $(\theta_1, \theta_2)$ only to the hard evidence provided by the IGs and does not base it on private values of the IGs. The third policy function we look at is $V(P, \theta) = \theta_i \ast v_i$, introduced by Gregor (2015). Here our decision maker relates the public value to both hard evidence and private values of the IGs. Lastly, we introduce a policy function $V(P, \theta) = \theta_i \ast v_i + v_i$, which behaves similarly to Gregor’s function for interest groups with a relatively small gap in wealth and which behaves similarly to the trivial example when the wealth disparity is above an arbitrary threshold.

4.2.1 $V(P_i, \theta) = v_i$

We start the discussion of the effects of the decision maker’s objectives by looking at a trivial example where the DM decides on the optimal policy in each of the four situations $\theta = (\theta_1, \theta_2)$ based on $V(P_i, \theta) = v_i$. In this setting, the DM’s policy choice is clearly constant in $\theta$ independently on $\sigma$ and the policy choice is affected by the size of IG’s value of her policy only.

The interpretation of this policy function within the context of our motivating example is that the quality (or legitimacy) of the projects does not interest the decision maker, she only values the scope of the projects, which are represented
by the private values.

The loss function under $V(P_i, \theta) = v_i$ is

$$l(P_i, \theta) = \max\{V_1(P_1, \theta), V_2(P_2, \theta)\} - V(P_i, \theta) = \max\{v_1, v_2\} - v_i$$

and it generates positive policy loss $l(P_i, \theta) > 0$ if and only if $v_i < v_{-i}$, which Cotton (2012) defines as $IG_i$ being “poor” and $IG_{-i}$ being “rich”. This means that the DM does not face any indifferences in each $(\theta_1, \theta_2)$. Immediately, we can tell that the ex ante advantaged IG is the rich one as the status quo policy is dictated by $P_\emptyset = \arg \max\{v_1, v_2\}$.\(^5\)

This policy function generates a doublet of outcomes

$$D_1 = \begin{pmatrix} P_1 & P_1 \\ P_1 & P_1 \end{pmatrix} \quad \text{and} \quad D_2 = \begin{pmatrix} P_2 & P_2 \\ P_2 & P_2 \end{pmatrix},$$

where $D_1$ is selected whenever $v_1 > v_2$ (or $\alpha > 1$) and $D_2$ is selected when $v_1 < v_2$ (or $\alpha < 1$). Figure 4.1 illustrates the outcomes.

$$O^* = \begin{cases} D_1 & \text{if } \alpha > 1 \\ D_2 & \text{if } \alpha < 1 \end{cases}$$

W.L.O.G we can assume that $\alpha > 1$. In this case, the optimal choice of outcome for the DM is $O^* = D_1$. If the DM invites $IG_1$, i.e. $\sigma = \{IG_1\}$, the information set is two events: $\theta_1 = 1$ where $IG_1$ seeks access and the DM wants to charge $f_1 > 0$ and $\theta_1 = 0$ where $IG_1$ abstains and the DM cannot collect access fees. However, because of the policy function choice, the DM selects $P_1$ in both events and the outside option for $IG_1$ (i.e. not seeking access) has the same value as the inside option. Thus in equilibrium, the DM charges $f_1 = 0$ and $IG_1$ is indifferent between abstaining and seeking access. Similarly, $\sigma = \{IG_2\}$ leads to $f_2 = 0$ as the DM selects $P_1$ according to $D_1$ and thus the inside and outside options for the interest group are of the same value $v_2(P_1) = 0$.

Finally if the DM invites both IGs ($\sigma = \{IG_1, IG_2\}$), the information sets

\(^5\)Notice that we disregard the event $v_1 = v_2$ as it stands as a point of indifference for the DM that is a result of a combination of parameters rather than a result of the policy loss function.
are the four events \((\theta_1, \theta_2)\) where in each event the DM still selects \(P_1\) according to \(D_1\). The DM again cannot set a positive \(f_i > 0, i = 1, 2\) as each IG’s outside option has the same value as the inside option.

Given the fact that under all three scenarios the access fee is \(f_i^* = 0, i = 1, 2\), the expected utilities of the politician are the same

\[
W(IG_1, IG_2) = W(IG_1) = W(IG_2) = 0 * v_1 + 0 * v_2 = 0
\]

and the DM will elect to be fully informed about \((\theta_1, \theta_2)\) and chooses \(\sigma^* = \{IG_1, IG_2\}\). The opposite case when \(\alpha < 1\) is analogous and the DM again cannot charge any positive fee to either interest group.

Clearly, with \(V(P_i, \theta) = v_i\), the DM’s policy preference is state-independent and the DM’s commitment not to listen to \(IG_i \notin \sigma\) does not create a more desirable inside option that would motivate at least one interest group to prefer access. Therefore, the DM cannot set a positive access fee \(f_i > 0, i = 1, 2\) and extract rent from an interest group whose policy is ex post chosen as optimal.

A way to remedy this (under the assumption \(V(P_i, \theta) = v_i\)) would be for the decision maker to set up an alternative mechanism of lobbying, a one that does not tie her hands the way our modeled commitment does. With the underlying assumption of \(v_i, i = 1, 2\) being common knowledge, the DM could make a different commitment that would enable the DM to trade-off information and financial contributions and therefore allow for an equilibrium with a positive
policy loss $l(P_i, \theta) > 0$.\textsuperscript{6}

### 4.2.2 $V(P_i, \theta) = \theta_i$

Another policy function is proposed by Cotton (2012), who models the function such that our decision maker values only the quality of the policy - a policy either is legitimate or it is not, i.e. $\theta_i \in (0, 1)$. This policy function is unique in our comparison as it does not include IGs’ policy values in the DM’s public value.

$$V(P_i, \theta) = \theta_i, \ i = 1, 2$$

In the context of our motivating example, this policy function implies that the decision maker only considers the projects based on their quality (legitimacy), but does not take the scope into consideration. We can interpret this policy function as preferences of a decision maker who, above all, wants to “play it safe”, i.e. she wants her decision to be backed by evidence.

$$V(P_i, \theta) = \theta_i, \ i = 1, 2$$

Under this $V(P_i, \theta)$, the policy loss function is

$$l(P_i, \theta) = \max\{V_1(P_1, \theta), V_2(P_2, \theta)\} - V(P_i, \theta) = \max\{\theta_1, \theta_2\} - \theta_i$$

and policy loss is zero if $\theta_1 = \theta_2 \in \{0, 1\}$, which means that the DM faces two events of indifference.

The status quo policy $P_\emptyset$ is here given by

$$P_\emptyset = \arg\min_{P_1, P_2} \lambda(P_i|\emptyset) = \arg\max_{P_1, P_2} \sum_{\theta \in \emptyset} \pi_i \theta_i = \arg\max_{P_1, P_2} \{\pi_1, \pi_2\},$$

which translates into the inequality $\beta \leq 1$.\textsuperscript{7}

Cotton addresses the indifference problem by introducing two commitments that the DM can make in addition to not listening to a group without access. The first way is by giving a “benefit of the doubt” (BoD) to the rich

\textsuperscript{6}Obviously, this undermines the assumption of non-contractible policy and in turn the whole model.

\textsuperscript{7}We defined $\beta$ earlier as $\beta = \frac{\pi_2}{\pi_1}$.\textsuperscript{7}
group, where our decision maker commits to select $IG_i$’s policy in both events of indifference if and only if $v_i > v_{-i}$. BoD generates a doublet of mappings

$$E_1 = \begin{pmatrix} P_1 & P_1 \\ P_2 & P_1 \end{pmatrix} \quad \text{and} \quad E_2 = \begin{pmatrix} P_2 & P_1 \\ P_2 & P_2 \end{pmatrix}$$

Let us assume that the DM commits to BoD and $\alpha > 1$, i.e. the interest group $IG_1$ is the rich one and the DM selects $P_i$ from $E_1$. Again for each group, her willingness to pay is dictated by the comparison of her options. Cotton assumes that under BoD, the DM does not exclude any interest group from the lobbying process and the equilibrium is $\sigma = \{IG_1, IG_2\}$. Here the rich group’s inside option is more desirable because it guarantees that the DM chooses $P_1$ independently on the action of the poor IG and the outside option brings a positive chance that the DM selects $P_2$. The rich group’s and the poor group’s willingness to pay is $\pi_2 v_1$ and $(1 - \pi_1) v_2$ respectively. In equilibrium, the DM sets $f_1$ and $f_2$ accordingly and her expected utility is

$$W(IG_1, IG_2) = \pi_1 \pi_2 v_1 + (1 - \pi_1) \pi_2 v_2$$

The case of BoD and $\alpha < 1$ is analogous, $O^* = E_2$, $(f_1^*, f_2^*) = ((1 - \pi_2) v_1, \pi_1 v_2)$ and

$$W(IG_1, IG_2) = \pi_1 (1 - \pi_2) v_1 + \pi_1 \pi_2 v_2$$

Second way to break the indifference is by issuing a “burden of proof” (BoP) to the rich group, where the commitment is to select rich group’s policy in the event $(\theta_1, \theta_2) = (1, 1)$ and to select poor group’s policy in the event $(\theta_1, \theta_2) = (0, 0)$. What is more, under the burden of proof, Cotton effectively assumes that the DM excludes the poor group from participation.

This commitment generates a doublet of mappings

$$E_3 = \begin{pmatrix} P_2 & P_1 \\ P_2 & P_1 \end{pmatrix} \quad \text{and} \quad E_4 = \begin{pmatrix} P_1 & P_1 \\ P_2 & P_2 \end{pmatrix}$$

Now let us assume that the DM chooses to commit to BoP and that $\alpha > 1$. What follows is that the DM chooses $P_i$ according to $E_3$ and clearly for any $\sigma$, the $IG_1$’s inside and outside options are independent on $\theta_2$, and $IG_2$’s options

---

8Notice that in neither BoD nor BoP does Cotton allow for the poor group’s policy to be chosen in the event where both groups seek access, even though it might have been expected because of the the DM’s value independence on IGs’ private policy values.
are of the same value. In equilibrium, the DM sets \( f_1^* = v_1 \) and and her expected policy utility is

\[
W(IG_1) = \pi_1 v_1
\]

The case where the DM commits to BoD and where \( \alpha < 1 \) is analogous. \( O^* = E_4, f_2^* = v_2 \) and

\[
W(IG_2) = \pi_2 v_2
\]

To derive optimal outcomes \( O^* \), we need to compare the DM’s expected policy utility under BoP and BoD given all combination of parameters \( (W_{BoD} \preceq W_{BoP}) \). First, we assume that \( \alpha > 1 \).

\[
W_{BoD} > W_{BoP}
\]

\[
\pi_1 \pi_2 v_1 + (1 - \pi_1) \pi_2 v_2 > \pi_1 v_1
\]

\[
\frac{1 - \pi_1}{1 - \pi_2} \pi_2 v_2 > \pi_1 v_1
\]

\[
\frac{1 - \pi_1}{1 - \pi_2} \pi_2 \frac{v_2}{\pi_1} > \frac{v_1}{v_2}
\]

Let \( \zeta = \frac{1 - \pi_1}{1 - \pi_2} \pi_2 \frac{v_2}{\pi_1} \) and we get

\[
W_{BoD} > W_{BoP} \iff \zeta > \alpha
\]

and

\[
W_{BoD} < W_{BoP} \iff \zeta < \alpha
\]

For \( \alpha < 1 \) we obtain the following

\[
W_{BoD} > W_{BoP} \iff \zeta < \alpha
\]

and

\[
W_{BoD} < W_{BoP} \iff \zeta > \alpha
\]

Figure 4.2 shows combinations of \((\alpha, \zeta)\) where the DM selects BoD and BoP. Finally we can derive the optimal outcome for the DM under \( V(P_i, \theta) = \theta_i \) (Figure 4.3).
Figure 4.2: Benefit of the doubt and burden of proof for $(\alpha, \zeta)$

\[
O^* = \begin{cases} 
E_1 & \text{if } \alpha > 1 \text{ and } \alpha < \zeta \\
E_2 & \text{if } \alpha < 1 \text{ and } \alpha > \zeta \\
E_3 & \text{if } \alpha > 1 \text{ and } \alpha > \zeta \\
E_4 & \text{if } \alpha < 1 \text{ and } \alpha < \zeta 
\end{cases}
\]

Figure 4.3: Equilibrium outcome for $V(P_i, \theta) = \theta_i$

4.2.3 $V(P_i, \theta) = \theta_i \ast v_i$

In his paper, Gregor (2015) introduces the following policy function

$V(P_i, \theta) = \theta_i \ast v_i, \ i = 1, 2$
Clearly, a policy $P_i$ generates a positive value to the DM if and only if $IG_i$ is a high type of lobbyist\(^9\), i.e. $\theta_i = 1$, which follows the previous case of $V(P_i, \theta) = \theta_i$. The difference is that now the decision maker weighs the quality of the project by its scope and thus she is able to escape the indifference that occurs in the previous case whenever both projects are legitimate.

The loss function then is

$$l(P_i, \theta) = \max\{V_1(P_1, \theta), V_2(P_2, \theta)\} - V(P_i, \theta) = \max\{\theta_1 v_1, \theta_2 v_2\} - \theta_i v_i$$

and the policy function choice generates a positive policy loss for policy $P_i$ if

$$l(P_i, \theta) > 0 = \begin{cases} 
\text{if } (\theta_i, \theta - i) = (0, 1) \\
\text{if } (\theta_i, \theta - i) = (1, 1) \text{ and } v_i < v_{-i}
\end{cases}$$

Therefore, the DM is indifferent over $P_1$ and $P_2$ in the event of $\theta_1 = \theta_2 = 0$. Gregor (2015) breaks the indifference by assuming that the the DM elects to choose $P_i \neq P_0$, where $IG_i$ is ex ante disadvantaged. The status quo policy $P_0$ is in this case given by

$$P_0 = \arg \min_{P_i, P_2} \lambda(P_i|\emptyset) = \arg \max_{P_i, P_2} \sum_{\theta \in \emptyset} \pi_i \theta_i v_i = \arg \max_{P_i, P_2} \{\pi_1 v_1, \pi_2 v_2\}$$

Given that $\alpha = v_1/v_2$ and $\beta = \pi_2/\pi_1$, $IG_1$ is ex ante disadvantaged whenever $\alpha < \beta$.\(^{10}\)

With the assumption in place, Gregor’s policy function $V(P_i, \theta) = \theta_i * v_i$, $i = 1, 2$ generates a quadruplet of possible mappings:

$$F_1 = \begin{pmatrix} P_1 & P_1 \\ P_2 & P_1 \end{pmatrix}, \quad F_2 = \begin{pmatrix} P_1 & P_1 \\ P_2 & P_2 \end{pmatrix}, \quad F_3 = \begin{pmatrix} P_2 & P_1 \\ P_2 & P_1 \end{pmatrix}, \quad F_4 = \begin{pmatrix} P_2 & P_1 \\ P_2 & P_2 \end{pmatrix}$$

First, we assume that $\alpha > 1$ and $\alpha > \beta$, i.e. $IG_1$ is “rich” and has an ex ante advantage. Since we have a procedure to break the $(\theta_1, \theta_2) = (0, 0)$ indifference, we know that the DM chooses $P_i$ according to $F_3$. If $\sigma = \{IG_1\}$, $IG_1$’s inside option is more valuable than her inside option ($v_1(P_1) > v_1(P_2) = 0$), she is willing to pay a positive access fee and the DM extracts the entire rent.

\(^9\)Notice here that we assumed earlier that $v_i(P_i) > 0$

\(^{10}\)Again, we disregard the knife edge indifference brought about by a specific combination of parameters.
by setting \( f_1 = v_1 \). The DM’s expected utility is \( W(IG_1) = \pi_1 * f_1 = \pi_1 * v_1 \).

If \( \sigma = \{IG_2\} \), \( IG_2 \)’s inside and outside options are the same and \( IG_2 \) is not willing to pay any positive access fee and \( f_2 = 0 \). The DM’s expected utility is then \( W(IG_1) = 0 \). If, finally, \( \sigma = \{IG_1, IG_2\} \), then neither interest group’s outside options change; \( f_1 = v_1 \) and \( W(IG_1, IG_2) = \pi_1 * v_1 \).

Second, we assume that \( \alpha > 1 \) and \( \alpha < \beta \), i.e. \( IG_1 \) is “rich” but now \( IG_2 \) has an ex ante advantage. In this case, the DM selects \( P_i \) according to \( F_1 \). If \( \sigma = \{IG_1, IG_2\} \), \( IG_1 \)’s inside option is still more valuable than her outside option though the different mapping for \((\theta_1, \theta_2) = (0, 0)\) makes the difference smaller. Now \( IG_1 \) is willing to pay only \( \pi_2 * v_1 \) and \( IG_2 \) is willing to pay \((1 - \pi_1) * v_2 \). The DM again extracts the full rent and the equilibrium outcome is that the DM invites both interest groups to be fully informed \( \sigma^* = \{IG_1, IG_2\} \), sets \( f_1^* = \pi_2 * v_1 \) and \( f_2^* = (1 - \pi_1) * v_2 \) and her expected policy utility is

\[
W(IG_1, IG_2) = \pi_1\pi_2v_1 + (1 - \pi_1)\pi_2v_2 > W(IG_1) = \pi_1v_1
\]

Finally, the two cases where \( \alpha < 1 \) are analogous. Together we get

\[
O^* = \begin{cases} 
    F_1 & \text{if } \alpha > 1 \text{ and } \alpha > \beta \\
    F_2 & \text{if } \alpha > 1 \text{ and } \alpha < \beta \\
    F_3 & \text{if } \alpha < 1 \text{ and } \alpha > \beta \\
    F_4 & \text{if } \alpha < 1 \text{ and } \alpha < \beta 
\end{cases}
\]

![Figure 4.4: Equilibrium outcome for \( V(P_i, \theta) = \theta_i * v_i \)](image-url)
4. Policy functions in pay-to-play lobbying

4.2.4 \[ V(P_i, \theta) = \theta_i * v_i + v_i \]

The fourth policy function we consider in our thesis is a combination of the trivial example and the policy function used in (Gregor 2015). As in the previous discussion, this policy function is a function of both the information and interest groups’ private values:

\[ V(P_i, \theta) = \theta_i * v_i + v_i \]

With the previous two policy functions, \( V(P_i, \theta) = \theta_i \) and \( V(P_i, \theta) = \theta_i * v_i \), the decision maker treated the two projects as equal whenever they were both illegitimate. The policy function we now introduce tweaks the decision makers preferences from \( V(P_i, \theta) = \theta_i * v_i \). A project is still more valuable when it is legitimate than when it is not and the quality is again weighed by the project scope, but as we will now show, the decision maker treats groups with similar scopes of their projects in a different manner than when there is a large gap in the private values.

The loss function in this case is

\[
l(P_i, \theta) = \max\{V_1(P_1, \theta), V_2(P_2, \theta)\} - V(P_i, \theta) \\
= \max\{\theta_1 v_1 + v_1, \theta_2 v_2 + v_2\} - (\theta_i v_i + v_i)
\]

and to determine its values, let us first look at each event \((\theta_1, \theta_2)\). In the event \((\theta_1, \theta_2) = (0, 0)\), The DM’s value from \(P_1\) is \(v_1\) and from \(P_2\) is \(v_2\) and the loss function is positive if the DM picks a policy of the poor group. The event \((\theta_1, \theta_2) = (1, 0)\) results in the comparison \(2v_1 \lessgtr v_2\), and the symmetric \((\theta_1, \theta_2) = (0, 1)\) leads to \(v_1 \lessgtr 2v_2\). Finally, in the event \((\theta_1, \theta_2) = (1, 1)\), the DM again compares \(v_1 \lessgtr v_2\) as in the \((\theta_1, \theta_2) = (0, 0)\) case.

What follows is that policy loss function generates positive values if

\[
l(P_i, \theta) > 0 = \begin{cases} 
if (\theta_i, \theta-i) = (0, 0) \text{ and } v_i < v_{-i} \\
if (\theta_i, \theta-i) = (1, 1) \text{ and } v_i < v_{-i} \\
if (\theta_i, \theta-i) = (1, 0) \text{ and } v_i < \frac{v_{-i}}{2} \\
if (\theta_i, \theta-i) = (0, 1) \text{ and } v_i < 2v_{-i} 
\end{cases}
\]
The only indifferences brought about by this policy function are a result of a knife-edge combination of parameters and we disregard them in accordance with our previous discussions. Our policy function \( V(P_i, \theta) = \theta_i v_i + v_i \) thus generates a quadruplet of outcomes:

\[
G_1 = \begin{pmatrix} P_1 & P_1 \\ P_1 & P_1 \end{pmatrix} \quad G_2 = \begin{pmatrix} P_1 & P_1 \\ P_2 & P_1 \end{pmatrix} \quad G_3 = \begin{pmatrix} P_2 & P_1 \\ P_2 & P_2 \end{pmatrix} \quad G_4 = \begin{pmatrix} P_2 & P_2 \\ P_2 & P_2 \end{pmatrix}
\]

Let us first assume that \( \alpha > 2 \). Then, our decision maker selects \( P_1 \) for each event \((\theta_1, \theta_2)\) and her policy choice corresponds to mapping \( G_1 \). This is the same situation as with our trivial policy function \( V(P_i, \theta) = v_i \) and thus for \( \alpha > 2 \), the access game collapses, no interest group is willing to pay access, \((f_1^*, f_2^*) = (0, 0)\) and the DM’s expected policy utility is \( W(\sigma) = 0 \).

More interesting is the case when \( \alpha \in (1, 2) \). The DM selects \( P_i \) according to \( G_2 \), which produces the same equilibrium outcome and fees as the case \( 1 < \alpha < \beta \) under \( V(P_i, \theta) = \theta_i v_i \). We obtain \((f_1^*, f_2^*) = (\pi_2 v_1, (1 - \pi_1) v_2)\) and the DM’s expected policy utility is \( W(IG_1, IG_2) = \pi_1 \pi_2 v_1 + (1 - \pi_1) \pi_2 v_2 \).

The cases where \( \alpha < 1 \) are analogous. For \( \alpha \in (\frac{1}{2}, 1) \) we obtain \( O^* = G_3 \), \((f_1^*, f_2^*) = ((1 - \pi_2) v_1, \pi_1 v_2)\) and \( W(IG_1, IG_2) = (1 - \pi_1) \pi_2 v_1 + \pi_1 \pi_2 v_2 \). And for \( \alpha < \frac{1}{2} \) we again get a trivial \( O^* = G_4 \) and \((f_1^*, f_2^*) = (0, 0)\), \( W(\sigma) = 0 \).

Therefore, the optimal outcome is given by:

\[
O^* = \begin{cases} 
G_1 & \text{if } \alpha > 2 \\
G_2 & \text{if } \alpha \in (1, 2) \\
G_3 & \text{if } \alpha \in (\frac{1}{2}, 1) \\
G_4 & \text{if } \alpha < \frac{1}{2}
\end{cases}
\]

A possible interpretation of the outcome mapping structure under \( V(P_i, \theta) = \theta_i v_i + v_i \) would be that our decision maker treats very rich groups and moderately rich groups differently. We found that the DM chooses the rich group’s policy independently on any event \((\theta_1, \theta_2)\) if it is rich enough, i.e. the relative wealth is above a threshold. This however hurts the the DM because, in our access mechanism, she is unable to set positive access fees. On the other hand,
if the relative wealth is below the derived threshold, access game occurs in the
way we would expect it and in equilibrium, the DM is able to charge positive
access fees and extract rent from the IGs.

4.2.5 Triviality and outcome mappings

An instant follow up to the analysis of the model under four different pol-
cy functions is that decision maker’s objectives have a large influence on the
mappings our DM chooses to assess policies. There is a visible difference be-
tween the trivial example and the three functions that followed, which we can
summarize with the following:

Definition 1 (Triviality of policy functions). A policy function \( V(P_i, \theta) \) is
called trivial if it is constant in \( \theta \). A policy function \( V(P_i, \theta) \) is called non-
trivial if it is not constant in \( \theta \).

We begin with a basic necessary condition for positive access fees:

Proposition 1 (Triviality and access fees). A necessary condition for an access
game equilibrium with at least one access fee positive is that the policy function
is non-trivial.\(^{11}\)

The non-triviality of the policy function is not a sufficient condition for the
existence of at least one \( f_i > 0 \), because, for example, a non-trivial function
\( V(P_i, \theta) = \theta v_i + v_i \) still results in trivial optimal outcome mapping for extreme

\(^{11}\)Proof is offered in the Appendix A.
values of $\alpha$ where $(f_1^*, f_2^*) = (0, 0)$.

Furthermore, the four forms of policy function resulted in only six of the 16 possible mappings. Obviously, we have the two trivial mappings

$$A_{13} = \begin{pmatrix} P_1 & P_1 \\ P_1 & P_1 \end{pmatrix}, \quad A_{14} = \begin{pmatrix} P_2 & P_2 \\ P_2 & P_2 \end{pmatrix}$$

but more importantly the four non-trivial mappings

$$A_{2} = \begin{pmatrix} P_1 & P_1 \\ P_2 & P_1 \end{pmatrix}, \quad A_{5} = \begin{pmatrix} P_1 & P_1 \\ P_2 & P_2 \end{pmatrix}, \quad A_{6} = \begin{pmatrix} P_2 & P_1 \\ P_2 & P_2 \end{pmatrix}, \quad A_{10} = \begin{pmatrix} P_2 & P_1 \\ P_2 & P_2 \end{pmatrix}$$

An interesting observation here is that the four non-trivial are a particular subgroup which we can define in the following manner:

**Definition 2** (Realistic mapping). *We call an outcome mapping realistic if for $\theta_i = 1 - \theta_{-i} = 1$ the DM selects $P_i$.***

This is a property we would expect from a policy function in the pay-to-play lobbying model - that if a group presents favorable evidence to the DM and the other group is expected to have unfavorable evidence, then the DM will prefer that group’s policy, i.e. disclosed evidence is fully influential when the other group is expected to have unfavorable evidence.

### 4.3 Curse of the ex ante advantage

The ex ante advantage of an interest group in our setting is a very interesting phenomenon. It poses the question of how much it is detrimental to the group’s ex post well-being and whether the ex ante disadvantaged group is always worse off.

Let $u_i(\sigma)$ be the payoff in subgame determined by $\sigma$. Since we defined an ex ante advantaged group as the one whose policy our decision maker chooses in light of no observation, i.e. ex ante advantaged interest group $IG_i$ prefers the status quo policy:

$$u_i(\emptyset) > u_{-i}(\emptyset)$$
In his paper, Gregor proposes that the relationship between the ex post and ex ante advantage is as follows:\textsuperscript{12}

**Proposition 2** (Curse of an ex advantaged lobbyist). *Ex ante advantage is equivalent to ex post disadvantage, and ex ante disadvantage is equivalent to ex post advantage.*

\[ u_i(\emptyset) > u_{-i}(\emptyset) \iff u_i(\sigma^*) < u_{-i}(\sigma^*) \]

Moreover, he formulates the “curse of access”, which states that a single invited interest group is ex post worse off than the non-invited one, and asserts that an interest group has an incentive to avoid being the only one invited to the DM. The underlying assumption for the both curse of the ex ante advantage as well as the curse of access is that the DM’s policy function is \( V(P, \theta) = \theta_i \ast v_i \), and Gregor proves that with his policy function, both curses are universal for any \((\alpha, \beta)\).

Figure 4.6 illustrates the ex ante advantage, where we know that \( IG_1 \) is ex ante advantaged if and only if \( \alpha > \beta \). As to the ex post advantage, we have to look at the ex post expected utilities \( u_1^* \) and \( u_2^* \).

\[ u_1^* = Pr(P = P_1|O^*)v_1 - \pi_1 f_1^* \]

and

\[ u_2^* = Pr(P = P_2|O^*)v_2 - \pi_2 f_2^* \]

We derived the equilibrium access fees in the previous section. For \( \alpha > \beta \) and \( \alpha > 1 \) as well as \( \alpha > \beta \) and \( \alpha < 1 \), the ex post utilities are \( u_1^* = 0 < (1 - \pi_1)v_2 = u_2^* \). For \( \alpha < \beta \), the ex post utilities are \( u_1^* = (1 - \pi_2)v_1 > 0 = u_2^* \). Figure 4.7 shows the ex post advantage and Figure 4.8 synthesizes both Figures 4.6 and 4.7 to illustrate the existence of the ex ante advantage for any \((\alpha, \beta)\).

So far in our discussions, we have shown that different policy functions produce a variety of equilibrium outcomes. What follows is that we want to determine whether the curse is universal for all policy functions we consider in our thesis and if not, we would like to know if there at least exists some combination of parameters in each case where the curse holds.

\textsuperscript{12}Gregor (2015, p.7)

\textsuperscript{13}The formed shown here is a simplification \( f_i^* = \bar{f}_i^* \ast 1(IG_i \in \sigma) \), where \( \bar{f}_i^* \) is the fee set by the decision maker and \( 1(IG_i \in \sigma) \in (0, 1) \) indicates whether the interest group paid the access fee.
4. Policy functions in pay-to-play lobbying

Figure 4.6: Ex ante advantage under $V(P_i, \theta) = \theta_i * v_i$

Figure 4.7: Ex post advantage under $V(P_i, \theta) = \theta_i * v_i$

Figure 4.8: Curse of the ex ante advantage under $V(P_i, \theta) = \theta_i * v_i$
4.3.1 Curse under $V(P_i, \theta) = v_i$

We will first consider the trivial policy function $V(P_i, \theta) = v_i$. We have shown that the ex ante advantaged group is $IG_i$ whenever $v_i > v_{-i}$. The equilibrium access fees are $(f_1^*, f_2^*) = (0, 0)$ as neither group has an incentive to pay a positive fee for both $\alpha > 1$ and $\alpha < 1$.

W.L.O.G. let $\alpha > 1$. The expected payoff of $IG_1$ is

$$u_1^* = Pr(P = P_1|O^*)v_1 - \pi_1 f_1^* = v_1$$

and of $IG_2$ is

$$u_2^* = Pr(P = P_2|O^*)v_2 - \pi_2 f_2^* = 0$$

Clearly, the curse of the ex ante advantage does not apply as the ex ante advantaged $IG_1$ is strictly better off than $IG_2$. This is the result of the DM state-independently preferring rich group’s policy and the DM’s inability to extract any rent from the rich group, because her inside and outside options are equal.

The opposite case ($\alpha < 1$) is analogous and again, ex ante advantage implies ex post advantage. Therefore, if our decision maker’s preferences are given by $V(P_i, \theta) = v_i$, the curse of the ex ante advantage does not hold for any combination of parameters. Figure 4.9 shows ex ante advantage, Figure 4.10 shows ex post advantage and Figure 4.11 illustrates the non-existence of the curse under $V(P_i, \theta) = v_i$.

![Figure 4.9: Ex ante advantage under $V(P_i, \theta) = v_i$](image-url)
4. Policy functions in pay-to-play lobbying

Figure 4.10: Ex post advantage under $V(P_i, \theta) = v_i$

Figure 4.11: Curse of the ex ante advantage under $V(P_i, \theta) = v_i$
4.3.2 Curse under $V(P_i, \theta) = \theta_i$

Now let us consider Cotton’s (2012) policy function $V(P_i, \theta) = \theta_i$. Here the identity of ex ante advantaged group depends on the inequality $\beta \leq 1$ (see Figure 4.12). W.L.O.G. we will assume that $\alpha > 1$, i.e. $IG_1$ is the rich group. First, if $\beta < 1$, $IG_1$ has an ex ante advantage and also

$$\beta < 1 \implies \alpha > \zeta$$  \hspace{1cm} ^{14} \hspace{1cm}

which means that the DM commits to issue the burden of proof to $IG_1$. We know that under BoP, the DM sets $(f_1^*, f_2^*) = (v_1, 0)$ and thus the expected utilities are

$$u_1^* = Pr(P = P_1|O^*)v_1 - \pi_1 f_1^* = \pi_1 v_1 - \pi_1 v_1 = 0$$

and

$$u_2^* = Pr(P = P_2|O^*)v_2 - \pi_2 f_2^* = (1 - \pi_1)v_2$$

And since $u_1^* < u_2^*$, the curse of the ex ante advantage holds when $\alpha > 1 > \zeta$.

Second, if $\beta > 1$, $IG_2$ is ex ante advantaged though the case is not as simple as the previous one, because it leaves the inequality $\alpha \leq \zeta$ unresolved. \hspace{1cm} ^{15}

For $\alpha > \zeta > 1$, the DM again commits to BoP and we get $u_1^* < u_2^*$, however, since $IG_2$ is ex ante advantaged, the curse does not hold. And for $\zeta > \alpha > 1$, the DM commits to BoD and sets $(f_1^*, f_2^*) = (\pi_2 v_1, (1 - \pi_1)v_2)$. Expected utilities for each interest group are

$$u_1^* = Pr(P = P_1|O^*)v_1 - \pi_1 f_1^* = (1 - (1 - \pi_1)\pi_2)v_1 - \pi_1 \pi_2 v_1 = (1 - \pi_2)v_1$$

and

$$u_2^* = Pr(P = P_2|O^*)v_2 - \pi_2 f_2^* = (1 - \pi_1)\pi_2 v_2 - \pi_2(1 - \pi_1)v_2 = 0$$

We arrive to $u_1^* > u_2^*$ and the curse of the ex ante advantage thus holds for $\zeta > \alpha > 1$.

\hspace{1cm} ^{14} \text{We have } \alpha > 1, \beta < 1, \zeta = \frac{1 - \pi_1}{1 - \pi_2} \text{ and thus } \frac{1 - \pi_1}{1 - \pi_2} < \frac{1 - \pi_1}{1 - \pi_1} \text{ and } 1 < \alpha. \hspace{1cm}

\hspace{1cm} ^{15} \text{We have } \alpha > 1, \beta > 1, \text{ which gives } \frac{1 - \pi_1}{1 - \pi_2} > \frac{1 - \pi_1}{1 - \pi_1} > 1 \text{ and we are left with } \alpha \leq \zeta.
Since the opposite case $\alpha < 1$ is symmetric, we can say that under $V(P_i, \theta) = \theta_i$, the curse of the ex ante advantage holds only for the following combinations of parameters $\alpha$ and $\zeta$.

$$
\begin{align*}
\alpha &> 1 > \zeta & \zeta > \alpha > 1 \\
\zeta &> 1 > \alpha & 1 > \alpha > \zeta
\end{align*}
$$

Figure 4.13 shows the ex post advantage under $V(P_i, \theta) = \theta_i$ and Figure 4.14 illustrates the four areas where the curse of the ex ante advantage holds.

**Figure 4.12:** Ex ante advantage under $V(P_i, \theta) = \theta_i$

**Figure 4.13:** Ex post advantage under $V(P_i, \theta) = \theta_i$
4. Policy functions in pay-to-play lobbying

4.3.3 Curse under $V(P_i, \theta) = \theta_i \ast v_i + v_i$

Finally, we consider the curse of the ex ante advantage under the policy function $V(P_i, \theta) = \theta_i \ast v_i + v_i$. In this case, an interest group $IG_1$ is ex ante advantaged whenever

$$v_1(\pi_1 + 1) > v_2(\pi_2 + 1)$$

Therefore, if we define $\kappa$ in the following manner

$$\kappa = \frac{\pi_2 + 1}{\pi_1 + 1}, \kappa \in \left(\frac{1}{2}, 2\right)$$

$IG_1$ is ex ante advantaged if and only if $\alpha > \kappa$ (see Figure 4.15).

First, we assume that $\alpha > 2$. Clearly, $IG_1$ is ex ante advantaged and we know from previous discussions that the DM sets $(f_1^*, f_2^*) = (0, 0)$ as the access game.
collapses, because no interest group has an incentive to pay any positive access fee $f_i$. The expected utilities are

$$u_1^* = Pr(P = P_1 | O^*) v_1 - \pi_1 f_1^* = v_1$$

and

$$u_2^* = Pr(P = P_2 | O^*) v_2 - \pi_2 f_2^* = 0$$

which means that the curse of the ex ante advantage does not hold.

Next, we consider $\alpha \in (\frac{1}{2}, 2)$. $IG_1$ is ex ante advantaged when $\alpha > \kappa$ and disadvantaged when $\alpha < \kappa$. For $\alpha \in (1, 2)$, our decision maker sets $(f_1^*, f_2^*) = (\pi_2 v_1, (1 - \pi_1) v_2)$ and the expected utilities are

$$u_1^* = Pr(P = P_1 | O^*) v_1 - \pi_1 f_1^* = (1 - (1 - \pi_1) \pi_2) v_1 - \pi_1 \pi_2 v_1 = (1 - \pi_2) v_1$$

and

$$u_2^* = Pr(P = P_2 | O^*) v_2 - \pi_2 f_2^* = (1 - \pi_1) \pi_2 v_2 - \pi_2 (1 - \pi_1) v_2 = 0$$

Therefore, we obtain that $u_1^* > u_2^*$ and the curse of the ex ante advantage holds for $\kappa > \alpha > 1$, but not for $\alpha > 1$ & $\kappa < \alpha$.

The opposite case $\alpha \in (\frac{1}{2}, 1)$ yields symmetric results and we learn that the curse of the ex ante advantage also holds for $1 > \alpha > \kappa$. And finally, for $\alpha < \frac{1}{2}$, the access game again collapses and curse does not hold. Figure 4.16 shows the ex post advantage.

The new policy function $V(P, \theta) = \theta_i * v_i + v_i$ therefore produces two areas where the curse of the ex ante advantage holds (see Figure 4.17). $IG_1$ suffers from the curse whenever $\kappa > \alpha > 1$ and $IG_2$ suffers whenever $1 > \alpha > \kappa$. We also get a similar result to the previous two policy functions - ex ante advantage does not always imply ex post disadvantage.

After a consideration of the three policy functions, we come to the conclusion that the curse of the ex ante advantage is not robust to the decision maker’s objective and that under a different $V(P, \theta)$, the curse holds only for some combination of parameters (and in the trivial case, it does not hold at all).
4. Policy functions in pay-to-play lobbying

Figure 4.16: Ex post advantage under $V(P_i, \theta) = \theta_i * v_i + v_i$

Figure 4.17: Combination of parameters $(\alpha, \kappa)$ where the curse holds under $V(P_i, \theta) = \theta_i * v_i + v_i$
The intuition behind our results is that the policy function in Gregor (2015), i.e. \( V(P_i, \theta) = \theta_i + v_i \), is structurally different from the other three. It behaves in such a way that the border between ex ante advantages is the same as the border of ex post advantages \((\alpha = \beta)\) and the identities of the interest groups are switched in the latter. The trivial \( V(P_i, \theta) = v_i \) also produces the same border but interest group is “on the same side” in both ex ante and ex post case, which means that the curse does not hold for any combination of parameters. Finally, the policy functions \( V(P_i, \theta) = \theta_i \) and \( V(P_i, \theta) = \theta_i + v_i \) produce two different borders for ex ante and ex post advantages, which necessarily implies existence of some combination of parameters where the curse does not hold.

4.4 Endogenous private values of interest groups

Up to this point, we have assumed that interest groups have exogenous private values and we have derived the equilibria for each policy function. In this section, we relax the assumption of exogenous private values\(^{16}\) and we want to find out whether the exogenous equilibria still hold even when we allow devaluation.

The discussion of endogenous valuations is also interesting from the standpoint of the role of the curse of the ex ante advantage. In (Gregor 2015), interest groups have an incentive to strategically devalue their \( v_i \) in order to escape the curse and increase their expected utility. The curse in this case always implies strategic devaluation and we want to extend the discussion to the other three policy functions in order to observe the effect of the curse. First, we need the following:

**Definition 3** (Structural break). We define structural break as parameters around which the optimal outcome mapping \( O^* \) changes.

**Proposition 3** (Devaluation and structural break). A necessary and sufficient condition for devaluation to happen in equilibrium is that decreasing IG\(_i\)’s private value \( v_i \) induces a structural break that strictly increases ex post utility \( u_i^* \).\(^{17}\)

Endogenous valuation can be thought of as a Stage 0 in our model, preceding the announcement of \((f_1, f_2)\) where both interest groups simultaneously

---

\(^{16}\)The discussion of endogenous priors \( \pi_i \) is beyond the scope of our thesis.

\(^{17}\)Proof is offered in Appendix A.
decide over their private value, i.e. they select $v_i \in [0, \bar{v}_i], i = 1, 2$. The timing is then as follows:

Stage 0 - IGs simultaneously decide over $v_i \in [0, \bar{v}_i]$.\(^\text{18}\)
Stage 1 - DM announces $f = (f_1, f_2)$ and commits to not listen to non-participating $IG_i$.
Stage 2 - IGs privately observe their own $\theta_i$.
Stage 3 - Each IG either pays her $f_i$ or does not participate.
Stage 4 - Participating IGs disclose their hard evidence.
Stage 5 - DM selects her ex post optimal policy.

4.4.1 Endogenous valuation under $V(P_i, \theta) = v_i$

We have shown that under $V(P_i, \theta) = v_i$ the curse does not hold for any combination of $(v_1, v_2)$ and we know that $IG_i$’s expected utility is 0 when it is the poor group and $v_i$ when it is the rich group. The expected utility is therefore non-decreasing and strictly increasing for $v_i$ when $v_i > v_{-i}$ (see Figure 4.18 for illustration).

\[ u_i^* \]
\[ v_{-i} \]
\[ 0 \]
\[ v_i \]

\[ v_i \]

\[ v_{-i} \]

\[ 0 \]

\[ v_i \]

Figure 4.18: Expected utility of $IG_i$ under $V(P_i, \theta) = v_i$

$IG_i$ does not have any incentive to devalue her $v_i$, as her utility would either decrease or stay the same. Therefore, devaluation does not occur under $V(P_i, \theta) = v_i$ and the equilibrium is robust to endogenous valuation.

\(18\) We assume that an interest group cannot increase her private value, it can only destroy a fraction or the whole value $v_i$. 
4.4.2 Endogenous valuation under $V(P_i, \theta) = \theta_i$

Under $V(P_i, \theta) = \theta_i$, our decision maker chooses between two approaches - benefit of the doubt and burden of proof, and this decision plays a significant role in exogenous valuation. In order to determine whether endogenous valuations lead to destruction of private values, we need to determine the expected utility for every private value $v_i$.

First, we will look at what happens for $\zeta > 1$ (recall Figures 4.2 and 4.14). We know that for $\alpha < 1$, the DM elects to issue burden of proof to $IG_2$ and the equilibrium expected utilities are $u_1^* = (1 - \pi_2)v_1 > 0 = u_2^*$. For $1 < \alpha < \zeta$, the decision maker gives benefit of the doubt to $IG_1$ and the expected utilities are again $u_1^* = (1 - \pi_2)v_1 > 0 = u_2^*$. And for $\alpha > \zeta$, the DM issues burden of proof to $IG_1$ and the equilibrium expected utilities are $u_1^* = 0 < (1 - \pi_1)v_2 = u_2^*$. Clearly, $u_1^*$ is increasing in $\alpha$ until $\alpha = \zeta$ and constant in zero for $\alpha > \zeta$ where as the case of $u_2^*$ is the exact opposite.

Next, in the case of $\zeta < 1$, the DM issues burden of proof to $IG_2$ for $1 > \zeta > \alpha$ and the equilibrium expected utilities are again $u_1^* = (1 - \pi_2)v_1 > 0 = u_2^*$. For $1 > \alpha > \zeta$, the DM gives benefit of the doubt to $IG_2$ and $u_1^* = 0 < (1 - \pi_1)v_2 = u_2^*$. Finally, for $\alpha > 1 > \zeta$, the DM issues burden of proof to $IG_1$ and again we have $u_1^* = 0 < (1 - \pi_1)v_2 = u_2^*$.

We get the same profile as in the previous case and we can summarize the expected utility of $IG_i$. The utility is increasing in $v_i$ until $v_i = v_{\zeta,i}$ such that $\alpha = \zeta$, where the utility stepwise drops to 0 (see Figure 4.19 for illustration).

As the shape of Figure 4.19 suggests, for all $\zeta$, an IG that is ex post disadvantaged has an incentive to decrease her private value to the point where the inequality $\alpha > \zeta$ for $IG_1$ (or $\alpha < \zeta$ for $IG_2$) becomes the opposite $\alpha < \zeta$ (\alpha > \zeta respectively). The best response for the other interest group is to counter that devaluation with a devaluation of her own to reverse the change. The equilibrium for endogenous valuations is then the state where $(v_1, v_2) = (0, 0)$ and the lobbying industry is destroyed.

Since we have shown that the curse of the ex ante advantage does not hold for every $(\alpha, \zeta)$, the devaluation here has a slightly less clear link to the
curse. With the policy function $V(P_i, \theta) = \theta_i$, the border of ex ante advantage is $\beta = 1$ and consequently $\zeta = 1$. For $\zeta > 1$, $IG_2$ is ex ante advantaged and by decreasing $v_2$ (and thus increasing $\alpha$), the interest group escapes the curse of the ex ante advantage as it becomes ex post advantaged. However, $IG_1$ is never ex ante advantaged under $\zeta > 1$ and her devaluation is explained by the desire to become ex post advantaged and not by escaping the curse. Symmetric conclusion holds for $\zeta < 1$, where $IG_1$’s devaluation is explained by the curse but $IG_2$’s is not.

The important outcome here is that devaluation is not exclusively triggered by the curse of the ex ante advantage and the exogenous equilibria are not robust to endogenous valuations. Moreover, the public value under $V(P_i, \theta) = \theta_i$ is constant in private values of the interest groups and the equilibrium destruction of lobbying does not destroy the public value.

### 4.4.3 Endogenous valuation under $V(P_i, \theta) = \theta_i \ast v_i$

In the third case where $V(P_i, \theta) = \theta_i \ast v_i$, we have shown that the curse of the ex ante advantage holds for every combination of parameters $(\alpha, \beta)$. For $\alpha > \beta$, $IG_2$ is ex post advantaged and $u_1^* = 0 < (1 - \pi_1)v_2 = u_2^*$, where as for $\alpha < \beta$ we get the opposite result $u_1^* = (1 - \pi_2)v_1 > 0 = u_2^*$.

The expected utility of $IG_1$ is increasing in $v_1$ until $v_2^\beta$, where $\alpha > \beta$ and the utility stepwise drops to 0 as $IG_1$ becomes ex ante advantaged and ex post disadvantaged in accordance with the curse of the ex ante advantage. The case
of $IG_2$ is analogous and we can illustrate the expected utility with Figure 4.20.

\begin{equation}
(1 - \pi_i) v_i - (1 - \pi_i - \pi_{-i}) v_i
\end{equation}

\begin{figure}[h]
\centering
\begin{tikzpicture}
\draw[->] (-2,0) -- (3,0) node[right] {$v_i$};
\draw[->] (0,-2) -- (0,3) node[above] {$u_i^a$};
\draw (0,0) -- (2,2);
\draw (0,0) -- (0,0) -- (2,0);
\draw [dashed] (2,0) -- (2,2);
\draw (0,0) node[below] {$v_{-i}^\beta$} -- (0,0) node[below] {$v_i^\beta$};
\end{tikzpicture}
\caption{Expected utility of $IG_i$ under $V(P_i, \theta) = \theta_i v_i$}
\end{figure}

Notice that the expected utility as a function of private value in Figure 4.20 has the same shape as in Figure 4.19. Similarly to $V(P_i, \theta) = v_i$, an ex post disadvantaged (and therefore ex ante advantaged) $IG_i$ has an incentive to decrease her private value such that she becomes ex ante disadvantaged and her expected utility stepwise increases from 0. However, the best response of the other interest group is to anticipate such devaluation by devaluing her own private value. Again, the equilibrium private values under endogenous valuations are $(v_1, v_2) = (0, 0)$ and the lobbying industry is destroyed.

Since the curse of the ex ante advantage holds everywhere with this particular policy function, the devaluation can be formulated in such way that an ex post disadvantaged group decreases her private value to escape the curse of the ex ante advantage. An interest group can make herself ex post advantaged if and only if she becomes ex ante disadvantaged.

What is more, the policy function $V(P_i, \theta) = \theta_i v_i$ is increasing in public value and the destruction of the lobbying industry also leads to the destruction of the public value of the decision maker, which is the opposite of the previous case.
4.4.4 Endogenous valuation under $V(P_i, \theta) = \theta_i \ast v_i + v_i$

Finally, we consider endogenous valuation under the policy function $V(P_i, \theta) = \theta_i \ast v_i + v_i$. The curse of the ex ante advantage for this function holds only for some combinations of parameters $(\alpha, \kappa)$, namely $1 > \alpha > \kappa$ and $\kappa > \alpha > 1$.

First, consider $\alpha > 2$, where $IG_1$ is ex ante advantaged as well as ex post advantaged. The expected utilities here are $u_1^* = v_1 > 0 = u_2^*$. Next, for $\alpha \in (1, 2)$, we distinguish $\alpha > \kappa$ and $\alpha < \kappa$. $IG_1$ is ex ante advantaged in the first case and ex ante disadvantaged in the second, but the key is that she is ex post advantaged in both cases given that the curse does not hold in the first case. Thus, the expected utilities in both are $u_1^* = (1 - \pi_2) v_1 > 0 = u_2^*$. The symmetric cases $\alpha \in \left(\frac{1}{2}, 1\right)$ and $\alpha < \frac{1}{2}$ result in $u_1^* = 0 < (1 - \pi_1) v_2 = u_2^*$ and $u_1^* = 0 < v_2 = u_2^*$, respectively.

What follows is that the expected utility of $IG_i$ is constant in zero up to the point where $IG_i$ becomes the rich group and the utility stepwise increases. The utility of a moderately rich group and an extremely rich group is increasing in the group’s private value with a stepwise increase when moderately rich becomes extremely rich (see Figure 4.21).

![Figure 4.21: Expected utility of $IG_i$ under $V(P_i, \theta) = \theta_i \ast v_i + v_i$](image)

In comparison with the previous two policy functions, this case produces non-decreasing expected utility function for both interest groups. Neither IG has an incentive to decrease her private value because the change would either have no effect or it would harm the group. Therefore, allowing for endogenous valuation
4. Policy functions in pay-to-play lobbying

does not change the exogenous equilibrium outcomes under \( V(P_i, \theta) = \theta_i v_i + v_i \) and the decision maker’s public value remains unchanged.

This policy function provides an interesting result with regard to the link between equilibrium devaluation and the curse - the curse of the ex ante advantage does not imply equilibrium devaluation, because escaping the curse does not improve the expected utility of the interest group.

### 4.4.5 Destruction of the lobbying industry

The endogenous valuations, as we have shown, have two very distinctive results. Either both groups completely devalue to zero and the lobbying industry is destroyed, or allowing endogenous devaluations does not change the exogenous equilibria.\(^{19}\) The destruction of the lobbying industry is then a major issue for a decision maker with public value not constant in private values.

Clearly, the decision maker’s objectives play a crucial role in determining the effects of endogenous valuations and we illustrate that in Table 4.1:

<table>
<thead>
<tr>
<th>Policy function</th>
<th>Curse of the ex ante adv.</th>
<th>Devaluation</th>
<th>Equilibrium endogenous ((v_1, v_2))</th>
</tr>
</thead>
<tbody>
<tr>
<td>( V(P_i, \theta) = v_i )</td>
<td>( \emptyset )</td>
<td>( \emptyset )</td>
<td>((\bar{v}_1, \bar{v}_2))</td>
</tr>
<tr>
<td>( V(P_i, \theta) = \theta_i v_i )</td>
<td>( \alpha &gt; 1 &gt; \zeta, \zeta &gt; 1 &gt; \alpha ), ( \zeta &gt; \alpha &gt; 1, 1 &gt; \alpha &gt; \zeta )</td>
<td>( \forall(\alpha, \zeta) )</td>
<td>((0, 0))</td>
</tr>
<tr>
<td>( V(P_i, \theta) = \theta_i v_i + v_i )</td>
<td>( \forall(\alpha, \beta) )</td>
<td>( \forall(\alpha, \beta) )</td>
<td>((0, 0))</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Policy function</th>
<th>Curse of the ex ante adv.</th>
<th>Devaluation</th>
<th>Equilibrium endogenous ((v_1, v_2))</th>
</tr>
</thead>
<tbody>
<tr>
<td>( V(P_i, \theta) = \theta_i v_i + v_i )</td>
<td>( \kappa &gt; \alpha &gt; 1, 1 &gt; \alpha &gt; \kappa )</td>
<td>( \emptyset )</td>
<td>((\bar{v}_1, \bar{v}_2))</td>
</tr>
</tbody>
</table>

Table 4.1: Policy functions and endogenous valuations

One of the motivations for the analysis of endogenous valuations was to observe the effect of the curse of the ex ante advantage on the existence of the destruction of the lobbying industry. As Table 4.1 shows, the existence of the curse does not necessarily imply the destruction of the lobbying industry, which is evident with the policy function \( V(P_i, \theta) = \theta_i v_i + v_i \), where the curse holds for some parameters \((\alpha, \kappa)\), but the exogenous equilibrium is robust to endogenous valuations. Furthermore, the destruction of the lobbying industry does not even imply the curse of the ex ante advantage, as is the case with

\(^{19}\)Notice the none of the four policy functions result in an equilibrium with partial devaluations where \( v_i \in (0, \bar{v}_i) \).
4. Policy functions in pay-to-play lobbying

\[ V(P_i, \theta) = \theta_i \] where the curse does not hold for some parameters \((\alpha, \zeta)\).

The destruction of the lobbying industry under \( V(P_i, \theta) = \theta_i v_i \) and \( V(P_i, \theta) = \theta_i \) is a direct result of the existence of a structural break that makes one group ex post disadvantaged and the other ex post advantaged, as we highlight in Proposition 3. In the case of \( V(P_i, \theta) = \theta_i v_i \), the equilibrium devaluation occurs on the boundary \( \alpha = \beta \) and each group changes the parameter \( \alpha \) to go from a region where she is cursed to a region where the other group is cursed, given that the curse holds everywhere, and the devaluation is competitive as it makes the other group strictly worse off.

The case of \( V(P_i, \theta) = \theta_i \) is slightly different as the curse does not hold everywhere. The devaluation happens on the boundary \( \alpha = \zeta \), but an interest group’s devaluation does not always need to be based on escaping the curse. For \( \zeta > 1 \), \( IG_1 \) is ex ante disadvantaged for any \( \alpha \) and is incentivized to escape the region where the curse does not hold for anyone (but \( IG_1 \) is ex post disadvantaged) to a region where the the curse holds for the other interest group. The devaluation makes the other group worse off and thus it is competitive. For \( \zeta < 1 \), \( IG_1 \) is ex ante advantaged and the group is incentivized to decrease her \( v_1 \) to move from a region where she suffers from curse to a region where the curse does not hold but \( IG_1 \) is ex post advantaged. Again, the devaluation is competitive, since it makes the other group worse off. The important conclusion here is that the interest group might have incentives to devalue even if she does not suffer from the curse of the ex ante advantage.

The difference between \( V(P_i, \theta) = \theta_i * v_i + v_i \) and the previous two is that crossing a boundary \( \alpha = \kappa \) does not induce a structural break. Notice that in both Figure 4.7 and Figure 4.13, the ex post advantage shifts from one group to another when we cross the boundary \( \alpha = \beta \) (and \( \alpha = \zeta \) respectively). That, however, does not happen with this particular policy function. As Figure 4.16 shows, the boundary is not \( \alpha = \kappa \) but rather \( \alpha = 1 \) which is a direct result of the structure of the policy function. Therefore, even though crossing \( \alpha = \kappa \) changes the identity of ex ante advantaged interest group (Figure 4.15), it does not affect who is ex post advantaged and escaping the curse of the ex ante advantage does not trigger the destruction of the lobbying industry in endogenous equilibrium.
Chapter 5

Conclusion

In our thesis, we build upon the existing models in the pay-to-play lobbying literature and show that the way the decision maker relates the two main instruments of interest groups, i.e. contributions and private hard evidence, to the public value has a crucial effect on the results of an access game where interest groups pay an access fee for the ability to disclose information.

We start by looking at the decision maker’s ability to charge access fees to the interest groups in equilibrium in the context of different policy functions. Using the framework of Cotton (2012) and Gregor (2015), we then turn our discussion to focus on the curse of the ex ante advantage, which asserts that the decision maker has a great power in her ability to extract rent from the interest groups and in turn she makes ex ante advantaged groups in equilibrium relatively worse off than their ex ante disadvantaged counterparts. We conclude in our analysis that the curse is not robust to the decision maker’s objective and that the ex post expected utilities of interest groups are not always determined solely by the relative ex ante utilities of the groups. Moreover, we learn that the curse then can be either limited to a certain set of parameters or that it does not have to hold at all, depending on the particular policy function.

Furthermore, we relax the assumption of exogenous private values and allow the interest groups to strategically decrease their valuations to improve their ex post utilities. The result of endogenous valuations in both Cotton (2012) and Gregor (2015) is that both interest groups competitively devalue to zero and destroy the lobbying industry. We therefore analyze the effects of the policy functions on the incentives of interest groups to devaluate and we present a
necessary and sufficient condition for the devaluation to occur in equilibrium. We find that the destruction of the lobbying industry is not robust and that, for some policy functions, endogenous valuation has no effect on the lobbying game and the exogenous equilibria hold.

We also look at the relationship between the curse of the ex ante advantage and the destruction of the lobbying industry. Specifically in Gregor (2015), the destruction is always associated with the curse as the ex ante advantaged group always devalues in order to escape it. We find that the the curse does not always imply the destruction of the lobbying industry under endogenous valuations and also that the destruction of the lobbying industry in endogenous equilibrium does not necessarily imply the existence of the curse for all parameters.

The limitation of our thesis is that we use a narrow framework where the decision maker deals with only two interest groups. Any further analysis of effects of decision maker’s objective on the phenomena we have analyzed, especially destruction of the lobbying industry, would greatly benefit from expanding the model to include the decision maker choosing from a continuous or multidimensional policy space. What is more, the policy functions we consider in our model are limited to various functional forms of interest group private values and private hard evidence, and an addition of issue importance as perceived by the decision maker’s constituents, either exogenous as defined in Cotton (2009) or endogenously in the discretion of the interest groups, could prove to be very useful in the future research.
Bibliography


Appendix A

Proofs

Proof of Proposition 1. By contradiction, i.e. trivial policy function enables positive access fee. If our policy function is trivial, then DM’s policy choice is independent on $\theta$. Therefore, DM selects a policy $P_i$ for all $(\theta_i, \theta_{-i})$. For $IG_i$, who values the policy $P_i$ at $v_i$, the outside option value is $v_i$ and inside option value is also $v_i$. $IG_i$ thus does not have an incentive to pay for access because her policy is chosen regardless of her action. For $IG_{-i}$, who values $P_i$ at 0, there is no chance that her favorite policy $P_{-i}$ is chosen and both her outside and inside options are worth 0. What follows is that in equilibrium, no interest group is willing to pay for access and DM cannot set positive access fee for any IG. However, we assumed that she can, ergo contradiction.

Proof of Proposition 3. We will first prove that equilibrium devaluation implies that there is a structural break for some $v_i^* < v_i$ that leads to strictly higher $u_i^*$. In equilibrium, the actions of $IG_i$ correspond to her best option available. Therefore, $IG_i$ would not devalue her private value in equilibrium if it meant that for all $v_i < \bar{v}_i$ the $u_i^*$ would never increase and thus there has to be a structural break that induces strictly higher $u_i^*$.

The second implication is that the existence of a structural break that induces a strict increase of $u_i^*$ for some $v_i^* < v_i$ implies devaluation in equilibrium. The proof is trivial. We assume that the interest group is rational and strictly prefers higher utility ergo it will devalue her $v_i$ in equilibrium.