Title: On Selected Geometric Properties of Brownian Motion Paths

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Abstract:

Our thesis is focused on certain geometric properties of Brownian motion paths.

Firstly, it deals with cone points of Brownian motion in the plane and we show some connections between cone points and critical points of Brownian motion. The motivation of the study of critical points is provided by a pleasant behavior of the distance function outside of the set of these points. We prove the theorem on a non-existence of $\pi+$ cone points on fixed line. This statement leads us to the conjecture that there are only countably many critical points of the Brownian motion path in the plane.

Next, the thesis discusses an asymptotic behavior of the surface area of r-neighbourhood of Brownian motion, which is called Wiener sausage. Using the properties of a Kneser function, we prove the claim about the relation of the Minkowski content and S-content. As the consequence, we obtain a limit behavior of the surface area of the Wiener sausage almost surely in dimension $d \geq 3$.

Finally, we study the asymptotic number of the connected components of the complement of a Wiener sausage in a plane. We found the motivation for this investigation in the article [**?**] where the authors ask the question concerning the mean Euler characteristic of the Wiener sausage. We prove a theorem on the limit behavior of the number of the connected components of the complement of a Wiener sausage with dependance on its radius.

Keywords:

Brownian motion, cone points, critical points, surface area of the Wiener sausage, Euler characterization of the Wiener sausage in a plane.

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