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Professor Jan Kratochvíl
Dean, Faculty of Mathematics and Physics
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Dear Prof. Kratochvíl,

This letter constitutes my referee's report of the doctoral thesis "Asteroid Models from Sparse Photometry", by Mr. Josef Hanuš. I am pleased to say without reservation that I find the thesis amply demonstrates the candidate's scientific creativity and abilities at a PhD level. I did not identify any major issues requiring further work or revision, and therefore I recommend that the thesis is ready for defense by the candidate. I will mention a few minor points that might serve as matters to be discussed by the candidate in his oral examination, but these points are not essential to the approval of the thesis.

I will start with a few words of background about this work and my past involvement in it. As Hanuš notes in his introductory section 2.1, the noted American astronomer Henry Norris Russell published a paper in 1906 in which he showed that the lightcurve of a rotating body cannot be inverted to derive an albedo map of the object. Indeed, any shape body can be "painted" to yield any physically possible lightcurve in an infinite number of ways, thus the inversion problem is impossible. This conclusion, while technically correct, set the field of lightcurve inversion back about 75 years, until it was realized that "asteroids paint themselves grey", with a dusty regolith of very uniform reflectivity. In effect, Nature has only one can of paint (per asteroid), and the light variation with rotation is almost entirely due to the changing cross sectional area, not to albedo variegation. Russell did not deal with this special case in detail, and it was simply overlooked for about 75 years that this special case of uniform reflectivity is in fact invertable, with enough measurements from different directions, to obtain at least an approximation of the object's shape. My Lowell Observatory colleague Edward Bowell and I, along with some of our Helsinki Observatory colleagues (Kari Lumme, Karri Muinonen and others) held a few small workshops in the early 1980's where it became apparent that the inversion problem, subject to uniform albedo, was in fact tractable, and a few people began working on the problem. Ostro and Connelly (1984) presented the first successful algorithm to obtain what they called a "convex profile", which was a 2-dimensional equatorial profile of the actual shape. It took almost ten more years until a full 3-D algorithm was developed by Kaasalainen et al. (1992a,b), and nearly another ten until the method was "tamed" into computer code and applied to a number of asteroids with extensive lightcurve data sets (Kaasalainen et al., 2001, 2002). It was my pleasure to serve as an outside referee of Johanna Torppa's PhD thesis in December, 2007, where she presented the results of these shape inversion studies.

In all the studies to that time, only so-called "dense lightcurves" were considered for shape inversions. A dense lightcurve is obtained usually by observing a single object continuously for the night, recording the brightness with sequential measurements, either by photoelectric photometer, or nowadays using CCD cameras. A typical lightcurve may have a hundred or more measurements taken over a single rotation cycle, and if the period of rotation is in a reasonable range, the full rotation cycle may be covered in a single night, or at most a few nights. The period is thus immediately determined to several significant figures, and typically lightcurves from several apparitions, hence different viewing directions, can be linked to determine the sidereal rotation rate and pole orientation, and thence derive a convex shape. This has been done now for more than a hundred asteroids. Ďurech maintains a very useful data base on the web with the shape models of all of these asteroids. However, sufficient dense lightcurve coverage is not available for many more objects, which leads to the principal thrust of this thesis. There is already a vast collection of measured magnitudes of asteroids collected as a part of astrometric (position) measurements. The problem is that most of these magnitude estimates are incidental and not very accurate. Some are useable, however, and the prospect is, with future surveys coming on line (Pan-STARRS, Gaia, LSST, etc.) that vastly more and better data will soon become available. Thus the main work of this thesis has been to extend the inversion technique to handle so-called "sparse data", consisting of only a few or even single measurements of a given asteroid on any one night, but collectively hundreds of measures taken over many nights and years. Already there are enough useable measurements of this type to approximately double the number of shape models that can be obtained, and these results are presented in the thesis. Much of the importance of this work is the prospect for the future. It is likely that within a decade or so there will be enough data of quality from the new surveys to generate a thousand or more shape models, thus greatly improving the statistics of pole orientations presented in this thesis. Thus, the importance of this work will only become greater with time.

Another line of study in asteroid science has converged with this work to the benefit of both, and that is the observation of occultations of stars by asteroids, from which one can obtain an instantaneous profile of the asteroid in the sky plane. If one has a correct model for the shape and orientation of the asteroid at the moment of the occultation, one can combine the two profiles and ideally extract two results. First, most shape and spin orientation models are ambiguous, with two possible solutions. Usually, only one model fits the occultation profile orientation, so it is possible to choose which of the solutions is correct. Secondly, and more importantly, the shape model derived from lightcurves is scale-independent, so one does not know the absolute dimension of the figure. The occultation profile is absolutely fixed-scale, with no significant model assumptions, so in combining the two, one can set an absolute scale to the 3-D shape model. This allows one to calculate the volume of the body, and if the mass is known, as it is for about 100 of the largest asteroids plus a number of binary systems, one can estimate the bulk density of the asteroid, a quantity of great physical importance in understanding the composition and physical state (for example, porosity) of the body.

I hope the above discussion suffices to indicate the importance of the research in its own special field and its importance for neighboring areas, i.e. physical studies of asteroids generally. I will now turn to mentioning a few items that might be appropriate for further study, or perhaps some slight revision of the thesis, although not calling for any additional research.

On the top of p. 12, the candidate writes that Russell showed that shape cannot be reconstructed from lightcurves. As I mentioned above, what Russell showed was that albedo variegations cannot be reconstructed, even if the shape is known or assumed, e.g., a sphere. Russell did not consider in detail the problem of reconstructing the shape assuming a uniform albedo, but he did not claim to show it was impossible. Some slight re-wording here is in order to accurately state what Russell showed, or did not show.

In the discussion of absolute versus relative photometry, and on scattering laws and phase functions (pp. 18-21), it would be good to make it clearer that the phase relation is essentially unimportant when only relative photometry is used, and also to mention more explicitly that in the case of absolute photometry, uncertainty in the phase relation adds to the uncertainty in absolute calibration between observatories when using mixed data in an absolute sense, especially sparse data where absolute photometry is necessary.

Chapter 4 on reliability tests is essential to the work, since it appears, especially for sparse data sets, that only a small fraction of solutions are successful, or "correct". Thus, a critical evaluation is an essential part of the solution procedure. The discussion in this chapter is appropriately thorough and well done. It should be noted that shape solutions based on relative photometry tend to very poorly constrain the polar dimension (flattening) of the figure. It will often be the case that the best solution in a χ^2 sense does not represent a shape that is in a stable rotation state about its maximum moment of inertia. In such cases, for dense lightcurves it may be better to include absolute light levels, even if the calibration between observatories is uncertain, or it may be that constraining the polar dimension to be flatter than the free solution by enough to result in a stable rotation state will still be acceptable in the χ^2 sense.

On p. 48, at the end of the chapter on reliability tests, it is mentioned that it is generally necessary to assume that objects are single bodies and have spins that are damped to principal axis rotation about the axis of maximum moment of inertia. As we look at smaller bodies, these assumptions tend to fail more often: among asteroids smaller than ~10 km diameter, perhaps 15% or more of them are binary, and a similar fraction, mostly slow rotators, may be "tumbling", that is in an excited state of complex rotation. A question that could be investigated is whether sparse data sets can identify cases of binaries or complex rotation, even if it is not possible to obtain unique solutions. If so, such data might serve to identify interesting objects for more detailed photometric study.

It is noted in Chapter 5 that the success rate of finding solutions for sparse data sets is only ~10%. This low success rate is obviously correlated with the asteroid spin and shape characteristics - asteroids of more spheroidal shape will be harder to model, as will asteroids with unusual spin rates, either very fast or very slow. Thus, the results of these shape studies cannot contribute to some statistical studies, most notably of spin rate distributions or shape versus size or spin rate. I explored this problem in some detail with regard to wide field photometric surveys (Harris et al., Icarus 221, 226-235, 2012). Nevertheless, Chapter 6 present very nice results relating spin axis direction to Yarkovsky drift in the asteroid belt, and YORP alignment of small asteroid spin vectors, matters that are not so strongly affected by the selection effects expected from the low success rate of the models. I commend the candidate for a very nice presentation of these results, and taking care not to overstate or over-interpret results that are possible from the shape models. I particularly like Fig. 6.6, which shows the evolution of spin axis alignments increasing with decreasing size of the objects.

On p. 86, just below eq. 6.5, it is stated "... reset orbital period *P*..." I believe this should be rotational period, not orbital. I also question the reason for resetting the period. It seems to me that following a mass-shedding event, the primary body should still retain a spin close to the critical rate, so perhaps it should be left at that value. What will change is the YORP coefficient, due to the changed shape. Indeed, this could even be reversed, resulting in the body slowing down over time, but it seems to me it should be tracked from the critical value, not from a randomly re-set rate.

A bit further down the page, it is noted that collisions are only important if the rotation rate has been driven to an extremely slow rate. Is it possible that collisions are in fact the source of "tumbling" excitation? Observationally, we see some slow rotators that are not tumbling, so it seems that the process of slowing *per se* does not induce tumbling. Perhaps collisions are the mode of excitation of tumbling. It could be interesting to investigate if this is so. Clearly, despinning does not continue all the way to spin-orbit synchronicity, as it does with tidal friction. Apparently tumbling sets in for one reason or another before rotation is reduced to sun-synchronous.

In Chapter 7, p. 97, the "volume equivalent diameter" is introduced. But in relating calibrated shape models to thermal IR or AO observations, what may be needed is an "area equivalent diameter". This of course depends on orientation, but to make a detailed comparison of a thermal observation, or of an AO observation, the area cross section at the time of observation should be used, not the area of the "volume equivalent diameter".

This concludes my detailed comments on the thesis. I congratulate the candidate on a work well done, and wish him the best in the upcoming thesis defense. It has been a pleasure for me to read and comment on this work.

With best regards,

Alan Harris