Title: Combinatorial problems in geometry

Author: Jan Kynčl

Department: Department of Applied Mathematics

Supervisor: Doc. RNDr. Pavel Valtr, Dr., Department of Applied Mathematics

Abstract: We prove that for every graph G with n vertices, m edges and no isolated vertices the number of weak isomorphism classes of simple topological graphs that realize G is at most $2^{O(n^2 \log(m/n))}$, at most $2^{O(mn^{1/2} \log n)}$ if $m < n^{3/2}$, and at most $2^{n^2 \cdot \alpha(n)^{O(1)}}$ if G is a complete graph. As a consequence we obtain a new upper bound $2^{O(n^{3/2} \log n)}$ on the number of intersection graphs of n pseudosegments. We show that the number of isomorphism classes of simple topological graphs that realize G is at most $2^{m^2+O(mn)}$. Improving a result of Károlyi, Pach and Tóth, we construct an arrangement of n segments in the plane with at most $n^{\log 8/ \log 169}$ pairwise crossing or pairwise disjoint segments. We also show that reachability in directed graphs embedded on a fixed surface of arbitrary genus is logspace-reducible to reachability in directed graphs embedded in the plane. Finally, we generalize a result of J. Fox and Cs. Tóth by proving that the crossing number of a graph decays continuously with respect to the fraction of suitably removed edges.

Keywords: simple topological graph, Ramsey-type theorem, directed graph reachability, decay of crossing number