

Ramsey-type results for ordered hypergraphs

Martin Balko

Abstract

We introduce *ordered Ramsey numbers*, which are an analogue of Ramsey numbers for graphs with a linear ordering on their vertices.

We study the growth rate of ordered Ramsey numbers of ordered graphs with respect to the number of vertices. We find ordered matchings whose ordered Ramsey numbers grow superpolynomially. We show that ordered Ramsey numbers of ordered graphs with bounded degeneracy and interval chromatic number are at most polynomial. We prove that ordered Ramsey numbers are at most polynomial for ordered graphs with bounded bandwidth. We find 3-regular graphs that have superlinear ordered Ramsey numbers, regardless of the ordering. The last two results solve problems of Conlon, Fox, Lee, and Sudakov.

We derive the exact formula for ordered Ramsey numbers of monotone cycles and use it to obtain the exact formula for geometric Ramsey numbers of cycles that were introduced by Károlyi et al. We refute a conjecture of Peters and Szekeres about a strengthening of the famous Erdős–Szekeres conjecture to ordered hypergraphs. We obtain the exact formula for the minimum number of crossings in simple x -monotone drawings of complete graphs and provide a combinatorial characterization of these drawings in terms of colorings of ordered complete 3-uniform hypergraphs.