Report on PhD Thesis "The generalized Dolbeault complexes in Clifford analysis" by Tomáš Salač

The thesis is devoted to a quite exciting area of modern geometric analysis using the Clifford algebra and Representation theory tools to understand various generalizations to classical analysis of holomorphic functions. More explicitly: The Dolbeault complexes are known as one of the main tools in complex analysis and it is generally known that the conformal invariance of the holomorphic functions is mainly responsible for many of their basic features. This had been the departure point for the multidimensional generalizations, where analogous complexes of invariant differential operators and the cohomological substance of many considerations in classical complex analysis played a crucial role.

The supervisor of the Thesis, Vladimír Souček, had been one of those introducing this development a few decades ago. But it was not feasible to handle the operators in question in great detail before the right blend between differential geometry and the corresponding representation theory found its expression in the widely emerging area of the so called parabolic geometry. This has happened in the last two decades only. Tomáš Salač has continued the very fruitful research of the Prague group related to the main center of this research area in Ghent. He has successfully attacked an extremely difficult task to understand the so called first BGG operators together with the entire complexes in the situations appropriate for the Clifford analysis.

The survey of the achievements of the Thesis is done very well in its Introduction. In its very beginning, the Euclidean versions of the so called *k*-Dirac operators are reminded and the computational complexity of their explicit description is indicated. The author himself follows the path coined by earlier papers dealing with other operators (the so called Cauchy-Fueter operators), based on the symmetries of differential operators in question (and another earlier Thesis from Prague devoted mainly to the first order operators in the same situation). We may think that the quite rigid restriction posted on the operators by their invariance in the realm of the appropriate parabolic geometry kills the ambiguity and thus also a part of the complexity of the Euclidean approach. Still, although the existence of the BGG complexes is granted in general for all parabolic geometries, the explicit description of the operators in the complexes is very difficult. In particular, this is the case because the modules in question have got singular character, as a rule. Thus it is necessary to use ad hoc techniques to construct them and this is the main achievement of the Thesis in several cases of interest.

Instead of the Euclidean Clifford scene, the operators are built on homogeneous spaces G/P of Grassmannians of isotropic planes of highest possible dimensions in appropriate pseudo-Euclidean spaces. The main tools are the curved Casimir operators and explicit splitting of the *P*- modules by

the canonical normal Weyl structure on the big open cell in the homogeneous space. This allows for fairly complicated combinatorial considerations which finally yield the formulae for the difficult second order operators.

All this is worked out in great detail starting in the Chapter 7. The preceding chapters provide a general quick survey on the general parabolic geometry theory and do not have a strict relation to the further text. After building the sequences of operators in Chapter 7, their exactness is verified in the next chapter and some consequences for the kernels of the first operators are drawn in the last chapter.

The Thesis reveals that the author had not only to master a lot of difficult Mathematics but, in particular, he had to gain the ability to combine different disciplines and also to show a lot of patience. The text is written in a very technical and condensed way, but I have not found any difficulties to read through. A have to admit, I would not be able to check all the computations easily and quickly, but they seem to be correct.

I am absolutely sure the presented PhD Thesis deserves to be defended.

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