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Report on the Ph.D. thesis manuscript

by

Mgr. Karel Tůma

entitled

Identification of rate type fluids suitable for modeling geomaterials

Contents of the manuscript

The manuscript of Mgr. Karel Tůma studies models of fluids allowing at the same time for the viscous and for the elastic behavior. A special emphasis is put on the case of asphalt. Some standard (linear rate) models are recalled, together with their derivation, and then several new (nonlinear rate) models are conceived. Then the fitting of these models to experimental data is verified. Finally, the considered models are casted into weak PDE (partial differential equation) formulations and numerically approximated by the finite element method. The text is divided into an introduction, 5 chapters, a conclusion, and 4 technical appendices.

Chapter 1. Non-Newtonian fluids

In this first chapter, the candidate starts by recalling basic quantities and principles of continuum mechanics. Then, four standard incompressible unsteady Navier–Stokes fluid flow models capturing (linearly) the viscoelastic behavior are presented. This is achieved by considering the Cauchy stress or the symmetric part of the velocity gradient that satisfy an additional linear evolutionary differential equation. The derivation of the equations in one spatial dimension is presented, with the help of simple mechanical analogs. This first chapter is nicely written; just a little more attention on the presentation could have been payed. For instance, I believe that on the first line of page 18, one finds the strain ε for the first time, without any previous explanation of this notation.

Chapter 2. Standard viscoelastic models

In chapter 2, the models of the previous chapter are generalized to multiple (two or three) space dimensions. Also, an alternative derivation of these models, based on microscopical principles, is presented.

Chapter 3. Thermodynamically compatible viscoelastic models

This is the first key chapter of the thesis bringing important new results. Using the thermodynamic framework proposed by Rajagopal and Srinivasa in 1998–2000, which is based on the principle of maximal rate of entropy production, several new nonlinear models are proposed. Importantly, all the developed models reduce to their standard linear counterparts by linearization. In particular, in Section 3.11, assuming the co-existence of two natural configurations, the most elaborated model is developed. Here two rate-type evolutive equations are employed. This model reveals as best since it can catch the two different relaxation mechanisms of the asphalt binder. This chapter is well-written and particularly inventive.

Chapter 4. Fitting of the experimental data with the models

Chapter 4 brings the confirmation that all the considered models enable to reproduce physical experiments in simpler cases. The newly developed nonlinear models then perform very well also in the cases where the standard linear ones present a significant misfit. I only have a very minor remark here: it would maybe be better to use the discrete integration formula specifying the continuous integration symbol on the top of the page 85 directly in equations (4.2)–(4.3).

Chapter 5. Weak formulations of the PDE models and their numerical solution

This is a second key chapter of the thesis manuscript. First, formal a priori stability estimates and formal weak formulations are derived for the models considered. Based on the weak forms, the finite element discretizations of the considered models are proposed and numerically assessed. Several time stepping schemes are considered: the 1st-order backward Euler scheme is not found sufficiently precise, so that it is rather supplemented by a higher-order Glowinski one. Quite importantly, also moving domains are allowed via the arbitrary Eulerian–Lagrangian formulation. This is a very valuable part of the thesis manuscript with nice and convincing results. I particularly appreciate the work on the choice of the time stepping scheme which reveals very important with rewarding results presented especially in Figures 5.22 and 5.23. The obtention of the numerical results must have been quite effort- and time-consuming and must have also involved advanced scientific programming. Here, I have a couple of observations. Most importantly, the temporal partial derivatives in (5.33) and at similar occurrences below are not properly defined with the actual choice of the functional spaces. This should be definitely rectified. My three forthcoming remarks are rather minor. Firstly, neither the choice of the Newton damping parameter nor the stopping criteria are specified on page 119, which is unfortunate. Secondly, for the test case of Section 5.3 (Poiseuille flow), I think that a numerical convergence of the finite element approximation could have been assessed, as the analytical solution is known. Finally, I would have appreciated reporting the experimental order of convergence on pages 156–157; I also think that the graphs of Figure 5.27 would have been more witnessing if the spatial and temporal mesh sizes were refined simultaneously. But once again, all these remarks are indeed very minor and they are by no means substantial for the manuscript results.

Conclusion

The dissertation manuscript of Mgr. Karel Tůma demonstrates that the candidate is able to master a wide scope of areas ranging from conception of physical models and their validation to PDEs and numerical simulations, passing through scientific programming. It contains many new and interesting results; in particular, the conceived nonlinear models clearly outperform the established linear ones and enable an unprecedented match with the physical experiments. I find especially valuable that the final numerical validation is performed both in illustrative scientific settings and in real-life applications settings. The presentation of the results is very clear and the English language is on a very good level. The manuscript definitely reaches international standards, which is demonstrated by the fact that one part of it has already been published in an international scientific journal and another one is being considered for publication.

In conclusion, Mgr. Karel Tůma is clearly able of creative scientific work and it is my pleasure to recommend his Ph.D. thesis manuscript for defense.

Rocquencourt, March 4, 2014 Martin Vohralík