Charles University in Prague
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RIGOROUS THESIS

Asset Price Bubbles: Housing Markets Data

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Declaration of Authorship

The author hereby declares that he compiled this thesis independently, using only the listed resources and literature.

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Abstract

This study deals with asset price bubbles and possible approaches for their identification. Afterwards focuses on real estate pricing mechanism and defines user cost or alternatively imputed rent as sum of all cash-flows connected to owning real estate property. These cash-flows consist of capital cost, depreciation rate, personal income and property taxes, additional asset risk and expected capital gain. Then we construct loglinear regression models for estimating recent price levels of purchase and renting for several segments on the real estate market. These estimated price levels after that compare with user costs. Comparison provides us with information about potential mispricing on Prague real estate market in examined segments. Study concludes that from long run point of view, all subjected segments are slightly underpriced.

JEL Classification C51, C58, G12, G14

Keywords price asset bubbles, real estate market, user cost, real estate price model

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Abstrakt

Tato studie se zabývá cenovými bublinami aktiv a možnými metodami jejich identifikace. Následně se zaměřuje na cenový mechanismus realitního trhu, kdy definuje uživatelské náklady či imputovaný nájem jako souhrnný odhad nákladů a příjmů spojených s vlastnictvím nemovitostí, jako jsou cena kapitálu, míra znehodnocení, daně z nemovitostí a z příjmu, dodatečné riziko aktiva a očekávané zhodnocení aktiva. Dále sestavujeme loglineární regresní modely, které odhadují aktuální cenové hladiny nájmů a cen vybraných segmentů nemovitostí. Tyto cenové hladiny následně porovnáváme s uživatelskými náklady. Toto srovnání nám poskytuje informaci o aktuálním stavu cen na Pražském realitním trhu v předmětných segmentech. Studie dochází k závěru, že všechny sledované segmenty jsou z dlouhodobého hlediska nepatrně podceněny.

Klasifikace C51, C58, G12, G14
Klíčová slova cenové bubliny, realitní trh, imputovaný nájem, nemovitostní cenový model
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Acronyms

**CDO**  Collaterized Debt Obligation
**NPV**  Net Present Value
**OLS**  Ordinary Least Squares
Master Thesis Proposal

Proposed Topic:
Asset Price Bubbles: Housing Markets Data

Topic Characteristics:
Based on global economic crisis 2008 – 2010 and possibly upcoming another one, is more and more obvious that not only our economic system is repetitively and continuously creating and bursting asset price bubbles, but also is considerably vulnerable to such bursts. Understanding how asset price bubbles are generated and how to recognize them in real time is essential to avoid devastating impacts of asset price bubbles bursts in the future. Housing market is due to its characteristics highly price bubbles prone and in combination with high-level involvement of wide population in developed countries is one of the most interesting markets for this topic. These are main reasons why I would like to focus on topic of assets price bubbles with real estate markets extension in my thesis. More specifically provide reader with short sum of nowadays theory and some historical examples. Then introduce and discuss several approaches of identification price bubbles in real estate assets and afterwards employ these models on empirical data.

Hypotheses:
1. Significant dependence of housing prices development on fundamental variables chosen for the model
2. Verification of real estate assets price bubbles existence
3. Identification of housing markets with possible ongoing price bubble

Methodology:
1. Basic descriptive analysis and time series comparison
2. Linear and loglinear regression models using ordinary least squares method
3. Stationarity and unit root tests

Outline:
1. Introduction
2. Theory and Historical examples of Asset Price Bubbles
3. Real Estate Assets Prices & Fundamental Factors
   a. Housing Pricing Methodology
   b. Fundamental Factors
4. Empirical modeling of Real Estate Price Bubbles
   a. Data
   b. Model
   c. Results
5. Conclusion
Core Bibliography:


1 INTRODUCTION

The topic of asset price bubbles is definitely not the brand new one. Many well-known economists paid a lot of attention on this topic for a long time and studied cases from our history even almost 400 years old, such as Dutch tulip mania from the 17th century or South Sea bubble from the 18th century. Despite all the time and interest, the topic is still not understood very well. At least in a practical manner, that could be used in common financial matters. Even though, it is getting more and more obvious, how important this topic really is. Two worst economic crises in modern mankind history, Great Depression 1929 and Global Financial Crisis 2007 were actually triggered by asset price bubbles bursts. Economical knowledge about asset price bubbles could be compared to knowledge about earthquakes. We have theories what cause them, we can ex post identify and somehow measure them, we know that consequences could be disaster, but still we cannot predict them with certainty in time, location nor magnitude.

In this thesis, we will focus on real estate bubbles and mispricing on housing markets. Real estate assets capitalization in developed countries is estimated to be 54% of all assets and around 40% out of all assets residential properties only. In developed countries, there are also from 40% to 80% (little over 60% in average) population living in their own properties. These facts combined with information that roughly 9 out of 25 most famous asset price bubbles were actually real estate bubbles, makes this particular field very interesting. We could state that the real estate market is the biggest market with highest level of wide population involvement, prone to asset price bubbles on the world. These are the main reasons that in this thesis we try to identify not only past price distortions, but also potential mispricing as trace of price bubble that could be present on selected housing markets in these days.

1 Actually there could be academic discussion about this as for example Kroszner (2003) argues that 1929 Great Depression was not actually positive asset price bubble as following gains justified top price but did not justified lower prices so market was actually underpriced.

2 http://www.housingfinance.org/publications/, Homeownership Trends Worldwide, Nasser Munjee

3 dtto
The thesis is structured as follows: after introduction, second chapter provide some most famous historical examples of price asset bubbles, then present several valuable definitions and general theoretical concepts. Third chapter introduces main theoretical concept of user costs that this study utilize for empirical assessment of market price level. In next chapter study deals with estimation of loglinear models needed for determining current market price levels for renting and purchase real estate property. Chapter five presents empirical testing of chosen real estate market segments price levels employing theoretical concepts from third chapter in combination with practical results from chapter four. Last chapter summarizes our main findings and conclusions.
2 THEORY AND HISTORICAL EXAMPLES

2.1 HISTORY

"Those who does not know history are destined to repeat it."

Edmund Burke

In order to better understand the asset price bubbles nature and topic in general, we will firstly introduce several famous historical cases of impressive price growths and declines. At the very beginning we find useful to observe what is topic about in real world and what consequences could follow. Also the message of this part is that, asset price bubbles exist all the time along with existence of the free markets as effective pricing mechanism failure. The time era when it appears only influence the transmission mechanism but not basic merit of the price bubbles. We describe chosen cases in chronological order.

2.1.1 TULIPMANIA

First chronicled case ever and maybe also historically most famous one was in Continental Europe in the first half of seventeenth century. More specifically it was situated in Holland from 1634 to 1637. It begun during 16th century, when tulip flowers were introduced by Ottoman Empire in the Western Europe. They became very popular in Netherlands during Dutch Golden Age for its fascinating bright colors. Uprising popularity in combination with the fact that it takes 7 years to grow the flower from the seed caused that at the moment there was an exceed of demand over supply making the price grow. At the price top in late 1636 and the beginning of 1637, the growth from 1634 was above 5 900 %. People even mortgaged their houses just to buy flowers in order to resell for higher price.

According to Charles Mackay’s book “Extraordinary Popular Delusions and the Madness of Crowds” following prices were paid for one single tulip bulb: two
lasts of wheat, four lasts of rye, four fat oxen, eight fat swine, twelve fat sheep, two hogsheads of wine, four tons of beer, two tons of butter, one thousand lbs. of cheese, a complete bed, a suit of clothes, a silver drinking cup. They were also signing contracts at the end of the season for the next season harvest. That could be considered as first future contracts in history. The bubble burst once one buyer denied to pay for such future contract he signed and all the market eclipsed very quickly after.

2.1.2 THE BULL MARKET OF THE ROARING TWENTIES

In the second decade of the 20th century, there were taking place many important changes to the United States markets. 1913 was established Federal Reserve institution. There were introduced new policies that helped to improve free trading, anti-inflation and anti-trust measures took place. Also due to new production improvements, workers productivity was growing and allowed higher focus on research and development field. There was one more factor important for rapid growth. That was debt and thanks to it allowed leverage financing. 1920s was the decade of massive spread of consumer as well as corporate credit. Americans used borrowed money for new mass production goods purchases such as radios and automobiles. But consumers also used borrow credit to stocks investments. So it happened that this credit support demand for stocks as well as for goods made by companies traded on stock markets. The wheels of stock market growth were rolling on maximum, especially in new technologically branches. Radio stocks raised 400 % during 1928.

On September 3rd 1929 the Dow Jones index reached its top for the decade and started to decline slowly. But on October 24th known also as “Black Thursday”, things speed up rapidly and started the period known as “Crash of 1929”. Markets were affected by panic and the trading of 13, resp. 16.5 millions shares per day with loss of 38 %, resp. 30 % in following days took place. Markets reached its first bottom in spring 1930, but than in early 1930s there was another decline and second bottom. This era was followed by well known years of the “Great Depression” that was considered as world biggest crisis until recent Global Financial Crisis 2007.
2.1.3 FLORIDA 1920S LAND BOOM

Miami in state Florida is actually favorite location for real estate price bubbles. First of them took place in 1920s. Location thanks to warm weather and beautiful nature scenery became very popular among the tourists and investors and thanks to that experienced great economic prosperity and development. Sunshine state, as called, even had the light billboards on New York’s Times Square advertising this Eden on earth. Prices were going nowhere than up so speculative trading came into place. At the peak there was 400 % growth from initial steady price. Then came a series of unpleasant incidents that caused the bubble to burst. First, on January 1925, Forbes magazine printed out an article that warned about ridiculously high prices of Florida real estates based on speculative expectations. Suddenly the growth disappeared. Adding the rising railroad shipping prices, two hurricanes in 1926 and 1928 and Wall Street Crash 1929 followed by the Great Depression, the bubble hit its bottom.

2.1.4 URANIUM SURGE

After long times that uranium commodity price slowly but steadily rose, in 2007 prices just rocked up exponentially. Possible explanation could be combination of following factors. In October 2006 was flooded the biggest undeveloped source of uranium on the world, Cigar Lake Mine of Saskatchewan. Strengthen of global warming phenomenon through beginning of the 21st century with nuclear power plants could be part of the answer for growing demand for electricity consumption and decreasing “greenhouse gases”. And finally last factor, depleting of huge stockpiles from 1980s that were created for planned new nuclear power plants that were actually never built because of political and public pressures.

At the top of the bubble, beginning 2007, price per kilogram of the uranium was 300 $. In comparison, for the same amount in June 2002, it was only 20 $. Final growth against initial price was at the peak 1 844 %. But when flood of Cigar Lake Mine of Saskatchewan ended and was resumed to works to deploy this source, new nuclear plants could not be built in that short time and actually most of power plants had long-term supply contracts signed in advance, it was more than obvious that real demand will not meet the expected one. Prices plummet during one year under 100 $ per kilogram and the bubble burst.
Dot-com bubble last roughly from 1995 to 2000 and it is closely connected to the fascination with new amazing technology called Internet in those times. All companies specializing in this or related industry were growing on stock markets with fascinating pace. Sometimes was enough to add e- prefix or .com suffix to the company name and market price doubled over-night. Also new interesting business model strategy was invented during this dot-com frenzy. Strategy was called “get large or get lost” and was based on aggressive capture as big market share as possible and as soon as possible no matter costs. Also quickly spread company awareness with belief that these assets will be profitable later in the future\textsuperscript{4}. Investors believed this strategy as well and also believed to the bright future of this new Internet technology. This particular combination allowed conditions, where firms that never made a profit and not even single revenue in some cases, were able to obtain incredible funds through initial public offers. This fact combined with very low interest rates in 1998-1999 encouraged massive growth of new companies in this new industry, even the absurd ones. For example investors were able to invest their money to firm called “Digiscents” that specialized for development of computer peripheral that would make the web sites smell.

During 1999 and early 2000, U.S. Federal Reserve raised six times interest rates and as a result economy finally slowed down. The NASDAQ index peaked on March the 10th 2000 with price more than double last year and more than 5 times higher than 1995. But already on the March 20th, which means in only ten days, NASDAQ lost 10 \% out of peak and by 2001 was Dot-com bubble deflating at full speed. Many of dot-com companies just burned out the remaining venture capital during one year and vaporized out of the market never made a single profit. Actually it is somehow surprising how was the end of dot-com companies surprising, because the high mortality existence is built-in feature in “get large or get lost” business strategy. It is easily understandable that only a few companies could reach high market share position for not that large number of web services available at the time. So it was an unavoidable and possibly predictable destiny for most companies just to “get lost”. But what makes it remarkable is the market value loss that was caused by that part of “get large or get lost” business model strategy and that is more than 5

\textsuperscript{4} In fact, Google and Amazon grew among biggest companies on the world using this business strategy.
trillion $ from March 2000 to October 2002. But in fact, that bubble burst did not cause too big economy crisis as could be guessed out of single loss number. There are also companies that survived through stock price turmoil and already broke the maximum prices from year 2000, for example well known company named Amazon.

2.1.6 REAL ESTATE AND LEVERAGE BUBBLE

The last example of historical price bubbles we are going to describe is also the most recent experienced one and we could say that we cope with its consequences until these days. It is originated in United States of America in year 2007 and during only one year spread to most of developed countries around the world. we will pay attention only for the initial part that took place in U.S., often also called sub-prime mortgages bubble.

The fundamentals were laid down during year 2000 when some essential aspects of way how industry worked so far were changed. To the system where lender institutions were supposed to hold the debt on their very own books until final maturity was introduced distribution mechanism that shifted actual debts from original lenders away. At the end, whole chain was actually quite long. Basic and simplified backbone included: homeowners – mortgages buyers, brokers – mortgages dealers, lending institutions – actual mortgages originators, investment bank institutions – mortgage packages buyers using leverage and after application of financial engineering magic seller of collateralized debt obligations (CDO) and last but not least pension funds, hedge funds etc. as investors buying CDO tranches. And this chain was working pretty well and everybody was happy. Homeowners had their houses, brokers had their commissions for dealing, lending institutions had earnings from sold mortgage packages (and even more money to lend), investment banks had pretty high earnings on sold CDOs tranches that they built from mortgage packages bought on high leverage for very cheap credit (as Federal Reserve set very low interest rates) and finally investor institutions had their relatively high return investment (relatively as bonds were no more interesting for very low interest rates).

In this basic chain we do not mention institutions as Insurance companies, Government-sponsored enterprises or Rating agencies... These institutions played their part in whole mechanism as well, but are not essential to understand the main driving forces that allowed bubble to be created so we will just omit them. The same stays for CDOs creation mechanism.
The real turning point came in the moment, when thanks to very cheap credit from low interest rates and huge inflow of foreign resources, there were so much money to lend but market of responsible and solvent homeowners with sufficient income was completely saturated. Supply exceeded demand and the only solution in this situation, besides slow down, was to create new demand. Because nobody in the chain wanted to give up his or her profit or commission, subprime mortgages time came. Subprime mortgages actually meant free money, because there was no initial payment needed, no proof of income what so ever, any property as collateral was enough. In fact, many American families took credit and collateralized their own houses just to use gained money for boosting their consumption. Nobody in that moment thought that there could be any problem, because even if homeowner default on his or her interest payments, there was still collateral to sell and as house prices were uprising almost forever, nothing bad could really happened. And even if something bad happened it really was not problem for the debt originator, because he was selling all the risks very quickly away, even though he was really the only one who knew who he selling mortgages to. So the bomb was ticking and it was only a matter of time when it is going to explode. As more and more homeowners default, there were more and more foreclosed properties to be sold on the market making supply exceeding demand. With supply over demand prices were after long time finally started to decrease and as result some of the homeowners who were able to pay their regular payments get to the situation, where value of their mortgage was far over the market price of the house. So it was irrational to continue with the payments and they quit paying. This created even more pressure on supply and prices were just falling. Suddenly whole chain stopped working. Lending institutions did not want any new homeowners from broker as they had low quality mortgages on their hands that nobody wanted to buy. Investment banks had on one hand the CDOs that in fact became nothing else than boxes with worthless houses and surprisingly nobody wanted to buy these anymore. But on the other hand they still had to repay borrowed credit, which they used for the leverage. And finally investors that did not want to buy anything new from investment banks, but already had full pockets of toxic CDOs losing its value as well. Market was frozen and toxic CDOs were actually spread all over the world.
2.2 TERMS AND DEFINITIONS

We start this part with introduction of two basic terms that needs to be clear for understanding next parts of the study properly.

Basic Term 2.1 Price is the value that will purchase or transfer ownership of a definite quantity, weight or other measurement of good or service.

Basic Term 2.2 Intrinsic value is the inherent worth of something independent of its value to anyone or anything else.

So the price is the amount of money for which the asset or anything else is sold on the market. The intrinsic value or just a value is the pure internal value of the asset if all psychological, emotional or other influences on final price that are not directly connected to ability of the asset future profit generation are deducted. It may seem very easy, but it is of essential importance to be able to distinguish between those two terms in order to understand what price bubbles means in their basics.

“Price is what you pay. Value is what you get.”

Warren Buffet

This quote of the famous investor Warren Buffet implies on the fact that the price of asset does not has to be always matched with its real value. These two very important characteristics could be in fact oscillating in time one around another, but never diverge in probability or long-run period, at least for most assets on the markets. It could be seen as cointegration case, both variables could be considered as random walks but the difference between them is linked one to another. In short time periods, due to information asymmetry, psychological aspects etc., there are gaps opened. Longer the gaps last and bigger they get means higher probability they will be closed or at least became smaller.

Such case inequilibrium provide some opportunities as well as threats. Some might make fortune some might lose it. Redistribution of wealth certainly takes the place. The question is, if this aspect is really the problem or just an application of

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Darwin concept – stronger (cleverer) survives (get richer). If one puts it in that easy and straightforward way, it is really difficult to argue with natural order. But in real world the regulations and bailouts comes in to the picture and as they may ease the economical impact of the bubble burst, they definitely damages the responsibility chain and breaks significantly upper mentioned Darwin concept. Another important thing is redistribution of political power. Bubbles bursts are often accompanied by social tensions that provide soil for political instability. But as far as these aspects are both very important and complicated, they are also out of main framework of this thesis. We find important to mention them, but will not discuss them in details within this work.

Another interesting thing about these gaps is the fact, that there are out of logic two basic kinds, negative and positive one. Negative gap is ended with growth closing it and positive vice versa. The amazing thing about this is, that if we speak about asset price bubble in general, we only mean the positive one terminated with negative drop.

Probable that it is caused by two main reasons. First is the way our generation is used to think. We are living in the “growth” generation\(^7\) that is expecting growth everywhere and in everything. The growth is the only thing that could interest us and draw our attention. So in case of negative bubble, because there is no growth on the beginning, we just do not pay attention to it. And afterwards, when the growth phase takes place, attention is somehow shifted because it was not important before. That means we do not notice the first phase and without this part it could not be bubble, it is just growth upon average during second phase. And anyway, growth is what we are expecting, so nothing strange here.

If we just switch the phases, this is what happens. During the first growth phase, everybody is interested by growth higher than the normal usual average one. That makes us pay attention to it and praise it. When second phase of correction and decrease takes place, everybody is scared and worried what happened to that upon average growth we all liked and admired. It has all the attention for both phases and that allows us to look at it as on the complete bubble with both phases.

Second reason is that it is generally accepted that people are more sensitive to declines than to growths, for the very same reason we just mentioned above probably. Bigger decline means more sensitive reaction. And we all know that if you want to

\(^7\) Tomas Sedláček, Good and Evil Economy (2009)
experience big deep fall, you should climb pretty high at first. This allows only the scenario with growth phase first and fall phase second.

Now finally let’s get to definitions of “Price Bubble” itself. Firstly in 18th century was used name “Mania” according to first recorded bubble “Tulipmania” in Netherlands. During the 19th century become more popular name “Bubble” and it lasts till present days. This name is actually very precise, because if we imagine natural bubble such as soap or gum bubble, it steadily grows when air inflows as far as pressure reach threshold and the bubble pops. That is, figuratively meant, exactly the very same what happens to asset price bubbles. In the second half of 20th century appear first economically serious definitions of this phenomena.

Definition 2.1 (Kindelberg, 1978). Bubble is an upward price movement over an extended range, that than implodes.

This one of the first definitions in economically literature is in fact very close to the example of natural bubble pop mentioned above. It only cares about price change itself and in positive direction with afterwards drop.

Definition 2.2 (Kindelberg, 1987). A bubble may be defined loosely as a sharp rise in the price of an asset or a range of assets in a continuous process, with the initial rise generating expectations of further rises and attracting new buyers – generally speculators interested in profits from trading in the assets rather its use or earnings capacity.

Nine years later the same author comes with a little bit extended definition. This time is definition more specific in the way of the growth nature. It implies the presence of internally unjustified price growth due to speculative pressure on demand. The question remaining is how to identify this substantial speculative growth.

Definition 2.3 (Stiglitz, 1990). If the reason the price is high today is only because investors believe that the selling price will be high tomorrow—when 'fundamental' factors do not seem to justify such a price—then a bubble exists.

In 1990 the Fed adopted the definition of “bubble” offered by Joseph Stiglitz. This definition suggests existence of fundamental factors that are important for backing up asset price development. That means, we should be able to compare these fundamental elements with asset price movements and find out, whether there is sufficiently strong co-movement or if there is none. Of course, we have to know what fundamentals are at first.
Second part included in this definition is also important, that is introduction of investor beliefs and their expectations about future. Does not suggest that it has to be necessarily speculating pressure. That fact allows possibly mispriced market due to its inaccurate expectations about future.

Definition 2.4 (Rosser, 2000). A speculative bubble exists when the price of something does not equal the market fundamentals for some period of time for reasons other than random shocks. Fundamental is usually argued to be long-run equilibrium consistent with a general equilibrium.

Even though this definition does not include any mathematical equation that could be used for analytical identification of price bubble, it contains interesting idea how to approach this topic. It suggests we should be able to separate price of the asset to three parts. One is fundamental price that captures solid and justifiable asset value, second is random shocks that exists for only short periods of time and the last part would be component capturing residuum between sum of first two parts and market asset price. We could call it asset price bubble indicator and if this particle grows to some extent, we might proclaim that the asset is in price bubble position.

The question now is how to determine these individual parts. Random shocks should be relatively easier to identify, as they are short-term variances that does not have long-term effect. Shocks also should not represent significant factor so it is definitely more important how to set fundamental value. One way is using other fundamental variables and derives from them this component, though that could be very difficult or even impossible in some cases. But there is also another widely accepted concept how to look at intrinsic value determination. Net present value (NPV) of future cash flows of the asset. This concept is very easy and straightforward to employ for determination of the real value of the asset and then with simple comparison we could say if the market price is in bubble position or not. But of course there are some shortcomings in this approach as well, otherwise there may not be any more price bubbles or even ineffective mispricing anymore as every investor could apply this approach.

The first problem is the uncertainty of future cash flows as well as discounting factor. Let say for stocks, dividends as well as inflation and cost of capital may vary in time and even a small change could have considerable influence on the results and thus real value. Another problem is, that high percentage of investors buy asset with expectation of future sell. That makes infinite perpetuity NPV formula not sufficient. Instead we are supposed to use the finite one, where the cash flow from selling the asset should be calculated as well. But then it is absolutely rational for individual
investor to buy asset even though the price is already on upward phase of the bubble, because there still could be expected, that the price of sell will be higher and the investor will make the profit. NPV would not be any lower than market price. The problem is on the edge where the growth breaks to decline. Any further NPV will be probably lower than the market price and that makes asset purchase irrational anymore, but we had to go through bubble market development first. But still, it is based only on expectations and even the buyer right before the edge could be rational in the way of expected growth. But this is already too far from intrinsic value we started with for now.

2.3 THEORY

Ever since markets exist, there is also possible to trade on such markets not only for reasons to get what people need in exchange for what they do not need that much, but also for simple profit itself. One of ways how to make this profit is speculative trading. It is the most straightforward possibility and simply it means to buy asset, commodity or etc. and sell after the price growth. Profit is relevant and understandable reason for trade. Problem occurs when this speculative trading becomes a significant share of market volume. This is the point, where self-fulfilling prophecy comes on the scene and where the price growths is not out of rational reasons anymore but for only one reason left and that is the price growth itself. Price allocation mechanism is broken. Speculative traders pushes the rational demand out of game, what is problem in longer horizon, because speculative traders buys the asset in order to sell it, not to use it or employ it any other way. And at the particular moment, when some speculative traders (or anybody else on demand side) breaks the chain and do not buy anymore, market participants are in a really bad position. Because speculative buyers does not have any other plan for their purchase than resell, and nobody is buying anymore as price is too high, they just hold asset and lose money as the price returns to the rational value path. Often the drop is even under the rational level resulting from the consequences impact of this scenario on the economy as a whole.

Very interesting about this speculative cycle is that even though from general macroeconomic point of view, final aftermath is definitively negative (due to broken price allocation mechanism and economy could not produce its potential output) and thus is desirable to avoid these situations to happen or lets say that it is generally
irrational to support such development. From microeconomic individual point of view, it is also opportunity soil as huge amount of wealth is redistributed. So in case some individual knows when enter and when to quit market position, it could be very rational and desirable to be part of bubble market and even actively participate on its formation. As interesting this topic area is, it is not main matter of our focus in this study so we will leave it. We only wanted to point out, that this is interesting field for further studies in this topic as if there would be some mechanism invented that would directly connect individual and macroeconomic motivations, it might be the end of most mispricing market situations.

So speculative forces could be one of the possible motivations and life-cycles of the price bubble. In fact there could be more initiations and variations, but the merit is always the same. Significant price growth that is not backed-up sufficiently by growth of important fundamentals and afterwards plunge of the prices. Following problems as ineffective allocation of resources due to price mechanism distortion, economy turmoil as credit system crashes due to collateral mechanism breaks etc.. These consequences are definitely worth to try identify price bubbles at the beginning of their life-cycle and undertake appropriate steps and actions to prevent bubble creation at first place or at least minimize its impacts. There is ongoing discussion between economists whether there should be taken any actions against forming bubbles or if it is generally more effective to let it burst and cope with bubble aftermath. However to make this discussion even relevant we need to be able to identify forming price asset bubble at the first place.

If we take into consideration all written above, we end up with three or maybe four basic possible approaches to asset price bubble identification. First is simply measurement of excessive changes over given level in asset price during given short time period. Next approach is to compare asset price development with fundamental variables development and assess whether asset price is backed up by those fundamentals sufficiently. Third general approach is to calculate (or estimate if there is no sufficient data available) future cash flows and thus net present value of such asset. The last fourth one is kind of additional method, that could be added for the second and third one and it is to care about investors future beliefs and their expectations. Case, Shiller (2004) successfully used this approach, combined with extensive fundamental analysis to point out upcoming real estate subprime bubble in three years advance. But as this is more like sociological approach method we would not discuss it in detail. Now lets have a closer look at those three main general approaches and discuss their advantages and shortcomings.
2.3.1 THEORETICAL CONCEPTS

First and most simple one is derived from Kindelberg definitions and cares about significantly excessive asset price changes in short time period, mainly growths. The idea is based on elementary notion that for every state of the nature in the economy, there is some natural threshold of possible growth beyond that must be some speculative forces added to overcome such values. And these speculative forces push the asset price value to price bubble position where burst is inevitable when speculators run out of fuel.

Definitely considerable advantage of this approach is its simplicity and straightforwardness. But the question is, where to set threshold that would indicate, which price changes are still in tact and which are not. It definitely would not be the same for all asset classes, also there should be different thresholds for different states of economical realities. Kroszner (2003) also shows, that asset mispricing position does not have to be always accompanied by the change of subjected asset market price. Sometimes it could be also situation when asset price remains on some level even though fundamentals would not support such price any longer. In situations like these approach fails completely. But actually for open markets with high trading volumes, this could be due to its simplicity valuable signaling instrument. We might be alarmed for situations where we should pay more attention and maybe also start more complicating procedures indicating whether the price change is justifiable or not. For all our historical examples, this signaling mechanism would probably work.

Second is the comparison of asset price development to its fundamental factors as Stiglitz definition suggests. The logic here is that price growth without fundamentals that would back up such price development is in time unsustainable. Thus for every asset we care about, we are supposed to find out those significant fundamental factors, estimate or derive long-run equilibrium relation and assess deviations from that equilibrium. For case of real estate markets are these fundamental factors very often represented by household income, rent level, construction costs, new construction level, population growth, mortgages rates, etc..

Advantage of this concept is its applicability in real time under condition we have relevant current data of fundamental factors. We could be even able to relatively accurately predict asset price development if we would be able to estimate future development of its fundamentals. But as fundamental factors could be advantage of
this approach, they also could be the problem for this method. Sometimes it could be very difficult to determine relevant fundamental factor as well as there could be just too many of them and it could be virtually impossible to compare them all together. Another thing is what happens if these factors are mispriced as well. We could end up in situation where we assess fundamental factors for fundamental factors to find out if they are priced precisely, which is strange idea.

The last concept, which is deduced from Rommer definition, is to determine assets intrinsic value or long-run general equilibrium consistent price. As we already mentioned, this could be done through calculating asset net present value of all its related cash flows.

This approach is very sound in its economical theory background. It is also generally applicable for every kind of asset as its value should be sum of its all incomes and costs in the future. This is also very understandable and straightforward concept. As long as we have sufficient and relevant information, it is very easy to take historical data, count all operational and capital gains and costs and discount them by realized inflation and compare obtained result with price of asset at the beginning of tested period. If backward looking analysis, it is definitely the most precise method, no matter any fundamental factor at all. Kroszner (2003) even calculated that 1929’s Great depression in his setup was not actually price bubble, because even very high prices at the top before great price drop, were in terms of capital costs justified by future earnings.

Problems emerge when trying to utilize this concept for assessment current price of asset. Dimension of future expectations could be very deceiving and what more, whole concept is highly sensitive for even small changes in expectations for important factors. Even if we would be able to predict very precisely future cash-flows, only a little change in discount factor would provide us with significantly different results.
In order to empirically test, if there is a possibly ongoing price bubble on real estate market, we need some methodological concept that will allow us to compare fundamental prices with current price level. For this we are going to employ method that is in some sense combination of fundamental factors comparison approach as well as estimating intrinsic value of the asset itself. It is based on exploring mutual relation between three main pillars, which are market price of purchase, market price of renting and user cost for the real estate asset. First element is simply the price of the property for which it is offered on the market, lets say it is the real exercised price on the present day market. The same holds for the second element, but for the case of long-term renting real estate property. So this element should stand for the renting price of the real estate property on the present day market. Last pillar is the most complicated one, and we are calling it “user cost” of housing. We could look at it as imputed rent or as sum of all cash-flows, costs as well as benefits, that are connected with owning a real estate property in present state of economic reality.

We will denote:

\[ P_t \quad \text{... purchase price level on real estate market in time } t \text{ (local currency)} \]
\[ R_t \quad \text{... renting price level on real estate market in time } t \text{ (local currency)} \]
\[ u_t \quad \text{... as user cost level for real estate properties in time } t \text{ (percentage)} \]

Then the basic idea is standing on theoretical background, that equilibrium state should be the one, where the renting market price as fraction of the market price of purchase is equal to the user cost of the owning the real estate property. Which is described by following relation:

\[ \frac{R_t}{P_t} = u_t \]
Or in other words, market price of renting should be equal to market price of purchase multiplied with user costs.

\[ (2) \quad R_t = u_t \cdot P_t \]

As already stated user costs stands for overall sum of costs and benefits resulting from owning the real estate property. As all its components are given in percentages, final value is also given as percentage. We could also look at user cost usage as in perpetuity bond, where owner of the bond receives infinitively cash flow – coupon and discounts at market interest rate. The very same for owner of the property as she infinitively receives rent and discounts at user cost. If we employ perpetuity formula, we obtain following:

\[ (3) \quad P_t = \frac{1}{u_t} \cdot R_t \]

Now from equation (3) is pretty clear relation of price and rent as inverse function of user cost. It also provides us with fraction determining willingness to pay for owning the property in rent price terms. If we put \( R_t = 1 \) and \( u_t = 5 \% \) than the purchase price is equal to 20. So for 5 \% level of user cost, it is supposed that purchase price is 20 fold of renting price in equality state (relation between user cost level and purchase price fold visible in figure 3.1).

If there is inequality between market price levels and user cost, there will be arbitrage opportunity for cheaper renting on the one side or cheaper owning on the other side. For example, if user costs are 5 \% again (suggesting 20 fold price), but price of purchase is only 15 multiple of rent price, and thus purchase price is lower (or renting price is higher) than market equality would suggest, we have arbitrage opportunity on the market. One could purchase real estate property and rent it for over six and half percent, but in situation where user costs are only 5 \%, there is left over one and half percent as arbitrage bonus. This arbitrage opportunity should naturally force market to shift the demand in favor of cheaper alternative and thus on long run close the gap and set equilibrium. The equilibrium in the system could be in general achieved through adjustment of any pillar. But purchase prices are, at least in nominal values, relatively downward rigid and for user cost, as we will discuss in further chapters, is relatively problematic if changes too frequently.

This approach is quite straightforward and should give very clear conclusions whether the real estate market is or is not in equality state. Also due to user cost structure, in comparison to simple rent/price ratio for example, it takes into account interest rates level, effective taxes and other important factors neglected by other ratios and thus it provides more precise and sensitive information about cost of
owning a real estate property. But again, the simplicity and clear logic structure is probably the biggest advantage of this approach, on the other hand, there are also some challenging issues contained. For results and their validity, is very crucial, how precise and valid those three main elements are. In case of user costs element, its underlying fundamentals are relatively easy to obtain more important is the way we adjust them and employ them to the calculation itself, but we pay more attention to that discussion further in the next chapters. The major problem as well as challenge is other two elements. It is of highest importance, that price of real estate property purchase and price of renting will be as precise as possible and also independent in structure of those two estimates. If one value will be derived from another one, concept we are using will be pointless as their ratio will be biased and thus will not provide us with information about their mutual relation. On the same hand it is intended to keep same level of quality and other characteristics of subjected real estate properties for reason of mutual consistency of both, purchase and renting price levels estimations. This is very important, as we need to remain consistent in these estimations, because this ratio is the key factor to be compared with user costs and it would be again of no use, if we would compare purchase price level for different kind of real estate properties than renting level. Despite those two very important demands, it is very difficult to gain data fulfilling these tight assumptions and requirements.

This is the reason why we decided to estimate price of purchase and renting indices on our own, using hedonic price models. Great advantage of estimation on our own is the fact, that models should provide relatively precise price of purchase and price of renting levels for detailed selection of parameters describing the property and also that these parameters selection would hold very same for both purchase and renting price levels. With those two pillars estimated separately, we could also state that these estimates are independent on each other and thus provide unbiased results. It also offers us opportunity for testing about mispricing in different segments of real estate properties that could be possibly estimated by our models. Drawback is absence of time-series data as information dataset needed for hedonic pricing models estimation is available only as static offer advertisements. Unfortunately that means, we have to focus only on static mispricing approach and we are not able to test asset price bubble, as it is dynamic concept. On the other hand we do not lose forecast possibilities completely. We still could have expectations about some underlying variables for user cost influencing its development and thus predict potential change in price levels as well, expecting that price levels follow user cost development in order to close potential gap and fulfilling “no arbitrage” condition. Another disadvantage is that price models we are using, works properly only for relatively
small local markets as city or similar. But as we intended to focus on Prague real estate market, which fulfills this condition well, it is not a limitation for us.

3.1 USER COSTS

We already stated that user cost should incorporate all cash-flows, costs as well as benefits, which are connected with owning a property in present state of economic reality. In alternative perspective we could look at it as on imputed rent calculated for the owner if occupied the property herself. Through this pillar we should be able to project influence of important factors as cost of invested capital, differences in risks, property and income taxes as well as tax deductibility benefits, maintenance expenses and expected capital gains from owning the real estate property. These influences, that many of other price assessment instruments neglect or fail to integrate to valuation process, will take important part in our approach.

We will start with formula used for estimating of imputed rent (Potreba 1984) where we will drop index i distinguishing individual real estate property cases, as we are only interested in market price level as aggregate:

\[
(4) \quad R_t = P_t - \frac{1-\delta_t}{1+r_t^m+y_t} E[P_{t+1}] + (1 - \tau_t)\omega_t P_t - \tau_t r_t^m P_t
\]

Where following symbols stands for:

- \( R_t \) … the imputed price of renting at time \( t \)
- \( P_t \) … the market price of purchase at time \( t \)
- \( E[P_{t+1}] \) … expected price of purchase at time \( t+1 \)
- \( \delta_t \) … reflects maintenance cost as fraction of the property value at time \( t \)
- \( r_t^m \) … stands for mortgage rate at time \( t \)
- \( y_t \) … represents additional risk compensation for property owning against renting
- \( \tau_t \) … stands for income taxes rate for property owners at time \( t \)
- \( \omega_t \) … tax property rate at time \( t \)
- \( u_t \) … real estate property user cost at time \( t \)
In order to obtain more suitable form for straightforward explanation of individual particles meaning, we slightly re-arrange the original formula and obtain following form:

\[ R_t = P_t - \frac{E[P_{t+1}]}{1+r_t^m+\gamma_t} + \frac{\delta_t E[P_{t+1}]}{1+r_t^m+\gamma_t} + \omega_t P_t - \tau_t \omega_t P_t - \tau_t r_t^m P_t \]

If we apply on this equation relation (1), we derive real estate property user cost as:

\[ u_t = 1 - \frac{E[P_{t+1}]/P_t}{1+r_t^m+\gamma_t} + \frac{\delta_t E[P_{t+1}]/P_t}{1+r_t^m+\gamma_t} + \omega_t - \tau_t \omega_t - \tau_t r_t^m \]

Now we could separate six main sections user cost consists of and assign their proper meaning to them:

1 + \tau_t^m + \gamma_t \ldots this sum stands for discounting factor employing cost of capital with additional risk compensation

1 - \frac{E[P_{t+1}]/P_t}{1+r_t^m+\gamma_t} \ldots section captures discounted expected capital gain (loss) from real estate asset price change

\frac{\delta_t E[P_{t+1}]/P_t}{1+r_t^m+\gamma_t} \ldots section stands for discounted maintenance (depreciation) costs

\omega_t \ldots reflects property tax cost

\tau_t \omega_t \ldots express property tax deductibility benefit on income taxes

\tau_t r_t^m \ldots represents mortgage rate deductibility benefit on income taxes

The last modification we will exercise in this part is presentation, how we could employ equation (6) for deciding if there is price equality state on the real estate market or not. As we know from formula (1), if the market is balanced, user cost is equal to fraction of price level of renting and price level of purchase. Let us denote \( r_t^l \) to be rent income put as percentage out of property price. Then we could substitute this rent income to equation (6) instead of user cost. For easier interpretation, we multiply equation by discount factor and neglect insignificantly small parts. Then we obtain following:

\[ r_t + (E[P_{t+1}]/P_t - 1) + \tau_t \omega_t + \tau_t r_t^m = \delta_t E[P_{t+1}]/P_t + \omega_t + r_t^m + \gamma_t \]
Where benefits, are on the left side, and costs are on the right side. To clear the market this equality must hold. Otherwise if the left side is greater than the right one, income exceeds costs and market is due to underlying fundamentals underpriced. Then with unchanged fundamentals, property price should grow or alternatively price of renting should decline. This situation holds for vice versa situation as well.

3.2 FUNDAMENTAL FACTORS AND USER COST SENSITIVITY

Although for further calculations we will employ formula (6), now for purposes of sensitivity presentation we accept negligible loss of accuracy and simplify equation (6) to even simpler form than equation (7), where individual components will be as separated as possible:

\[ u_t = \gamma_t + r_t^m + \delta_t + \omega_t - \tau_t (\omega_t + r_t^m) - g_{t+1} \]

With this type of formula, where \( g_{t+1} \) holds for expected capital gain (loss) from holding asset, we can easily observe, how sensitive user costs are for changes in individual parameters. All cases described are for ceteris paribus situation. With one percentage point move of additional risk compensation, maintenance cost or expected capital gain (loss), user cost move together with this change for one percentage point as well. For first two parameters user costs move in the same direction, for the third one in opposite direction. For the case of changes in mortgage rate as well as property tax rate, the change in of user cost moves in the same direction, but in proportionally smaller amount depending on income tax rate. Higher income taxes cause smaller impact on user cost and vice versa. And finally income taxes itself, changes go in opposite direction but the magnitude is decided by sum of property tax and mortgage rate. As it is usual for these two parameters to be relatively low, the proportion of change cause by income tax change is generally low as well. On the other hand it is also true that income taxes could change significantly in time, even two digit percent differences, which makes it not negligible part of user cost calculation.

In matter of user cost sensitivity topic, we should also emphasize that this sensitivity to fundamental changes relatively grows for situations with generally low user costs. For example, one percentage point growth in maintenance cost cause one percentage point growth in user cost, which could be 33% growth if original user costs were 3 % or alternatively 20% growth if original user costs were 5 %. This
might not seem on first sight that crucial, but let us remind that user cost also could be used as discount factor for perpetuity determining price level of property or simply price to income ratio determined by user cost. Following chart demonstrate this relation between user cost size and its impact on property price level if changed.

![Graph](image)

**Figure 3.1: User cost and price level relation**

*Source:* author’s computations.

For example if original user costs were 1% and they grow on 2%, property price level is supposed to lose 50% of its value from 100 fold of user costs to only 50 fold (fall of price to rent ratio from 100 to 50). Of course this is extreme example, but this monumental price level drop was caused only by a little higher than one percentage point growth in interest rates (mortgage rate) or alternatively one percentage point decline of expected capital gain from real estate asset, which are definitely not extreme changes.

This property of user cost sensitivity for fundamental factors changes makes extremely interesting situations, with relatively low levels of user costs. As income and property taxes impact could be considered as relatively low. Additional risk premium and maintenance costs are not the highest components as well, but more importantly we consider them as quite stable in time. That leaves us with two fundamentals of high interest. Especially for situations with low interest rates and
high expected capital gains from real estate assets. These situations pushes on lowering user costs and thus creates environment of high sensitivity of real estate prices on changes in fundamental factors via user costs.

3.2.1 INTEREST RATES

The real interest rates are one of two key factors as already mentioned. But it is also important to point out to keep in mind, that its importance holds only for real rates. It is because the inflation part of nominal rate is offset by presence of the very same inflation in expected capital gain determinant, that goes in opposite direction against interest rates. Real interest rates thus represent capital costs for the property owner. Lower rates reduce overall user cost because of making debt financing cheaper and opportunity costs for investing to real estate is lower as well.

There is still one question which interest rates best suit for our purpose. Poterba (1984) employs mortgage rate, as more practical explanation of capital cost is preferred. This suggests that price of capital for real estate property investment is best estimated by direct mortgage rate. On the other hand Himmelberg, Mayer and Sinai (2005) argue for favor of long-term interest rates rather than short-term ones. Firstly they look at this determinant more in way of opportunity costs for invested capital. So they prefer risk-free rate instead, actually long-term interest rates to be accurate, as there is important to consider practical impact of expected future real interest rates on expected appreciation rate of future real estate prices. This argument could be demonstrated on example, when real short-term interest rates are well below real long-term interest rates, bond market then anticipate real short-term interest rates to rise in the future. As real short-term rates are expected to grow, user costs are expected to grow as well and thus lower future real estate prices. That will be projected to expected capital gain (loss) factor by lowering these expectations. Decline in future expectations should roughly offset the real long-term and real short-term rates spread. So interest rates choice should be indifferent if we take in consideration influence on future expectations. On the other hand we could argue, that spread between real long-term and real short-term rates might in some special

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8 Of course we mean here mortgage rates that are derived from short-term rates. For Czech republic generally in recent years holds that average mortgage rate index is approximately 2 percent points over real short-term rates. Source: cnb.cz, hypoindex.cz, own calculations.
cases prevail for a very long time and also, some gap will remain naturally, because of basic economic logic about time risk premium, unclosed forever. Another interesting fact about decline in real short-term interest rates (or mortgage rates) is that it does not only make debt financing cheaper and thus more affordable, but also cause growth of maximal debt limit borrower could reach. We demonstrate this on example, where household have gross income EUR 2,000 per month. That makes EUR 24,000 per year and if we estimate the solvency clarification process simply by multiplying their income by ratio 0.5 we obtain maximal possible installments EUR 12,000 per year. Now if we derive from that fix installments maximal debt limit as dependent on mortgage rate, we conclude significant growth. Let say, when mortgage rate is 4.5 %, maximal debt limit for such household is little over EUR 156,000, but if mortgage rate drop to 3.5 %, maximal debt exceed EUR 170,500 limit standing for 9.3 % growth.

Another effect of this limit shift is that some originally insolvent applicants become solvent now. This effect of real short-term rate definitely should have important influence on property price level. But it is also very important to care about stability of this effect. Because it works more like a swing effect multiplying overall effect of real short-term rates change in both directions. If rates grow again, limit drops, but more importantly some of borrowers could become insolvent and thus quit repaying their debt. Foreclosed properties would boost supply side and lead to another market price level contraction. At the end of the day, this effect might support choice of both rates. Long-term one in case of unstably low short-term interest rates situations, where it is expected return to higher and more stable level and this correction might result to significant price level drop. Or in case of possibly stable real short-term interest rates, when forces on price level resulting from short-term rates are more decisive than the long-term ones and then it is more accurate to employ those short-term rates.
We conclude here, that appropriate choice of interest rates depends mainly on their stability and spread. If the spread is high, we could expect short-term rates to be less stable and because of reasons mentioned above using short-term rate will provide us with overpriced estimations. On the other hand using long-term interest rate could underestimate effect of more affordable debt financing for cases of small and stable spread.

Appropriate choice of interest rate for calculating of user cost is especially crucial for situations with low interest rates, as we are currently experiencing. We decided then to use weighted average of long-term interest rates and current market mortgage rates index, with growing weight of long-term rates for cases with higher spread. This approach would also well cover situation, where spread between short-term and long-term rates is very small and stable. In such situation mortgage rates might be above long-term rates and because of stability we could not expect to change this state soon, then it would be logic to use mortgage rate with higher weight as this rate represent better capital cost for real estate property.

Figure 3.2: Mortgage rate and maximal debt limit

Source: author’s computations.
3.2.2 EXPECTED CAPITAL GAIN

The second key factor, influencing user cost most, is expected capital gain. Combination of its high influence on overall user cost and the fact of uncertainty included in future expectations dimension makes this fundamental central point of debate when constructing user cost estimation for particular markets. We should keep in mind that expected changes of all fundamental factors should influence expectations about future price of real estate asset and thus is incorporated already before expected change takes place. Another important aspect about this fundamental factor is that expectations construction subject to some basic psychology aspects such as memory effect or negative news asymmetry impact.

Another interesting thing is, if we follow basic economic theory, appreciation expectations should be especially high in locations with inelastic supply of real estates. This relation is also empirically proved for example by Himmelberg, Mayer and Sinai (2005). This is very common in cities or other locations with some concentration factor, where inelasticity is mainly the product of bounded land supply and construction regulations. These locations are highly interesting for us because of supply inelasticity support high expectations about future appreciation and pushes user cost down, making the market price level more sensitive to fundamental changes. Completely other story is on the market, where supply is highly elastic. On such market, mechanism of user cost needs to introduce other important fundamental factor, which is construction costs. Then two general scenarios could emerge. First, price to user cost ratio suggest lower price level than construction costs allows, so the assumption about elastic supply will not be employed in practice, because no one should pay price for newly constructed properties. Second and more interesting scenario is, when price to user cost ratio suggest higher price level than construction costs and if the market is sufficiently competitive thus arisen difference is not exploited, construction costs should be predominant price level setting factor. On such market, no one has to pay purchase price that price to user cost ratio suggest. In this kind of situation, user cost approach fail to assess rational price level correctly. It is permanently signaling underpriced situation, but market does not converge to equality state as construction costs does not let this happened. But we do not have to care about this shortcoming much, because these situations are in practice very rare, definitely when it comes to heavily populated locations such as big cities, which is the case of our empirical testing.
3.2.3 OTHER FUNDAMENTALS

From other fundamentals we start with depreciation rate or alternatively maintenance costs. Depends on point of view, because we could perceive this fundamental as both as value lost because of using the asset in time or as costs we need to pay for keeping the asset in initial condition state. In practice, both explanations are equally valid. If these costs are high, it causes real estate to lose its value quickly and makes holding asset more costly in time. And if rising, results in renting relatively cheaper to owning. This factor is perceived to be relatively stable in time and its annual value, is by Harding, Rosenthal and Sirmans (2004) estimated as 2.5 %. We will add to this estimation 0.3 % for insurance of the property and also add premium 0.5 % for panel structures as their life expectancy is considered to be shorter.

Next factor is income tax rate. Its growth lower user cost and makes owning relatively less expensive than renting. This is because of owner-occupied real estate provide higher tax deductions.

Property taxes are simply taxes imposed on the ownership of the property for owning the asset itself. So it is cost added to user costs and as property taxes rise, overall user cost rise as well. This makes owning of property more costly and if other remains unchanged, makes renting cheaper than owning. The little confusing about this fundamental is that these taxes are not only different internationally, but very often also regionally, for type and usage purpose of real estate property etc. We might feel lucky, as in the Czech Republic are these taxes very low. Generally around 0.03 % from property market price.

Last fundamental factor for user costs is additional risk premium. This is cost factor for capital invested in asset and thus its growth makes user cost to grow as well as investor demand to be compensated more for risk included in such investment. According to Flavin and Yamashita (2002) this additional risk premium is calculated as 2 %. However this estimate seems a bit too high. Real estate market, if considered long run horizon, is not most risky asset as well as we could look at property owning as kind of insurance against future undesirable changes in renting price levels.
3.2.4 USER COSTS VALUES

We will now discuss values of individual components for user costs. First is depreciation rate or maintenance cost, which is set as 2.8% as we already mentioned before. This value should be relatively stable in time.

Second is expected appreciation level. We are going to apply most recent data from CZSO about Prague flat prices development. These data suggest average annual appreciation from 2nd quarter 2005 to 2nd quarter 2012 to be 4.8%. This number consist of average 2.6% inflation in the same time interval and 2.2% left as real annual appreciation. But we should include psychology factor as recent information has bigger impact on expectations than historical ones. We achieve this by employing weighted average where every one period older information loss 10% of its importance for overall expectations. As we have 7 observations, this is given by equation:

\[
E[g_{t+1}] = \frac{\sum_{i=0}^{7} 0.9^i (g_{t-i})}{\sum_{i=0}^{7} 0.9^i}
\]

Now we obtain expected appreciation 3.29%, which should reflect present day situation better as market is experiencing stagnation or even prices decline for last four years. We did not care about negative news asymmetry impact, because it is due to time-series development naturally included in weighted average formula, as bad news years are the last four with highest weight.

Additional risk premium we are going to set at 1.75% level, as we lowered original Flavin and Yamashita (2002) estimation for reason of considering owning real estate property as insurance against undesirable growth in rent prices.

Income tax is set according to current personal income tax in Czech Republic, which is flat tax of 15%. Another taxing fundamental factor of property tax would be 1.75% premium.

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9 Harding, Rosenthal and Sirmans (2004) estimation + 0.3% for property insurance.

10 http://www.czso.cz/csu/redakce.nsf/i/ceny_bytu


12 Under tax law n. 586/1992 modified 1.1.2010
be a bit more complicated because this tax is for personal housing real estate properties calculated as multiple of regional coefficient, size of city coefficient, tax coefficient, tax tariff per square meter and floor size of property in squared meters\textsuperscript{13}. As all coefficients are the same for all segments in our selection, differences will be observable only due to floor size. Also we would like to express this yearly paid tax as fraction of overall price, as we obtain percentage value consistent with our user cost construction. In general, this value is the most insignificant one component of user costs as for our properties selection is average around 0.03 \%.

Last fundamental is interest rate. As we already discussed, probably most suitable would be weighted average of long-term rate and current mortgage rate, where weights depends on spread between those two rates. Weights should move in favor of long-term rate as spread grows and short-term rates become unstable and vice versa. In our case, as we calculate this average on only one date, it should be sufficient arithmetic average of these two rates. According to Fincentrum Hypoindex\textsuperscript{14}, average mortgage rate in 1st quarter 2013 was 3.2 \%. For long-term rate we put 3.875 \% as it is coupon on 10 year Eurobond 6th emission in February 2012\textsuperscript{15}. Average is then equal to 3.54 \%.

If we employ all these values to relation (8) we obtain following:

\[
u_{\text{ESR}} = 1.75 \% + 3.54 \% + 2.8 \% + 0.03 \% - 15 \% (0.03 \% + 3.54 \%) - 3.29 \% = 4.2945 \%
\]

We see that result of 4.3 \% is relatively low and thus relation between purchase and rent prices would be highly sensitive to any changes in fundamental variables. Mainly to those potentially volatile ones with relatively high impact – interest rate and expected capital gain. For example of its sensitivity, average mortgage rate\textsuperscript{16} in 05/2007 was 4.27 \% then one year later 05/2008 5.52 \%, leaving year to year difference 1.25 \% in mortgage rate and 0.4375 \% in user cost if other variables remain unchanged. In our case, user cost level would growth to 4.732 \%.

\textsuperscript{13} http://www2012.mfcr.cz/cps/rde/xchg/mfcr/xsl/dc2_dane.html

\textsuperscript{14} Weighted average of personal mortgage rates during current month. Weights are total debt volumes for individual banks included to calculation.

\textsuperscript{15} http://www2012.mfcr.cz/cps/rde/xchg/mfcr/xsl/podle_splatnosti.html#3-2013

\textsuperscript{16} http://www.hypoindex.cz/hypoindex-vyvoj/
and thus purchase price would drop for almost 10%. Such drop is not very common on year-to-year bases, on the other hand growth of mortgage rate around 1% is no exception, mainly if average mortgage rate is around 3.0% level as nowadays. Even worse might be situation, if we look at some values influencing capital gain expectations. For example Prague flats exercised price index\textsuperscript{17} grew from Q3/2006 to Q3/2007 by 32%. Of course we know today, that from Q3/2008 to Q3/2011 index lost almost 20%, so the exceptional growth pace was not sustainable. But as we apply losing memory approach, if we add such exceptional growth year to data as the last one, we obtain total value of expected growth over 6.5%, which is more than 3% over original value and also ceteris paribus pushing overall user cost to 1.3% value. This is absolutely unthinkable as influence on prices would be astronomical. More even we know that after such exceptional growth would be probably followed by downward price level correction.

Based on discussion in chapter 3.2.1 and 3.2.2 we might try to find a solution to such bottleneck by trading off some precision for stability of those two variables and thus for stability of overall user cost. For expected capital gain we might set a long-run real house price index growth from data of Bank for International Settlements (Hilbers, 2008) from 1985 to 2007 and connect calculations for next 5 years till 2012\textsuperscript{18}. Over those 27 years was real price index annual growth for European average performers\textsuperscript{19} 2.06%.

For interest rate we would accept hypothesis of main importance for long-term rates as difference to short-term one is covered by expectations in capital gain. This allows us to focus on long-run difference between inflation level and long-term interest rates. Eurobond with 10 years maturity\textsuperscript{20} had on secondary markets from 2002 to 2012 average yield 4.07% and for the same time series is average inflation\textsuperscript{21} in Czech Republic 2.35% leaving long-term real yield 1.72%. Now if we reapply stable values to user cost equation (8) we obtain:

\textsuperscript{17} http://www.czso.cz/csuz/redakce.nsf/i/ceny_bytu

\textsuperscript{18} http://www.bis.org/statistics/pp.htm

\textsuperscript{19} Sweden, Norway, Finland, Greece, Italy and Denmark. We are not using here data for Czech Republic as there is not comparably long time series.

\textsuperscript{20} http://epp.eurostat.ec.europa.eu/portal/page/portal/statistics/search_database

\textsuperscript{21} http://www.czso.cz/csuz/redakce.nsf/i/mira_inflace
\[ \mu_{LR} = 1.75\% + 4.07\% + 2.8\% + 0.03\% - 15\%(0.03\% + 4.07\%) - (2.06\% + 2.35\%) = 3.625\% \]

It is even logical substitution for long-term stable averages as we expect that property is hold by owner for infinity. We might lose some important psychological aspects that definitively are important for setting price levels in short-run, on the other hand this special user cost option could be seen as long-run anchor around which short-run user cost would due to those psychological aspects oscillate in time.

If we compare our final values of user costs for long and short run, we observe that long-run user costs are even indicating higher purchase prices than short-run user costs. It seems that lower expected capital gains more than offset very low mortgage rate values for short-run user costs estimation.
4 EMPIRICAL MODELING OF REAL ESTATE PRICE LEVELS

In this part we are going to empirically model price levels of purchase and renting on Prague real estate market. More precisely in medium flat segment excluding new built structures. We need those two price levels to be able to assess whether exist mispricing on Prague real estate market, which might indicate ongoing price bubble. As we already mentioned, significant advantage of estimation on our own is the fact, that models should provide very precise price levels for detailed selection of parameters describing the property and also that these parameters will be same for both purchase and rent price levels and thus will provide consistent relation between those two levels.

4.1 DATA

For purposes of estimation price of purchase and renting by hedonic models, we use as data source public website sreality.cz with the widest range of public offers for both, purchases as well as renting real estate properties in the Czech Republic. On this server are very easily accessible data for every single offered real estate property as price of purchase or alternatively price of rent, floor size, GPS location, condition of property, if garage or other services are included etc. These are the data we will employ in our models, but there is also one inconvenience we should not forgot about. These data are mainly representing supply side of the market. Some advertisement offers could not be exercised for very long time as they are overpriced, some of them even never. The general estimation by CZSO is that this difference moves in time around 5 – 10 percent\(^2\). As it is very problematic to obtain realized data, the main aim of this study is not price estimation itself, but its relation with price of renting, which are probably dealing with the very same situation. That means both overpricing premiums will be mutually put off in fraction and have no problem to compare such value to user costs.

For strengthen the consistency of estimated models we decided to put some restrictions on data selection. These restrictions are imposed in order to keep as much heterogeneity as possible only within model variables, but exclude all other factors that are not covered in model or are against main model logic. These restrictions on offer advertisements are:

- No older than 7 days from date of data gathering – 09.05.2013 – for data time consistency.
- Only offers located in Prague, for flat segment with floor size from 50 to 80 squared meters in inhabitable condition. Because of property type consistency.
- We exclude new buildings, built in 2012 and later.
- We include only personal owned properties to leave out ownership type influence.
- Rents offered only for long term rent possibility. Short-term accommodation has totally different price level and aspects influencing price of rent, so we strictly exclude those offers.

At the end, after data trim and preparation, there left 172 cases for price of purchase estimation model and 380 cases for rent price estimation. Now, we will describe individual explanatory variables, as they are with one exception common for both models.

4.1.1 CONDITION

This variable captures quality condition of property as whole. Mainly the state of wear rate as well as quality of equipment with special importance paid to kitchen and bathroom. This variable is graded in percent scale with following levels:

- 100 % means after overall reconstruction with well looking kitchen and bathroom and none or almost none wear rate.
- 75 % is still pretty good shape, well looking kitchen and bathroom with some noticeable rate of wear.
• 50% inhabitable, but with considerable rate of wear and prevailing older equipment.

• 25% properties with these grading are those with really high rate of wear and generally needs some kind of reconstruction or other construction treatment.

4.1.2 DISTANCE

This variable is, as its name suggests, distance measure from property location to city center indicated by Old Town Square chosen as historical, geographical and cultural central point of the Prague city. Logic of this variable is based on Von Thunen property location theory that concludes two important features we employ:

• Land and property values tend to decrease with increasing distance from city center due to growing transportation costs.

• The land value is one of concentric zones pattern from the city center.

For measuring distance itself we employ great-circle distance method. This method measures shortest distance between two points on a sphere based on GPS location. For data itself it means that firstly we have to convert GPS coordinates to decimal degrees, than calculate angle $c$ in radians between two points using formula:

$$ c = \arccos (\sin(\text{latitude } A) \cdot \sin(\text{latitude } B) + \cos(\text{latitude } A) \cdot \cos(\text{latitude } B) \cdot \cos (|\text{longtitude } A - \text{longtitude } B|)) $$

To obtain final points distance we need to multiply angle $c$ with sphere radius. In our case is sphere Earth, which is actually due to its rotation oblate ellipsoid with radius from 6,357 km on poles to 6,378 km on equators. Around latitude 50 degrees where Prague lies, radius should be approximately around 6,369 km and this is also number we use to multiply angle $c$. But the precise radius of Earth is not that much important for us anyway, because for us is important difference between measures for individual cases and for that great-circle formula provide us very good estimation. For models itself we use distances in meters.
4.1.3 FLOOR SIZE AND DUMMY VARIABLES

Rest of explanatory variables are very easy and straightforward. Floor size stands for internal floor size of the property and is expressed in squared meters. All other variables are dummy type, that means they signal whether individual property has this characteristics or not. These dummy variables are:

- **Garage** – signaling if the garage place is part of the property, which could have significant positive impact on total price as garages have their own value itself.

- **Prefabrcated structure** – shows if the building in which is individual property is build from prefabricated panels parts. Other buildings are mostly built from bricks or other materials. Generally is viewed at panel structures as less quality buildings with shorter life expectancy. So it should have negative effect on total value.

- **Ground level** – this variable stands for the ground level or first lower floor level properties. Flats on these lowest floor levels provide less intimacy and generally provide almost none view. Also often there is less sun-time during the day and other factors as more often higher humidity etc. These are the reasons why properties with this characteristic should have slightly lower price or rent than other equivalents on higher floor levels.

- **Furniture** – this last dummy variable is employed for rent price model only. We expect positive effect of such variable on price of renting property, as owner has to be compensated for equipment usage.

4.1.4 PRICE OF PURCHASE

Price of purchase is dependent variable in our first type model. It is generally the price demanded for selling of the ownership rights for real estate property. But there were two important adjustments that had to be done. First, some of the prices were listed with and some without real estate agency fees. For those prices with real estate agency fees we subtracted them as estimated 5 % from listed price. Second adjustment is because prices distribution has naturally right skewness. That is because prices could not be negative on the left side, but on the right side, there is
generally no limit. This characteristic needs to be dealt with because of using ordinary least square estimation. This skewness might and actually cause heteroskedasticity to model residuals, which is unpleasant. Natural logarithm of the variable would help us significantly as it modifies data in wanted direction. Logarithm regression cause loss of simple and straightforward results explanation, but that is reasonable price for more precise estimations.

4.1.5 PRICE OF RENT

Price of rent is dependent variable in our second type model. This is the very same case as price of purchase, it is the price listed in offers as price for renting the property. For easier application in further analysis we multiply rent values that are listed as monthly figure by 12 to obtain rent price for a year. The other adjustments are the same as for purchase price. Some of the prices were listed including with fees for energy and other service payments. Luckily in almost all cases, where rent prices were not listed alone, information about value of these fees was included in description. So it was easy to deduct them directly. And again as for purchase price, distribution of rents has right skewness property. Natural logarithm again should solve this problem for us.

4.2 MODELS

In this chapter we describe the methodology that lies behind modeling two main pillars that we are going to compare at last. An estimation approach for market price of purchase level is the same as for market price of renting level and this is linear and loglinear regression by ordinary least square method.

4.2.1 ASSUMPTIONS

As already mentioned, we are going to apply linear and loglinear regression models using ordinary least square method allowing us to identify decisive factors
influencing real estate price levels and finding out their coefficients that we will be able to employ later in next steps. But first to apply ordinary least square method correctly, following assumptions need to be fulfilled:

I. Model is linear in parameters

II. The data are a random sample of the population. The errors are statistically independent from one another

III. The expected value of the errors is always zero

IV. The independent variables are measured precisely

V. The independent variables are not too strongly collinear

VI. The residuals have constant variance

VII. The errors are normally distributed

First and third assumptions are fulfilled from model execution. With second assumption, for selected segment of data, we generally used all cases available, so we cover all population. Also other part of second assumption is no concern for us as we estimate cross-sectional data. We already mentioned problem with fourth assumption (supply data), but unfortunately there is nothing to do about it and also for the cause we are going to need models results, it is not a big deal. From fifth to seventh assumptions we will test ad hoc for each and every model individually.

4.2.2 PRICE MODEL 1

First model is defined by following equation:

\[
\text{priceCZK} = \alpha + \beta_1 \text{condition} + \beta_2 \text{distance}_m + \beta_3 \text{floor}\_\text{size}\_m^2 \\
+ \beta_4 \text{garage}_D + \beta_5 \text{prefabricated}\_\text{structure}_D \\
+ \beta_6 \text{ground}\_\text{level}_D + u
\]
Below we could observe results for model 1. We have obtained very high R-squared over 68%. This high number indicates that this model covers great deal of variability of dependent variable. Also p-value of ANOVA testing suggests that model covers variability well. We also observe significance on 99% significance level for all coefficients excluding coefficient for variable garage and ground level, which is the only insignificant variable in the model. Interesting is negative value for constant, but when we take closer look at the results, we could observe that the role of constant for model appropriated dummy variables with variable describing condition of property. The coefficient for the last named variable suggests over 13,000 CZK change in total price for every one percent point of condition score. That makes more than 1.3 million CZK constant for property with 100% condition. For distance variable resulting coefficient inform us, that every meter step from Old Town Square will cost us almost 76 CZK on the total price. Price for every square meter also seems very reasonable, as 42,290 CZK more or less corresponds with current market price. Price for garage place is approximately 225,000 CZK. Discount for prefabricated panel building is more than half of million and discount for ground floor almost ninety thousands. In fact, for all variables there is direction of slope as we expected and also price tags seem very reasonably as well. But we should check the assumptions as we might have a problem with skewness of explained variable.

Model 1: OLS, using observations 1-172

Dependent variable: priceCZK

Table 4.1: Model’s 1 results

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Std. Error</th>
<th>t-ratio</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>const</td>
<td>-279546</td>
<td>320285</td>
<td>-0.8728</td>
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<tr>
<td>condition</td>
<td>13815.8</td>
<td>2002.27</td>
<td>6.9001</td>
</tr>
<tr>
<td>distance_m</td>
<td>-75.811</td>
<td>15.0743</td>
<td>-5.0292</td>
</tr>
<tr>
<td>floor_size_m2</td>
<td>42290.8</td>
<td>3625.2</td>
<td>11.6658</td>
</tr>
<tr>
<td>garage_D</td>
<td>225175</td>
<td>131535</td>
<td>1.7119</td>
</tr>
<tr>
<td>prefabricated_structure_D</td>
<td>-575217</td>
<td>83534.1</td>
<td>-6.8860</td>
</tr>
<tr>
<td>ground_level_D_D</td>
<td>87095.8</td>
<td>85437.4</td>
<td>-1.0194</td>
</tr>
<tr>
<td>Parameter</td>
<td>Value</td>
<td></td>
<td></td>
</tr>
<tr>
<td>---------------------------</td>
<td>----------------</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean dependent var</td>
<td>2838313</td>
<td></td>
<td></td>
</tr>
<tr>
<td>S.D. dependent var</td>
<td>737385.7</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sum squared resid</td>
<td>2.90e+13</td>
<td></td>
<td></td>
</tr>
<tr>
<td>S.E. of regression</td>
<td>419000.1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>R-squared</td>
<td>0.688451</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Adjusted R-squared</td>
<td>0.677122</td>
<td></td>
<td></td>
</tr>
<tr>
<td>F(6, 165)</td>
<td>60.76857</td>
<td></td>
<td></td>
</tr>
<tr>
<td>P-value(F)</td>
<td>2.78e-39</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Log-likelihood</td>
<td>-2467.132</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Akaike criterion</td>
<td>4948.264</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Schwarz criterion</td>
<td>4970.296</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Hannan-Quinn</td>
<td>4957.203</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Source: author’s computations, Gretl SW

As we tested assumptions number 5 to 7. Results are unfortunately as we expected negative. Both tests for heteroskedasticity, White’s test (A) as well as Breusch-Pagan (B) gives us information, that null hypothesis of homoskedasticity is rejected. Also null hypothesis of normally distributed residuals is strongly rejected (C). Only collinearity test (D) gives us positive information of none collinearity presence. Heteroskedasticity and not normally distributed residuals could be problem for estimated coefficients precision as well for their validity. We need to solve this situation before we use estimated model for determination of price of purchase level.

(A)

White's test for heteroskedasticity -

Null hypothesis: heteroskedasticity not present

Test statistic: LM = 41.4258

with p-value = P(Chi-square(23) > 41.4258) = 0.0105818

(B)

Breusch-Pagan test for heteroskedasticity -

Null hypothesis: heteroskedasticity not present

Test statistic: LM = 16.0314

with p-value = P(Chi-square(6) > 16.0314) = 0.0135866

(C)
Test for normality of residual –

Null hypothesis: error is normally distributed

Test statistic: Chi-square(2) = 11.2207

with p-value = 0.00365973

(D)

Variance Inflation Factors

Minimum possible value = 1.0

Values > 10.0 may indicate a collinearity problem

Condition 1.127
Distance_m 1.543
Floor_area_m2 1.103
Garage_D 1.100
Prefabricated_structure_D 1.676
Ground_level_D 1.083

VIF(j) = 1/(1 - R(j)^2), where R(j) is the multiple correlation coefficient between variable j and the other independent variables.

4.2.3 PRICE MODEL 2

Second model is very similar to the first one. In fact it is defined with the same equation with only one slight difference, on explained variable. As almost all assumptions tested for previous model were violated, we need to pay attention to that topic. Residuals suffered from unstable variance and non-normal, right skewed, distribution. We try to repair these shortcomings by introducing natural logarithm of explained variable. So the equation that holds this time is following:
\[ \ln\text{price}_CZK = \alpha + \beta_1 \text{condition} + \beta_2 \text{distance}_m + \beta_3 \text{floor\_size}_m^2 \]
\[ + \beta_4 \text{garage}_D + \beta_5 \text{prefabricated\_structure}_D \]
\[ + \beta_6 \text{ground\_level}_D + u \]

Again, in table below, we obtained results of OLS method for equation above. We have even higher R-squared values with adjusted over 70%. This number suggests that we have explained a great deal in modeled variance. ANOVA test result also provides us with positive information about model ability to capture variability of dependent variable. Even all variables strengthen their significance values including garage variable, which is now significant on higher significance level, and ground level variable, that remained insignificant, but came closer to significance threshold.

In comparison to first model, there is one important difference, this time constant regained its natural position of relatively high positive base number. That happened mainly due to rescaling of explained variable. All other coefficients roughly remained same in direction as well as their relative size. This time is for us more important if the model accomplished to stick to assumptions better than the one before and thus we could declare its results consistent.

Model 2: OLS, using observations 1-172

Dependent variable: ln\_priceCZK

**Table 4.2: Model’s 2 results**

<table>
<thead>
<tr>
<th></th>
<th>Coefficient</th>
<th>Std. Error</th>
<th>t-ratio</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>const</td>
<td>13.7727</td>
<td>0.10233</td>
<td>134.581</td>
<td>&lt;0.0000</td>
</tr>
<tr>
<td>condition</td>
<td>0.00478</td>
<td>0.00063</td>
<td>7.4752</td>
<td>&lt;0.0000</td>
</tr>
<tr>
<td>Distance_m</td>
<td>-2.63842</td>
<td>4.81655</td>
<td>-5.4778</td>
<td>&lt;0.0000</td>
</tr>
<tr>
<td>floor_size_m2</td>
<td>0.01426</td>
<td>0.00115</td>
<td>12.3146</td>
<td>&lt;0.0000</td>
</tr>
<tr>
<td>garage_D</td>
<td>0.08710</td>
<td>0.04202</td>
<td>2.0725</td>
<td>0.03977</td>
</tr>
<tr>
<td>prefabricated_structure_D</td>
<td>0.19484</td>
<td>0.02669</td>
<td>-7.3000</td>
<td>&lt;0.0000</td>
</tr>
</tbody>
</table>
We test assumptions 5-7 about collinearity, constant error variance and normal distribution of errors. First assumption tested by (D) was not a problem in previous model and is not a problem now as well. According to two executed tests for assessing constant variance in errors, White’s test (A) as well as Breusch-Pagan (B), we could not reject null hypotheses of homoscedasticity. That is definitively desirable change against model 1. Also last assumption about normally distributed errors holds. Test (C) cannot reject null hypothesis about normality. All the tests came out as we desired, so we ended up with estimated coefficient that are consistent with used method of estimation and thus we could employ them in further analysis.

(A)

White's test for heteroskedasticity -

Null hypothesis: heteroskedasticity not present

Test statistic: $LM = 29.0815$

with $p$-value $= P(\text{Chi-square}(23) > 29.0815) = 0.177622$

(B)

Breusch-Pagan test for heteroskedasticity -
Null hypothesis: heteroskedasticity not present

Test statistic: $LM = 8.24299$

with p-value $= P(\text{Chi-square}(6) > 8.24299) = 0.220836$

(C)

Test for normality of residual -

Null hypothesis: error is normally distributed

Test statistic: $\text{Chi-square}(2) = 3.25606$

with p-value $= 0.196316$

(D)

Variance Inflation Factors

Minimum possible value $= 1.0$

Values $> 10.0$ may indicate a collinearity problem

Condition 1.127

Distance_m 1.543

Floor_area_m2 1.103

Garage_D 1.100

Prefabricated_structure_D 1.676

Ground_level_D 1.083

$\text{VIF}(j) = \frac{1}{1 - R(j)^2}$, where $R(j)$ is the multiple correlation coefficient between variable $j$ and the other independent variables.
4.2.4 RENT MODEL 3

With this model we will try to gain instrument for price level of renting modeling. Model 3 is defined by following equation:

\[
\frac{rent}{Y CZK} = \alpha + \beta_1 \text{condition} + \beta_2 \text{distance}_m + \beta_3 \text{floor}_\text{size}_m^2 \\
+ \beta_4 \text{garage}_D + \beta_5 \text{furniture}_D \\
+ \beta_6 \text{prefabricated}\_\text{structure}_D + \beta_7 \text{ground}\_\text{level}_D + u
\]

The results are listed in table below. $R^2$-squared value suggests that this model explains less variability of dependent variable than previous models, but in general it is still very good model with over 55 % of variance explained. Loss of some variance explanation power could be caused by the fact, that landlords could have slightly different tactics about pricing in case of rent and fees separation. They in some cases input some rent fraction to fees, in order to make the rent price more competing and others just announce rent price as whole, fees included, with no need to bias rent and fees values. But ANOVA p-value also provides us with positive information about model competence. Most of the explanatory variables show themselves to be significant on highest level, which is also positive information.

Similarly to model 1, it is a bit surprising that constant is negative, but there is the same explanation, as before. Constant natural position in model took over dummy variables and condition variable. Condition coefficient concludes that for every percentage point in condition grade score tenant should pay almost 900 CZK per year, this makes 100 % condition property base of 89,490 CZK rent per year. Distance from city center is expressed with relation of rent decline around 3,300 CZK for every kilometer in year terms. Another coefficient suggests that for every rented square meter we should be yearly paying extra 1,239 CZK. Furniture and garage variable indicate that for extra services such as renting furniture or a garage parking place will approximately cost you extra 1,000 CZK, respectively 2,000 CZK per month. For renting a flat in panel prefabricated building or on ground floor level we should get discount as expected, but for those two variables, influence is not significant. Again as in model 1, we obtain reasonable results with directions as expected, but we still need to test assumptions whether the model is consistent with estimation method employed.
Model 3: OLS, using observations 1-380

Dependent variable: Rent/YCZK

**Table 4.3: Model’s 3 results**

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Std. Error</th>
<th>t-ratio</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
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<td>9493.8</td>
<td>-0.7736</td>
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<tr>
<td>condition</td>
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<td>66.9954</td>
<td>13.3578</td>
</tr>
<tr>
<td>distance_m</td>
<td>-3.30842</td>
<td>0.44940</td>
<td>-7.3617</td>
</tr>
<tr>
<td>floor_size_m</td>
<td>1239.26</td>
<td>111.711</td>
<td>11.0934</td>
</tr>
<tr>
<td>garage_D</td>
<td>25681.2</td>
<td>6060.08</td>
<td>4.2378</td>
</tr>
<tr>
<td>furniture_D</td>
<td>13442.5</td>
<td>2115.99</td>
<td>6.3528</td>
</tr>
<tr>
<td>prefabricated_structure_D</td>
<td>4106.85</td>
<td>3200.28</td>
<td>-1.2833</td>
</tr>
<tr>
<td>ground_level_D</td>
<td>-</td>
<td>3047.24</td>
<td>-0.0520</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Mean dependent var</th>
<th>S.D. dependent var</th>
<th>30313.22</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sum squared resid</td>
<td>1.49e+11</td>
<td>20012.97</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.572178</td>
<td>0.564128</td>
</tr>
<tr>
<td>F(7, 372)</td>
<td>71.07442</td>
<td>9.45e-65</td>
</tr>
<tr>
<td>Log-likelihood</td>
<td>-4298.726</td>
<td>8613.451</td>
</tr>
<tr>
<td>Schwarz criterion</td>
<td>8644.973</td>
<td>8625.959</td>
</tr>
</tbody>
</table>

*Source: author’s computations, Gretl SW*
If we observe results for assumptions tests, the only one we have no problem with is collinearity testing one (D). Both test (A) and (B) strongly rejects null hypothesis of heteroskedasticity not present. Also, test for normally distributed residuals strongly rejects this assumption and we have to deal with those violations in following model for the second time.

(A) 

White's test for heteroskedasticity -

Null hypothesis: heteroskedasticity not present

Test statistic: \( LM = 47.1748 \)

with p-value = \( P(\text{Chi-square}(30) > 47.1748) = 0.023923 \)

(B) 

Breusch-Pagan test for heteroskedasticity -

Null hypothesis: heteroskedasticity not present

Test statistic: \( LM = 24.9642 \)

with p-value = \( P(\text{Chi-square}(7) > 24.9642) = 0.000769954 \)

(C) 

Test for normality of residual -

Null hypothesis: error is normally distributed

Test statistic: \( \text{Chi-square}(2) = 13.0186 \)

with p-value = 0.00148955

(D) 

Variance Inflation Factors

Minimum possible value = 1.0

Values > 10.0 may indicate a collinearity problem

Condition 1.090
EMPIRICAL MODELING OF REAL ESTATE PRICE LEVELS

Distance_m  1.337
Floor_area_m2  1.077
Garage_D  1.066
Furniture_D  1.037
Prefabricated_structure_D  1.476
Ground_level_D  1.041

VIF(j) = 1/(1 - R(j)^2), where R(j) is the multiple correlation coefficient between variable j and the other independent variables.

4.2.5  RENT MODEL 4

In this model we are trying to get rid of assumptions violations by using loglinear regression. Model 4 is thus defined by following equation:

\[ \ln\frac{rent}{Y CZK} = \alpha + \beta_1 \text{condition} + \beta_2 \text{distance}_m + \beta_3 \text{floor}_size_m2 + \beta_4 \text{garage}_D + \beta_5 \text{furniture}_D + \beta_6 \text{prefabricated}_structure_D + \beta_7 \text{ground}_level_D + u \]

The results of OLS method for equation above are listed in following table. We have even higher R-squared values with adjusted over 56%. This number suggests that we have explained over half of modeled variance. ANOVA test result is also positive as it suggests that model has high ability to capture variability of dependent variable. Even all variables strengthen their significance values including prefabricated structure and ground level variables, which on the other hand remained insignificant, but came closer to significance threshold. Needed to say, that the second named variable remained very strongly insignificant.

For case of constant, there is very similar development as for the first adjustment from the model 1 to model 2, where within loglinear regression model intercept regained its natural position of relatively high positive base number. That again happened mainly due to rescaling of explained variable. In comparison with model 3, all other coefficients roughly remained the same in direction as well as their relative size. To the comparison of results for model 2 and model 4 we will pay more
attention in next chapter. Now more importantly for us we need to check assumptions if the model is consistent this time.

Model 4: OLS, using observations 1-380
Dependent variable: ln_rent/YCZK

Table 4.4: Model’s 4 results

<table>
<thead>
<tr>
<th></th>
<th>Coefficient</th>
<th>Std. Error</th>
<th>t-ratio</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>const</td>
<td>10.7248</td>
<td>0.068225</td>
<td>157.1972</td>
<td>&lt;0.00001***</td>
</tr>
<tr>
<td>condition</td>
<td>0.00665246</td>
<td>0.000481447</td>
<td>13.8177</td>
<td>&lt;0.00001***</td>
</tr>
<tr>
<td>distance_m</td>
<td>-2.28193e-05</td>
<td>3.22957e-06</td>
<td>-7.0657</td>
<td>&lt;0.00001***</td>
</tr>
<tr>
<td>floor_size_m2</td>
<td>0.00924848</td>
<td>0.000802782</td>
<td>11.5205</td>
<td>&lt;0.00001***</td>
</tr>
<tr>
<td>garage_D</td>
<td>0.154413</td>
<td>0.0435494</td>
<td>3.5457</td>
<td>0.00044***</td>
</tr>
<tr>
<td>furniture_D</td>
<td>0.0915367</td>
<td>0.0152061</td>
<td>6.0198</td>
<td>&lt;0.00001***</td>
</tr>
<tr>
<td>prefabricated_structure_D</td>
<td>-0.031178</td>
<td>0.0229981</td>
<td>-1.3557</td>
<td>0.17601</td>
</tr>
<tr>
<td>ground_level_D</td>
<td>0.00632776</td>
<td>0.0218983</td>
<td>0.2890</td>
<td>0.77277</td>
</tr>
</tbody>
</table>

Mean dependent var: 11.78689
S.D. dependent var: 0.218032

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Sum squared resid</td>
<td>7.694363</td>
<td>S.E. of regression</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.572936</td>
<td>Adjusted R-squared</td>
</tr>
<tr>
<td>F(7, 372)</td>
<td>71.29485</td>
<td>P-value(F)</td>
</tr>
<tr>
<td>Log-likelihood</td>
<td>201.7432</td>
<td>Akaike criterion</td>
</tr>
<tr>
<td>Schwarz criterion</td>
<td>-355.9650</td>
<td>Hannan-Quinn</td>
</tr>
</tbody>
</table>

Source: author’s computations, Gretl SW

The tested assumption (D) about linear independence of explanatory variable is fulfilled as in all other models. We mainly care about other two assumptions and their tests. The assumption about constant variance is tested by White’s (A) and Breusch-Pagan (B) tests. This time we could not reject null hypotheses in both tests by far. These results suggest residuals to be homoskedastic. In last test (C) we also
could not reject null hypothesis about normality of residuals and that result concludes that for this model are all assumptions fulfilled as we desired.

(A)
White's test for heteroskedasticity -  
Null hypothesis: heteroskedasticity not present 
Test statistic: $LM = 26.9989$  
with p-value $= P(\text{Chi-square}(30) > 26.9989) = 0.62333$

(B)
Breusch-Pagan test for heteroskedasticity -  
Null hypothesis: heteroskedasticity not present 
Test statistic: $LM = 6.67566$  
with p-value $= P(\text{Chi-square}(7) > 6.67566) = 0.463413$

(C)
Test for normality of residual -  
Null hypothesis: error is normally distributed 
Test statistic: $\text{Chi-square}(2) = 1.08284$  
with p-value $= 0.581922$

(D)
Variance Inflation Factors  
Minimum possible value $= 1.0$  
Values $> 10.0$ may indicate a collinearity problem  
Condition $1.090$  
Distance_m $1.337$  
Floor_area_m$^2$ $1.077$
Garage_D  1.066
Furniture_D  1.037
Prefabricated_structure_D  1.476
Ground_level_D  1.041

\[ \text{VIF}(j) = \frac{1}{1 - R(j)^2}, \] where \( R(j) \) is the multiple correlation coefficient between variable \( j \) and the other independent variables.

### 4.2.6 MODELS COMPARISON

Now we are about to compare differences between models 2 and 4, as we count with employing these models for estimating price levels of chosen real estate segments. These differences could have important influence on distinction and explanation of differences of total price levels estimated.

Starting with most important difference between models, we compare prefabricated structure variable as it was highly significant in price model, but it is not in rent model. This difference suggests, that the fact if the flat is located in panel prefabricate structure has negative impact if we are purchasing this property, but it has no significant effect if we are renting it. This could be explained by shorter life expectancy of the panel structures and thus introducing additional risk in case of ownership, owner wants to be compensated for. But in case of renting, tenant is not bearing such risk of shorter life expectancy, she just could move away if such situation happens so there is no need to be compensated for this risk. Similar situation is in case of garage variable where changed significance level in favor of higher importance is for renting case. This is a bit more difficult difference to explain. Probably it would be caused by the fact that renting garage is in some way luxury service and thus it stands for luxury aspect of property and is priced accordingly. But when we are purchasing property with garage, this decision stands before more rational consideration if we are really willing to pay the sum for this feature. Panel prefabricated structure and garage where two main differences in pricing models, but also other factors have relatively different impact on price level estimation in case of purchase and renting, as condition of property has relatively higher impact in case of renting than purchasing, this could be explained with the fact that in case of owning the property its reconstruction is possible in most cases for relatively smaller fraction
of purchase price so it has relatively smaller importance than in case of renting, when it is practically unthinkable to reconstruct the premises as we are not the owners. Also in case of renting the condition state is more important for us, because we will be probably living in the rented property (long run re-renting of noncommercial properties is very scares, because there is very small value added from agent side giving only little service premium). But in case of purchase, we could alternatively rent the property to tenant, makes the condition state personally less important for us.

In the case of distance, the relative impact on price is more or less the same, which is interesting, because one could expect that there might be in different locations different expectations about capital gains from owning the property and thus different compensation for owning the property reflected in rent, but this effect could be offset by relatively higher rents to purchase price on high distance location, where is expected capital gain from property lower as there is more free space for new constructions. Also this approximately equality is consistent with Von Thunen location theory about price differentiation in distance due to transport costs, that are the same for owning or renting case. The last variable to compare is floor size in squared meters. This variable is more relevant for purchase pricing case. Again the explanation will be in logic similar to prefabricated panel structure but vice versa case. If we are renting floor size, our motivation is only usage of this space and we also needs to pay for whole floor size no matter if we utilize it or not, so more floor space than needed is undesirable. On the other hand in purchase case we could expect additional capital gains from more floor size. From that point of view, all floor size is utilized at least for that one reason for the owner. So more floor size would be more desirable for the owner.
5 EMPIRICAL ANALYSIS

Now as we have estimated models for price of purchase and price of renting levels. We also have final formula for calculating user costs. Finally we employ these procedures to estimate values of those three pillars in order of comparing them and assess whether market is mispriced or not.

We will process estimations, calculations and comparisons for three different Prague real estate market segments, which could be covered with our models:

A. “standard quality periphery flat”
B. “low quality, medium city small flat”
C. “high quality inner city larger flat”

We prepared two user costs using same formula but with slightly different logic for its components. First one, we call short-run user cost, which cover some psychological aspects as memory and negative information bias. Also incorporate growing debt limit with decreasing mortgage rates. Disadvantage is relatively too high sensitivity to changes in its components. In case of this sensitive user cost we try to take in consideration individual differences between segments, so we add 0.5 % premium appreciation for high quality city center segment, because the supply should be strongly inelastic\textsuperscript{23}. On the other hand we will subtract 0.5 % for standard quality periphery segment, as they have to compete to new higher quality developers projects in the area\textsuperscript{24}. Last segment we leave as it is with no premium or discount. Next we add 0.5 % to maintenance cost for prefabricated panel constructions as we expect shorter life expectancy.

\textsuperscript{23} Himmelberg, Mayer, Sinai 2005
\textsuperscript{24} dtto
Second one is long-run user cost representing long-term anchor for user costs. This one does not incorporate psychological aspects or sentiments in exchange for high stability as we fixed two mostly volatile and influencing components with long-run averages. Its sensitivity is only for changes in tax rates, which have relatively small impact. We would not customize this cost for different segments at all.

For final assessment whether there is considerable inequality on the market segment, we calculated approximately 90% confidence interval boarders by adding and subtracting 1.5 of rents standard deviation. These standard deviations were calculated by standard procedure for around 40 most similar segment type cases in dataset for rents modeling. We consider this confidence interval as signaling of statistical significant price inequality if user cost is outside critical values.
5.1 STANDARD QUALITY PERIPHERY FLAT

This is the type representing most common segment of real estate housing property in Prague. It is defined as medium size – 68 m², normal quality – 75 % condition state with standard wear rate, in high distance from city center – 10.000 meters, this distance roughly includes huge residence settlements as Háje, Letňany, Zličín, Černý Most and similar, where live highest share of Prague population. We set most common type of building for these locations - prefabricated panel structure without garage and furniture. And it is not located on ground floor.

Table 5.1: Type’s A results

<table>
<thead>
<tr>
<th>Real Estate Flat Price &amp; Rent Level Calculator - 9.05.2013 data</th>
<th>Inputs</th>
</tr>
</thead>
<tbody>
<tr>
<td>CONDITION</td>
<td>75%</td>
</tr>
<tr>
<td>FLOOR (50-80 m²)</td>
<td>68</td>
</tr>
<tr>
<td>DISTANCE (metres)</td>
<td>10,000</td>
</tr>
<tr>
<td>GARAGE (dummy)</td>
<td>0</td>
</tr>
<tr>
<td>FURNITURE (dummy)</td>
<td>0</td>
</tr>
<tr>
<td>PREFABRICATED STRUCTURE (dummy)</td>
<td>1</td>
</tr>
<tr>
<td>GROUND LEVEL (dummy)</td>
<td>0</td>
</tr>
<tr>
<td>Results</td>
<td></td>
</tr>
<tr>
<td>PRICE Estimation</td>
<td>CZK 2,286,558</td>
</tr>
<tr>
<td>RENT per year Estimation</td>
<td>CZK 108,384</td>
</tr>
<tr>
<td>RENT per month Estimation</td>
<td>CZK 9,032</td>
</tr>
<tr>
<td>Rent to Price ratio</td>
<td>4.74%</td>
</tr>
<tr>
<td>RtP + 1.5 standard deviation</td>
<td>5.95%</td>
</tr>
<tr>
<td>RtP - 1.5 standard deviation</td>
<td>3.78%</td>
</tr>
<tr>
<td>Price to Rent ratio</td>
<td>21.1</td>
</tr>
<tr>
<td>User Cost Calculator</td>
<td></td>
</tr>
<tr>
<td>Depreciation rate or Maintenance cost SR/LR</td>
<td>3.3%</td>
</tr>
<tr>
<td>Expected appreciation level SR/LR</td>
<td>2.79%</td>
</tr>
<tr>
<td>Additional risk premium</td>
<td>1.75%</td>
</tr>
<tr>
<td>Personal income tax</td>
<td>15.0%</td>
</tr>
<tr>
<td>Property tax</td>
<td>0.04%</td>
</tr>
<tr>
<td>Cost of capital SR/LR</td>
<td>3.54%</td>
</tr>
<tr>
<td>Results</td>
<td></td>
</tr>
<tr>
<td>User cost - SR</td>
<td>5.10%</td>
</tr>
<tr>
<td>Inverse User cost - SR ratio</td>
<td>19.6</td>
</tr>
<tr>
<td>User cost - LR</td>
<td>4.01%</td>
</tr>
<tr>
<td>Inverse User cost - LR ratio</td>
<td>24.9</td>
</tr>
<tr>
<td>Mispricing SR User cost</td>
<td>7.5%</td>
</tr>
<tr>
<td>Mispricing LR User cost</td>
<td>-15.4%</td>
</tr>
</tbody>
</table>

Source: author’s computations
From results in table 5.1 we see that recent market price of purchase for this type of property is little under 2.3 millions CZK. Rent is 108,000 CZK per year so the Price to Rent ratio is 21.1.

In comparison with user costs we conclude that this market segment for chosen input parameters is little underpriced as well as overpriced, but within critical boarders for both user cost types. If we account psychological aspects about expectations, consider recently very low mortgage rates and account existence of new development projects in the area pushing supply up we end with market overpriced 7.5 %. In case of long-run more stable user costs, market is underpriced around 15 %, but still not statistically significant mispricing. Long-run user cost suggest Price to Rent ratio up to 24.9, in CZK 2,676,272, leaving almost four hundred thousand growth potential. Results for this type of real estate property shows how important is preference about user cost approach, as spread is relatively high and results are in opposite directions.
5.2 LOW QUALITY INNER CITY SMALL FLAT

This segment is defined as smaller size – 55 m², lower quality – 50% condition state with significant wear rate, in medium distance from city center – radius 4.500 meters including locations as Vinohrady, Dejvice, Žižkov, Smíchov and similar. Flat is in other type of building than prefabricated panel structure and does not include garage or furniture. It is not located on ground floor.

Table 5.2: Type’s B results

<table>
<thead>
<tr>
<th>Real Estate Flat Price &amp; Rent Level Calculator - 9.05.2013 data</th>
<th>Inputs</th>
</tr>
</thead>
<tbody>
<tr>
<td>CONDITION</td>
<td>50%</td>
</tr>
<tr>
<td>FLOOR (50-80 m²)</td>
<td>55</td>
</tr>
<tr>
<td>DISTANCE (metres)</td>
<td>4,500</td>
</tr>
<tr>
<td>GARAGE (dummy)</td>
<td>0</td>
</tr>
<tr>
<td>FURNITURE (dummy)</td>
<td>0</td>
</tr>
<tr>
<td>PREFABRICATED STRUCTURE (dummy)</td>
<td>0</td>
</tr>
<tr>
<td>GROUND LEVEL (dummy)</td>
<td>0</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Results</th>
</tr>
</thead>
<tbody>
<tr>
<td>PRICE Estimation</td>
</tr>
<tr>
<td>RENT per year Estimation</td>
</tr>
<tr>
<td>RENT per month Estimation</td>
</tr>
<tr>
<td>Rent to Price ratio</td>
</tr>
<tr>
<td>RtP + 1.5 standard deviation</td>
</tr>
<tr>
<td>RtP - 1.5 standard deviation</td>
</tr>
<tr>
<td>Price to Rent ratio</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>User Cost Calculator</th>
<th>SR</th>
<th>Inputs</th>
<th>LR</th>
</tr>
</thead>
<tbody>
<tr>
<td>Depreciation rate or Maintenance cost SR/LR</td>
<td>2.8%</td>
<td>2.8%</td>
<td></td>
</tr>
<tr>
<td>Expected appreciation level SR/LR</td>
<td>3.29%</td>
<td>4.41%</td>
<td></td>
</tr>
<tr>
<td>Additional risk premium</td>
<td>1.75%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Personal income tax</td>
<td>15.0%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Property tax</td>
<td>0.03%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cost of capital SR/LR</td>
<td>3.54%</td>
<td>4.07%</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Results</th>
</tr>
</thead>
<tbody>
<tr>
<td>User cost - SR</td>
</tr>
<tr>
<td>Inverse User cost - SR ratio</td>
</tr>
<tr>
<td>User cost - LR</td>
</tr>
<tr>
<td>Inverse User cost - LR ratio</td>
</tr>
</tbody>
</table>

| Mispricing SR User cost | 3.0% |
| Mispricing LR User cost | -12.7% |

Source: author’s computations

From results in table 5.2 we observe that market price estimation for this type of real estate property is little over 2.3 millions CZK. Floor size is almost 20% smaller than in Type A property, also condition is significantly lower, but combination of better location and fact, that it is not situated in panel structure,
pushes overall purchase price little higher than in first case. On the other hand, same conclusion does not hold for rent situation, which is per year 13,000 CZK lower. That makes 95,000 CZK per year. Price to Rent ratio is 24.9 for Type B property.

With user costs comparison we see that this market segment for chosen input parameters is both, 3.0 % overpriced in more sensitive user cost case and 13 % underpriced for long-run stable user costs. In sensitive case of user costs we have to admit, that market is priced very well. But again both values fit within confidence interval boarders. We conclude there is no significant mispricing on this market.
5.3 HIGH QUALITY CITY CENTER FLAT

Last type of property is defined as larger floor size – 75 m², highest quality – 100 % condition state with almost none wear rate and generally located in city center as radius is set to 1.000 meters from Old Town Square. Flat is in other type of building than prefabricated panel structure and as all other types, does not include garage or furniture and is situated on some upper floor level.

Table 5.3: Type’s C results

<table>
<thead>
<tr>
<th>Real Estate Flat Price &amp; Rent Level Calculator - 9.05.2013 data</th>
<th>Inputs</th>
</tr>
</thead>
<tbody>
<tr>
<td>CONDITION</td>
<td>100%</td>
</tr>
<tr>
<td>FLOOR (50-80 m²)</td>
<td>75</td>
</tr>
<tr>
<td>DISTANCE (metres)</td>
<td>1,000</td>
</tr>
<tr>
<td>GARAGE (dummy)</td>
<td>0</td>
</tr>
<tr>
<td>FURNITURE (dummy)</td>
<td>0</td>
</tr>
<tr>
<td>PREFABRICATED STRUCTURE (dummy)</td>
<td>0</td>
</tr>
<tr>
<td>GROUND LEVEL (dummy)</td>
<td>0</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Results</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>PRICE Estimation</td>
<td>CZK 4,388,125</td>
</tr>
<tr>
<td>RENT per year Estimation</td>
<td>CZK 172,968</td>
</tr>
<tr>
<td>RENT per month Estimation</td>
<td>CZK 14,414</td>
</tr>
<tr>
<td>Rent to Price ratio</td>
<td>3.94%</td>
</tr>
<tr>
<td>RtP + 1.5 standard deviation</td>
<td>5.34%</td>
</tr>
<tr>
<td>RtP - 1.5 standard deviation</td>
<td>2.91%</td>
</tr>
<tr>
<td>Price to Rent ratio</td>
<td>25.4</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>User Cost Calculator</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Depreciation rate or Maintenance cost SR/LR</td>
<td>2.8%</td>
</tr>
<tr>
<td>Expected appreciation level SR/LR</td>
<td>3.79%</td>
</tr>
<tr>
<td>Additional risk premium</td>
<td>1.75%</td>
</tr>
<tr>
<td>Personal income tax</td>
<td>15.0%</td>
</tr>
<tr>
<td>Property tax</td>
<td>0.02%</td>
</tr>
<tr>
<td>Cost of capital SR/LR</td>
<td>3.54%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Results</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>User cost - SR</td>
<td>3.67%</td>
</tr>
<tr>
<td>Inverse User cost - SR ratio</td>
<td>27.2</td>
</tr>
<tr>
<td>User cost - LR</td>
<td>3.50%</td>
</tr>
<tr>
<td>Inverse User cost - LR ratio</td>
<td>28.6</td>
</tr>
</tbody>
</table>

| Mispricing SR User cost                                     | -6.9%    |
| Mispricing LR User cost                                     | -11.2%   |

*Source: author’s computations*

In table 5.3 we see results for last type of real estate property, which is the most luxury one in our selection. Market price is estimated at value of almost 4.4 millions CZK. Rent per year estimation shows 173,000 CZK figure. That gives us highest Price to Rent ratio 25.4.
Based on user costs comparison we conclude that this market segment for chosen input parameters is underpriced for both user cost approaches. In case of short-run sensitive user costs are market prices 7.0 % lower and in case of long-run stable user cost 11 % lower than theory suggests. Theoretical price values would be then 4,704,730 CZK, respectively 4,946,885 CZK. According to these numbers, capital gain potential is over 300,000 CZK or 500,000 CZK depending on user costs approach.
6 CONCLUSION

In time period with poor economical performance for several last years, triggered by real estate bubble burst on U.S. market in 2007, we might observe that when housing bubble bursts, a great amount of value is lost and directly or indirectly are affected living standards of whole population. That is why an appropriate pricing of real estate assets became more important topic than ever before.

We began this paper with reminding some most famous cases of asset price bubbles and then we discuss basic theoretical concepts and approaches about assets mispricing identification and assessment. We found three main useful concepts. First is asset price change threshold. That means if asset price exceed predefined change threshold during some short time period, it should ring an alarm and signal that there might be some pricing problem and attention is needed. Main shortcoming of this approach is, what if market price does not change that much or maybe even remains fixed, but fundamental value change. In such case this approach fails completely. On the other hand, for all historical cases presented in this paper, this signaling method would probably work just fine. Next approach is fundamentals comparison method. If we are able to find fundamental elements with essential influence on the asset price, then we might observe mutual development of such variables. That allows us to assess whether price changes are in tact or not. The main shortcoming here is fundamentals, which could be sometimes difficult to identify or even measure them precisely and what if these fundamentals are not priced effectively as well. Last concept is assessment of economical potential of the asset and thus calculating net present value of all its future cash flows. This is very straightforward concept saying that price of the asset should be discounted its future earning capacity. Problem here could be of course, what would be expectations about future and also the fact that even little change in such expectations could have dramatic influence on the total asset value.

For purposes of assessment real estate market price levels, we derive theoretical concept of user costs as combination of fundamentals and NPV of future cash flows approaches. These user cost could be seen as imputed rent as we define them as sum of all cash flows related with owning a property, such as capital cost, depreciation rate, personal income and property taxes, additional asset risk and expected capital gain. These components could be also seen as fundamentals, because...
their development in time has crucial influence on overall price of the asset. NPV of all components cash flows provide us with theoretical price level for defined real estate property market. We defined two slightly different user cost concepts as first one is precisely reflecting recent economy setup and also accounts some psychological aspects about expectations, but is also very sensitive in short run to fundamental components changes. That is why we also defined long run user costs as stable historical anchor based on long-run values and averages.

In order to obtain precise market price levels, which would be compared to theoretical price levels set by user costs concept, we construct loglinear models. By ordinary least squares estimation method, loglinear regression models allow us to determine structurally consistent market price levels for purchase and renting in different real estate segments. As we need those models primarily for estimating market price levels. They also provide us with several interesting conclusions, such as the fact that if the flat is located in prefabricated panel structure it has significant negative impact on purchase price, but is insignificant for rent level estimation.

Finally this study set three basic market segments to cover different property types distinctions and compare estimated market price levels with price levels suggested by user cost formulas. Based on this comparison we conclude, that for all segments is pricing within confidence intervals and thus generally priced well. Interesting is the fact, that according to short-run user cost, two out of three real estate segments are slightly overpriced, but using long-run user costs are all segments more than 10 % underpriced. That means if psychological aspects accounted, by stagnation on recent real estate market prices influenced expectations more than offset historically low interest rates. But if we employ long run historical development values as equilibrium state, we might expect prices to grow in future.

At last we conclude that Prague real estate market is not overpriced and definitely there should not be ongoing positive price bubble. Actually it seems more likely that market is slightly underpriced and if there would not be another type of economic crisis soon, price level should rise again in the near future.


HIMMELBERG, CH.; MAYER CH.; SINAI T. 2005: Assessing high house prices: bubbles, fundamentals, and misperceptions, Federal Reserve Bank of New York Staff Reports, SR no.218.


Appendix A: Model 2

Figure A.1: Dependent vs. Explanatory variable

Source: sreality.cz, author’s computations. Gretl SW
Appendix A: Model 2

Figure A.2: Reg. Residuals vs. Dependent variable

Source: sreality.cz, author’s computations. Gretl SW
Appendix A: Model 2

Figure A.3: Reg. Residuals Normality

Source: sreality.cz, author’s computations. Gretl SW
Figure A.4: Individual Cases Influence and Leverage

*Source:* sreality.cz, author’s computations. Gretl SW
Figure A.5: Actual vs. Fitted, MODEL 2 PRICE

Source: sreality.cz, author’s computations. Gretl SW
Appendix B: Model 4

Figure B.1: Dependent vs. Explanatory variable

Source: sreality.cz, author’s computations. Gretl SW
Figure B.2: Reg. Residuals vs. Dependent variable

*Source:* sreality.cz, author’s computations. Gretl SW
Figure B.3: Reg. Residuals Normality

Source: sreality.cz, author’s computations. Gretl SW
Appendix B: Model 4

Figure B.4: Individual Cases Influence and Leverage

Source: sreality.cz, author’s computations. Gretl SW
Figure B.5: Actual vs. Fitted, MODEL 4 RENT

Source: sreality.cz, author’s computations. Gretl SW
Appendix C: Price & Rent Estimated Density Function

Figure C.1: Estimated density function Price

Source: sreality.cz, author’s computations. Gretl SW
Figure C.2: Estimated density function Ln_Price

Source: sreality.cz, author’s computations. Gretl SW
Appendix C: Price & Rent Estimated Density Function

Figure C.3: Estimated density function RentY

Source: sreality.cz, author’s computations. Gretl SW
Figure C.4: Estimated density function Ln_RentY

Source: sreality.cz, author’s computations. Gretl SW