UNIVERSITAT POLITÈCNICA DE CATALUNYA
BARCELONATECH
Departament de Matemàtica Aplicada IV

Edifici C3,Campus Nord UPC
C/Jordi Girona, 1-3
E-08034 Barcelona, Spain
Tel: 34 93 401 0959
Fax: 34 93 401 5981

Report on the Thesis
Geometric and algebraic properties of discrete structures
By Pavel Rytír

The Thesis deals with geometric representations of linear codes and its consequences for the connection among the weight enumerator of codes, cycle spaces and matchings in graphs. This connection is inspired in turn by connections with partition functions of Ising models in statistical physics. The framework of the work is thus well motivated and shows a nice interplay between discrete geometry, graph theory, statistical physics and combinatorial enumeration.

The first chapter is motivated by a question posed by Martin Loebl some 15 years ago on the possible representation of binary linear codes by simplicial complexes. The question arose on the fruitful conection established by Loebl between the linear codes which correspond to cycle spaces of graphs which allows to write the weight enumerator of the code in terms of Pfaffians. It asks about the larger class of binary codes which admit a geometric representation as (punctured) cycle spaces of simplicial complexes.

In the first chapter of the Thesis this question is answered in full generality by establishing such a geometric representation for all binary codes. This is called in the Thesis a *triangular* representation, and it is carried over through a specifically designed technique (tunneling) which allows to transfer the weight enumerator of the cycle space to the weight enumerator of the code. In a second step, a related triangular configuration is built which transfers the weight enumerator of the cycle space of even configurations to the matching enumerator of the new one, thus fulfilling one of the goals which prompted the original question.

The second chapter of the Thesis moves one step further to the computational aspect of the matching enumerator. One of the motivations of the geometric constructions was to effectively compute the weight enumerator. By setting a correspondence with a matching enumerator the problem is only half solved, because the latter is given by the permanent of a matrix, still a computationally hard problem. Kasteleyn proved that, for planar bipartite graphs, the problem can be reduced to the computation of a determinant, turning the problem to an efficiently computable one. However, this essentially can only be done for matrices which arise precisely from biadjacency matrices of bipartite planar graphs, as shown by Seymour, Robertson and Thomas. The main contribution in the second chapter of the Thesis is to show that, in the more general framework of geometric representations by simplicial complexes, a parallel reduction to the permanent of 3-matrices (via tripartite triangular

configurations) and then to the determinant of related 3-matrices can be achieved. The main result states that, for every non negative matrix, there is a 3-matrix with the same permanent which admits a resigning for which the permanent can be computed as a determinant. I think that this is a beautiful and exciting result.

The third chapter considers the minimum dimension d of an Euclidian space where a geometric representation of a binary linear code can be embedded to. It is known that every 2-dimensional simplicial complex can be embedded in the five dimensional Euclidian space. In this chapter the author shows that triangular configurations arising as geometric representations of binary linear codes are four dimensional embedable, and the ones which can be embedded in three dimesnional ones are characterized. The characterization paralels the one by McLane for planar graphs.

The fourth and final chapter considers geometric representations for general linear codes, not only binary, by triangular configurations. These representations in this general setting still stablish a correpsondence between the weight enumerator of the code and the one for the (punctured) cycle space of the simplicial complex. Such representations are shown to exist in prime fields, and it is also shown that for nonprime fields there are codes that do not admit such geometric representations.

In all the Thesis presents substantial contributions to a sound and ambitious research project. Both, for the significance of the results and for the techniques and methods brought into the scene, this is a high quality research work. The writing is very good showing an excellent mathematical maturity and research level. I found the proofs correct and the arguments neat and clear.

Summarizing, the work satisfies every high standard criteria for an excellent quality Doctoral Thesis, and I positively recommend the author is awarded the title of PhD.

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Oriol Serra
Professor of the Departament of Applied Mathematics
Universitat Politècnica de Catalunya, Barcelona