Posudek

na Disertační práci Jana Vršovského

„Pologrupy operátorů a jejich orbity"

Práce se zabývá orbitami spojitých lineárních operátorů na Banachových (a Hilbertových) prostorech. Diskrétní případ je T^n, $n \in \mathbb{N}_0$, spojitý pak orbity C_0-pologrup.

Práce má kromě úvodu tři části. V kapitolách II a IV je zpracován materiál z článků [62] a [61] (v tomto pořadí), které jsou společné práce autora disertace a vedoucího disertační práce V. Müllera. Tento materiál je přepracován a doplněn o další vlastní výsledky autora – nejvíc z nich je v kapitole II. Výsledky z kapitoly II se použijí jak v kapitole IV tak i v kapitole VII. V kapitole IV jsou také uvedeny dvě technicky náročné konstrukce prostoru a operátoru na něm, z článku [61]. Tyto příklady jsou experty uznány jako jednodušší než dosud známé. Oba články [62] a [61] jsou vyšlé a mezinárodně uznané. Kapitola III je z větší části kompilací množství netriviálního materiálu a je doplněna o některá pozorování – vlastní přínosy autora. Práce je napsána anglicky, proto podrobnější popis uvedu v angličtině.

Description of Chapter II.

Ball's plank theorem states that it is not possible to cover the unit ball with a sequence of planks if the widths of the planks are too small (summability condition on widths). There is also a known version for functionals. In the thesis there is a generalization for operators (Propositions 1.3 and 1.5). These general statements are used in the proof of Proposition 3.2 which is finally employed in Chapter IV. This strategy is published in the joint paper of the author and his supervisor, reference no. [62]. I point out two differences between [62] and the thesis. One of them is that all concerned theorems and propositions of the thesis contain also the additional statement about the density of vectors with the required property, whereas only in one place in the paper [61] does such an additional statement appear. The second one is that in the thesis a full general statement in the spirit of plank theorem is formulated and proved for operators and this deserves a particular attention.

Apart of the plank-type theorems and Proposition 3.2, which is a kind of discrete orbit case, Chapter II considers also the continuous orbit case. A central result is Proposition 1.9 due to the author, which is then used to prove proposition 3.5, a continuous analogue of Proposition 3.2. Although Propositions 3.2 and 3.5. appear in the paper [62], but Proposition 1.9 is new. It is to point out that the proofs in the thesis are different from those in [62] in that they rely on continuous arguments whereas the proof in [62] relied on passing to the discrete case. So the approach in the thesis is more elegant since „continuous" statements are proved by „continuous" methods there.

The aim of Chapter II is to investigate under which conditions do the orbits tend to infinity - this is the statement of Propositions 3.2 and 3.5. Near this, also orbits tending to zero are studied (Section 2), the set of vectors whose orbit tends to ~~zero~~ infinity is investigated (Observation 3.6 and Example 3.8 are own statements of the author in this direction) and weak orbits are also considered. This chapter contains also two examples from [62]: an operator on ℓ^p such that $\|T^n\| \to \infty$ and $\|T^n x\| \not\to \infty$ for all x, and a second one which is analogue but on a Hilbert space and for weak orbits $\langle T^n x, y\rangle \not\to \infty$.

Description of Chapter III.

The middle part of the thesis, Chapter III, deals with irregular orbits, for example which neither tend to 0 nor to ∞, or even worse, when the orbit of some vector is dense in the whole space. Such a vector is called hypercyclic vector, the operator hypercyclic operator.

First, the hypercyclicity criterion is discussed in technical details, then the attention is turned to the set of hypercyclic vectors. If there is one then there is a dense set of them - this is easy to see. But there are other, deep statements about the set of hypercyclic vectors, and about the set of hypercyclic operators.

On the other hand, if for a given operator there is a vector with nonzero, non-dense orbit (thus, this vector is not hypercyclic), then this operator admits a a nontrivial closed invariant subset: namely the closure of the above orbit. Whether this can be done for each operator on a given space, is the so-called invariant subset problem. If the word „subset" is replaced by „subspace" and the word „orbit" by „linear span of orbit", one speaks about the famous invariant subspace problem.

Further technical concepts of Chapter III are ε-hypercyclicity, ε-hypercyclicity with exponent p. In this context there are some improvements of the author of known results (see Theorems 9.2 and 9.3). There is an unclear condition $\varepsilon < 5^{-p}$ in the statement of Theorem 9.1. It is not clear from the proof why exactly this condition is posed.

Chapter III ends with a statement for operators having no nontrivial weakly closed invariant subset (Corollary 10.4). Such an operator must have spectral radius 1. If one passes to the reading of Chapter IV, Corollary 11.5 will imply that only such an operator can be non-orbit-reflexive. This is also a motivation to look for examples of operators which are not orbit reflexive – such examples are constructed in Chapter IV. I consider this last Corollary 10.4 of Chapter III, which is an improvement of known results by the author, also of interest with respect to the whole structure of the thesis. Its proof uses plank-type theorems for operators from Chapter II. It is a pity that there are missprints in its proof.

Description of Chapter IV.

Chapter IV deals with the notion of orbit reflexivity of an operator. It starts with different sufficient conditions. One of them (Proposition 11.4) relies on a plank-type theorem for operators (powers of operator). Another known results in Hilbert space setting are presented by the author from his own point of view - in particular with a short new proof for the case $r(T) = \|T\|$ (part iv) of Theorem 11.6). After this it is proved, using the general results, that weighted shifts are orbit reflexive. I wonder whether one could prove orbit reflexivity of concrete shift operators without using the above general results.

The second part of Chapter IV deals with description of operators which are not orbit reflexive. A known abstract result is Theorem 12.2 presented from the viewpoint of the author with his new Observation 12.1. But most of this part of Chapter IV is devoted to the constructions of a non-orbit-reflexive operator on a Hilbert space, and on the Banach space ℓ^1 of an operator which is not orbit reflexive but reflexive. Both constructions are very technical and are taken verbatim from the paper [61].

Po tomto odbornom popise v angličtine pokračuji v posudku na práci.

Práce má 71 stránek, 79 citací a její výsledky jsou publikovaní ve dvou článcích. Styl je spíš zhuštěný. Obsáhlost materiálu a hloubka zpracovaného tématu je přiměřena doktorské disertační práci.

Z globálního hlediska je práce správná. Obsahuje sice několik menších chyb v detailech které se dají snadno upravit (např. v důkazu Proposition 1.3 má být použita předchozí věta pro opatrnější zvolení koeficientů, tj. tak aby $\sum \alpha_n < 1$; podobně i v důkazu Corollary 10.4), několik nepřímočarých ale pravdivých matematických argumentů (např. v důkazu Corollary 11.5 přechod k posloupnosti $(n^2)^{\frac{1}{n}}$ kde přirozenější by byla posloupnost q^n, $q > 1$) a jen málo překlepů. Na dvou nebo tří místech je mi detail důkazu nejasný (Theorem 11.6 část (b) existence s; Theorem 9.1 použití předpokladu $\varepsilon < 5^{-p}$), zde se ale jedná konečně o již známé výsledky.

Autor poukazuje na svoji vlastní výsledky. Práce je psaná anglicky na velmi dobré úrovni.

Práce dokazuje, že autor danou matematickou disciplínu a její aktuální stav nastudoval, o své vlastní pozorování doplnil a samostatně zpracoval. Proto doporučuji, aby práce byla uznána jako disertační práce v oboru matematická analýza na matematicko-fyzikální fakultě Univerzity Karlovy v Praze.

V Drážďanech a Norimberku
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